Decision making under uncertainty and ambiguity

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Abstract

This paper presents and analyzes the results of two experiments including small decision problems. In the experiments, subjects are asked to choose one of two alternatives for a few thousand times. First experiment, where payoff structure is clearly told, is carried out to explore choice under uncertainty by examining whether expected utility theory holds or not. Second experiment, where subjects have no prior information as to payoff structure, is performed to explore search under uncertainty by investigating whether the ambiguity model holds or not. The results capture expected utility theory by setting extremely concave utility functions, while the ambiguity theory is supported.

1 Introduction

The theory of decision making under uncertainty could be considered one of the "success stories" of economic analysis: it rested on solid axiomatic foundations, it had seen important breakthroughs in the analytics of uncertainty and their applications to economic issues. Today decision making under uncertainty is a field in flux: the standard theory, *expected utility theory* proposed by von Neumann and Morgenstern [36], is being challenged on several grounds from both within and outside economics. The nature of these challenges is the topic of this paper.

It has been recognized that *the expected utility theorem* is an essential and important tool for making a decision under uncertainty. However, the Allais example [1] has provoked a great deal of controversy. Kahneman & Tversky [19] and Barron & Erev [3] claim that some violations of expected utility theory was observed in various choice problems.

The analysis of decision making, where the outcome individuals face is not certain, can be divided into two treatments: *the search treatment*, where individuals are not informed of the probability of each possible outcome, and *the choice treatment*, where individuals are informed of the probability of each possible outcome.

Over the past decades, a number of studies have been made on search under uncertainty (e.g. Einhorn & Hogarth [8] and Barron & Erev [3]) and choice under uncertainty (e.g. Kahneman & Tversky [19] and Levi [21]). The expected utility model of preferences over uncertain outcome is almost always provided as a description of choice under uncertainty; *the ambiguity model* of preferences over uncertain outcome, which is presented by Einhorn and Hogarth [8], is often provided as a description of normative model of search under uncertainty. However, very few attempts have been made at an experimental study examining systematically both search under uncertainty and choice under uncertainty. This paper explores it with a series of computerized economic experiments.

The purpose of this paper is to explore how people make a decision under uncertainty in terms of both theory and experimental study. To put it concretely, this paper describes the results of small decision making experiments Barron and Erev [3] conducted, where subjects are asked to choose one of two alternatives 400 times.

Barron and Erev's small decision making experiments are done under the same and different conditions. Two experiments were conducted: Experiment 1 and Experiment 2 are performed to explore search under uncertainty and choice under uncertainty respectively. Subjects are not informed of the payoff structure in Experiment 1, while they are informed in Experiment 2. A search model as a description of search under uncertainty is presented in Experiment 1. Examined in Experiment 2 are expected utility theory and Allais

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examples as a description of choice under uncertainty. The ambiguity theory is checked by comparing the results of both experiments.

The new findings of this paper are as follows. We conclude that the probability that subjects misestimate the probability of uncertain outcomes in Experiment 1 is fairly large in just hundreds of rounds. In fact, the results of experiments suggest that many subjects must have considered the alternative with higher (lower) expected value as the one with lower (higher) expected value in the experiments.

¿From this view point, although Barron and Erev claim that Allais paradox is not observed (the "reversed certainty effect" is observed in their terms) in our choice problems, it does not necessarily imply our subjects are risk averters; they may merely choose the alternative more frequently that has produced higher posterior average points so that they misestimated each alternative.

Most subjects in Experiment 2 choose both alternatives during a session for the same problem. This can be explained within the framework of expected utility theory, only with rather risk averse utility functions.

The comparison of the results of Experiment 1 and those of Experiment 2 shows the mental simulation process, which is consistent with the adjustment process proposed by the ambiguity model. In short, the results of experiments suggest that expected utility theory can explain choice under uncertainty only with extremely unrealistic utility function, while the ambiguity theory could explain search under uncertainty.

The paper is organized as follows. Section 2 outlines the two existing experimental studies: Kahneman and Tversky's study of choice under uncertainty and Barron and Erev's study of search under uncertainty. Section 3 presents our experiments: Experiment 1 and Experiment 2. The former is examined in Section 4 while the latter is examined in Section 5. Section 6 compares Experiment 1 and Experiment 2.

