第6回日大理工・益川塾連携シンポジウム, 15th October, 2016

AdS/CFT and Chaos

Koji Hashimoto (Osaka u)

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi}_i \left(i\gamma^\mu D_\mu - m_i \right) \psi_i$$

Which "QCD" is more chaotic?

$$S_{\text{QCD}}^{\text{SU}(3)} \qquad S_{\text{QCD}}^{\text{SU}(5)}$$

Chaos hidden in strongly coupled theories

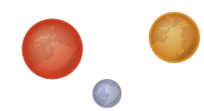
1. Chaos : sensitive to initial conditions 3p
2. Problem : chaos in QFT? 5p
3. Chaos in QCD linear sigma model 4p
4. Chaos in large N super QCD 3p

1605.08124 (hep-th)
w/ K. Murata, K. Yoshida

1-1 Chaos : sensitive to initial conditions

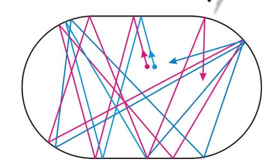
Classical chaos (in deterministic dynamical systems)
= Non-periodic bounded orbits
sensitive to initial conditions

Long history



- Three-body planetary system [Poincare, 1892]

- Atmospheric model, butterfly effect [Lorenz, 1963]

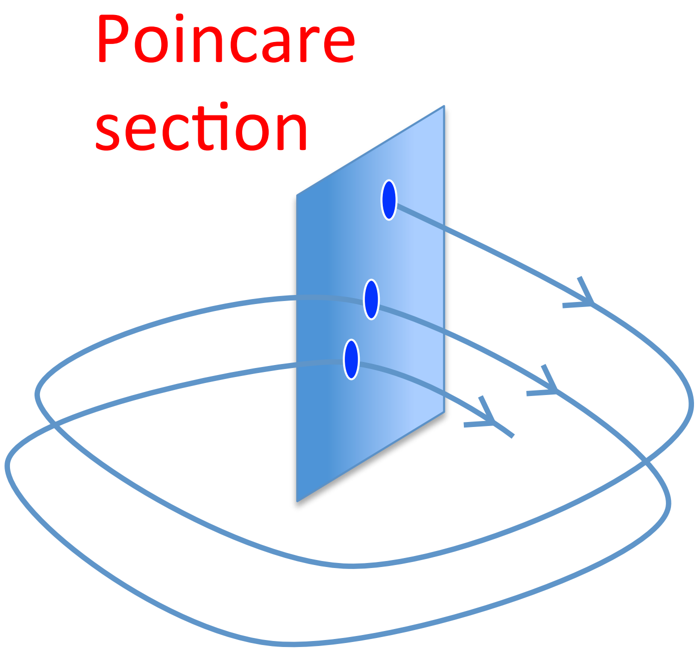


- Billiard ball [Bunimovich, 1974]
- Yang-Mills [Savvidy 1981, Muller et al. 1992]
- Black hole merger?

