Beyond the Standard Model in Multi-Spinor Field Formalism

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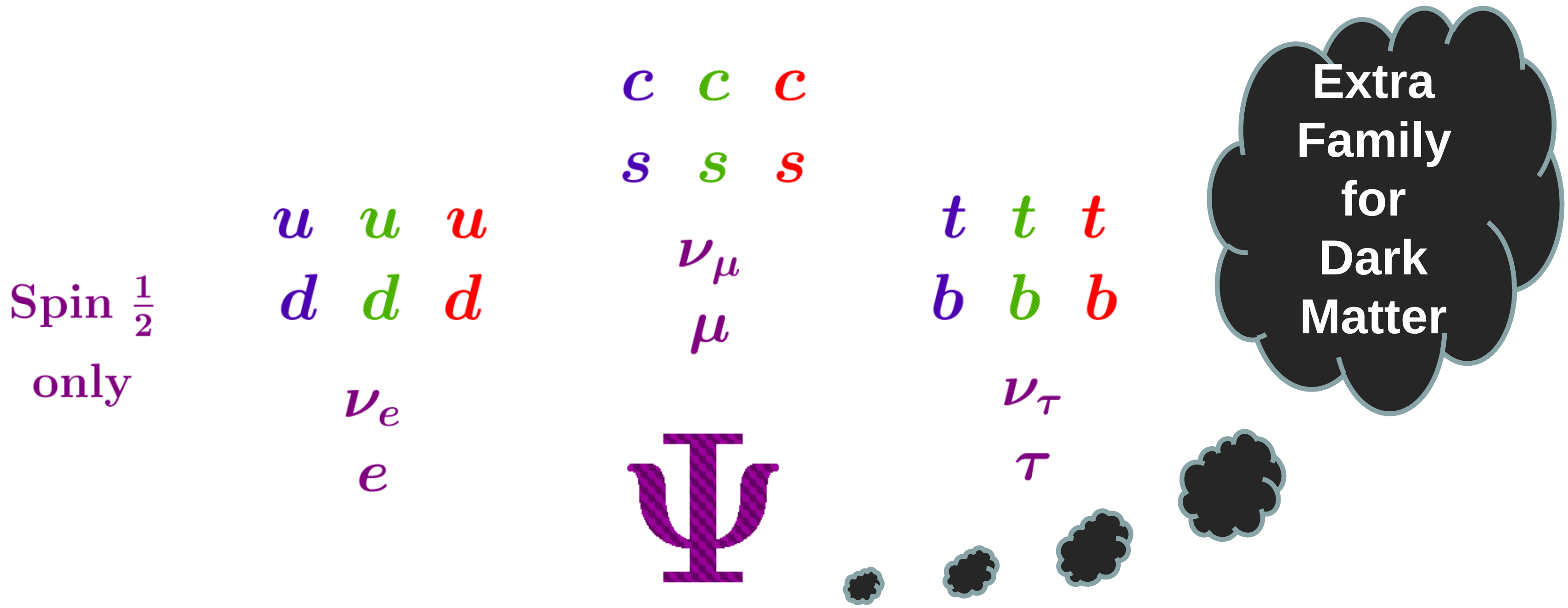
We are still
at phenomenological stage

We have to look for
new key concepts and formalism
step by step

Remind the era of "Eightfold Way"

Motivation

Rich spectra of quarks & leptons



Tree of fermionic master field

Multi-spinor local field
A substitute for composite scheme

Basic postulate

Dirac's spinor field for electron

$$\psi_a \qquad\qquad 4$$

$$\Downarrow$$

Multi-spinor local field

$$\Psi_{abc} \propto \psi_a \psi_b \psi_c \qquad\qquad 4x4x4 = 64$$

Local field behaving as triple-tensor-products of Dirac's spinor fields

Naïve extension of WS to triplet fields

Chiral spinor fields

EW doublet

$$\psi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

EW singlets

$$\psi_{\nu R} = (\nu_e)_R$$

$$\psi_{eR} = (e)_R$$

$\psi \quad \psi$

4 + 4 = 8

$\Rightarrow$

Chiral triplet fields

EW doublet

$$\Psi_L = \begin{pmatrix} \Psi_u \\ \Psi_d \end{pmatrix}_L$$

EW singlets

$$U = (\Psi_u)_R$$

$$D = (\Psi_d)_R$$

$\Psi \quad \Psi$

64 + 64 = 128

Sequential 4 family scheme : Excluded!

L-R twisted scheme with (3+1) families

4th family for dark matter

$$\Psi_L = {}^t\left(\boxed{\Psi_{(v)}} : \boxed{\begin{matrix} U_{(d)} \\ D_{(d)} \end{matrix}} \right)_L$$

$$\Psi_R = {}^t\left(\boxed{\begin{matrix} U_{(v)} \\ D_{(v)} \end{matrix}} : \boxed{\Psi_{(d)}} \right)_R$$

Visible $G = SU_c(3) \times SU_L(2) \times U_Y(1)$

Dark $G_\star = SU_{c^*}(3) \times SU_R(2) \times U_{Y^*}(1)$

$$2 \times 2 \times (3+1) \times (3+1) = 64$$

$$2 \times 2 \times (3+1) \times (3+1) = 64$$

Triplet algebra for triplet field

Dirac algebra

$$A_\gamma \;=\; \{1,\, \gamma_\mu,\, \sigma_{\mu\nu},\, \gamma_5\gamma_\mu,\, \gamma_5\} \;=\; \langle\gamma_\mu\rangle$$