¿From what has been mentioned above, we conclude that it is important to distinguish search under uncertainty from choice under uncertainty in exploring human decision making, where possible outcomes are uncertain.

2 Existing experiments

2.1 Kahneman and Tversky's experiments

The best known counter-example to expected utility theory, which exploits the certainty effect, was introduced by Allais [1]. Slovic and Tversky [35] examine it from both normative and descriptive standpoints.

Kahneman and Tversky [19] performed the following pair of choice problems along with other problems:

Problem A. Choose between:
H: 4 with probability .8 ; 0 otherwise N=95
L: 3 with certainty

Problem B. Choose between:
H: 4 with probability .2 ; 0 otherwise N=95
L: 3 with probability .25 ; 0 otherwise,

which are variations of the Allais examples, and differ from the original in that it refers to moderate rather than to extremely large gains. The outcomes represented hypothetical payoffs in thousand Israeli Lira, and N denotes the number of respondents in each choice problem. Notice that Problem B was created by dividing the probability of winning in Problem A by four.

Kahneman and Tversky show that while 80% of subjects preferred L in Problem A, only 35% preferred L in Problem B. However, their results violate the tenet of expected utility theory. Let $X_y = (\alpha, a)$ be a prospect, which yields α points with probability a and does 0 point with probability $(1-a)$ in Problem y. To show that the modal pattern of preferences in Problem A and B is not compatible with the theory, note that the prospect H_B=(4,000, .20) can be expressed as (H_A, .25), while the prospect L_B=(3,000, .25) can be rewritten as (L_A, .25). The independence axiom of expected utility theory asserts that if L_A is preferred to H_A, then any (probability) mixture (L_A, p) must be preferred to the mixture (H_A, p).

This "Allais pattern" is a violation of the independence axiom of expected utility theory that implies that decision makers should have the same preferences in the two problems. In expected utility theory, the utilities of outcomes are weighted by their probabilities. The comparison of Problem A and Problem B describes a series of choice problems in which people's preferences systematically violate such axiom. The results show that people overweight outcomes that are considered certain, relative to outcomes which are merely probable–a phenomenon which Kahneman and Tversky label the certainty effect. [19]

What has to be noticed is the following. First, Kahneman and Tversky's subjects are asked to answer a questionnaire including choice problems only once. Second, all benefits the subjects face were denoted in Israeli Lira, however, were in fact hypothetical payments. The subjects recieved no real money and they are correctly informed of the payoff structure.

2.2 Barron and Erev's experiments

Barron and Erev [3] focus on an important subset of the small decision problems that can be referred to as "small feedback-based" decisions. These problems are defined by three main properties. First, they are repeated; decision makers face the same problem many times in similar situations. Second, each single choice is not very important; the alternatives tend to have similar expected values that may be fairly small. Finally, the decision makers do not have objective prior information concerning the payoff distributions. In selecting among the possible options, they have to rely on the immediate and unbiased feedback obtained in similar situations in the past.

Barron and Erev conducted experiments including the same choice problems as Kahneman and Tversky's [19]. Barron and Erev did experiments with 48 undergraduates including 400 rounds for each choice problem. Then the information available to subjects is limited to feedback concerning the outcomes of their previous decisions. They claim a *reversed certainty/Allais effect*: While the mean proportion of H choice (having a higher expected value but more risky) over subjects was .63 for Problem A, it decreased significantly to .51 for Problem B.

However, there are two objections which can be raised against Barron and Erev's claim. Remember that in Kahneman and Tversky's experiments, subjects are correctly informed of the payoff structure and they did only one round with a hypothetical payoff. On the other hand, Barron and Erev carried out experiments, where subjects are not informed of the payoff structure, asked to choose 400 times, and paid real money according to their performance.

First, it has not been examined whether subjects correctly estimate each alternative or not in hundreds of rounds. As they repeatedly choose an alternative to get points, they will gradually form a subjective payoff structure of the

problem, which may or may not the same as the objective one. As a result, a subject may choose H (L), supposing or not supposing that it has a higher (lower) expected value. In the circumstances, Barron and Erev's results are not directly comparable with Kahneman and Tversky's, where their subjects have the exact knowledge of the payoff structure . In words, multi-decision making is not a mere repetition of single decision making.

