超弦の場の理論における "多重Half-brane" Solution について

(On “Multiple Half-brane” Solutions
in Modified Cubic String Field Theory)

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第６回日大理工・益川塾連携シンポジウム 2016年10月15日

0. Introduction

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String Field Theory

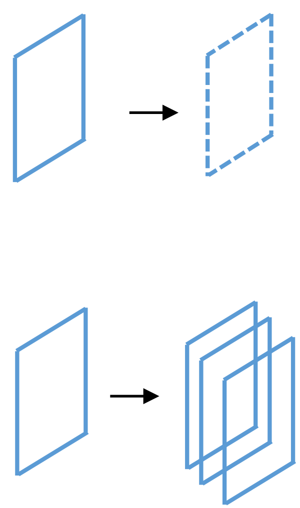
Non-perturbative string theory

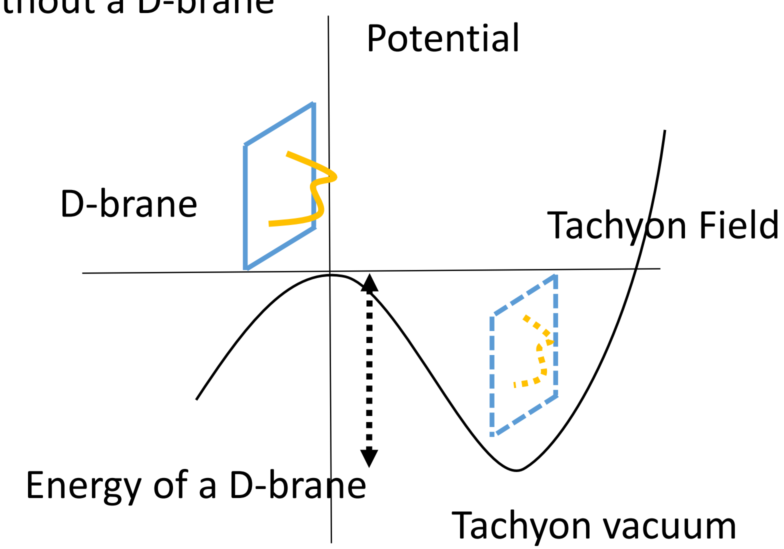
Sen's conjecture

Tachyon vacuum = the solution of string field theory without a D-brane

# of D-brane → -1

# of D-brane → +n ??





1. Bosonic Multiple Solution

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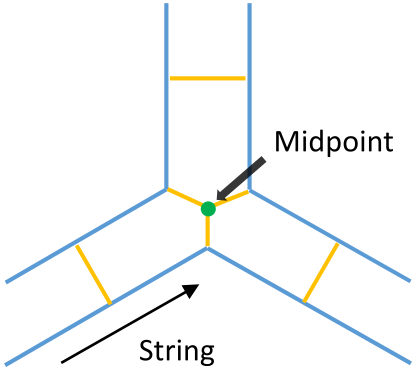
Action

$$-\int[\frac{1}{2}\Psi * Q\Psi + \frac{1}{3}\Psi * \Psi * \Psi]$$

[Witten '86]



Ψ String Field : Open String, gh# = 1, grassmann odd, defined by a superposition of all string states

$$\Psi = T(X)\hat{c}_1|0\rangle + A_\mu(X)\hat{\alpha}^\mu_{-1}\hat{c}_1|0\rangle + \frac{i}{\sqrt{2}}B(X)\hat{c}_0|0\rangle + \dots$$

$\int$ BPZ inner product : String Field → c#

Q BRS operator : String Field → String Field

$*$ Star product (Midpoint int.): String Field $*$ String Field → String Field

EOM

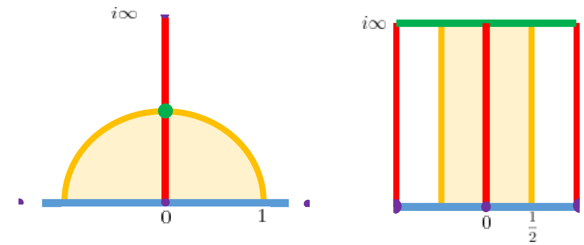
$$Q\Psi + \Psi^2 = 0$$

Gauge transformation

$$\delta_\Lambda \varphi = Q\Lambda + [\varphi, \Lambda]_\pm$$

(finite : $\varphi' = U^{-1}(Q+\varphi)U$)

