Spontaneous Supersymmetry Breaking, Negative Metric and Vacuum Energy

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1 Introduction

SUSY $\rightarrow$ positive definite vacuum energy:

$$H = \{\, Q,\ \bar{Q}\, \} = \sum_{\alpha} (Q_\alpha Q_\alpha^\dagger + Q_\alpha^\dagger Q_\alpha) \tag{1}$$

$\rightarrow$ potential is bounded from below, so

Dynamical breaking of SUSY, or any other symmetries is difficult!

In order for the broken symmetry vacuum to be stable, negative matric is necessary. $\longrightarrow$ Is this true?

Soft SUSY breaking terms (origin of many FREE parameters in MSSM) can be generated by dipole ghost mechanism (a kind of spontaneous SUSY breaking) Ohta and Fujii '82

Recently, Y. Cheng, Y. M. Dai, G. Faisel and O. C. W. Kong, arXiv:1507.01514 [hep-ph], arXiv:1603.00724 [hep-th]. proposed a supersymmetric NJL model which, they claim, realizes the dynamical SUSY breaking.

Is this healthy model, not suffering from negative metric modes?

Is SUSY broken vacuum really stable, realizing lower energy than the SUSY vacuum?

If this is a good model, it is the first direct and simple model realizing (dynamical) spontaneous SUSY breaking.

(cf. High blow model using Seiberg duality is very indirect.)

2 Cheng-Dai-Faisel-Kong Model

Y. Cheng, Y. M. Dai, G. Faisel and O. C. W. Kong, arXiv:1507.01514 [hep-ph], arXiv:1603.00724 [hep-th]. Supersymmetric NJL-like model:

$$\boxed{\mathcal{L} = \int d^4\theta \Big(\bar{\Phi}\Phi - \frac{G}{2N} \big(\bar{\Phi}\Phi\big)^2 \Big), \qquad \bar{\Phi}\Phi \equiv \sum_{i=1}^{N} \bar{\Phi}_i \Phi^i} \tag{2}$$

For the chiral matter fields $\Phi^i = [A^i,\ \psi^i,\ F^i]$. Note that this has a dangerous kinetic term like

$$\left(1 - \frac{G}{N} A^\dagger A\right) \left(-\partial_m A^\dagger \cdot \partial^m A - i\bar{\psi}\bar{\sigma}^m \partial_m \psi\right) \tag{3}$$

which is negative metric for $(G/N)A^\dagger A > 1$.

This is equivalently rewritten by adding a Gauussian term:

$$\mathcal{L} = \int d^4\theta\Big(\bar{\Phi}\Phi - \frac{G}{2N}\big(\bar{\Phi}\Phi\big)^2 + \frac{N}{2G}\big(U + \frac{G}{N}\bar{\Phi}\Phi\big)^2\Big)$$
$$= \int d^4\theta\Big(\bar{\Phi}\Phi\big(1+U\big) + \frac{N}{2G}U^2\Big) \tag{4}$$

The auxiliary vector superfield U is

$$U = -\frac{G}{N}\bar{\Phi}\Phi \qquad \text{so that} \quad \langle U\rangle = 0 \text{ at } G = 0 \tag{5}$$

Write the vector superfield $U + 1$ as

$$U + 1 = \bar{\Sigma}e^{2V}\Sigma \tag{6}$$

by chiral Σ and vector superfield V. This is redundant so that it is invariant under the followinghidden U(1) gauge-trf:

$$\begin{cases} \Sigma \ \rightarrow \ e^{-i\Lambda}\Sigma, \qquad \bar{\Sigma} \ \rightarrow \ e^{+i\Lambda}\bar{\Sigma}, \\ 2V \ \rightarrow \ 2V + i(\Lambda - \bar{\Lambda}) \end{cases} \tag{7}$$

Further, if we redefine the chiral matter as

$$\Sigma\Phi^i \equiv \phi^i, \qquad \bar{\Sigma}\bar{\Phi}^i \equiv \bar{\phi}^i, \tag{8}$$

then, the Lagrangian becomes

$$\boxed{\mathcal{L} = \int d^4\theta \Big(\bar{\phi}_i e^{2V} \phi^i + \frac{N}{2G} \big(\bar{\Sigma} e^{2V} \Sigma - 1 \big)^2 \Big)} \tag{9}$$

which is U(1)-gauge invariant under

$$\begin{cases} \phi^i & \rightarrow \; e^{-i\Lambda} \phi^i, \qquad \bar{\phi}_i \; \rightarrow \; e^{+i\Lambda} \bar{\phi}_i, \\ \Sigma & \rightarrow \; e^{-i\Lambda} \Sigma, \qquad \bar{\Sigma} \; \rightarrow \; e^{+i\Lambda} \bar{\Sigma}, \\ 2V & \rightarrow \; 2V + i(\Lambda - \bar{\Lambda}) \end{cases} \tag{10}$$