Second, although they do not know the exact number of rounds, they are able to expect they will make their decision for a number of times. The optimal behavior for a case, where the same problem is repeatedly asked, is not necessarily to repeat the optimal choice for the problem. Suppose that one chooses an alternative if she is asked to choose only once. This does not necessarily imply that she will choose the alternative 400 times, if she is asked to choose an alternative 400 times.

We must note that Barron and Erev's experiments are distinct from Kahneman and Tversky's in the following. First, their subjects are asked to perform choice problems 400 times. Second, the subjects recieved cash contingent upon their performance and they are not informed of the payoff structure. Therefore, it is not safe that we compare Barron and Erev's results with Kahneman and Tversky's, and Barron and Erev's claim must be carefully interpreted.

3 Experimental design

Our economic experiments, which consist of Experiment 1 and Experiment 2, were performed at Kyoto Sangyo University Economic Experiment Laboratory (KSUEEL) on the 20th of November in 2002.

Sixteen undergraduates at Kyoto Sangyo University served as paid subjects in the experiments. Subjects received payoff contingent upon performance and no initial (showing up) fee is paid. The translation from points to monetary payoffs was according to the exchange rate: 1 point= .3 Yen (.25 US cent).

Experiment 1 and Experiment 2 are conducted in order. Each experiment consists of four sessions.

Each session consists of 400 rounds, where subjects are repeatedly faced with the same problem 400 times.

Problem 1
H: 3.2 points with probability 1
L: 3 points with probability 1

Problem 2
H: 4 points with probability .8 ; 0 otherwise
L: 3 points with probability 1

Problem 3
H: 4 points with probability .2 ; 0 otherwise
L: 3 points with probability .25 ; 0 otherwise

Problem 4
H: 32 points with probability .1 ; 0 otherwise
L: 3 points with probability 1

Here, for example, if a subject chooses H in Problem 2, then she gets 4 points with probability .8 and 0 point with probability .2. At each round they are asked to choose one of the two alternatives.

3.1 Apparatus and procedure

Each subject performs Problem 2, 3 and 4 in different order in the first three sessions and then Problem 1 in the last session. For example, subject 1 performed Problem 2, 3, 4 and 1, while subject 2 did Problem 2, 4, 3 and 1 in order. (Six is the total number of the combination of the order in each session.) Subjects are aware of the expected length of the experiments, so they know that it includes many rounds. They are not informed that one session includes exactly 400 rounds [1].

Subjects are informed that they were playing on a "computerized money machine" in the experiments. As shown in Figure 1 and Figure 2, in Experiment 1, they are not informed of payoff structure; in Experiment 2, they are clearly informed of payoff structure

The screen subjects face in Experiment 1 and in Experiment 2 are shown in Figure 1 and Figure 2 respectively. They are asked to choose one of the two unmarked buttons shown in Figure 1 in Experiment 1, while they are asked to choose one of the buttons shown in Figure 2, on which corresponding payoff and its probabilities appear in Experiment 2.

Two types of feedback immediately follow each choice: (1) the payoff for the choice, that appears on the selected key for the duration of one second, and (2) an update of an accumulating payoff counter, which is constantly displayed.

[1] Not knowing the length of the experiments also prevents subjects from using probability-based reasoning (the focus on the likelihood of achieving a particular aspiration level) (Lopes [22]). This type of reasoning bases choice on the probability of coming out ahead, which is a function of the number of choices to be made. A second reason for not telling subjects the game's length is that this better approximates the real-world small decisions that interest us. In such situations, the number of future choices to be made is often unknown.