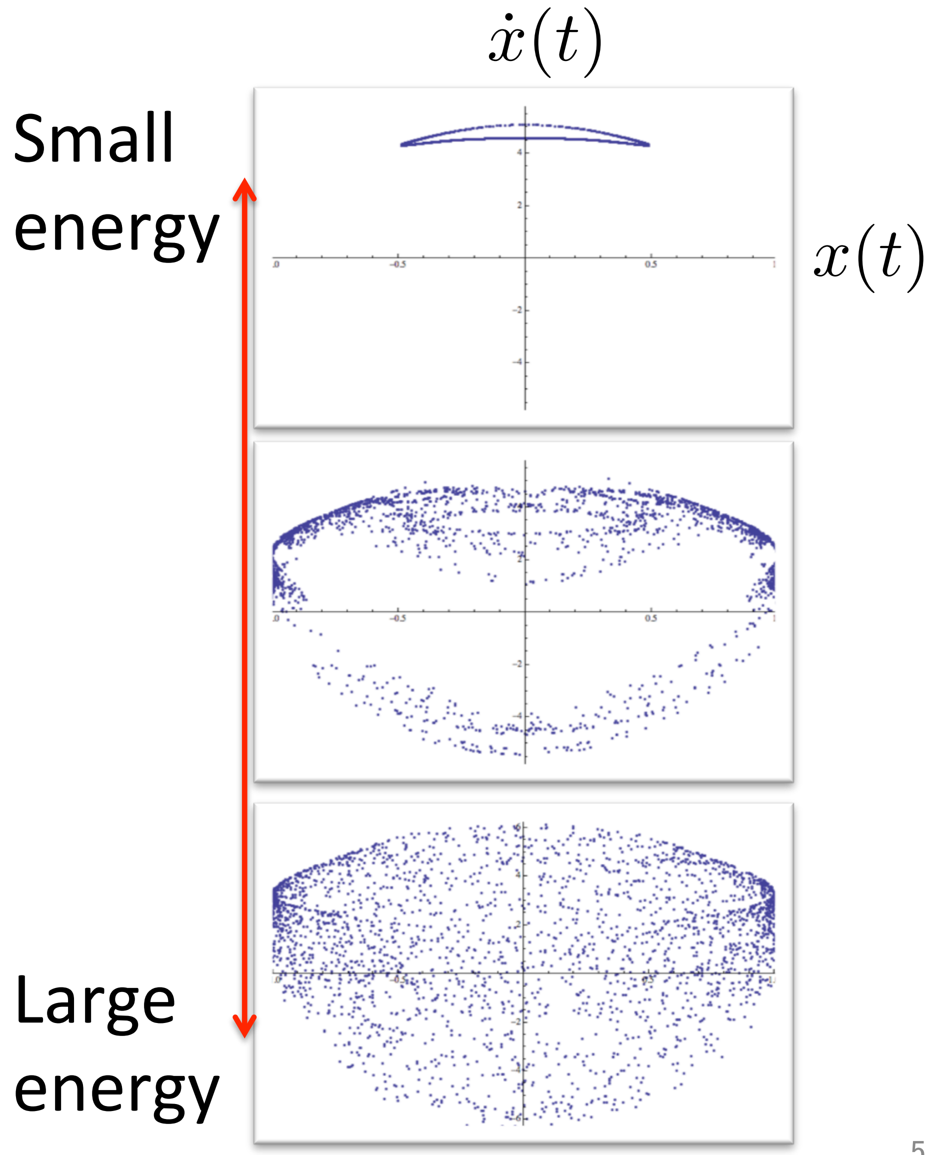
Lots of applications

Information theory (Kolmogorov-Sinai entropy), math modeling in chemical reactions, biology, economy, sociology, traffic forecast, financial crisis, cryptography, Thermalization of heavy ion collisions [Kunihiro et al., 2009], etc.

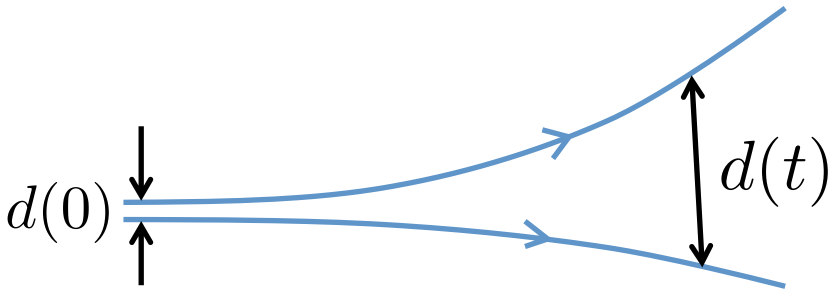
1-2 Poincare section captures chaotic phase



Motion in the phase space



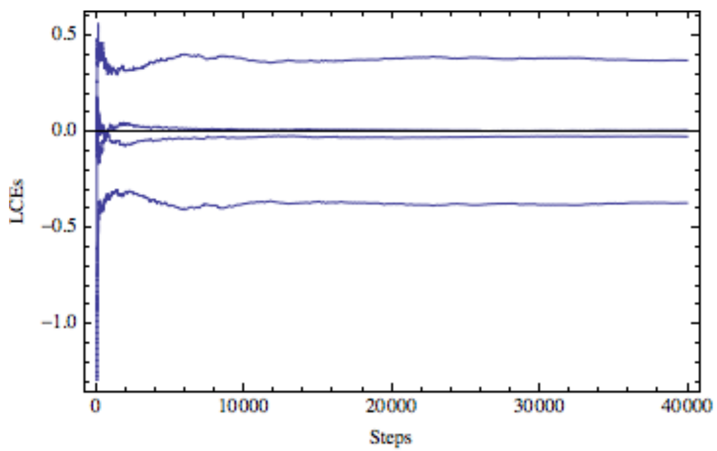
1-3 Lyapunov exponent is the chaos index



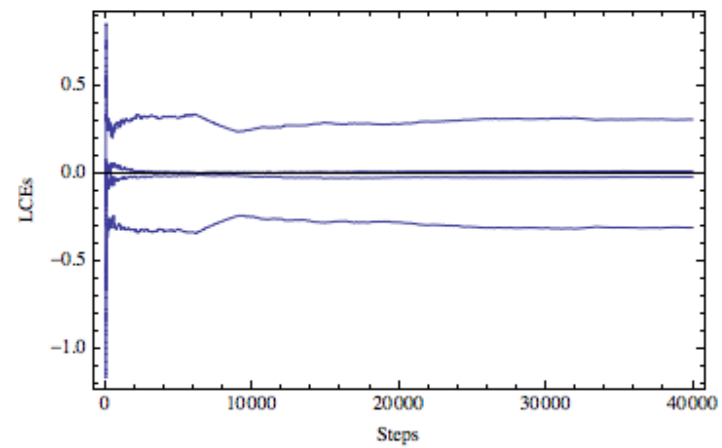
$$d(t) \sim d(0)\exp[Lt]$$

Lyapunov exponent

$$L = \lim_{t\to\infty}\lim_{d(0)\to 0}\frac{1}{t}\log\frac{d(t)}{d(0)}$$



$L \simeq 0.38 \; (l_1/l_2 = 1)$



$L \simeq 0.30 \; (l_1/l_2 = 2)$

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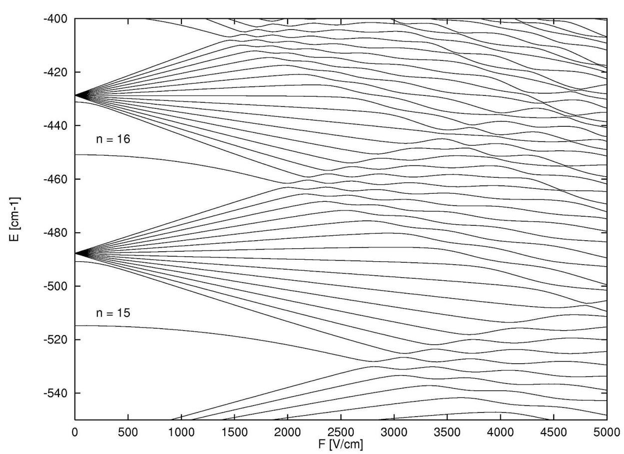
2-1 Quantum Problem: Lyapunov washed out?

Schrodinger equation is linear, thus no chaos!?