Using this, we can take the Wess-Zumino gauge in which

$$V = [\ 0,\ 0,\ 0,\ 0,\ v_m,\ \lambda,\ D\ ] \tag{11}$$

and Σ becomes a normal chiral ‘matter’:

$$\Sigma = [\ z,\ \chi,\ h\ ] \tag{12}$$

Using the covariant derivative $\mathcal{D}_m \equiv \partial_m + i v_m$, the chiral matter $\phi^i =$

$[\,A^i,\ \psi^i,\ F^i\,]$ part of the Lagrangian reads

$$\int d^4\theta \bar{\phi}_i e^{2V} \phi^i = \left(A_i^\dagger\ \bar{\psi}_i\ F_i^\dagger\right) \underbrace{\begin{pmatrix} \mathcal{D}_m\mathcal{D}^m - D & \sqrt{2}i\lambda & 0 \\ -\sqrt{2}i\bar{\lambda} & -i\bar{\sigma}^m\mathcal{D}_m & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\equiv\Delta} \begin{pmatrix} A^i \\ \psi^i \\ F^i \end{pmatrix} \tag{13}$$

In the leading order in $1/N$ expansion, the effective action is:

$$NS = N\left[+i\,\mathrm{sTr}\,\mathrm{Ln}(-\Delta) + \int d^4x d^4\theta \frac{1}{2G}\left(\bar{\Sigma}e^{2V}\Sigma - 1\right)^2\right] \tag{14}$$

Only the bosonic scalar fields can take the constant VEV

$$\langle\Sigma\rangle = [\,z,\ 0,\ h\,], \qquad \langle V\rangle = [\,0,\ 0,\ D\,] \tag{15}$$

The effective potential is

$$V(z,h,D) = \int \frac{d^4k}{(2\pi)^4}\left(\ln(k^2+D) - \ln(k^2)\right) + \int d^4\theta \frac{1}{2G}\left(\langle\bar{\Sigma}\rangle\, e^{2\langle V\rangle}\,\langle\Sigma\rangle - 1\right)^2 \tag{16}$$

$$G\,V(z,h,D) = \frac{G}{32\pi^2}\Big[\Lambda^4 \ln(1+\frac{D}{\Lambda^2}) - D^2 \ln(1+\frac{\Lambda^2}{D}) + D\Lambda^2\Big] + (1-2\,|z|^2)\,|h|^2 + (|z|^2-1)\,|z|^2\,D \qquad (17)$$

$$\frac{\delta V}{\delta h} = 0 \Rightarrow (2\,|z|^2-1)h^* = 0 \;\Rightarrow\; h = 0 \text{ or } \;|z|^2 = 1/2 \qquad (18)$$

$$\frac{\delta V}{\delta z} = 0 \Rightarrow \big[2\,|h|^2 - (2\,|z|^2-1)D\big]\,z^* = 0 \qquad (19)$$

$$\Rightarrow\; h = 0 \text{ and } (D = 0 \text{ or } \;|z|^2 = 1/2) \qquad (20)$$

$$\frac{\delta V}{\delta D} = 0 \Rightarrow \boxed{\frac{G}{32\pi^2}\Big[2\Lambda^2 - 2D\ln(1+\frac{\Lambda^2}{D})\Big] = (1-|z|^2)\,|z|^2} \qquad (21)$$

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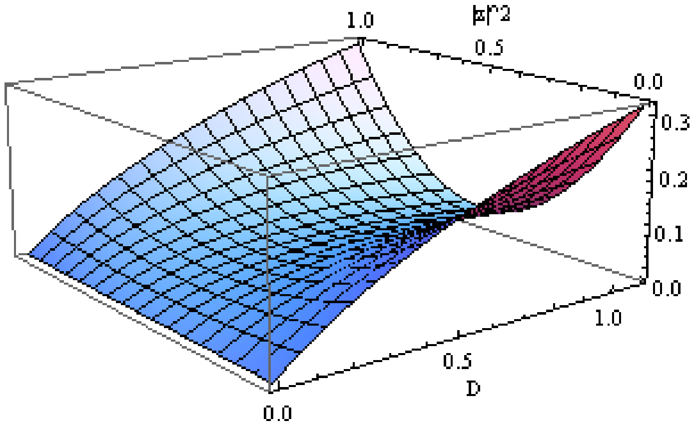
In[1]:= $GV[x\_, Z\_, G\_] := G\left(\text{Log}[1 + x] - x^2\,\text{Log}\left[1 + \frac{1}{x}\right] + x\right) + x\,(Z - 1)\,Z$

