History of Science in South Asia

A journal for the history of all forms of scientific thought and action, ancient and modern, in all regions of South Asia

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MLA style citation form: Michio Yano. "An Indian Sine Table of 36 entries." *History of Science in South Asia*, 7 (2019): 42–51. DOI: 10.18732/hssa.v7i0.43.

Online version available at: http://hssa-journal.org

HISTORY OF SCIENCE IN SOUTH ASIA
A journal for the history of all forms of scientific thought and action, ancient and modern, in all regions of South Asia, published online at http://hssa-journal.org

ISSN 2369-775X

Publisher:

History of Science in South Asia

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The electronic versions were generated from sources marked up in LaTeX in a computer running GNU/LINUX operating system. PDF was typeset using XeTeX from TeXLive. The base font used for Latin script and oldstyle numerals was TeX Gyre Pagella developed by GUST, the Polish TeX Users Group.

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1 INTRODUCTION

Trigonometry is an indispensable tool of Indian mathematical astronomy. The concept of trigonometry originated in Greece and it was transmitted to India together with astronomy. The Greek concept of a chord was skilfully changed into the Indian sine.[1] The earliest Sanskrit text that deals with the sine function is the *Āryabhaṭīya* of Āryabhaṭa (b. 476 CE).[2] Twenty-four sine differences (K_i in Table 1) in minutes are given here by Āryabhaṭa's own method of expressing numbers.[3] The radius (R or sin 90°[4]) of the table is 3438 minutes. The first arc 225 in minutes was regarded as equal to its sine ($J_1 = K_1$). By adding K_i successively we get

$$\sin \alpha_i (= J_i)$$

Due to Āryabhaṭa's method of successive approximation,[5] five values, i.e. J_6, J_7, J_{16}, J_{17}, and J_{18} (underlined in Table 1), are off by one minute from the geometrically computed correct values, but Āryabhaṭa's numbers were faithfully preserved in the *Sūryasiddhānta*, and we find the same values of K_i and J_i in the Chinese text *Jiuzhi li* (九執暦, 718 CE) during the Tang Dynasty.[6] This type of sine table with 24 entries was standard.

In Table 2 I have summarized different kinds of sine tables before the time of Mādhava (fl. 1380). In the last column I have added table numbers from Pingree (1978).

2 *GARGASAṂHITĀ*

It is noteworthy, therefore, that a text entitled *Gargasaṃhitā* (GS) offers a sine table of 36 entries in a quadrant. On this strange text I contributed "The *Gargasaṃhitā*: One

1 Toomer (1973) claimed the use of a chord table (with $2R = 6875'$) by Hipparchus, but later he cast doubt on his claim (Toomer 1984). Klintberg (2005) throws doubt on the origin of the Indian sine in the Greek chord, criticizing Toomer's argument. See also van Brummelen (2009: 43, n. 30). But I would like to propose that Sanskrit term *jīva* is from the Greek word βιὸσ (*biós*, bow) which, by the shift of accent, becomes βῖοσ (*bíos*, life).

2 Pingree (1967–8) argued that the *Paitāmahasiddhānta* of the *Viṣṇudharmottarapurāṇa* predates the *Āryabhaṭīya*. But I do not agree with this view.

3 *Āryabhaṭīya* 1.12:

मखि-भकि-फखि-धखि-णखि ञखि-ङखि-हस्झ-स्ककि-किष-श्घकि-किघ्व।

घ्लकि-किग्र-हक्य-धकि-किच-स्ग-झश-ङ्न्व-क्ल-प्त-फ-छ-कलार्धज्याः

4 I use capital S to express R sin.

5 See Hayashi 1997.

6 See Yabuuti 1979.

DOI: 10.18732/HSSA.V7I0.43

i	α_i	K_i	J_i	versin α_i
1	3;45	225	225	7
2	7;30	224	449	29
3	11;15	222	671	66
4	15;00	219	890	117
5	18;45	215	1105	182
6	22;30	210	$\underline{1315}$	261
7	26;15	205	$\underline{1520}$	354
8	30;00	199	1719	460
9	33;45	191	1910	579
10	37;30	183	2093	710
11	41;15	174	2267	853
12	45;00	164	2431	1007
13	48;45	154	2585	1171
14	52;30	143	2728	1345
15	56;15	131	2859	1528
16	60;00	119	$\underline{2978}$	1719
17	63;45	106	$\underline{3084}$	1918
18	67;30	93	$\underline{3177}$	2123
19	71;15	79	3256	2333
20	75;00	65	3321	2548
21	78;45	51	3372	2767
22	82;30	37	3409	2989
23	86;15	22	3431	3213
24	90;00	7	3438	3438

Table 1: Āryabhaṭa's K_i

of the Texts of Jyotiḥśāstra Ascribed to Garga" as section 5 of the joint paper "Garga and Early Astral Science in India" published in this journal (Geslani et al. 2017: 173–82). Since my contribution was only a very brief summary of the text, I would like to offer here a more detailed account.