Triplet algebra

$$\begin{aligned} A_T \; &= \{\, p\otimes q\otimes r : p,\, q,\, r \in A_\gamma \,\} \\ &= \langle\, \gamma_\mu\otimes 1\otimes 1,\, 1\otimes\gamma_\mu\otimes 1,\, 1\otimes 1\otimes\gamma_\mu \,\rangle \end{aligned}$$

Criterion for physical subalgebra

Closed and irreducible under permutation group S_3

$$p\otimes q\otimes r \;\rightarrow\; q\otimes r\otimes p \quad \text{etc}$$

A_Γ algebra

$$A_\Gamma = \langle\, \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \,\rangle$$

spacetime

$$\Gamma_\mu = \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \longleftrightarrow x^\mu$$

$$\Gamma_\mu \Gamma_\nu + \Gamma_\nu \Gamma_\mu = 2\eta_{\mu\nu} I$$

$$I = 1 \otimes 1 \otimes 1$$

$$\Sigma_{\mu\nu} = -\frac{i}{2}(\Gamma_\mu \Gamma_\nu - \Gamma_\nu \Gamma_\mu) = \sigma_{\mu\nu} \otimes \sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$$

$$\Gamma_5 = -i\Gamma^0\Gamma^1\Gamma^2\Gamma^3 = \gamma_5 \otimes \gamma_5 \otimes \gamma_5$$

$$\Gamma^\mu = \eta^{\mu\nu}\Gamma_\nu$$

$$A_\Gamma = \{1,\, \Gamma_\mu,\, \Sigma_{\mu\nu},\, \Gamma_5\Gamma_\mu,\, \Gamma_5\} = \langle \Gamma_\mu \rangle \leftrightarrow A_\gamma$$

Lorentz transformation for triplet field $\Psi(x) \equiv (\Psi_{abc})(x)$

$$x'^{\mu} = \Omega^{\mu}{}_{\nu} x^{\nu} \ : \ \Omega_{\lambda\mu}\Omega^{\lambda}{}_{\nu} = \eta_{\mu\nu}$$

Dirac spinor field

$$\psi'(x') = s(\Omega)\psi(x) \qquad s(\Omega) = \exp\left(-\frac{i}{4}\sigma_{\mu\nu}\omega^{\mu\nu}\right)$$

Triplet fields

$$\Psi'(x') = S(\Omega)\Psi(x) \qquad S(\Omega) = \exp\left(-\frac{i}{4}\Sigma_{\mu\nu}\omega^{\mu\nu}\right)$$

$$\Sigma_{\mu\nu} = -\frac{i}{2}(\Gamma_{\mu}\Gamma_{\nu} - \Gamma_{\nu}\Gamma_{\mu}) = \sigma_{\mu\nu} \otimes \sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$$

External subalgebra $A_{ex} = \{\, \Sigma_{\mu\nu} \,\} \subset A_{\Gamma}$

Chirality for triplet fields

$$L = \frac{1}{2}(I - \Gamma_5), \quad R = \frac{1}{2}(I + \Gamma_5)$$

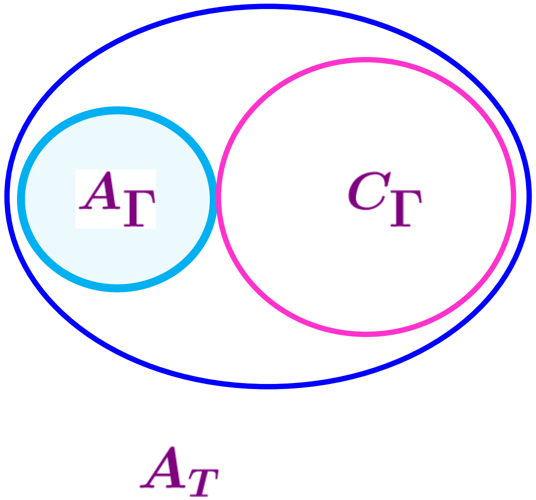
Centralizer of A_Γ algebra

$$C_\Gamma = \{X \in A_T : [\,X,\ \Gamma_\mu\,] = 0\} \qquad \Gamma_\mu = \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu$$

$$C_\Gamma = \langle\, 1 \otimes \gamma_\mu \otimes \gamma_\mu,\ \gamma_\mu \otimes 1 \otimes \gamma_\mu \,\rangle$$

$$A_\Gamma = \langle\, \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \,\rangle$$

