Defored extra dimension

-Non-local fields out of non-commutative spacetime-

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第 6 回 日大理工・益川塾連携シンポジュウム

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はじめに

第 6 回 日大理工・益川塾連携シンポジュウム

素粒子と時空／現象から探る素粒子（2011∼）

1. 2011.10.30-31 日大研究会と時空に拡がった素粒子模型
2. 2012.11.02-03 Non-Local vs. Non Commutative
3. 2014.03.15-16 A Dream of Yukawa
4. 2014.11.08-09 衝撃波型背景時空に置ける bi-local 模型
5. 2015.10.24-25 Bi-Local model in AdS_5 Spacetime and Higher-spin Gravity

日大物理学科：湯川秀樹を顧問として 1957 年創設

非局所場（湯川）理論

湯川の視点（当時）

- 局所場の困難（発散の問題）
- 素粒子の多様性

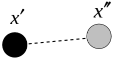
湯川の試み

- bi-local fields
- domain

- bi-local fields (1947∼)

$$[p^\mu,[p_\mu,U]] - m^2U = 0$$
$$[x^\mu,[x_\mu,U]] + \lambda^2U = 0$$
$$[p^\mu,[x_\mu,U]] = 0$$

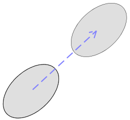
$$\Phi(X,\bar{x}) = \langle x'|U|x''\rangle$$
$$X = \frac{1}{2}(x'+x'')$$
$$\bar{x} = x' - x''$$



two particle system ?

- domain (1968)

$$\exp\left(\sum_{\alpha=1}^{4}\lambda_\alpha\epsilon_\mu^\alpha\partial/\partial X_\mu\right)\Psi = \exp(-i\lambda S)\Psi$$



非局所の seeds ？

- noncommutative ?
  - 量子群型 $[A, B] \to [A, B]_q = AB - qBA$
    - q-deformed phase space
      $$[x, p]_q = i\hbar f(q, xp)$$
    - q-deformed oscillator
      $$[a, a^\dagger]_q = f(q, a^\dagger a)$$
  - Lie 群型 $[x^\mu, x^\nu] = 0 \to [x^\mu, x^\nu] = T^{\mu\nu}{}_\rho x^\rho$
    $\to$ κ-Minkowski spacetime
- extra dimensions (+noncommutative) ?
  $\to$ κ-Minkowski type of spacetime

S.Naka, H.Toyoda and T.Takanashi, PTP **124**(2010), 1019

q-deformed phase space

$$[\hat{\partial}_y, y]_q = 1 \to \hat{\partial} = \partial_y \frac{q^{(y\partial_y)} - 1}{(q-1)(y\partial_y)}$$

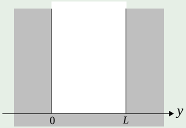
$$\hat{p} = -\frac{i\hbar}{2}(\hat{\partial}_y + \hat{\partial}_y^\dagger) = -i\hbar \frac{q+1}{2q} D_y$$

$$D_y f(y) = \frac{f(qy) - f(q^{-1}y)}{(q-q^{-1})y}$$

$\Downarrow$ 非可換 $\sim$ 非局所!

$$(\partial_\mu \partial^\mu - D_y^2 + m^2)\psi(x^\mu, y) = 0$$

$$\psi|_{0,L=0} \to \psi \propto \bar{\sin}_q(y),\ n\pi \to \pi_q(n)$$



κ-Mikowski spacetime

κ-deformation

$$[x^i, x^j] = 0$$

$$[x^0, x^i] = -i\kappa^{-1} x^i$$

$\Leftrightarrow$

dS_4

$$(y^A) = (y^0, y^i, y^4) = (y^i, y^+, y^-)$$

$$y^2 = -\kappa^2, \;\; M_{AB} = i\kappa^{-1} y_{[A}\partial_{B]}$$

$$x_0 = 2M_{-+}, \quad x_i = 2M_{i+}$$

$$C1 \text{ inv.} \rightarrow \;\; \left[\frac{1}{2} e^{\kappa^{-1} p^0} \mathbf{p}^2 - \kappa^2 \cosh(\kappa^{-1} p^0) - m^2\right] \Psi = 0$$

- domain like
- break Lorentz symm.