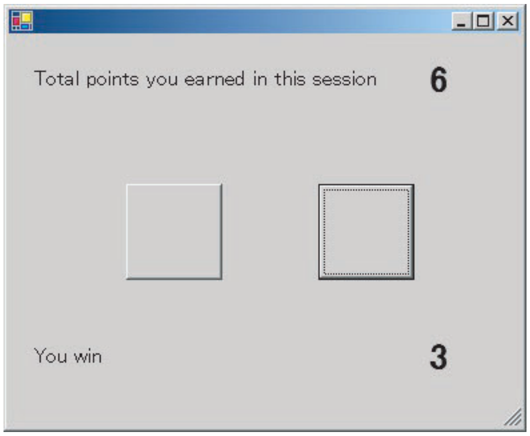

Figure 1: Experiment 1

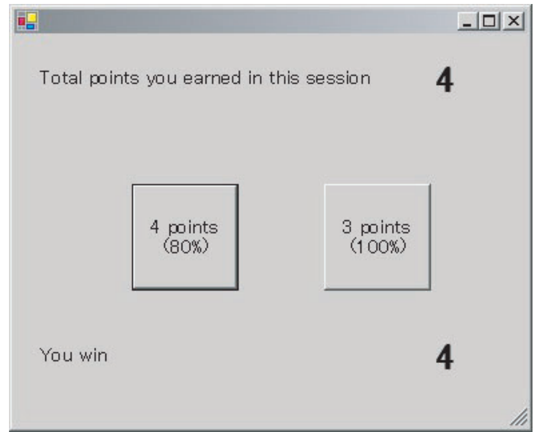

Figure 2: Experiment 2

	Problem 1	Problem 2	Problem 3	Problem 4
Kahneman and Tversky		0.20 (*N*=95)	0.65 (*N*=95)	
Barron and Erev	0.90 (*N*=48)	0.63 (*N*=48)	0.51 (*N*=48)	0.24 (*N*=48)
Experiment 1	0.92 (*N*=16)	0.48 (*N*=16)	0.60 (*N*=16)	0.16 (*N*=16)
Experiment 2	0.99 (*N*=16)	0.75 (*N*=16)	0.65 (*N*=16)	0.45 (*N*=16)

Table 1: Average proportion of H choices

4 Experiment 1

4.1 Results

The average proportion of H choices in each experiment and the proportion of H in each subject are shown in Table 1 and Table 2 respectively. Denoted by *N* is the number of subjects in each experiment.

	Problem 1	Problem 2	Problem 3	Problem 4
Subject 1	0.89	0.5025	0.335	0.0325
Subject 2	0.905	0.7875	0.7875	0.89
Subject 3	0.9625	0.0425	0.515	0.005
Subject 4	0.9775	0.8925	0.4975	0.0275
Subject 5	0.9925	0.655	0.89	0.0075
Subject 6	0.9825	0.615	0.5625	0.0075
Subject 7	0.8	0.555	0.61	0.725
Subject 8	0.6825	0.3575	0.495	0.06
Subject 9	0.985	0	0.8225	0
Subject 10	0.9175	0.66	0.5175	0.2425
Subject 11	0.9775	0.3275	0.775	0.0175
Subject 12	0.8375	0.39	0.5075	0.0825
Subject 13	0.945	0.7975	0.565	0.04
Subject 14	0.995	0.0025	0.815	0.0025
Subject 15	0.9825	0.84	0.3175	0.3575
Subject 16	0.9625	0.31	0.575	0.015
Average	0.9246875	0.4834375	0.59921875	0.15703125
STD	0.086471937	0.295227137	0.171873788	0.273774875
Max	0.995	0.8925	0.89	0.89
Min	0.6825	0	0.3175	0

Table 2: Proportion of H choices

The posterior average payoff for the first n rounds is defined as the points the subject earned for the first n rounds divided by n. Table 3 and Table 4 summarizes the posterior average payoff. Note "-" in Table 4 indicates that subject 9 does not choose H at all so that the posterior average for H choices cannot be defined.

Table 1 shows that the proclivity for the proportion of H choices in Experiment 2 is similar to that in experiments carried out by Barron and Erev.

However, we must be careful to claim the reversed certainty/Allais effect is observed in our experiment.