$$A_T = A_\Gamma C_\Gamma, \quad A_\Gamma \cap C_\Gamma = I$$

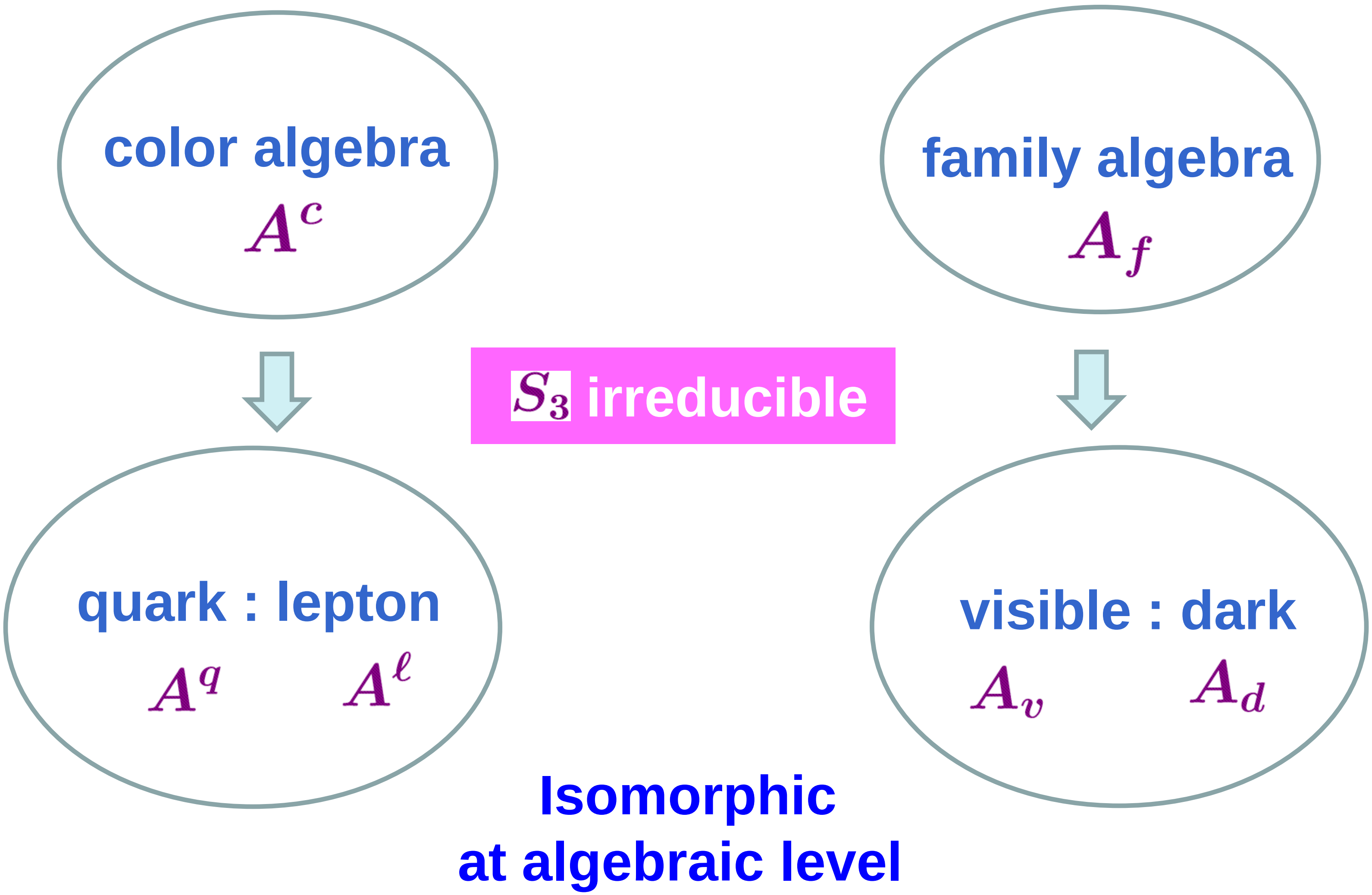


Coleman-Mandula theorem

External subalgebra $A_{ex} = \{\,\Sigma_{\mu\nu}\,\} \subset A_\Gamma$

Internal subalgebra $A_{in} \subset C_\Gamma$

Internal subalgebra $A_{in} \subset C_\Gamma$



Model building with triplet fields

L-R twisted model

$G = SU_c(3) \times SU_L(2) \times U_Y(1)$ **for visible sector**

$G_\star = SU_{c^*}(3) \times SU_R(2) \times U_{Y^*}(1)$ **for dark sector**

Fundamental representation

L field $\Psi_L = {}^t\begin{pmatrix} \Psi_{(v)} & : & \begin{matrix} U_{(d)} \\ D_{(d)} \end{matrix} \end{pmatrix}_L$

EW doublet EW* singlets

R field $\Psi_R = {}^t\begin{pmatrix} \begin{matrix} U_{(v)} \\ D_{(v)} \end{matrix} & : & \Psi_{(d)} \end{pmatrix}_R$

EW singlets EW* doublet

G $G_\star$

L field

$$\Psi_L = {}^t\left(\Psi_{(v)} \; : \; \begin{matrix} U_{(d)} \\ D_{(d)} \end{matrix} \right)_L$$

Quark states

$$\Psi^{(q)}_{(v)} = \begin{pmatrix} uuu & ccc & ttt \\ ddd & sss & bbb \end{pmatrix}_L$$

Lepton states

$$\Psi^{(\ell)}_{(v)} = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{pmatrix}_L$$

G

Star symbol

$$U^{(q)}_{(d)} = \left(u_\star u_\star u_\star \right)_L$$

$$D^{(q)}_{(d)} = \left(d_\star d_\star d_\star \right)_L$$

$$U^{(\ell)}_{(d)} = \left(\nu_\star \right)_L$$

$$D^{(\ell)}_{(d)} = \left(e_\star \right)_L$$

$G_\star$

R field

$$\Psi_R = {}^t\left(\begin{matrix} U_{(v)} \\ D_{(v)} \end{matrix} \; : \; \Psi_{(d)} \right)_R$$