The text of the GS is preserved in a single manuscript belonging to VVRI in Hoshiarpur, Panjab, MS Hoshiarpur VVRI 2069, written in the Malayālam script. I first worked on the text with Prof. David Pingree in 1973. The manuscript we used is a modern copy in Devanagari script possessed by Pingree. After a long interruption, I recently I resumed the work. The manuscript is full of mistakes and some verses do not allow reasonable interpretation. The Sanskrit text I am preparing is only provisional. Our present topic is discussed in Chapters 7, 8 and 10 of the GS.

Chapter 7 begins with the reason why the sine function is needed. I quote GS 7.4–6 (verse numbering is my own):

Date	i	R	Sources	DSB
499 earliest	24	3438	*Āryabhaṭīya* (ca. 499), *Jiuzhi-li* (718), *Sūrya-siddhānta* (9th century?), *Siddhāntaśiromaṇi* (12th cent. with corrections)	V.6
ca. 550	24	120	*Pañcasiddhāntikā* of Varāhamihira	III.20
628	24	3270	*Brāhmasphuṭasiddhānta* of Brahmagupta	V.16
665	6	150	*Khaṇḍakhādyaka* of Brahmagupta	
904	96	3437;44	*Vaṭeśvarasiddhānta* of Vaṭṭeśvara	
966	9	200	*Karaṇatilaka* of Vijayananda	VIII.13
ca. 1050	24	3415	*Siddhāntaśekhara* of Śrīpati	V.33
ca. 1092	6	120	*Karaṇaprakāśa* of Brahmadeva	VI.27
1281	6	43	*Vākyakaraṇa* of anonymous author	VI.33
fl. 1380	24	3437;44,48	Mādhava quoted by Nīlakaṇṭha (Sāmbaśivaśāstri 1930–57: v. 1, 55)	

Table 2: Different Kinds of Sine Tables

> 7.4 By their power, the sky-moving (planets) never move uniformly there (in their orbit). Sometimes they move slowly, (sometimes) swiftly, likewise (sometimes) retrograde.
>
> 7.5 Thus they have different motions. Without correction, O twice-born, all the planets would not come to (the position) equal to observation.
>
> 7.6 Therefore (I will explain), in the beginning, the division of sines which is the means of its accomplishment [and] which was produced in the old time on the sky having an evenly round form.[7]

Then our text (GS 7.7) says that the circumference of a circle is 21,600 minutes and the diameter is 6877 minutes.[8] GS 7.8–10 mentions some other possible divisions of a quandrant besides 24, i.e. 'its half' (48) and "its half" (96). The division of a quadrant into 96 equal parts is attested in the *Vaṭeśvarasiddhānta* as shown in Table 2.

The first half of GS 7.11 says that chord 60° is the radius of a circle.[9] But the second half abruptly changes the topic and says:

> [The minutes] of the Chord of 1/18 th [of a circle] are [those of its] arc diminished by its own 200th part.[10]

7 तद्वशात्खेचरास्तत्र नैव सामन्यमागताः।
कदाचिन्मन्दगाः शीघ्रगतयो वक्रगास्तथा॥ (7.4)
इति संभिन्नगतयो ते स्फुटीकरणं द्विज।
अन्तरेण ग्रहाः सर्वे न च दृक्तुल्यतां गताः॥ (7.5)
तदस्य साधनीभूतामादौ जीवाप्रकल्पनाम्।
या पुरा समवृत्तस्वरूपव्योम्नि संपद्यते॥ (7.6)

8 वृत्तविग्रहिणस्तस्य खखषट्कश्विनः कलाः।
अस्या विष्कंभमानानि सप्तसप्तगजर्तवः॥ (7.7)

9 समवृत्तस्य कर्णार्धं षडंशज्यासमं भवेत्। (7.11ab)

10 धृत्यंशजीवायास्तस्य खखाश्व्यंशोनितं धनुः। (7.11cd) Our manuscript is difficult to read, and I used Pingree's reading.