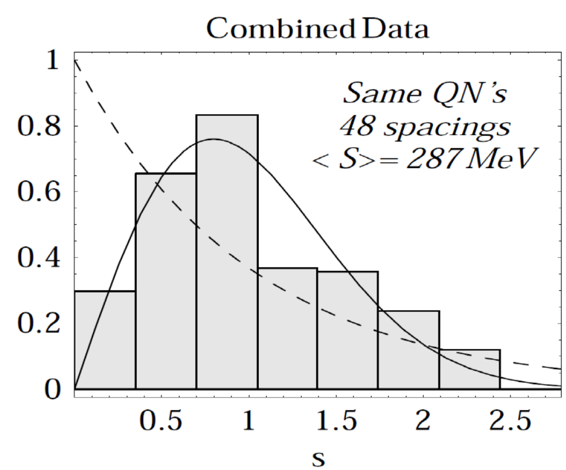
Quantum chaos

= Quantizing classically chaotic system

Character: Energy level spacing is Wigner, not Poisson



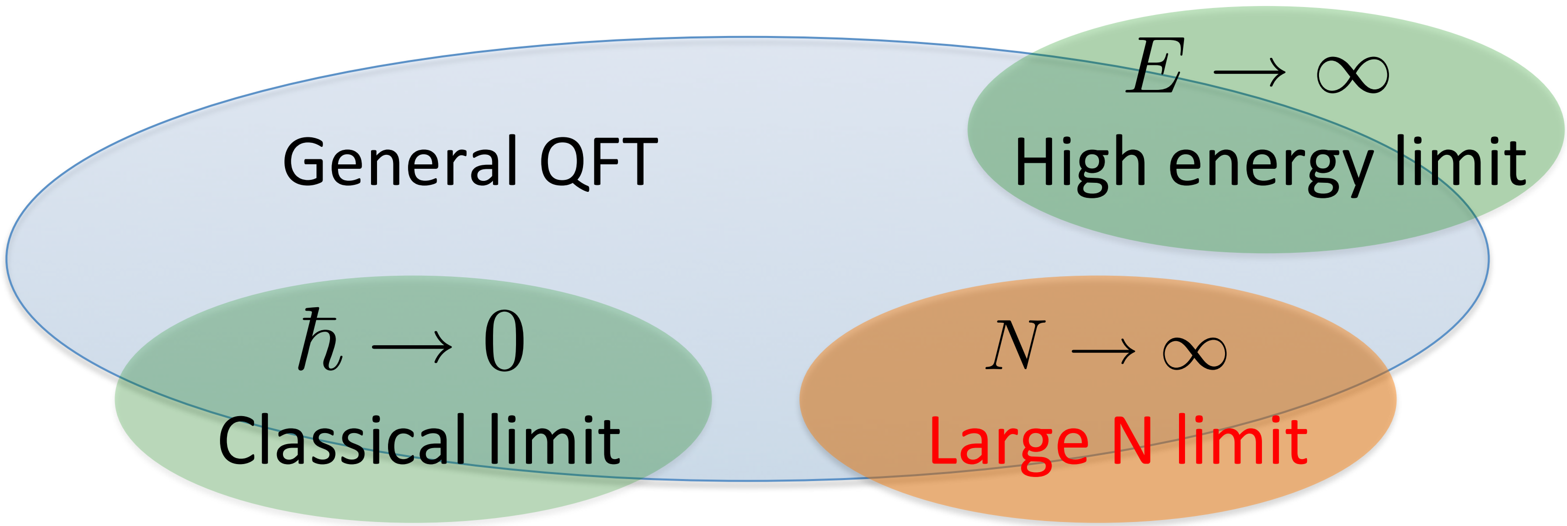
Atomic spectra of Lithium under electric field
[Courtney, Spellmeyer, Jiao, Kleppner, 95]



Spacing of hadron spectra
[Pando-Zayas, 00]

2-2 Solution for Lyapunov exponent of QFT

Our solution: Employing large N limit



Another solution: Out-of-Time-Ordered Correlator

$$\langle Q(t)P(0)Q(t)P(0)\rangle \sim \left(\frac{\delta Q(t)}{\delta Q(0)}\right)^2$$

[Larkin, Ovchinnikov `69]
[Kitaev `14] [Maldacena, Shenker, Stanford `15]

2-3 SYK model : dual to black hole

1) Lyapunov upper bound (conjecture) for thermal OTO

$$L \leq 2\pi T$$

[Maldacena, Shenker, Stanford `15]

Suggested from AdS/CFT with black holes

2) SYK (Sachdev-Ye-Kitaev) model [Kitaev `15][Sachdev,Ye `95]

(1+0 dim., N Majorana fermions, disordered interaction)

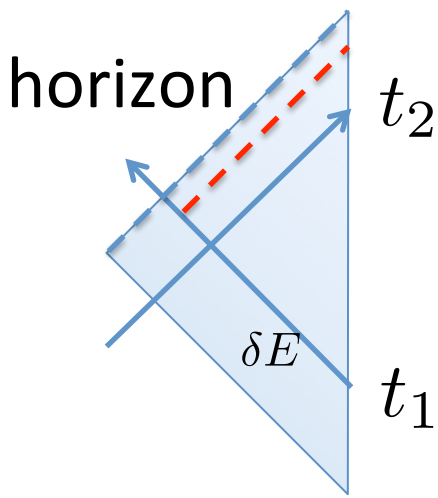
$$H = \frac{-1}{4!} \sum_{i,j,k,l=1}^{N} j_{[ijkl]} \psi_i \psi_j \psi_k \psi_l \qquad \left(\sum_{j,k,l=1}^{N} \langle j_{ijkl} j_{ijkl} \rangle = 6J^2 \right)$$

Solvable at strong coupling $\beta J \to \infty$

Shown to saturate the bound [Kitaev `15] [Maldacena, Stanford `16]

2-4 Black hole is a nest of chaos

Shock wave delay [Shenker, Stanford `13,`14]



Black hole is a fast scrambler?