1. Bosonic Multiple Solution



[Rastelli-Zwiebach '01]

Sliver frame

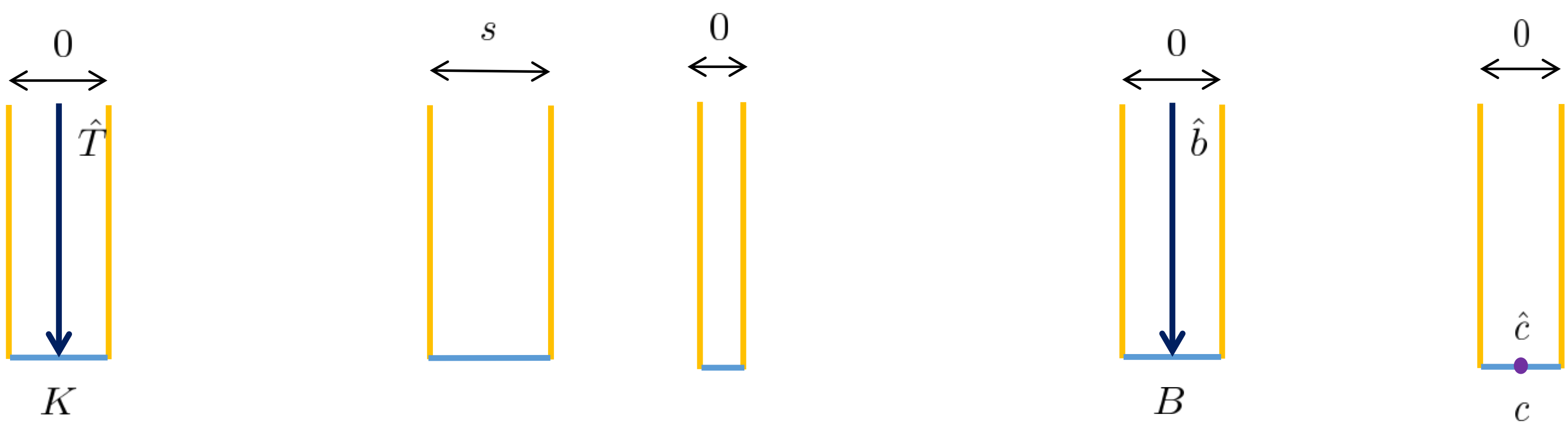
Convenient coordinate of CFT
to describe the * product

$$z = \frac{2}{\pi} \arctan w \qquad w \ : \ \mathrm{UHP}$$

[Okawa '06]

String field *K,B,c*

K,B,c are defined by the sliver frame
and used to construct solutions.



$$K \rightarrow \int_{i\infty}^{-i\infty} \frac{dz}{2\pi i} \hat{T}(z)$$

$\hat{T}$: energy momentum tensor

$$e^{sK}$$

$s \in \mathbb{R}_{\geq 0}$

Wedge state

$$e^{0K} \rightarrow 1$$

Identity
under the $*$ product

$$B \rightarrow \int_{i\infty}^{-i\infty} \frac{dz}{2\pi i} \hat{b}(z) |\mathrm{I}\rangle$$

$$c \rightarrow \hat{c}(z)$$

$\hat{b},\ \hat{c}$: ghost

1. Bosonic Multiple Solution

KBc alg. closed under the following relations:

[Okawa '06]
[Nicholas-Schnabl '16]

$$[K,B]=0 \quad \{B,c\}=1 \quad B^2=c^2=0$$
$$QB=K \quad QK=0 \quad Qc=cKc$$

BPZ inner product by using *K,B,c* in the sliver frame

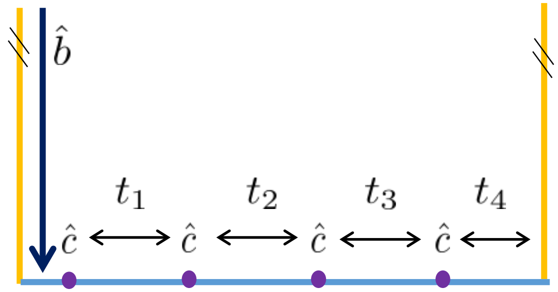
$$\int Bc\,e^{t_1K}c\,e^{t_2K}c\,e^{t_3K}c\,e^{t_4K}=\int_{-i\infty}^{i\infty}\frac{dz}{2\pi i}\langle b(z)c(0)c(t_1)c(t_1+t_2)c(t_1+\dots)\rangle_{C_L}=-\frac{L^2}{4\pi^3}[t_3\sin 2\theta_1+\dots]$$

$$L=t_1+\cdots+t_4$$

$$\theta_i=\frac{\pi t_i}{L}$$

cf.) $\langle c(w_1)c(w_2)c(w_3)\rangle_{\mathrm{UHP}}=w_{12}w_{13}w_{23}$

$(w_{ij}=w_i-w_j)$



1. Bosonic Multiple Solution

Pure-gauge form solution

[Okawa '06]

$$\Psi = U^{-1} Q U \qquad U = Bc + cBg(K)$$

＊ Formally, any solutions can be written in pure-gauge form by using the homotopy opr. .

$$U = 1 + A\Psi \qquad QA = 1$$

[Ellwood '09]

Tachyon vacuum solution

NO D-brane = NO open string vacuum

[Schnabl '05
Erler-Schnabl '09]

$$\Psi_0 = U_1^{-1} Q U_1 = -(Q(cB) + c)\frac{1}{1-K}$$

$$U_1 = Bc + cB\left(\frac{-K}{1-K}\right)$$

＊ we can treat 1/(1-K) as the superposition of wedge state
by using the Schwinger parameter

$$\frac{1}{1-K} = \int_0^\infty dx e^{-x} e^{xK}$$

1. Bosonic Multiple Solution

Double-brane Solution

[Murata-Schnabl '11
Hata-Kojita '12]

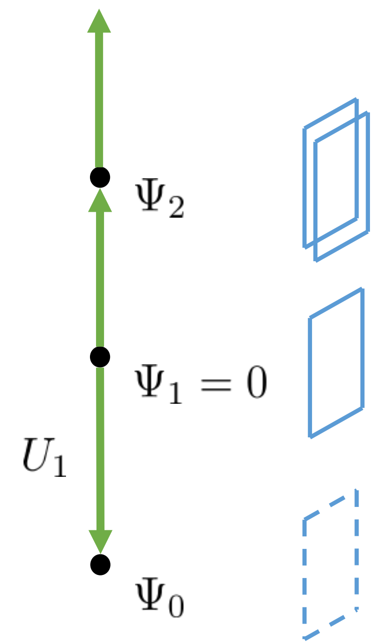
$$\Psi_2 = U_1 Q U_1^{-1} = -cB\frac{K^2}{1-K}c\frac{1}{K}$$

K_ϵ Regularization

$\frac{1}{K}$ is too singular to treat in the wedge state

$$\frac{1}{K} \to \frac{1}{K_\epsilon} = -\int_0^\infty dz \mathrm{e}^{-\epsilon z}\mathrm{e}^{zK}$$

$$[\Psi_2]_\epsilon = -cB\frac{K_\epsilon^2}{1-K_\epsilon}c\frac{1}{K_\epsilon}$$



EOM in the strong sense

$$\int [[\Psi_2]_\epsilon (Q[\Psi_2]_\epsilon + [\Psi_2]_\epsilon{}^2)] = 0$$

$$\int [[\Psi_3]_\epsilon (Q[\Psi_3]_\epsilon + [\Psi_3]_\epsilon{}^2)] \neq 0$$

$$\Psi_3 = U_1^2 Q U_1^{-2}$$

2. Half-brane Solution

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Modified cubic super SFT

[Preitschopf-Thorn-Yost
Arefeva-Medvedev-Zubarev ‘90]

$$S = -\int_{Y_{-2}} [\frac{1}{2}\Psi Q \Psi + \frac{1}{3}\Psi^3]$$

Inverse PCO

$$\int_{Y_{-2}} \varphi = \langle \hat{Y}(i\infty)\hat{Y}(-i\infty)\hat{\varphi}_1(0)\rangle_{C_1}$$