This means: chord $20^\circ = 1200(1 - \frac{1}{200}) = 1194'$. Therefore $\sin 10^\circ = 597'$. The same value can be obtained by the linear interpolation using Table 1:[11]

$$671 - 222 \times \frac{(11;15 - 10)}{225} = 671 - \frac{222 \times 75}{225} = 671 - 74 = 597$$

It is strange that the author explained the derivation in a different way. Chord 20° is found again in GS 7.46–47 and GS 7.71. The use of chord is not common in India, where usually sine or "half chord" (*jīvārdha* or *jyārdha*) is used. It is possible that the author of our text knew some Arabic sources. Anyway, this value of $\sin 10^\circ = 597'$ is used as one of the three starting points for the derivation of 36 sines (see below). From the three starting points all the sines are derived, as GS 7.12ab says.[12] For the derivation two rules are used. One is to compute the complementary sine of a given sine. GS 17cd-18ab defines the sine of the complementary arc (*saṃpūrṇajyā*), which is nothing but cosine (*koṭijyā*):

> 7.17cd–18ab O inspired one, whatever is the result of the subtraction of that (i.e., the square of a sine) from the square of the radius, that is the complementary sine of the 36 arcs of the three signs.[13]

This means

$$J_{36-i} = \sqrt{R^2 - J_i^2} \tag{1}$$

Another rule is to compute the sine of half arc. The wording of GS 7.14cd–15ab[14] is not very clear, but it tells how to obtain versed sine (*sāyaka*, "arrow"). Probably versed sine was used in the following formula:

$$J_{\frac{i}{2}} = \frac{\sqrt{J_i^2 + \mathrm{Vers}J_i^2}}{2} \tag{2}$$

The rule is not explicitly stated anywhere in our text, but a similar formula is found in Brahmagupta's *Brāhmasphuṭasiddhānta* 21.23 and it was a common knowledge. Then GS describes the derivations of sines starting from three different points. In the following diagrams the vertical arrow means the use of (1) and the horizontal arrow that of (2).

The three starting points are: (A) J_{36}, (B) J_{12}, and (C) J_4.

(A) GS 7.16–25: Starting from $\sin 90^\circ = J_{36}$, a total of 4 sines are derived.

$$\begin{array}{l} J_{36} \\ \downarrow \\ J_{18} \\ \downarrow \\ J_9 \rightarrow J_{27} \end{array}$$

11 I owe this explanation to an anonymous referee.

12 एतेनार्थत्रयेणैव सर्वज्यानां प्रकल्पना। (7.12ab)

13 व्यासार्धवर्गतो विपुर विश्लेषाद्यत् फलं भवेत्॥ (17cd)
संपूर्णज्या त्रिराशीनां षड्विंशच्चापि देहिनाम्। (18ab)

14 यद्धनुर्ज्यार्धतः कर्णदलाद्यदुपलभ्यते॥ (14cd)
तद्विशुद्धं कर्णार्धं तद्धनुर्ज्यार्धसायकम्। (15ab)

As results, our text gives $J_{36} = 3438$, $J_{18} = 2431$, $J_9 = 1316$[15], and $J_{27} = 3178$.

(B) GS 7.29cd-45: Starting from $\sin 30^\circ$ (J_{12}), a total of 8 sines are derived.

$$\begin{tikzcd}
J_{12} \arrow[r] \arrow[d] & J_{24} & & \\
J_6 \arrow[rr] \arrow[d] & & J_{30} \arrow[d] & \\
J_3 \arrow[r] & J_{33} & J_{15} \arrow[r] & J_{21}
\end{tikzcd}$$

Our text states that chord $60^\circ = R$, therefore $\sin\ 30^\circ = J_{12} = \frac{R}{2} = 1719$. The remaining values are given as: $J_{24} = 2978$, $J_6 = 890$, $J_{30} = 3321$, $J_{15} = 2093$, $J_{21} = 2728$, $J_3 = 499$, $J_{33} = 3372$[16], in this order.

(C) GS 7.47–71: Our text starts from $J_4 (= \sin\ 10^\circ)$, which is, as mentioned above, 597. A total of 24 sines are derived.

$$\begin{tikzcd}
 & & J_4 \arrow[rr] \arrow[d] & & J_{32} \arrow[d] & & & & & & & \\
 & J_{34} \arrow[d] & J_2 \arrow[l] \arrow[d] & & J_{16} \arrow[rrrr] \arrow[d] & & & & J_{20} \arrow[d] & & & \\
J_{19} & J_{17} \arrow[l] & J_1 \arrow[r] & J_{35} & J_8 \arrow[r] & J_{28} \arrow[d] & & & J_{10} \arrow[rr] \arrow[d] & & J_{26} \arrow[d] & \\
 & & & & & J_{14} \arrow[r] \arrow[d] & J_{22} \arrow[d] & & J_5 \arrow[r] & J_{31} & J_{13} \arrow[r] & J_{23} \\
 & & & & J_{29} & J_7 \arrow[l] & J_{11} \arrow[r] & J_{25} & & & &
\end{tikzcd}$$