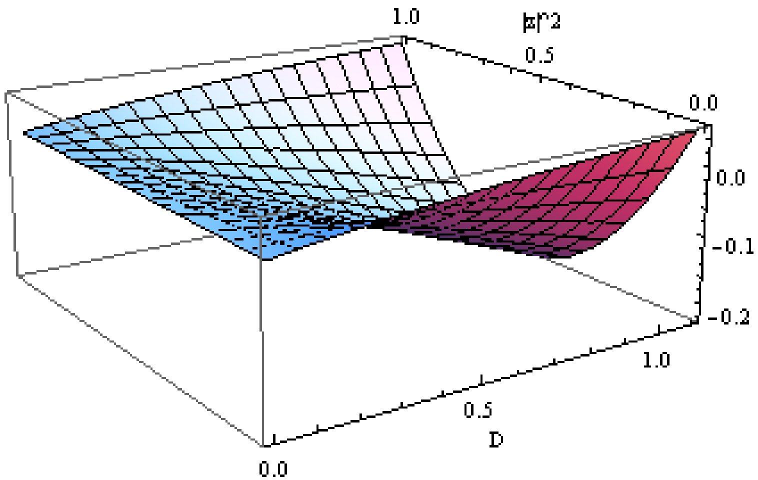
In[9]:= Plot3D[GV[x, Z, 0.3], {x, 0, 1.1}, {Z, 0, 1}, AxesLabel → {"D", "|z|^2"}]

Out[9]=



In[10]:= Plot3D[GV[x, Z, 0.05], {x, 0, 1.1}, {Z, 0, 1}, AxesLabel → {"D", "|z|^2"}]