GSO+ → GSO+,- : “internal” CP factor

[Aref'eva-Belov-Giryavets ‘02]

$$\varphi \to \tilde{\varphi} \otimes \sigma_\mu$$

EOM

$$Q\Psi + \Psi^2 = 0$$

2. Half-brane Solution

[Erler '11]

$KBcG\gamma$ alg. Extension of the *KBc* alg.

$$G \to \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \hat{G}_{\text{supercurrent}}(z) \otimes \sigma_1 \qquad \gamma \to \hat{\gamma}(z) \otimes \sigma_2$$

$$K \to \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \hat{T}(z) \otimes \mathbb{I} \qquad B \to \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \hat{b}(z) \otimes \sigma_3 \qquad c \to \hat{c}(z) \otimes \sigma_3$$

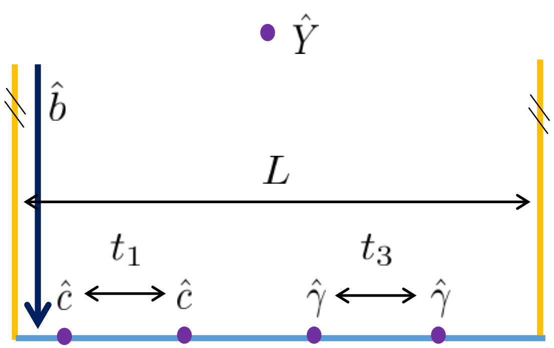
KBc alg +

$$\{G,G\} = 2K,\ [G,B] = 0,\ [G,c] = 2\gamma,\ \{G,\gamma\} = \frac{1}{2}\partial c$$

$$\{\gamma, B\} = \{\gamma, c\} = 0$$

$$QG = 0,\ Qc = cKc + \gamma^2,\ Q\gamma = c\partial\gamma + \frac{1}{2}\gamma\partial c$$

BPZ inner product by using K,B,c,γ in the sliver frame



$$\int_{Y_{-2}} Bce^{t_1}ce^{t_2}\gamma e^{t_3}\gamma e^{t_4} = \langle \int \frac{dz}{2\pi i} \hat{Y}(i\infty)\hat{Y}(-i\infty)\hat{b}(z)\hat{c}(0)\hat{c}(t_1)\hat{\gamma}(t_1+t_2)\hat{\gamma}(t_1+\dots)\rangle_{C_L}$$

$$= -\frac{t_1 L}{\pi^2}\cos[\frac{t_3\pi}{L}]$$

2. Half-brane Solution

Half-brane Solution

The solution constructed by using *K,B,c,G,*γ [Erler '11]

$$\Psi_{\frac{1}{2}} = U_{\frac{1}{2}}^{-1} Q U_{\frac{1}{2}}$$
$$= -(Q(cB) + cBGc)\frac{1}{1-G}$$

$$U_{\frac{1}{2}} = Bc + cB\frac{-G}{1-G}$$

$$\text{cf.)}\ U_1 = Bc + cB\frac{-K}{1-K}$$
$$\Psi_0 = -(Q(cB) + c)\frac{1}{1-K}$$

The energy of this solution is half of the tachyon vacuum.

$$\frac{\pi^2}{3}\int_{Y_{-2}} [\Psi_{\frac{1}{2}} Q \Psi_{\frac{1}{2}}] = -\frac{1}{2}$$

∗ Since 1/K does not exist, we use the EOM.

2. Half-brane Solution

$U_{\frac{1}{2}}{}^2$ and U_1

$$U_{\frac{1}{2}}^{-1}QU_{\frac{1}{2}}^2 = (B\gamma^2 - cB(1-G)^2c)\frac{2G-1}{(1-G)^2}$$

= **Tachyon Vacuum**?

⇓

$$\tilde{U} = U_{\frac{1}{2}}^{-2}U_1 = Bc + cB\frac{G-1}{1+G}$$

= **regular** gauge transformation?