[Sekino, Susskind `08]

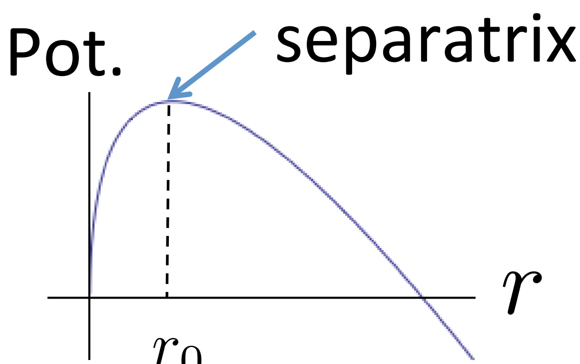
2d dilaton gravity dual to SYK

[Almheiri, Polchinski `14] [Engelsoy, Martens, Verlinde `16]

$$\delta t_2 = \frac{\delta E}{8\pi T M} e^{2\pi T(t_2 - t_1)}$$

Universal chaos in particle motion near BH

[Tanahashi, KH to appear]



$$\mathcal{L} = -m\sqrt{-g_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu} - V(X)$$

$$\sim C\left[\dot{r}^2 + \frac{1}{(2\pi T)^2}(r - r_0)^2\right]$$

2-5

Red shift giving a chaos

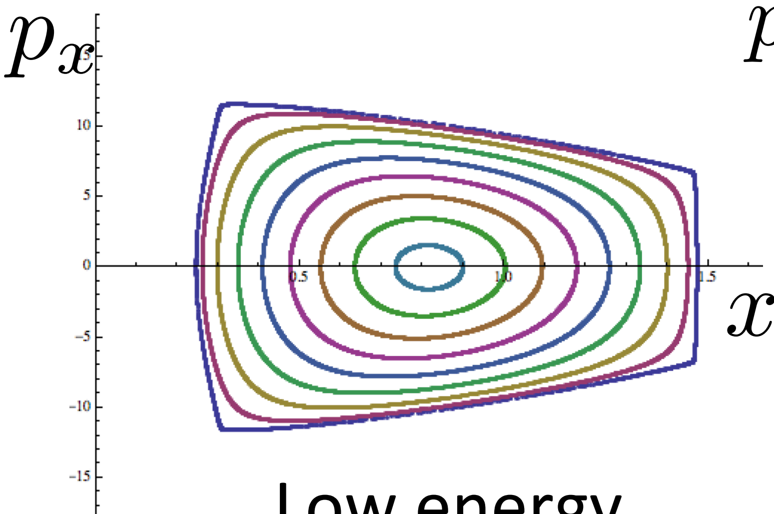
[Tanahashi, KH to appear]

Simple model: harmonic oscillator with a red shift

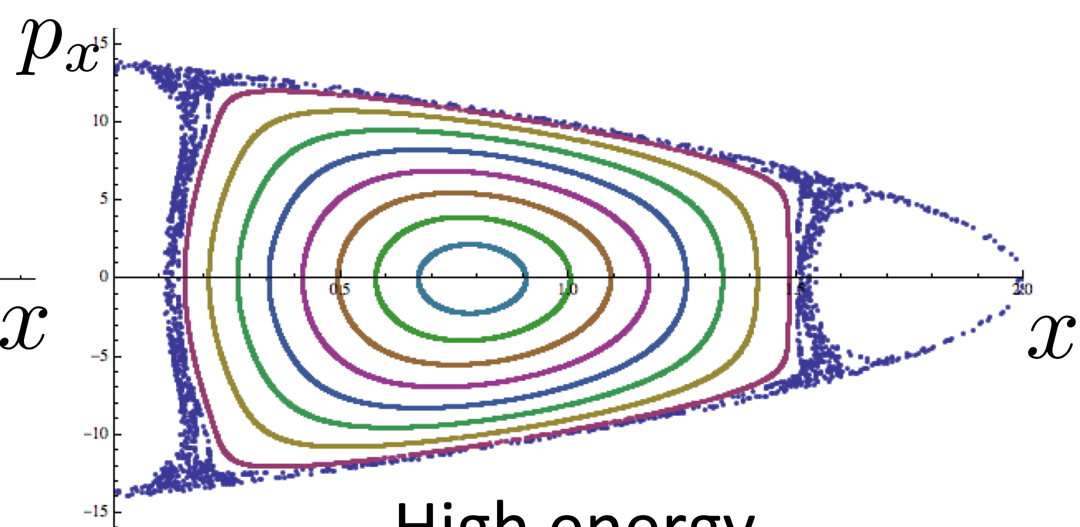
$$\mathcal{L} = \frac{g(x,y)}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{\omega^2}{2}\left((x - x_c)^2 + y^2\right)$$

``metric'' $g(x,y) \equiv \frac{1}{x}$

Poincare sections



Low energy



High energy

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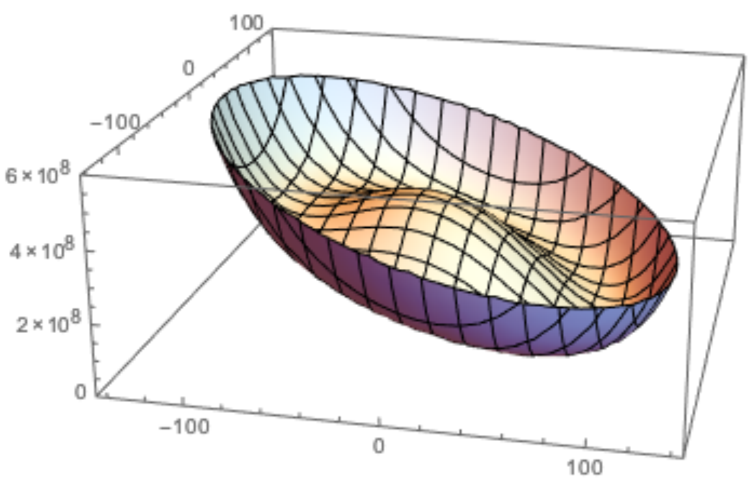
3-1