The 24 sine values derived in the third process are shown in Table 3.

i	α°	J_i	ΔJ_i	versin α
1	2;30	150	150	3
2	5;0	300	150	13
3	7;30	449	149	29
4	10;0	597	148	54
5	12;30	744	147	83
6	15;0	890	146	117
7	17;30	1034	144	170

15 In Chapter 8, J_9 is given as 1315.

16 This is wrong because $J_{33} (= \sin\ 82;30^\circ)$ should be 3409 as in Āryabhaṭa's J_{22}. The figure 3372 is Āryabhaṭa's J_{21}.

i	α°	J_i	ΔJ_i	versin α
8	20;0	1175	142	218
9	22;30	1316	140	270
10	25;0	1452	137	334
11	27;30	1586	134	400
12	30;0	1719	130	471
13	32;30	1846	127	550
14	35;0	1971	124	634
15	37;30	2093	121	721
16	40;0	2209	116	816
17	42;30	2322	113	915
18	45;0	2431	109	1018
19	47;30	2534	103	1127
20	50;0	2633	99	1240
21	52;30	2728	94	1356
22	55;0	2815	89	1478
23	57;30	2899	84	1603
24	60;0	2978	79	1730
25	62;30	3049	71	1863
26	65;0	3115	66	1997
27	67;30	3178	63	2134
28	70;0	3230[17]	55	2274
29	72;30	3278	46	2416
30	75;0	3321	43	2559
31	77;30	3355	34	2705
32	80;0	3385	30	2852
33	82;30	3409	25	3000
34	85;0	3425	16	3149
35	87;30	3435	10	3288
36	90;0	3438	3	3438

Table 3: Sine table of 36 entries

17 GS 7.56 reads: गुणवह्निरद (3233), which is also the reading of JM (see below, Sarma 1977: 48), but GS 8.8 reads खद्वित्रिकृष्णवर्त्म (3320), which I have changed to खत्रिद्विकृष्णवर्त्म (3230).

i	α°	J_i	versin α
1	7;30	449	29
2	15;0	890	117
3	22;30	1315	260
4	30;0	1719	460
5	37;30	2093	710
6	45;0	2431	1007
7	52;30	2728	1345
8	60;0	2978	1719
9	67;30	3178	2123
10	75;0	3321	2548
11	82;30	3409	2989
12	90;0	3438	3438

Table 4: Concise sine table

In Chapter 8 of GS all the 36 sines are stated again, this time in the order from J_1 to J_{36} (GS 8.2–10). Some values are slightly different from those given in Chapter 7. The difference of sines (*khaṇḍajyā*, ΔJ_i) and versed sines are also given in GS 8.10–15 and GS 8.16–22 respectively. I have shown these values in Table 3. The values of ΔJ_i and their sum (J_i) are good and mostly agree with Āryabhaṭa's values (Table 1). What is very strange is the values of the versed sines. GS 8.15cd gives definition of versed sine:

> If these (sine differences) are added in reverse order, they in order are the sum of the versed sines.[18]

and the following verses just give numerical values . I quote only the beginning part (GS 8.16):

> *vahni* (3) *viśve* (13) *randhradasrā* (29) *sindhupañca* (54) *trihastinaḥ* (83)/
> *parvateśāḥ* (117) *khasaptaikā* (170) *dhṛtidasrā* (218) *khabhāni* (270) *ca*// (8.16)

Only four values up to "54" for $i = 4$ are correct, but after that differences from the correct values conspicuously increase. I do not understand the reason. Even stranger is that correct values are found in the concise sine table in Chapter 10 (Table 4).

All the 12 values in the concise table are same as those of Āryabhaṭa, except sin $67;30^\circ = 3178'$ (also in the detailed table above), which is better than Āryabhaṭa's 3177.

18 एते क्रमोत्क्रमात्युक्ताः क्रमज्योत्क्रमपिण्डकाः ॥ (8.15cd).