	Posterior avg for H	Posterior avg for L	Number of H choices	Number of L choices
Subject 1				
Problem 1	3.2	3	356	44
Problem 2	3.2039801	3	201	199
Problem 3	0.537313433	0.890977444	134	266
Problem 4	0	3	13	387
Subject 2				
Problem 1	3.2	3	362	38
Problem 2	3.238095238	3	315	85
Problem 3	0.914285714	0.635294118	315	85
Problem 4	2.786516854	3	356	44
Subject 3				
Problem 1	3.2	3	385	15
Problem 2	3.058823529	3	17	383
Problem 3	0.873786408	0.618556701	206	194
Problem 4	0	3	2	398
Subject 4				
Problem 1	3.2	3	391	9
Problem 2	3.238095238	3	357	43
Problem 3	0.743718593	0.76119403	199	201
Problem 4	2.909090909	3	11	389
Subject 5				
Problem 1	3.2	3	397	3
Problem 2	3.297709924	3	262	138
Problem 3	0.752808989	0.340909091	356	44
Problem 4	0	3	3	397
Subject 6				
Problem 1	3.2	3	393	7
Problem 2	3.447154472	3	246	154
Problem 3	0.8	0.805714286	225	175
Problem 4	0	3	3	397
Subject 7				
Problem 1	3.2	3	320	80
Problem 2	3.153153153	3	222	178
Problem 3	0.737704918	0.769230769	244	156
Problem 4	3.2	3	290	110
Subject 8				
Problem 1	3.2	3	273	127
Problem 2	3.328671329	3	143	257
Problem 3	0.747474747	0.683168317	198	202
Problem 4	4	3	24	376
Subject 9				
Problem 1	3.2	3	394	6
Problem 2	0	3	0	400
Problem 3	0.911854103	0.591549296	329	71
Problem 4	0	3	0	400
Subject 10				
Problem 1	3.2	3	367	33
Problem 2	3.242424242	3	264	136
Problem 3	0.792270531	0.839378238	207	193
Problem 4	4.618556701	3	97	303
Subject 11				
Problem 1	3.2	3	391	9
Problem 2	3.236641221	3	131	269
Problem 3	0.761290323	0.7	310	90
Problem 4	0	3	7	393
Subject 12				
Problem 1	3.2	3	335	65
Problem 2	3.256410256	3	156	244
Problem 3	0.807881773	0.578680203	203	197
Problem 4	5.818181818	3	33	367
Subject 13				
Problem 1	3.2	3	378	22
Problem 2	3.172413793	3	319	81
Problem 3	0.778761062	0.74137931	226	174
Problem 4	0	3	16	384
Subject 14				
Problem 1	3.2	3	398	2
Problem 2	0	3	1	399
Problem 3	0.871165644	0.689189189	326	74
Problem 4	0	3	1	399
Subject 15				
Problem 1	3.2	3	393	7
Problem 2	3.285714286	3	336	64
Problem 3	0.598425197	0.769230769	127	273
Problem 4	4.475524476	3	143	257
Subject 16				
Problem 1	3.2	3	385	15
Problem 2	3.322580645	3	124	276
Problem 3	0.782608696	0.811764706	230	170
Problem 4	0	3	6	394

Table 3: Posterior average for both alternatives and its proportion

	Problem 1		Problem 2		Problem 3		Problem 4	
	Posterior avg for H	Posterior avg for L	Posterior avg for H	Posterior avg for L	Posterior avg for H	Posterior avg for L	Posterior avg for H	Posterior avg for L
Subject 1	3.2	3	3.20398	3	0.537313	0.890977	0	3
Subject 2	3.2	3	3.238095	3	0.914286	0.635294	2.786517	3
Subject 3	3.2	3	3.058824	3	0.873786	0.618557	0	3
Subject 4	3.2	3	3.238095	3	0.743719	0.761194	2.909091	3
Subject 5	3.2	3	3.29771	3	0.752809	0.340909	0	3
Subject 6	3.2	3	3.447154	3	0.8	0.805714	0	3
Subject 7	3.2	3	3.153153	3	0.737705	0.769231	3.2	3
Subject 8	3.2	3	3.328671	3	0.747475	0.683168	4	3
Subject 9	3.2	3	-	3	0.911854	0.591549	-	3
Subject 10	3.2	3	3.242424	3	0.792271	0.839378	4.618557	3
Subject 11	3.2	3	3.236641	3	0.76129	0.7	0	3
Subject 12	3.2	3	3.25641	3	0.807882	0.57868	5.818182	3
Subject 13	3.2	3	3.172414	3	0.778761	0.741379	0	3
Subject 14	3.2	3	0	3	0.871166	0.689189	0	3
Subject 15	3.2	3	3.285714	3	0.598425	0.769231	4.475524	3
Subject 16	3.2	3	3.322581	3	0.782609	0.811765	0	3
Average	3.2	3	3.256626	3	0.793742	0.735673	3.311443	3

Table 4: Posterior average for the finals in each alternative.