Star symbol

Quark states

$$U^{(q)}_{(v)} = (\; uuu \quad ccc \quad ttt \;)_R$$

$$D^{(q)}_{(v)} = (\; ddd \quad sss \quad bbb \;)_R$$

Lepton states

$$U^{(\ell)}_{(v)} = (\; \nu_e \quad \nu_\mu \quad \nu_\tau \;)_R$$

$$D^{(\ell)}_{(v)} = (\; e \quad \mu \quad \tau \;)_R$$

G

$$\Psi^{(q)}_{(d)} = \begin{pmatrix} u_\star u_\star u_\star \\ d_\star d_\star d_\star \end{pmatrix}_R$$

$$\Psi^{(\ell)}_{(d)} = \begin{pmatrix} \nu_\star \\ e_\star \end{pmatrix}_R$$

$G_\star$

Lagrangian density of L-R twisted model

Kinetic and gauge parts

$$\mathcal{L}_{kg} = \bar{\Psi}_L i\Gamma^\mu \mathcal{D}_\mu \Psi_L + \bar{\Psi}_R i\Gamma^\mu \mathcal{D}_\mu \Psi_R$$

Yukawa parts

$$\mathcal{L}_Y \quad = \quad \bar{\Psi}_L \mathcal{Y}(\,\mathrm{Higgs}\,)\Psi_R + \mathrm{h.c.}$$

$\mathcal{Y}$: Kernel for Yukawa couplings of elements of family algebra

Two Higgs doublets

Extra Higgs singlet

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \quad \varphi_\star = \begin{pmatrix} \varphi_\star^+ \\ \varphi_\star^0 \end{pmatrix}$$

$$\phi_\star$$

Visible Higgs

G

Dark Higgs

$G_\star$

Necessary to make the dark photon massive

Higgs potential

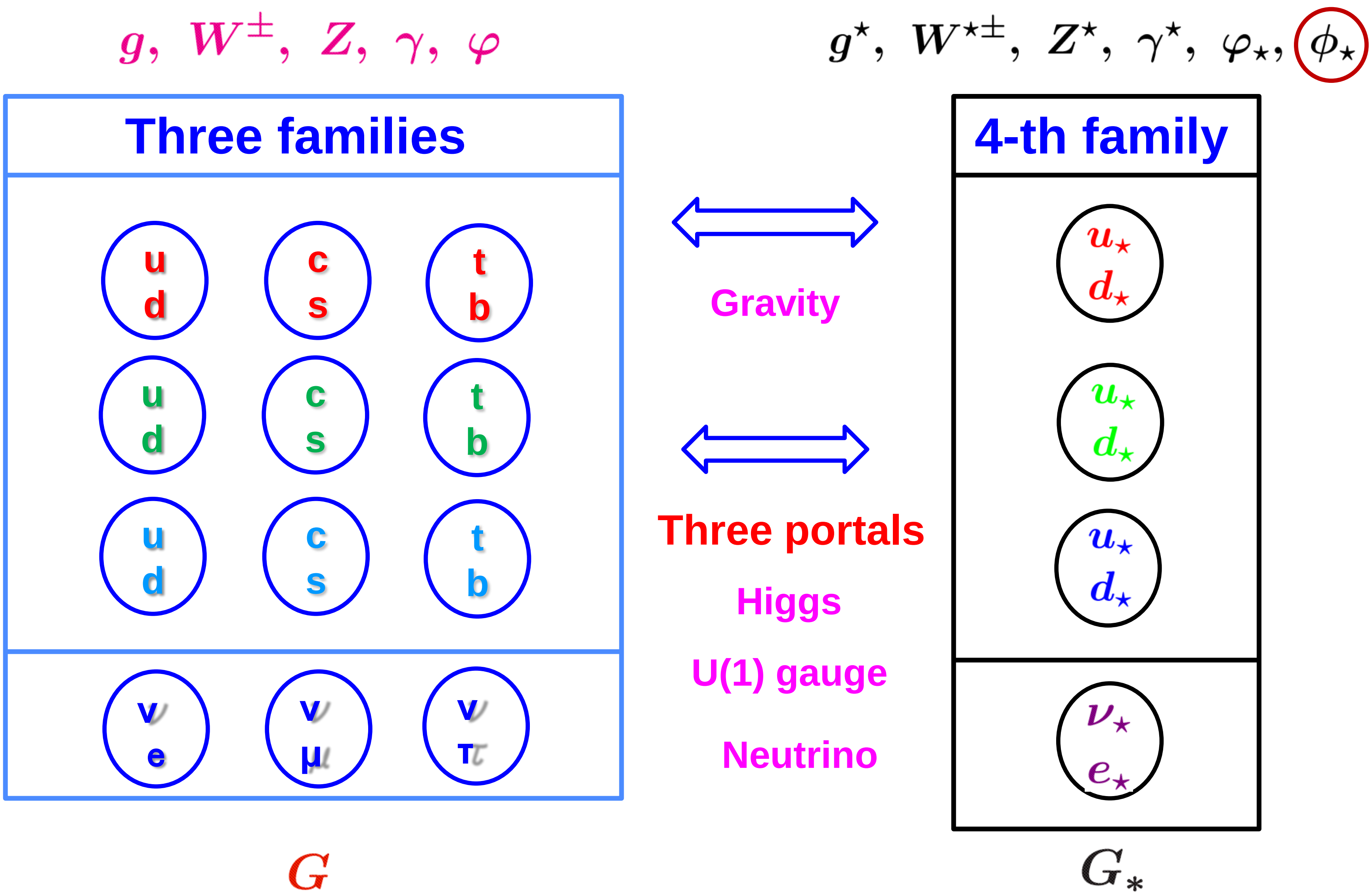
$$V_H = V_0 - \mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2 - \mu_\star^2 \varphi_\star^\dagger \varphi_\star + \lambda_\star (\varphi_\star^\dagger \varphi_\star)^2$$