⇓

$$\int_{Y_{-2}} \tilde{\Psi}Q\tilde{\Psi} = 0$$

… ?? → Yes

$$\tilde{\Psi} = \tilde{U}^{-1}Q\tilde{U} = -2(B\gamma^2 + cB\frac{1+G}{1-G}Kc)\frac{1}{1+G}$$

$U_{\frac{1}{2}}{}^{-2}QU_{\frac{1}{2}}{}^2$ and $U_1^{-1}QU_1 = \Psi_0$ are gauge equivalent.

2. Half-brane Solution

Classification by the sliver frame level expansion in [Erler '11]

$$U = Bc + cBg(K,G) \qquad g(K,G) = g_+(K) + g_-(K)G$$

$$\begin{aligned} \text{Perturbative Vacuum} &: \quad g_+(0) \neq 0 \\ \text{Half Solution} &: \quad g_+(0) = 0,\ g_-(0) \neq 0 \\ \text{Tachyon Vacuum} &: \quad g_+(0) = 0,\ g_-(0) = 0,\ g'_+(0) \neq 0 \end{aligned}$$

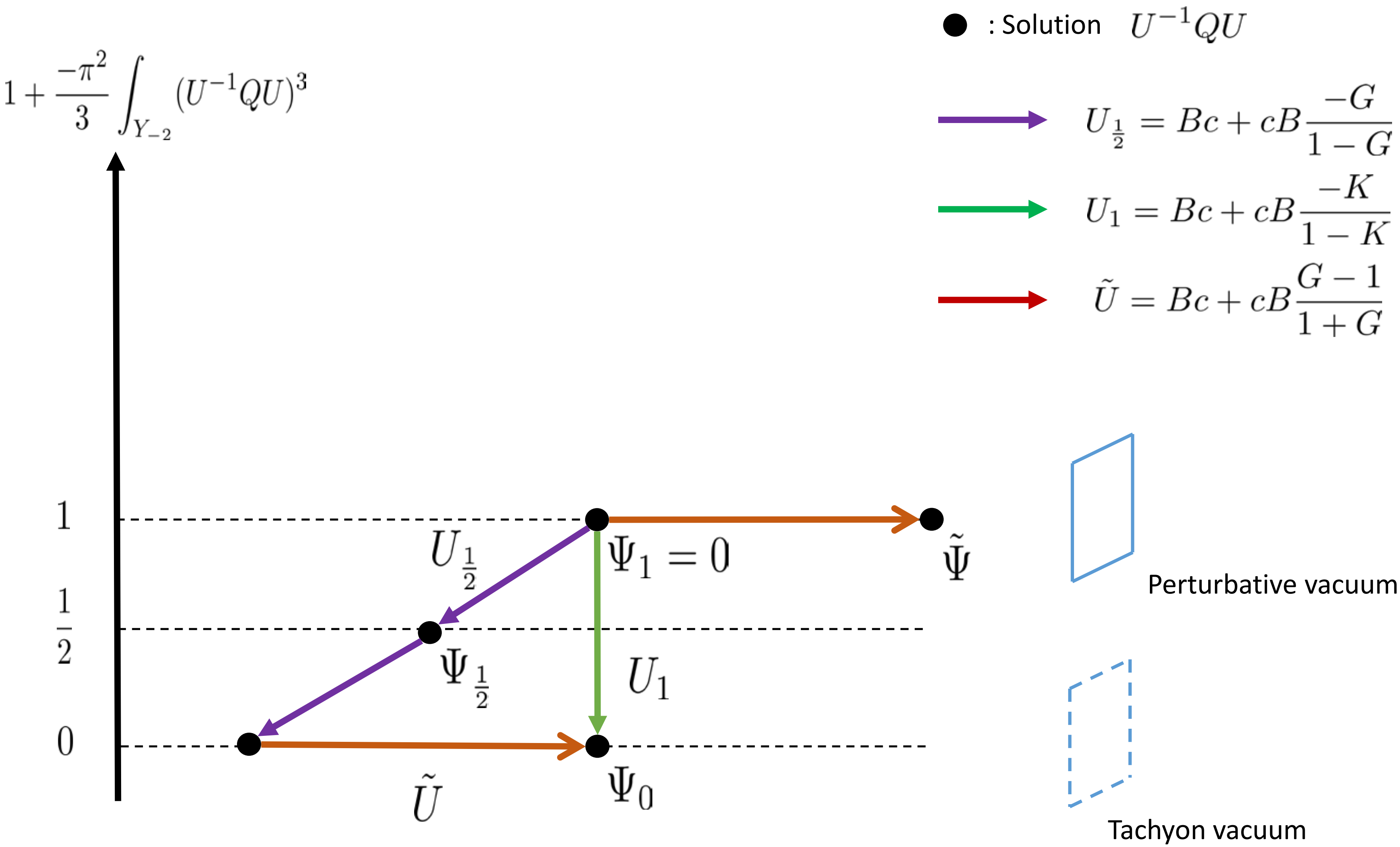
$$U_{\text{half}} = Bc + cB[g_{\text{h}+}(K) + g_{\text{h}-}(K)G]$$