We see from Table 2 that apart from Problem 1, there are substantial differences in the proportion of H choices among the subjects. It can result from the subjects' mistaken estimation of the payoff structure in each problem. In fact, although it is found from Table 4 that the posterior average point for all subjects (i.e. 3.256626, 3, .793742, .735673, 3.311443, 3) well mirrors the expected points of each problem (i.e. 3.2, 3, .8, .75, 3.2, 3), there are considerable differences of the posterior average points of H among the subjects. In particular, the posterior average points of L for some subjects, (e.g. subject 2 in Problem 4) did exceed the one of H choices. We shall explore it in the next section.

4.2 Analysis

4.2.1 Search model

Experiment 1 includes the situations, in which the information available to subjects is limited to feedback concerning the outcomes of their previous decisions. In the situations, subjects must discover payoff structure.

First, let us examine Problem 2 and Problem 4, where only one of the alternatives includes uncertain prospect. To examine Problem 2 and Problem 4, we have only to examine the following choice problem. Each subject is asked to choose one of the following two alternatives (H and L) at each round:

$H : x \quad (p) \qquad ; \quad 0 \quad (1-p)$
$L : 1 \quad (1),$

where
$0 < p < 1, \quad px > 1.$

This choice problem applies to Problem 2 in Experiment 1 by setting $p = .8$ and $x = \frac{4}{3}$: it applies to Problem 4 by setting $p = .1$ and $x = \frac{32}{3}$. In words, if the subject chooses H, she gets x points with probability p, and 0 point with probability $(1-p)$: if she chooses L, she gets 1 point for sure.

Assuming that the subject chooses H m times, she gets x points k times with the probability

$$ {}_mC_k\,(p)^k(1-p)^{m-k}. \tag{1}$$

Hence, if she chooses H m times, her average points are greater than or equal to 1, which is the point she can get if she always chooses L, with the probability

$$P(H_m) = \sum_{\text{all } m:\, \frac{kx}{m} \geq 1} {}_mC_k\, p^k\,(1-p)^{m-k} = \sum_{k=[\frac{m}{x}]+1}^{m} {}_mC_k\, p^k\,(1-p)^{m-k}. \tag{2}$$

This allows us to analyze the number for H choices needed for judging that an alternative H has higher expected value than an alternative L. Suppose that a subject chooses H 200 times in Problem 2, then her

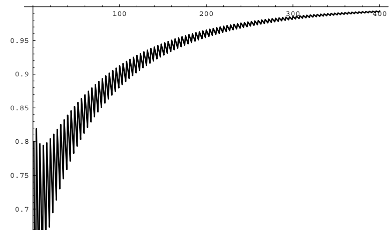

Figure 3: Problem 2

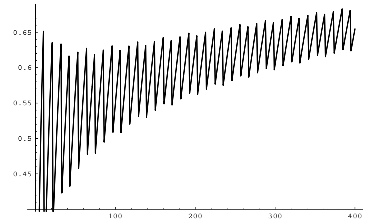

Figure 4: Problem 4

posterior average for H choices exceeds 3 with probability .97 as shown in Figure 3 . Similarly, if she chooses H 200 times in Problem 4, then her posterior average for H choices exceeds 3 with probability .63. In addition, interestingly, its probability does not exceed .98 until she chooses H 10,000 times.