$$+ \boxed{2\lambda_I (\varphi^\dagger \varphi)(\varphi_\star^\dagger \varphi_\star)} \boxed{- \mu_{0\star}^2 \phi_\star^\dagger \phi_\star + \lambda_{0\star} (\phi_\star^\dagger \phi_\star)^2}$$

Portal coupling

Extra Higgs singlet

Visible and dark sectors



Simple scheme for dark dynamics : without $\phi_\star$

Breakdown of $G_{WS\star} = SU_R(2) \times U_{Y*}(1)$ symmetry

$$m_{u_\star} \gg m_{d_\star} + m_{e_\star}$$

$$u_\star \to d_\star + \bar{e}_\star + \nu_\star$$

Only one stable hadron Spin $\frac{3}{2}$

Dark hadron $\Delta_\star^- = [d_\star\, d_\star\, d_\star] = \frac{1}{\sqrt{6}} \epsilon_{ijk} d_\star^i\, d_\star^j\, d_\star^k$

Dark atom $H_\star = (\Delta_\star^- \; + \; \bar{e}_\star)$

Dark molecule $(H_\star)_2 = H_\star H_\star$

Candidates of DM $(H_\star)_2,\; H_\star,\; \Delta_\star^-,\; e_\star,\; \nu_\star : \nu_{iR} : \gamma^\star$

No nuclear reaction : Simple thermal history

Simplest scenario of dark dynamics: with $\phi_\star$

Breakdown of $G_{WS\star} = SU_R(2) \times U_{Y*}(1)$ symmetry

$$m_{u_\star} \gg m_{d_\star} + m_{e_\star}$$

$$u_\star \to d_\star + \bar{e}_\star + \nu_\star$$

Only one stable hadron

Spin $\frac{3}{2}$

Dark hadron $\Delta_\star^- = [d_\star\, d_\star\, d_\star] = \frac{1}{\sqrt{6}} \epsilon_{ijk} d_\star^i\, d_\star^j\, d_\star^k$

No dark atom $H_\star \not= (\Delta_\star^- + \bar{e}_\star)$

No dark molecule $(H_\star)_2 \not= H_\star H_\star$

Candidates of DM $\Delta_\star^-,\ e_\star,\ \nu_\star : \nu_{iR} : \gamma^\star,\ \phi_0^\star\ ?$

No electromagnetism, No nuclear reaction
Very simple thermal history

Breakdown of two symmetries

$$G_{WS} = SU_L(2) \times U_Y(1)$$

$$G_{WS\star} = SU_R(2) \times U_{Y*}(1)$$

$$V_H = V_0 - \mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2 - \mu_\star^2 \varphi_\star^\dagger \varphi_\star + \lambda_\star(\varphi_\star^\dagger \varphi_\star)^2$$

$$+ 2\lambda_I (\varphi^\dagger \varphi)(\varphi_\star^\dagger \varphi_\star)$$

Higgs portal coupling

Unitary decomposition

$$\varphi(x) = \frac{1}{\sqrt{2}} U(\vartheta(x)) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Visible Higgs

$$\varphi_\star(x) = \frac{1}{\sqrt{2}} U_\star(\vartheta_\star(x)) \begin{pmatrix} 0 \\ v_\star + h_\star(x) \end{pmatrix}$$

Dark Higgs

$$V_H(h,\, h_\star) = \boxed{\lambda v^2 h^2} + \boxed{\lambda_\star v_\star^2 h_\star^2} + \boxed{2\lambda_I v v_\star h h_\star}$$

Interaction mode

$$+\ \lambda v h^3 + \lambda_\star v_\star h_\star^3 + \lambda_I v h h_\star^2 + \lambda_I v_\star h^2 h_\star$$

$$+\ \cdots$$

$$\boxed{m_h^2 = 2\lambda v^2 (\simeq \Lambda^2)} \qquad \boxed{m_{h\star}^2 = 2\lambda_\star v_\star^2 (\simeq \Lambda_\star^2)}$$