Effective QCD: linear sigma model

$$S = \int d^4x \left[\frac{1}{2}((\partial_\mu \sigma)^2 + (\partial_\mu \pi)^2) - V \right],$$

$$V \equiv \frac{\mu^2}{2}(\sigma^2 + \pi^2) + \frac{g_4}{4}(\sigma^2 + \pi^2)^2 + a\sigma + V_0$$



The model describes QCD with:

- 1-flavor, ignoring anomaly
- 2-flavor, neutral pion sector

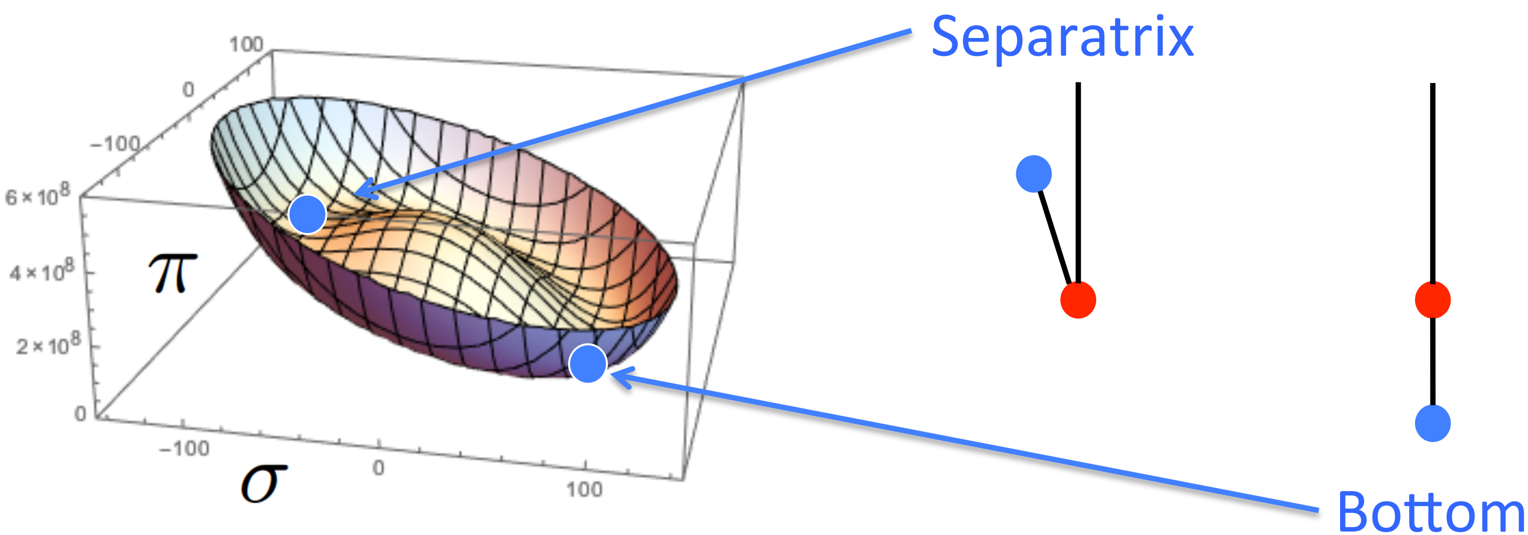
Breaking of U(1) (or sigma_3 of SU(2))

- spontaneously by chiral condensate
- explicitly by quark mass

3-2 Pendulum hidden in the sigma model

$$S = \int d^4x \left[\frac{1}{2}((\partial_\mu \sigma)^2 + (\partial_\mu \pi)^2) - V \right],$$

$$V \equiv \frac{\mu^2}{2}(\sigma^2 + \pi^2) + \frac{g_4}{4}(\sigma^2 + \pi^2)^2 + a\sigma + V_0$$

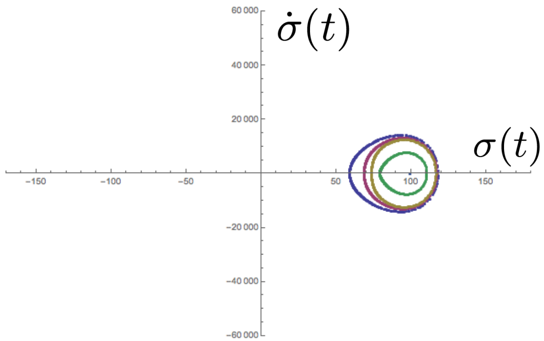


$$(\langle \sigma \rangle, \langle \pi \rangle) = (f_\pi, 0)$$

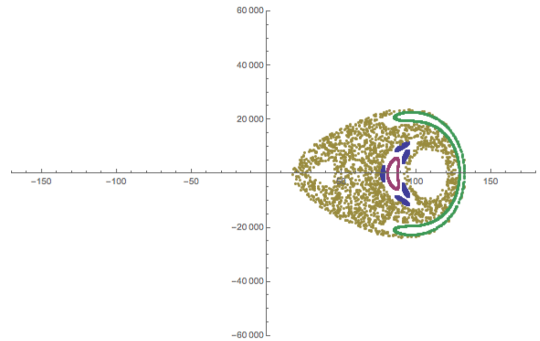
3-3 Chaos of chiral condensate

Poincare sections for $\pi(t) = 0$

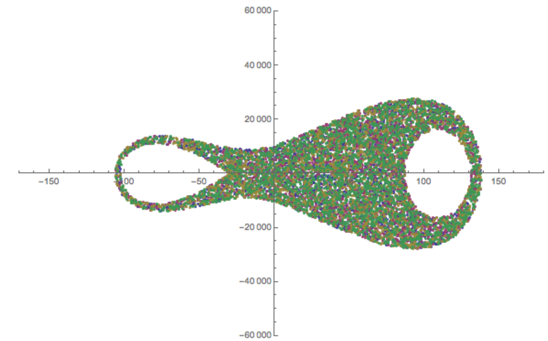
E=100[MeV]