$$U_{\text{half}}^2 = Bc + cB[g_{\text{h}}{}_+^2(K) + g_{\text{h}}{}_-^2(K)K + 2g_{\text{h}+}(K)g_{\text{h}-}(K)G]$$

$$\begin{aligned} &(g_{\text{h}}{}_+^2(K) + g_{\text{h}}{}_-^2(K)K)|_{K=0} = 0 \\ &(2g_{\text{h}+}(K)g_{\text{h}-}(K))|_{K=0} = 0 \\ &(g_{\text{h}}{}_+^2(K) + g_{\text{h}}{}_-^2(K)K)'|_{K=0} = (g_{\text{h}}{}_-^2(K))|_{K=0} \neq 0 \end{aligned}$$

$$\Rightarrow \quad U_{\text{half}}^2 \sim U_{\text{tachyon vacuum}}$$

2. Half-brane Solution



3. Double-brane Solution in Modified Cubic

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Double-brane Solution

$$\Psi_2 = U_1 Q U_1^{-1} = (B\gamma^2 + cB\frac{K^2}{1-K}c)\frac{1}{-K}$$

$$\text{cf.)}\ \Psi_0 = U_1^{-1} Q U_1$$

EOM in the strong sense

$$\int_{Y_{-2}} [\Psi_2]_\epsilon (Q[\Psi_2]_\epsilon + [\Psi_2]_\epsilon{}^2) = \frac{-2}{\pi^2\epsilon} \neq 0$$

↔ In the **bosonic** case, the double-brane solution **satisfy** the EOM in the strong sense.

"winding number"

The double-brane solution
done not satisfying the EOM in the strong sense, **but**

$$1 + \frac{-\pi^2}{3}\int_{Y_{-2}} [\Psi_2]_\epsilon^3 = 2$$

Cf.) $1 + \frac{-\pi^2}{3}\int_{Y_{-2}} \Psi_0{}^3 = 0$

4. “Multiple Half-brane” Solution

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“Multiple Half-brane” Solution

$$\Psi_{?} = U_{\frac{1}{2}} Q U_{\frac{1}{2}}^{-1} = -(-cBKc + B\gamma^2 + cBK\frac{1}{1-G}c)\frac{1}{G}$$

G_ϵ Regularization

$$(U_{\frac{1}{2}} = Bc + cB\frac{-G}{1-G})$$

$$G_\epsilon \equiv \tilde{G} \otimes \sigma_1 - \sqrt{-\epsilon} \otimes \sigma_3$$

$$\frac{1}{2}\{G_\epsilon, G_\epsilon\} = \tilde{G}^2 \otimes \sigma_1^2 - \sqrt{-\epsilon}\tilde{G} \otimes \sigma_3\sigma_1 - \sqrt{-\epsilon}\tilde{G} \otimes \sigma_1\sigma_3 - \epsilon \otimes \sigma_3^2$$

$$= \tilde{G}^2 \otimes 1 - \epsilon \otimes 1 = \tilde{K} \otimes 1 - \epsilon \otimes 1 = K_\epsilon$$

$$\Psi_{?} \to [\Psi_{?}]_\epsilon = -(-cBK_\epsilon c + B\gamma^2 + cBK_\epsilon \frac{1}{1-G_\epsilon}c)\frac{1}{G_\epsilon}$$