Mass eigen-modes

Visible Higgs

Dark Higgs

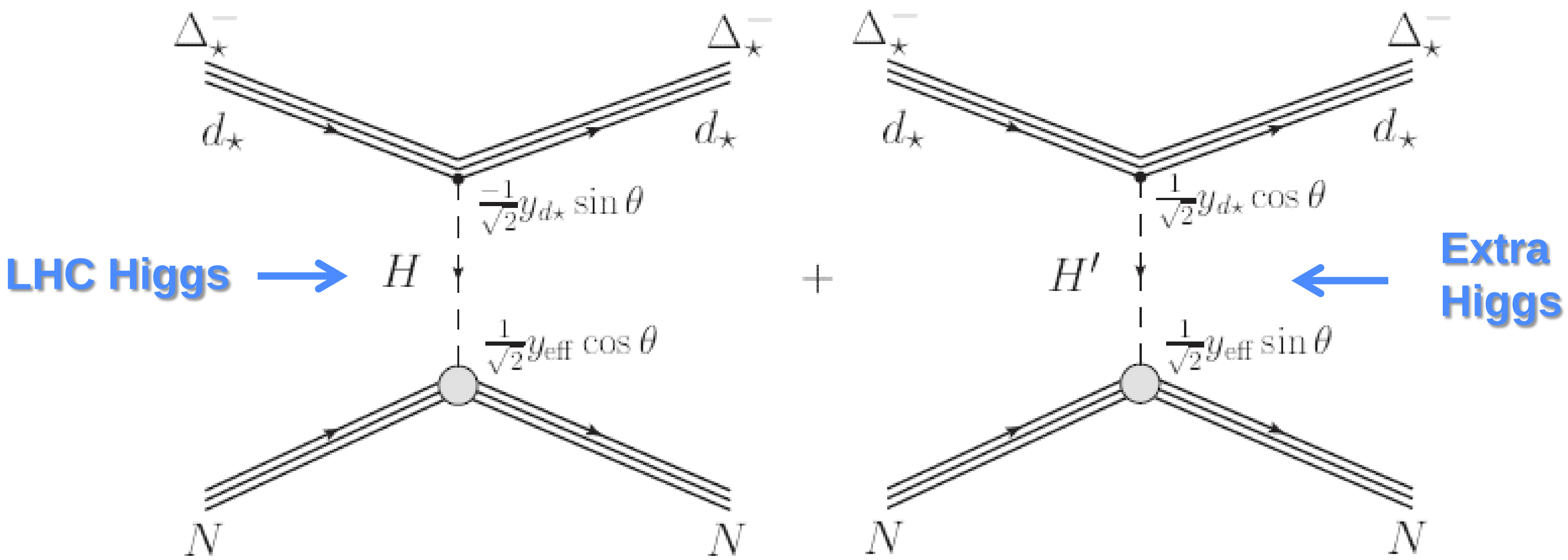
LHC Higgs

Extra Higgs

$$\begin{cases} H = \cos\theta\, h - \sin\theta\, h_\star \\ H' = \sin\theta\, h + \cos\theta\, h_\star \end{cases}$$

A portal to the dark sector

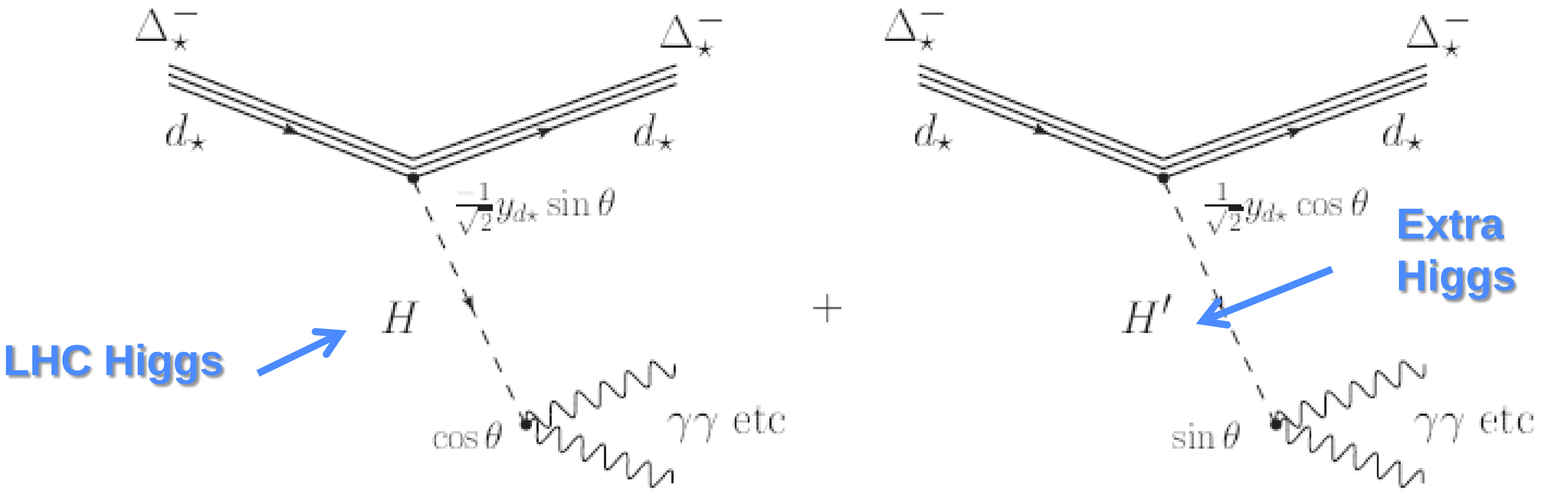
Direct detection of dark matter



$$\frac{1}{2} y_{d\star} y_{\rm eff} \sin\theta\cos\theta \left[\frac{1}{m_H^2} - \frac{1}{m_{H_\star}^2} \right] = \frac{1}{4} \frac{\lambda_I}{\lambda_\star \lambda - \lambda_I^2} \frac{y_{d\star} y_{\rm eff}}{v_{h\star} v_h}$$

LUX & PandaX & Xenon 1t

Indirect detection of dark matter



Difficult to identify the process from decay products

Fermi & AMS-2

Mass of stable dark hadron

$$\Delta_\star^- = [d_\star\, d_\star\, d_\star] = \frac{1}{\sqrt{6}}\epsilon_{ijk} d_\star^i\, d_\star^j\, d_\star^k$$