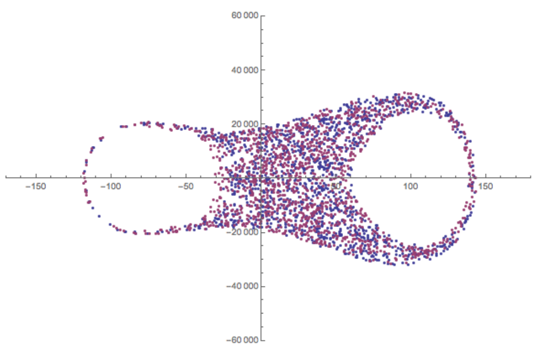
E=130[MeV]



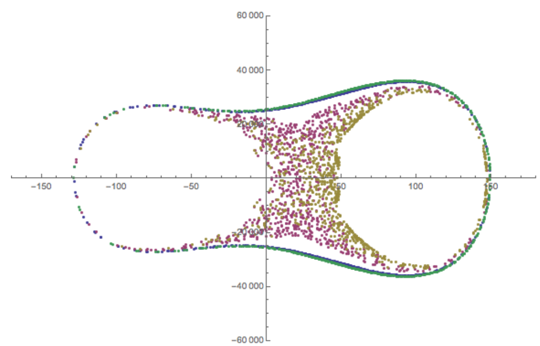
E=140[MeV]



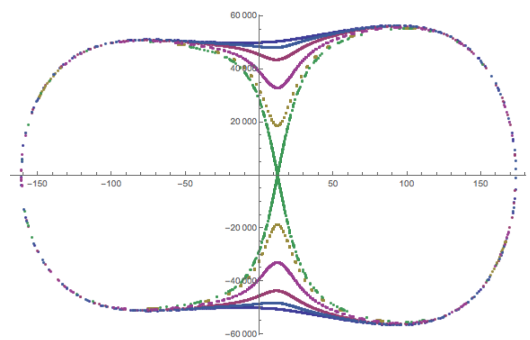
E=150[MeV]



E=160[MeV]



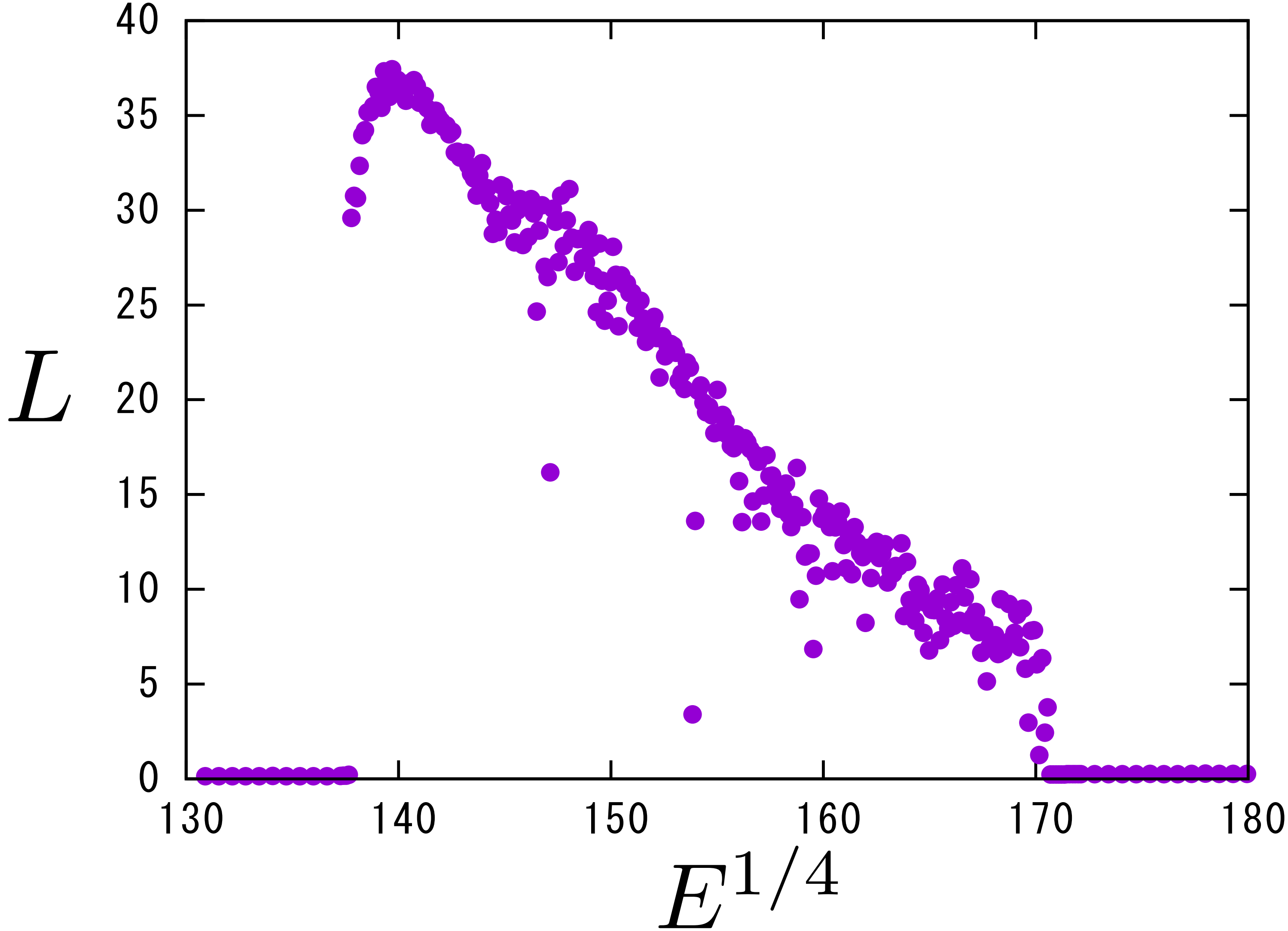
E=200[MeV]