4. “Multiple Half-brane” Solution

EOM in the strong sense

$$\int_{Y_{-2}} [\Psi_?]_\epsilon (Q[\Psi_?]_\epsilon + [\Psi_?]_\epsilon{}^2) = 0$$

“winding number”

$$1 + \frac{-\pi^2}{3} \int_{Y_{-2}} [\Psi_?]_\epsilon^3 = \frac{3}{2}$$

Cf.) $$1 + \frac{-\pi^2}{3} \int_{Y_{-2}} \Psi_{\frac{1}{2}}{}^3 = 1 + \frac{\pi^2}{3} \int_{Y_{-2}} \Psi_{\frac{1}{2}} Q \Psi_{\frac{1}{2}} = \frac{1}{2}$$

EOM in the strong sense + “winding number”

⇒ energy of this solution = (energy of a D-brane) × 3/2

4. “Multiple Half-brane” Solution

＊Detail of the calculation

Ex.)

$$\int_{Y_{-2}} [\Psi_?]_\epsilon^3 = -3\int B\gamma^2 \frac{1}{G_\epsilon} cB \frac{K_\epsilon G_\epsilon}{1-G_\epsilon} c \frac{1}{G_\epsilon} cB \frac{K_\epsilon G_\epsilon}{1-G_\epsilon} c \frac{1}{G_\epsilon} + \ldots$$

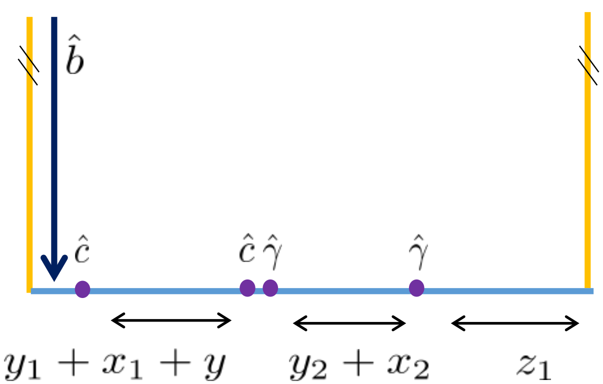
$$= 3\int BcK_\epsilon \frac{1}{1-K_\epsilon} Kc\gamma K_\epsilon \frac{1}{1-K_\epsilon}\gamma \frac{1}{K_\epsilon} + \ldots\ldots$$

$$= \ldots$$

$$\int_{Y_{-2}} G_\epsilon \varphi = \frac{1}{2}\int_{Y_{-2}} \delta(\varphi)$$

(δ : super transformation)

$$-\frac{\partial}{\partial y_1}\frac{\partial}{\partial y_2}\frac{\partial}{\partial y}\int_0^\infty dx_1 dx_2 dz\, e^{-x_1-\epsilon-\epsilon(y_1+y_2)-\epsilon z_1}$$