S.Naka, H.Toyoda, T.Takanashi, and E.Umezawa, PTEP 043B03

κ-Minkowski type of spacetime based on AdS_{n+1}

$$(y^A) = (\underbrace{\overbrace{y^0, y^1, y^2, y^3}^{y^\mu}, \overbrace{y^4, \cdots, y^{n-1}}^{y^i}}_{y^{\hat{\mu}}}, y^n, y^{n+1})$$

$$\hat{x}_{\hat{\mu}} = 2M_{\hat{\mu}+},\ \hat{x}_n = -2M_{-+}\ ,\ (y^{\pm} = y^{n+1} \pm y^n)$$

$$[\hat{x}_{\hat{\mu}}, \hat{x}_{\hat{\nu}}] = 0$$

$$[\hat{x}_n, \hat{x}_{\hat{\mu}}] = i\kappa^{-1}\hat{x}_{\hat{\mu}}$$

$\Downarrow$ $\quad k^n = \frac{1}{\mu} k^{\hat{\mu}} k_{\hat{\mu}}$ at a projective boundary of AdS_{n+1}

$$C1 \text{ inv.} \rightarrow \left[\tilde{k}^{\hat{\mu}}\tilde{k}_{\hat{\mu}} + 2\sinh\left(\frac{\kappa}{\mu}\tilde{k}^{\hat{\mu}}\tilde{k}_{\hat{\mu}}\right) - \tilde{m}_0^2\right]\Psi = 0$$

- domain like
- holds Lorentz symm.

S.Naka and H.Toyoda, PTP **109** (2003) 103

q-deformed extra dimension based on AdS_5

Randall-Sandrum type of spacetime

$$ds^2 = e^{-2\sigma(\eta)}\eta_{\mu\nu}dx^\mu dx^\nu - d\eta^2,\ (d\eta = e^{-\sigma}dy)$$

$$e^{-2\sigma} \sim \kappa^2 y^2,\ (\Lambda_5 \sim 0)\cdots \text{no IR boundary !}$$

$$\Downarrow$$

$$g^{AB}p_A p_b - m^2 e^{-2\sigma} = p^2 - \{p_y^2 + (m\kappa)^2 y^2\} = 0$$

$$\left(p^2 - m\kappa\{a, a^\dagger\}\right)\Psi = 0$$

$$\Downarrow\ q-\text{deformation}$$

$$a_q a_q^\dagger - q^\alpha a_q^\dagger a_q = q^{-\alpha(N+\beta)},\ (\beta = \gamma p^2)$$

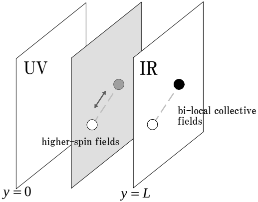
$$m\kappa\{a_q, a_q^\dagger\} = m\kappa\frac{\sinh\left[\alpha(\frac{1}{2}\{a, a^\dagger\} + \gamma p^2)\log q\right]}{\sinh(\frac{1}{2}\alpha\log q)}$$

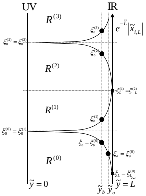
$$\cdots\kappa\text{-Minkowski based on } AdS_5\ ?$$

naive expectation

Bi-Local Fields in AdS_5 κ-Minkowski Spacetime

K.Aouda, S.Naka and H.Toyoda, arXiv:1603.09542/JHEP

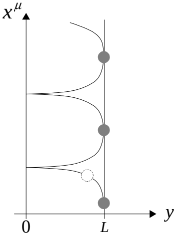




geodesic interval:

$$\sigma_{ba} \simeq \frac{1}{2}\tilde{L}^2 e^{-2\tilde{L}} |x_{ba}|^2$$

$$U_{x_b,x_a} = \kappa^2(2\sigma_{ba}) + \omega$$



$\Longleftarrow$ case of particle motion:

discrete picture at IR brane ($y = L$) ?