Second, let us consider Problem 3, where both an alternative H and L include uncertain outcomes. Each subject, in such a situation, faces the following two alternatives at each round:

$H : x \quad (\theta p) \qquad ; \quad 0 \quad (1-\theta p)$
$L : 1 \quad (\theta) \qquad ; \quad 0 \quad (1-\theta),$

where
$0 < p < 1, \quad \theta p x > \theta$

By the same token as the previous example, when choosing H once, a subject in the above choice problem gets x points with probability θp and 0 point with probability $(1-\theta p)$: when selecting L, she gets 1 point with probability θ and 0 point with probability $(1-\theta)$. If she chooses H m times and gets x points k times, then her average points are $\frac{kx}{m}$: if she chooses L n times and gets 1 point l times, then her average points are $\frac{l}{n}$. The probability that the former average points are equal to or greater than the latter, in other words, $\frac{kx}{m} \geq \frac{l}{n}$, is

$$\sum_{j=0}^{[\frac{nkx}{m}]} {}_nC_j \, (\theta)^j \, (1-\theta)^{n-j}. \tag{3}$$

Therefore, assuming that a subject chooses H and L m and l times respectively, her posterior average of H choices is greater than or equal to the average of L with the probability $P(L_n)$.

$$P(L_n) = \sum_{k=0}^{m} \left[{}_mC_k \, (\theta p)^k (1-\theta p)^{m-k} \times \sum_{j=0}^{[\frac{nkx}{m}]} {}_nC_j \, (\theta)^j \, (1-\theta)^{n-j} \right]. \tag{4}$$

Suppose that a subject chooses H and L each 200 times in Problem 3, she judges that an alternative H has

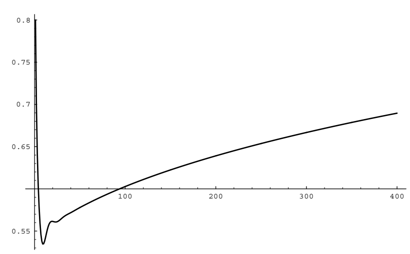

Figure 5: Problem 3

higher expected value than an alternative L with probability .64 as shown in Figure 5. The search model well captures the results shown in Table 2.

4.2.2 Experimental

In this subsection, we explore an analysis under the assumption that subjects take 300 rounds to search their decision, and they continue making their decision the following 100 rounds. Table 5 indicates comparison of a posterior average for H at the 300 round and that for L, and which button is chosen more frequently for the following 100 rounds. Table 6 summarizes the data on Table 5. It is found from Table 6 that posterior average of H at the 300 round exceeds the one of L in the 29 (60%) of 48 examples and H is frequently chosen the following 100 rounds in 19 of 29 examples. The results indicate that each subject makes her decision for the following 100 rounds to follow her posterior average at the 300 round. First, classified

Problem 2	H	14	(88)	H	9	(64)
				L	5	(36)
	L	2	(13)	H	0	(0)
				L	2	(100)
Problem 3	H	10	(63)	H	9	(90)
				L	1	(10)
	L	6	(38)	H	3	(50)
				L	3	(50)
Problem 4	H	5	(31)	H	1	(20)
				L	4	(80)
	L	11	(69)	H	1	(9)
				L	10	(91)

Table 5:

H	29	(60)	H	19	(66)
			L	10	(34)
L	19	(40)	H	4	(21)
			L	15	(79)

Table 6:

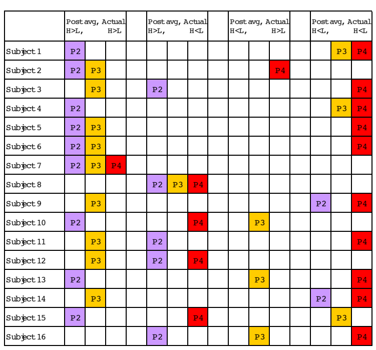

Figure 6: Posterior average at the 300 round, and which button was chosen more frequently for the following 100 rounds.

into higher and lower one is the posterior average payoff of each choice per subject at the 300 round. Next, those are compared with the number of H (L) choices for the following 100 rounds. It is found from the results that each subject makes her decision for the following 100 rounds to follow her posterior average at the 300 round.

It seems that the results violate expected utility theory indicating that average subjects choose H 192 of 400 times in Problem 2 and 240 in Problem 3 in Experiment 1. The results, however, do not necessarily lead us to the conclusion that subjects behave violating the independence axiom of expected utility theory. Rather, it seems reasonable to conclude that subjects' decisions depended on the results of searching their subjective probabilities.