$$\Big|_{y=0, y_1=0, y_2=0}$$

||

$$-\frac{(y_1+x_1+y)L}{\pi^2}\cos\left[\frac{z_1\pi}{L}\right]$$

$$L = y_1 + x_1 + y + y_2 + x_2 + z_1$$

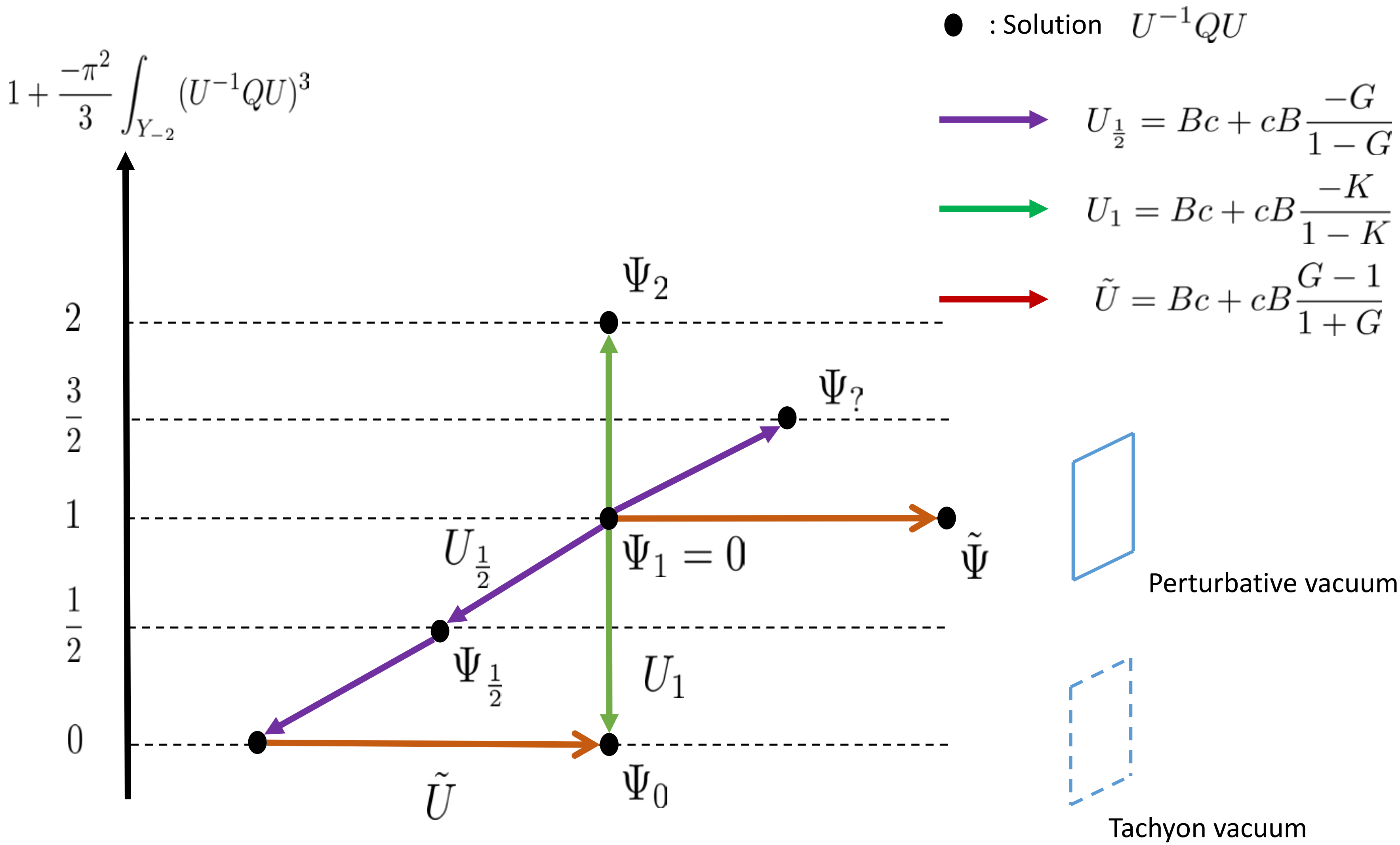
$$\int_0^\infty da \int_0^1 db \int_0^1 dc\, a^2$$

$$a = x_1 + x_2 + z_1$$

$$b = \frac{z_1}{x_1+x_2+z_1}$$

$$c = \frac{z_1+x_1}{x_1+x_2+z_1}$$

4. “Multiple Half-brane” Solution



5. Summary

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Summary

As in bosonic case, we check **the EOM in the strong sense for the double-brane**, in modified cubic string field theory.
However, it does **not satisfy** the EOM and its energy is not twice of a D-brane.
On the other hand, "winding number" is expected value.

We think the **"multiple half-brane" solution** constructed by the $KBcG\gamma$ alg.
and we introduce the G_ϵ **regularization**,
and we check that it satisfy the **EOM in the strong sense**
and its energy is **(energy of a D-brane) × 3/2** .

Future works

Multiple brane solution as the Erler-Maccaferri solution,
which uses Boundary Condition Changing Operator
in modified cubic string field theory

Solution of modified cubic string field theory
→ Solution of Berkovits' string field theory