Interpretations of Transfinite Recursion and Parametric Abstraction in Types

Mariko Yasugi[1)] and Susumu Hayashi[2)]

1) Faculty of Science, Kyoto Sangyo University, Kyoto 603, Japan

2) Department of Applied Mathematics and Informatics,
Ryukoku University, Ohtsu 520-21, Japan

1 Introduction

In our previous work Yasugi[3], the first author defined a system of generalized types, which has transfinite recursion and parametric abstraction among formation rules of types. A system of terms with these types (now called **TRM**) was then introduced. It contains parametric abstraction,lambda-abstraction and bar recursion among formation rules of terms. Our purpose there was to realize the modified realizability interpretation of a constructive system of arithmetic, now called **TRDB**, which contains universal quantification over functions, definition of predicates by transfinite recursion and bar induction. The significance of these systems is explained in Yasugi[3].

Strong normalizability and the Church-Rosser property of these types (called type-forms) as well as of the terms (called term-forms) have been established in Yasugi-Hayashi[4]. There, a map from **TRDB** to **TRM** is defined, by which the modified realizability interpretation of **TRDB** is better presented, and also the computational meaning of **TRDB** is explicated. In this article, we continue to investigate some properties of **TRM**. First, a set model of type-forms is presented, and the term-forms are interpreted as functions in the image sets of type-forms (Theorem 1). Since a map from **TRDB** to **TRM** is defined in Yasugi-Hayashi[4], this means that **TRDB** also

has a set model. Another theme is to translate **TRM** into **TRDB** (Theorem 2). This is the reversal of a result in Yasugi-Hayashi[4].

The article is more or less self-contained, although familiarity with Yasugi[3] will help understanding its background. Howard-Kreisel[1] and Troelstra[2] are useful in learning bar induction and systems of constructive arithmetic.

2 Set-model of type-forms

Type-forms are types with parametric variables. They are briefly described below. We do not elaborate with details, since they are thoroughly discussed in Yasugi[3] and Yasugi-Hayashi[4].

Definition 2.1. Let $\mathcal{I} \equiv (I$