(1) Era of common soup of visible & dark quanta

1 dark family : 3 visible families

(2) Same mechanism for baryogenesis in two sectors

(3) Era with stable nucleons N and dark hadrons Δ_*

4 **4**

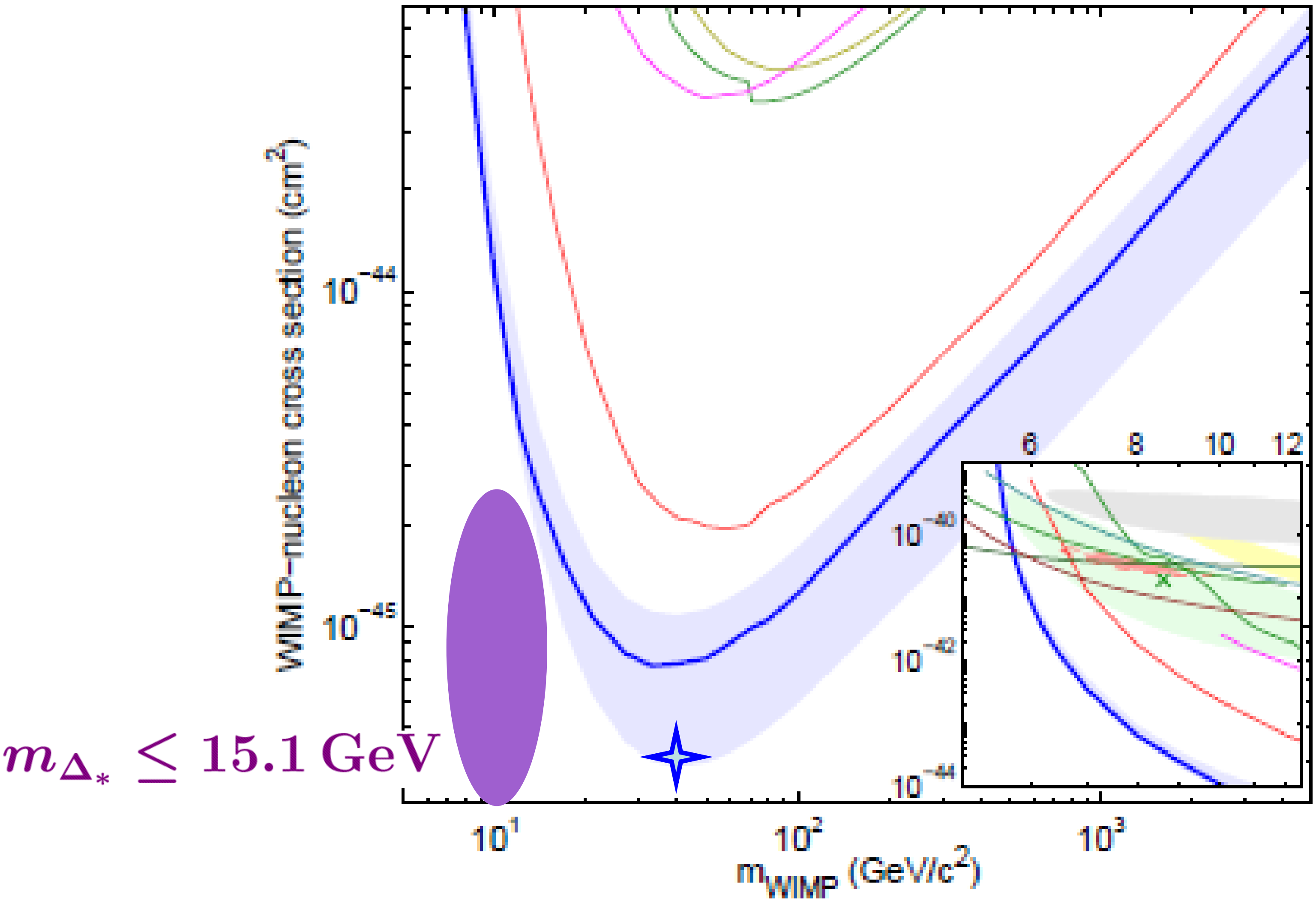
$$2m_{\Delta_\star} : 6m_N = \Omega_c h^2 : \Omega_b h^2 = 0.11889 : 0.022161$$