3 *Jyotirmīmāṃsā*

It is remarkable that recently I found that Nīlakaṇṭha (b. 1444), the most celebrated astronomer/mathematician in the Mādhava school in Kerala, knew the peculiar sine table of the *Gargasaṃhitā*. In his edition of the *Jyotirmīmāṃsā* (Sarma 1977), there is a section (according to Sarma, "18. *Ṣaṭtriṃśajjyānayana* (?)"), called "Derivation of the 36 Rsines".[19] The beginning of this section is missing and Sarma notes that, "The ms. has a gap, which has been suitably filled up".[20]

The Sanskrit text of this section is very corrupt and Sarma's restoration is not successful because he did not know that the parallel passage is found in the *Gargasaṃhitā*. Actually this section begins with a verse that is identical with GS 7.51cd–52ab. I quote here from the beginning of GS 7.51.

> तच् चाङ्कशून्याकृतिकस्ततो विंशतिमो गुणः । /
> तस्मात्त्रिवह्न्युत्कृतिकाद् [कृतिकाद्][21] दशज्याखण्डसंभवः ॥ (7.51)
> दस्त्रेषुसिन्धुभूमानं ततः षड्विंशतेर्गुणः ।
>
> That is 2209 (minutes). From this is the sine of the 20th (arc). From this which is 2633 (minutes) there arises the (sum of) the ten sine differences, of which the measure is 1452[22] (minutes). From it (there originates) the sine of the 26th (arc).

This is the derivations of $J_{16} = 2209$, from which is J_{20} (2633), from which is $J_{10} = K_1 + \ldots + K_{10}$ (=1452), and from which again is J_{26} (=3115), which is in the next line (7.52cd). In this way the whole section is borrowed from GS 7.51cd–64ab. We can restore the corrupt text of the *Jyotirmīmāṃsā* with the help of the GS. And vice versa, sometimes we can check the readings of the GS by those of the *Jyotirmīmāṃsā*.

We need to know how Garga's sine table was included in the *Jyotirmīmāṃsā*. In his commentary on the *Āryabhaṭīya*, Nīlakaṇṭha calls himself Gārgyakerala, which means that he belonged to the Gārgya clan (*gotra*) of Kerala. He sometimes quotes Garga's verses that are found in Varāhamihira's *Bṛhatsaṃhitā*,[23] an encyclopedic book on divination. This Garga can be different from the author of our GS. Nīlakaṇṭha had at least two kinds of texts ascribed to Garga as he says:

> It is evident that there were two Gargas, one is Vṛddhagarga and the other is Garga[24].

In the same passage, Nīlakaṇṭha refers to a mathematical book called *Gargasaṃhitā*[25]. He quotes a half verse from this *Gargasaṃhitā* in his commentary on the "Chapter on Mathematics" (*Gaṇitapāda*) of the *Āryabhaṭīya*:[26]

19 Sarma 1977: viii
20 Sarma 1977: 48, note 1.
21 Sarma (1977: 48) reads कृतिकाद् and puts 2233 above the line, but this is wrong.
22 Sarma (1977: 48) puts 1752 above the line, but this is wrong.
23 For example, in the edition of Sāmbaśivaśāstri (1930–57: v. 3, p. 160) he quotes five verses from "Garga," but actually they are found in the mūla text of the *Bṛhatsaṃhitā*, Chapter 2. But it is possible that the *Bṛhatsaṃhitā* at his hand was different from that of Dvivedin's modern edition.
24 Nīlakaṇṭa ad. *Āryabhaṭīya*, Kālakriyāpāda 10 (Sāmbaśivaśāstri 1930–57: v. 2, p. 16, line 13): वृद्धगर्गः पुनर्गर्गश्चेति गर्गद्वयं प्रसिद्धम्.
25 Idem, line 17: स्वप्रणीते गर्गसंहिताख्ये गणितशास्त्रे.
26 Sāmbaśivaśāstri 1930–57: v. 1, p. 11.

> पृथग्दोःकोटिवर्गाभ्यां कर्णवर्गोऽनुषज्यते।
>
> The square of the hypotenuse (=diameter) is derived from separately the squares of sine (*bāhu*) and cosine (*koṭi*).

This is exactly the line we find as GS 7.10cd.

Thus we can conclude that Nīlakaṇṭha had access to at least two texts ascribed to Garga, one astrological and the other astronomical. What he refers to as "mathematical *Gargasaṃhitā*" was used when he wrote the *Jyotirmīmāṃsā* and the commentary on the *Āryabhaṭīya*. We do not know whether he actually used this unusual sine table in his computation. Lastly, let us remember that Nīlakaṇṭha quotes the very accurate sine table of Mādhava as mentioned in Table 2 above.

4 ACKNOWLEDGEMENTS

The original version of this paper was read at the second International Conference on the History of Mathematics and Astronomy, December 2–8, 2018, Xi'an.

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