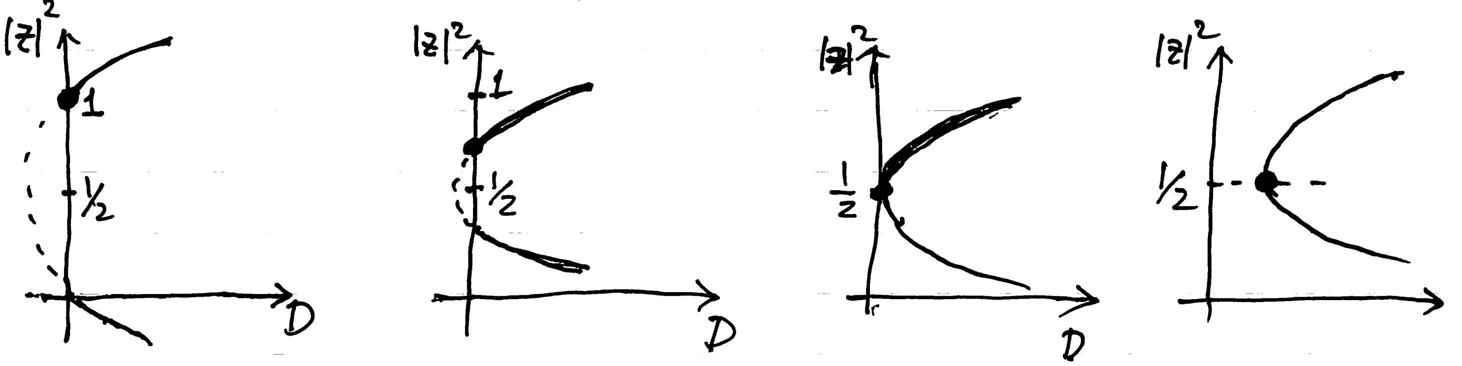
Out[10]=



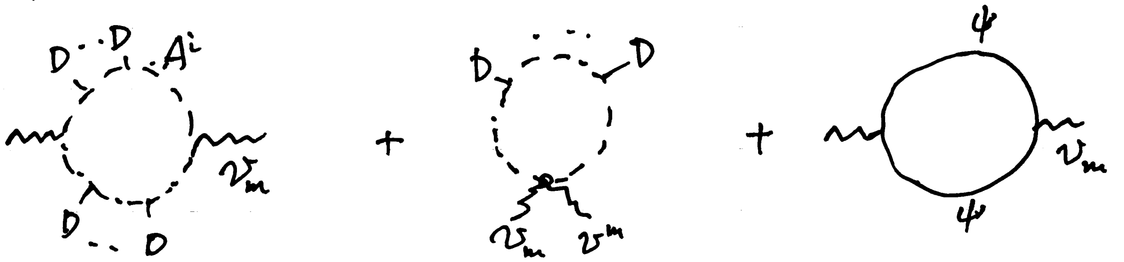
$V(z, h=0, D)$

停留点 の plot in $(|z|^2, D)$ plane $\frac{\partial V}{\partial D} = 0$

$G=0$ < G < G_{cr} < G

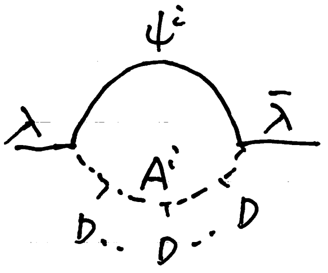


v_m の 2点関数



λ の 2点関数

$\longrightarrow (p^2 g_{\mu\nu} - p_\mu p_\nu)\,\Gamma^{(2)}(p^2)$



A, ψ が massless だと

$\Gamma^{(2)}(p^2) \propto \log p^2$

The right hand side $0 \le \mathrm{RHS} \le 1/4$. At free theory $G = 0$,

$$|z|^2 = 1 \quad \text{and} \quad D = 0. \tag{22}$$

As G becomes larger and larger, the stationary point keeps $D = 0$ and $|z|^2$ becomes smaller from 1 towards $|z|^2 = 1/2$, such that

$$\frac{G}{32\pi^2} 2\Lambda^2 = (1 - |z|^2)\,|z|^2 \tag{23}$$

Beyond the critical coupling

$$\frac{G_{\rm cr}}{32\pi^2} 2\Lambda^2 = 1/4 \quad \Rightarrow \quad G_{\rm cr} = 4\pi^2/\Lambda^2, \tag{24}$$

$|z|^2$ stays at the maximum point of RHS, $|z|^2 = 1/2$, so that D can no longer be zero, as determined by

$$\frac{D}{\Lambda^2} \ln(1 + \frac{\Lambda^2}{D}) = 1 - \frac{G_{\rm cr}}{G} \tag{25}$$

That is, the SUSY is spontaneously broken.

Note that there is no stationary point which keeps SUSY!

Moreover, always $|z| \neq 0$ for $G > 0$. That is, the hidden U(1) gauge symmetry is always spontaneously broken. $\mathrm{Im}\, z$ is the NG boson absorbed in vector v_m in $V = [\,v_m,\ \lambda,\ D\,]$.

For $0 < G < G_{\rm cr}$, the SUSY is not broken so that $({\rm Re}\, z,\ \chi) \in \Sigma$ form a massive vector multiplet $0 \oplus 1/2 \oplus 1$ with $(v_m,\ \lambda) \in V$. (But actually this massive vector multiplet is unstable.)

$$\int d^4\theta \big(\bar{\Sigma} e^{2V} \Sigma - 1\big)^2$$
$$= 2(2\,|z|^2 - 1)\big(|h|^2 - \underbrace{\mathcal{D}_m z^* \mathcal{D}^m z}_{\text{vector mass}} - \frac{i}{2}\bar{\chi}\bar{\sigma}^m \overleftrightarrow{\mathcal{D}}_m \chi + \sqrt{2} i(\underbrace{z^* \lambda \chi - z \bar{\lambda}\bar{\chi}}_{\text{Dirac mass}})\big)$$
$$+2\,|z|^2\,(|z|^2 - 1) D - 2i(\bar{\chi}\bar{\sigma}^m \chi)(z^* \overleftrightarrow{\mathcal{D}}_m z) - 2(z h \bar{\chi}^2 + z^* h^* \chi^2) + \chi^2 \bar{\chi}^2$$

For $G > G_{\rm cr}$, however, $|z|^2 = 1/2$ implies that the kinetic term for the chiral multiplet Σ vanishes! This means that there appears no mass term for the vector multiplet V. Namely, V gives massless vector multiplet. And actually λ

These both massless and massive vector multiplets are not true stable particles since the matter multiplets $\phi^i = [\,A^i,\ \psi^i,\ F^i\,]$ are massless to which the vector multiplet can decay or resonate. So in order to make the vector multiplet truly stable particles, it is necessary to put the mass term for the original matter multiplets Φ^i.

In conclusion, the model is very healthy, and SUSY broken vacuum is stable, I think. There are no SUSY vacuuum existing. This is owing to the absense of stationary point with respect to D variation. Unboundedness of the potential in the direction of D does not imply the instability of the model.