$$m_{\Delta_*} = \frac{3 \times 0.11889}{0.022161} m_N$$

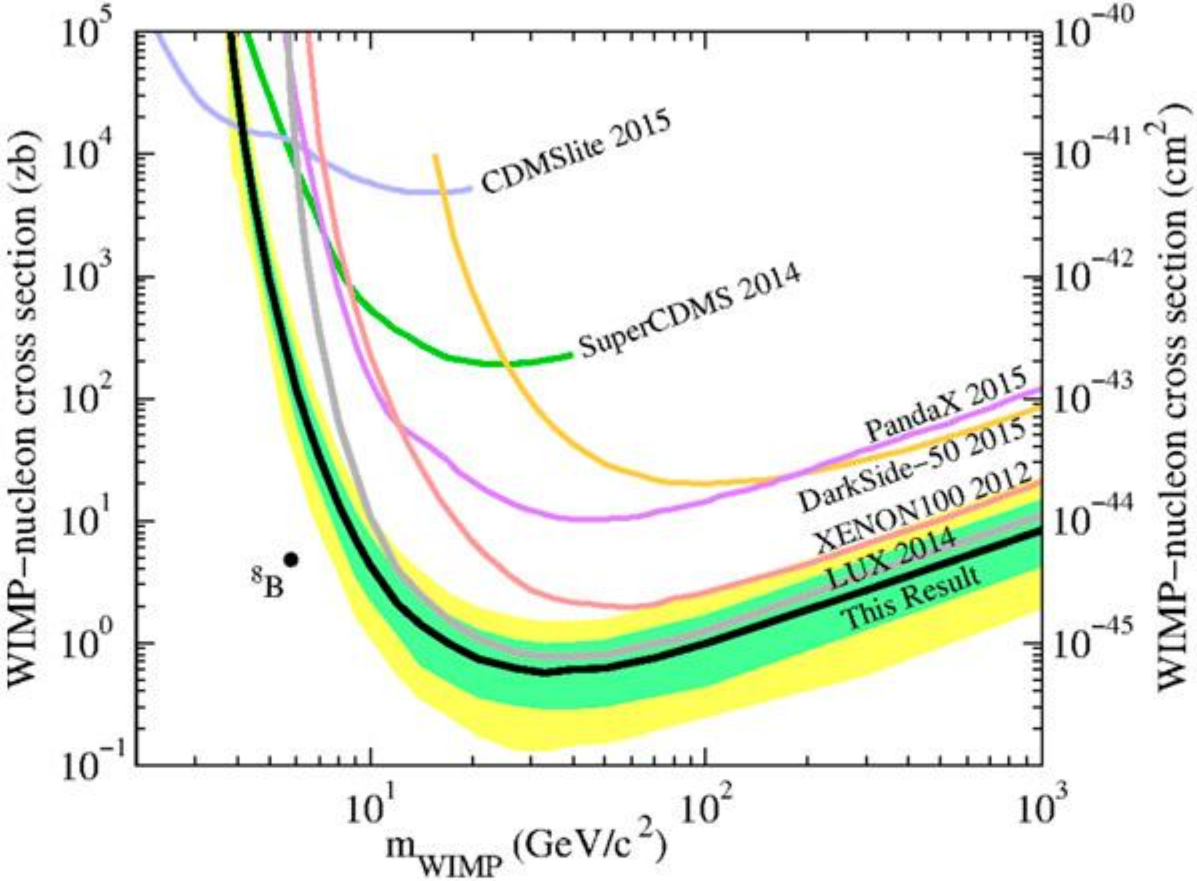
Data from Planck

$$m_{\Delta_*} \leq 15.1\,\mathrm{GeV}$$

LUX detection



7.6×10^{-46} cm^2 at a WIMP mass of 33 GeV/c^2



LUX & PandaX & Xenon 1t : 2016

Early reheating period : Inseparable phase

G and $G_\star$ symmetric stage

Friedmann equation ⇦ All visible & dark fields

Effective number of relativistic d. o. f.

$$
\begin{aligned}
g_* &= \left(28 + \tfrac{7}{8} \times 90\right) + \left(28 + \tfrac{7}{8} \times 30 + 2\right) \\
&= 106.75 + (54.25 + 2) = 161 + 2 = 163
\end{aligned}
$$

⇩ **Many scales of symmetry breakings**

No evidence for extra relativistic species except ordinary photons + (almost) massless neutrinos

Dark photon γ_* should be massive

Results

Fundamental representation of chiral triplet fields

$$\Psi_L(x) = {}^t\begin{pmatrix} \Psi_{(v)} & : & \begin{matrix} U_{(d)} \\ D_{(d)} \end{matrix} \end{pmatrix}_L \quad \Psi_R(x) = {}^t\begin{pmatrix} \begin{matrix} U_{(v)} \\ D_{(v)} \end{matrix} & : & \Psi_{(d)} \end{pmatrix}_R$$

Three families of visible Q's and L's

$$G = SU_c(3) \times SU_L(2) \times U_Y(1)$$

Additional single family of dark Q's and L's

$$G_\star = SU_{c^*}(3) \times SU_R(2) \times U_{Y^*}(1)$$

Monotone world of dark matter

⇩

Careful study of thermal history : Required

Problems remained

Massive dark photon & massive dark scalar $\gamma_\star \ \phi_0^\star$

Dark U(1) Higgs mechanism

U(1) gauge portal coupling

$$\mathcal{L} = g_{\rm mix} F_{\mu\nu} F_\star^{\mu\nu}$$

Neutrino sector: SM + portal

$$\begin{array}{cccc|c} l_1 & l_2 & l_3 & & N_4^L \\ N_1^R & N_2^R & N_3^R & & r_4 \end{array}$$

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\rm SM} + \lambda_{ij} \bar{N}_i^R l_j + \tfrac{1}{2} M_i \bar{N}_i^R N_i^R \\ &+ \mathcal{L}_{\rm DARK} + \lambda_{44} \bar{N}_4^L r_4 + \tfrac{1}{2} M_4 \bar{N}_4^L N_4^L \\ &+ \lambda_{i4} \bar{N}_i^R N_4^L + \text{h.c.} \end{aligned}$$

Seesaw mechanism **Leptogenesis**

Thank you
for
your attention

We are still
at phenomenological stage

We have to look for
new key concepts and formalism
step by step