Abstracts: CC-seminar 2004-2

December 5, 2004

T. Mori: *Effective convergence of Fine computable functions*

We reformulate the Fine-computabilily of functions on [0,1), which was introduced by Brattka as (ρ_F, ρ_E)-computable functions. We intorduce effective Fine-convergence of functions and prove that Fine-computability is preserved under this convergence.

V. Brattka: *The Hahn-Banach Theorem and its Impact on Computable Analysis*

We will discuss the Hahn-Banach Theorem, its computational status and the impact the theorem has on the possibilities to handle computability on non-separable spaces. Surprisingly, it turns out that the results depend on the underlying axiomatic setting.

S. Hayashi: *LCM and games*

A new idea which relates LCM (limiting computable mathematics) and games.

P. Hertling-1: *Topological Complexity of Zero Finding for Continuous Functions*

The topological complexity of a problem over the real numbers is the minimum number of tests or comparisons that one needs to perform in order to solve the problem. One can consider topological complexity in various settings. It has turned out that the topological complexity of a problem can be very different depending on the set of operations which, besides comparisons, are allowed for its solution: either only algebraic operations, or also other operations like exp, log, absolute value, or even all continous operations, which is the purely topological setting. In the purely topological setting, the topological complexity can be characterized also as a degree of discontinuity. It is closely related to the Schwarz genus, and one is led to problems involving algebraic topology; see work by Smale and Vassiliev. Computational problems in various fields give rise to an investigation in topological complexity, namely algebraic complexity theory, numerical computation, and computational geometry. For example in computational geometry the topological complexity of a problem can be considered as a measure of the degree of degeneracy of the degenerate input configurations, those configurations where the output values or some values computed during the solution process show a discontinuity.

In this talk we give an introduction to topological complexity and present recent results and open problems. After presenting fundamental definitions and results, we will focus on results concerning the topological complexity of various versions of the problem to approximate zeros of univariate functions on the unit interval, studied by Novak and Wozniakowski and by the author.

M. Yasugi, Y. Tsujii and T. Mori: *Effective sequence of uniformities*

In order to give a property of computability to a certain sequence of functions which jump at different points, we introduce the notion of an effective sequence of uniformities. The computability of a sequence of functions under concern can then be regarded as computable in the "limit uniformity", the union of all the uniformities in the sequence.

P. Hertling-2: *Is the Mandelbrot set computable?*

We discuss the question whether the Mandelbrot set is computable. The computability notions which we consider are studied in Computable Analysis and will be introduced and discussed. We show that the exterior of the Mandelbrot set, the boundary of the Mandelbrot set, and the union of its hyperbolic components satisfy certain natural computability conditions. We conclude that the two–sided distance function of the Mandelbrot set is computable if the Hyperbolicity Conjecture is true.

T. Yamazaki: *Some results on higher order reverse mathematics*

Kohlenbach has advocated extending the context of reverse mathematics to the language of higher order arithmetic. In accordance with his " higher order reverse mathematics ", we make some investigation into several old results of reverse mathematics.

A. Yoshikawa: *Questions and problems from a naive observer of the field*

Some relflections by a working analyst on computable analysis.

H. Kamo: *Computability of Urysohn's Universal Metric Space*

We introduce a computability structure of Urysohn's universal metric space. With the computability structrure, every computable metric space can be embedded isometrically and computably into Urysohn's universal metric space.