$\rightarrow$ $\times$

toy model:(1) infinite square y well in flat spacetime

$$G_{ba} = \langle b | (\hat{p}_\mu \hat{p}^\mu + \hat{p}_y^2 + m^2 + i\epsilon)^{-1} | a \rangle$$

$$= \frac{m^3}{2^5 \pi^2 i} \sum_{n=-\infty}^{\infty} (-1)^n \left(\frac{d}{d\Delta_{ba}^{(n)}} \right)^2 \left(e^{-2\sqrt{\Delta_{ba}^{(n)}}} \right)$$

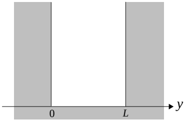
$$\Delta_{ba}^{(n)} = \left(\frac{m}{2} \right)^2 \left\{ (x_b - x_a)^2 + (y_b^{(n)} - y_a)^2 \right\}$$

$$y^{(n)} = \begin{cases} nL + y_b & (n = \text{even}) \\ (n+1)L - y_b & (n = \text{odd}) \end{cases}$$

$$\Downarrow$$

$$\Delta x_{ba} \sim m^{-1}, \quad \Delta y_{ba}^{(n)} \sim m^{-1}$$

$\cdots$ ゆるい離散構造



(2) q-deformed extra dimension+booundary condition

$$\Pi_L : y \to L,\ \theta_L : y \to 0 \le y \le L \ \cdots \text{projection operators}$$

$$(\Pi_L \theta_L)^\dagger \{\kappa_m \{a_q, a_q^\dagger\}(\theta_L \pi_L) = \hat{M}^2 (\theta_L \Pi_L)\,, \quad (\kappa_m = m\kappa)$$

$$\Downarrow$$

$$\hat{M}^2 = \frac{m\kappa}{2\sinh(\frac{1}{2}\alpha \log q)} \left[e^{\alpha(\frac{1}{2}+\gamma\hat{p}^2)\log q} K_\alpha - (\alpha \to -\alpha) \right]$$

$$K_\alpha = \sqrt{\frac{\pi}{m\kappa(1-q^{2\alpha})}} \mathrm{erf}\left(\sqrt{\frac{m\kappa}{2} \frac{1-q^{2\alpha}}{1+q^{2\alpha}}} L \right)$$

非局所構造 $\to$ deformation 由来 ($\neq$ 離散構造)

まとめ

- 非局所場理論に結びつく様々な要因
- domain（高階微分）型の非局所は非可換と関連
  - 1) AdS_5 に結び付く κ-Minkowski 型時空
    $[\hat{x}_\mu, \hat{x}_\nu] = 0, \ [\hat{x}_5, \hat{x}_\mu] = i\kappa^{-1}\hat{x}_\mu$
    $$\hat{x}_\mu = 2M_{\mu+}, \ \hat{x}_5 = -2M_{-+}, \ (y^\mu, y^+, y^-) \in AdS_5$$
  - 2) warp 計量で表現される AdS_5 時空の q-変形
    $(x_\mu, y; p_\mu, p_y) = (x_\mu, p_\mu; a, a^\dagger) \to (x^\mu, p^\mu; a_a, a_q^\dagger)$

$$1), 2) \ \Rightarrow \quad [p^2 + A\sinh(Bp^2 + C) - m^2]\Psi = 0$$

bi-local 場の q 変形も同形の方程式を導く

S.Naka,H.Toyoda, A,Kimishima, PTP **113**(2005),645

- 4 次元と余剰次元の間の deformation
  $\rightarrow$ 非局所性を引き起こす要因として有効に働く
  - IR brane 上の非局所場 $\leftrightarrow$ bulk の構造を反映 ？
  - deformation を導入する指導原理は？

- κ-Minkowski 型非可換構造と量子群型非可換構造のより深い関係を探る必要がある