$f_\pi \sim 93[\mathrm{MeV}]$ $m_\pi \sim 135[\mathrm{MeV}]$ $m_\sigma \sim 500[\mathrm{MeV}]$

3-4

Positive Lyapunov exponent



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4-1 Exact classical meson theory of SQCD

Classical action

$$S = \int d^4x \, \mathrm{Tr} \left[\frac{1}{2}\dot{\phi}_a^2 - \frac{8\pi^2 m_q^2}{\lambda}\phi_a^2 + \frac{36\pi^2}{5N_c}[\phi_8, \phi_9]^2 \right] \quad (a, b = 8, 9)$$

Meson masses: [Kruczenski, Mateos, Myers, Winters 03]

$$\left(\phi_8^{ij}(t)\, , \phi_9^{ij}(t)\right) \propto \left(\langle \bar{q}^i q^j(t)\rangle \, , \langle \bar{q}^i \gamma_5 q^j(t)\rangle\right)$$

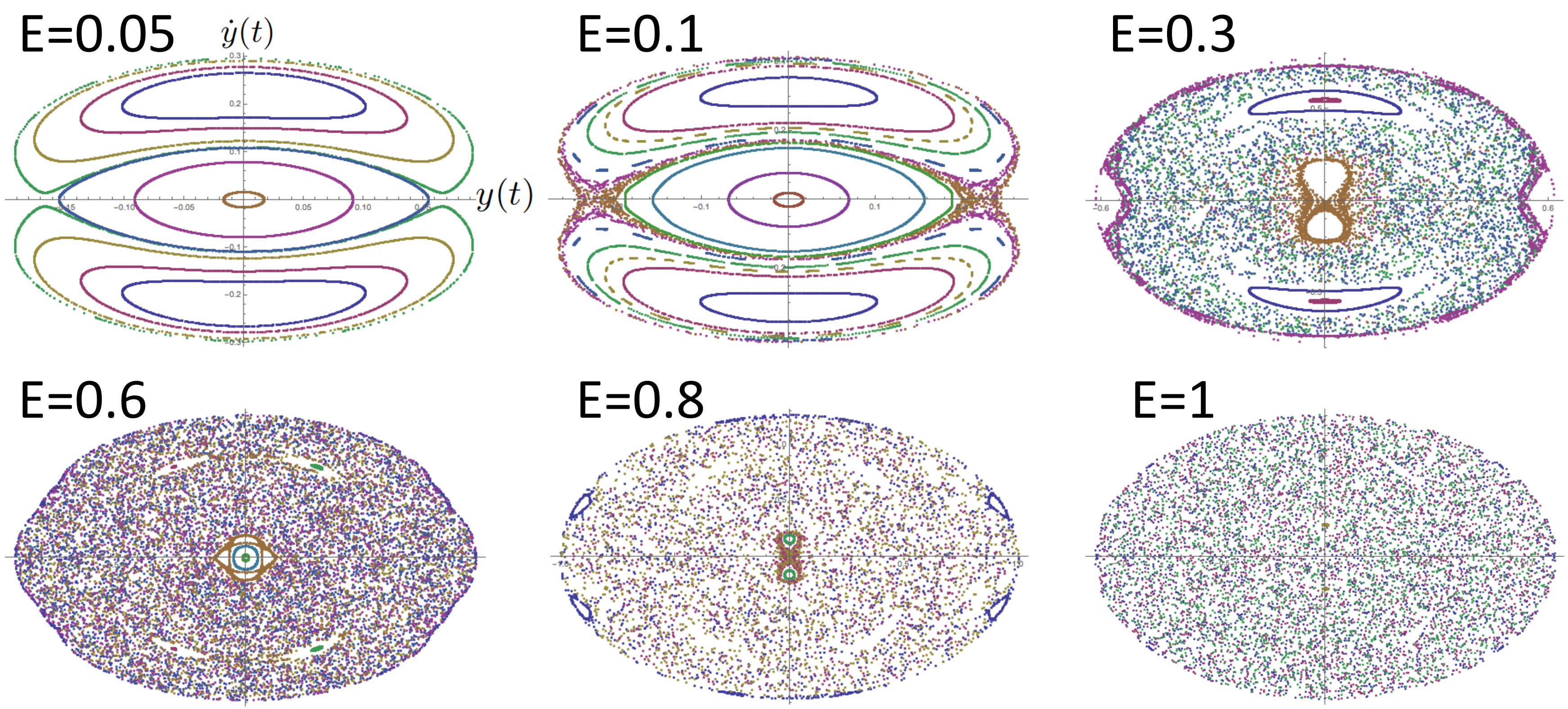
Effective theory of mesons of "N=2 supersymmetric QCD"

- N=4 Super Yang-Mills plus N=2 quark hypermultiplets
- Parameter of the theory: λ, N_c and m_q
    - 2-flavor, quark mass m_q
    - SU(N_c) gauge group with large N_c
    - Large 't Hooft coupling λ

4-2 Chaos of quark condensate

Chaos-Order phase transition: $E_{\text{chaos}} \sim (6 \times 10^2) \times m_q^4 \frac{N_c}{\lambda^2}$

Poincare sections for $x(t) = 0$

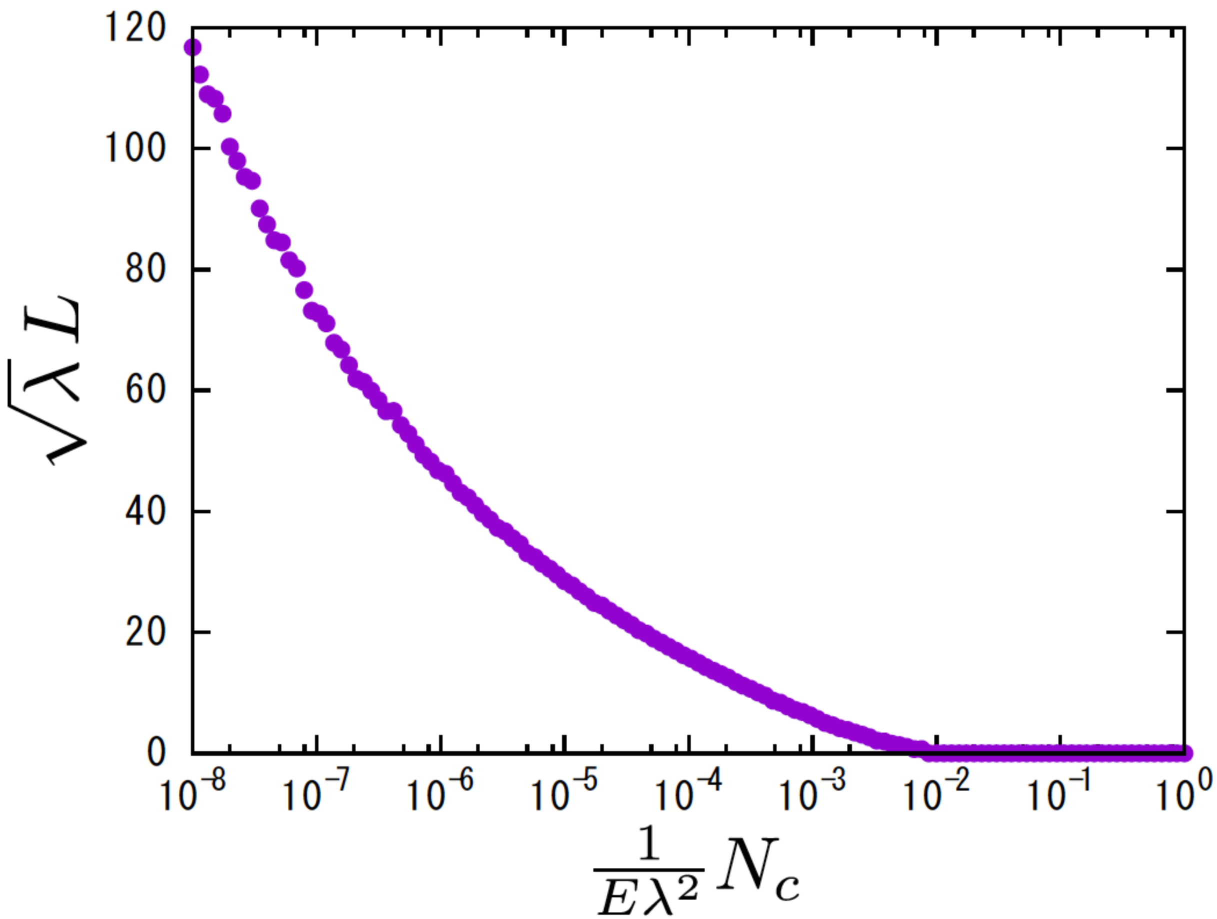


$\lambda = 100 \quad N_c = 10 \quad m_q = 1$

Ref. [Matinyan, Savvidi, Savvidi, 81]

4-3 Smaller Nc, more chaotic

Lyapunov exponent $$L(\lambda, N_c, E) \equiv \lim_{t\to\infty} \lim_{d(0)\to 0} \frac{1}{t} \log \frac{d_{\langle \bar{q}q \rangle}(t)}{d_{\langle \bar{q}q \rangle}(0)}$$



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Discussion: chaotic QFT

1) Holography?

Integrability versus chaos.

Black holes? Information loss?

Upper bound of Lyapunov?

[Aref'eva, Medvedev, Rytchkov, Volovich 99]
[Asano, Kawai, Yoshida 15]
[Hawking 14] [Farahi, PandoZayas 14]
[Maldacena, Shenker, Stanford 15]

2) Entropy production? Phase transition?

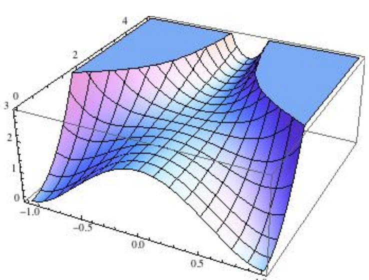
Kolmogorov-Sinai entropy = Shannon entropy rate [Latora, Branger, 99]

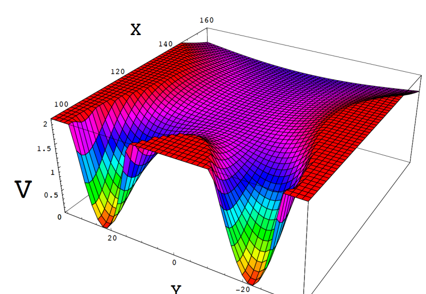
$$S_{\mathrm{KS}} \equiv \sum_{L>0} L \qquad\qquad H(P) \equiv -\sum_{A} P(A) \log P(A)$$

Thermalization from color glass? [Kunihiro, Muller, Ohnishi, Schafer, 10]

3) Cosmology?

Separatrix.





Hybrid inflation [Linde 93]
Brane inflation [Dvali, Tye 99]
Racetrack inflation
[Blanco-Pillado et al. 04]