Table 3 summarizes the proportion of H and L choices and the posterior average of them in each subject. It is found from Table 3 that some of subjects often choose the alternative L. For example, subject 4 chooses L 389 times in Problem 4. One, however, does not guarantee that she is risk averse or she has a concave utility-of-wealth function. It is reasonable to conclude that she chooses L many times because her posterior average of L choices is greater than that of H choices.

We see from Table 2 and Table 3 that subject 1, for example, chose H 134 times (33.5%), and L 266 times (66.5%) in Problem 3: her posterior average of H and L choices in Problem 3 are .537313 and .890977 respectively. Figure 7 and 8 indicates her posterior average and cumulative number of both choices. It is found from Figure 7 and 8 that posterior average of L exceeds the one of H because she tried out H only 134 times.

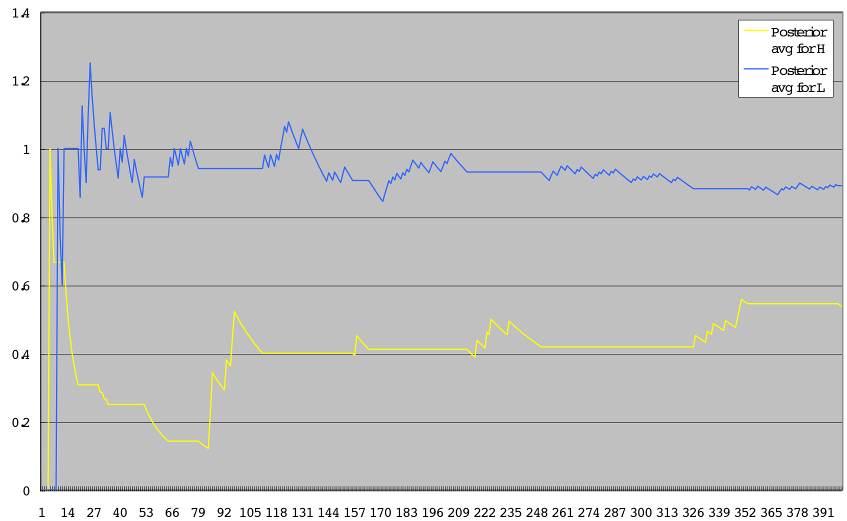

Figure 7: Subject 1 in Problem 3: This shows her posterior average of L at the 300 round exceeds the one of H.

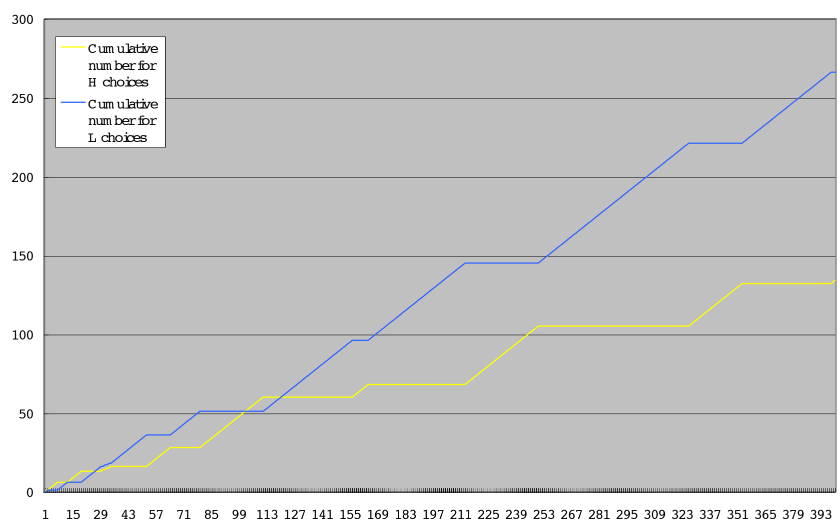

Figure 8: Subject 1 in Problem 3: This shows she often chooses L for the last 100 rounds.

We see from Table 2 and Table 3 that subject 5 chose H 356 times (89%), and L 44 times (11%) in Problem 3: her posterior average of H and L choices in Problem 3 are .752809 and .340909 respectively. Figure 9 and 10 indicates her posterior average and cumulative number of both choices. It is found from

Figure 9 and 10 that posterior average of H exceeds the one of L because she tried out H many times.

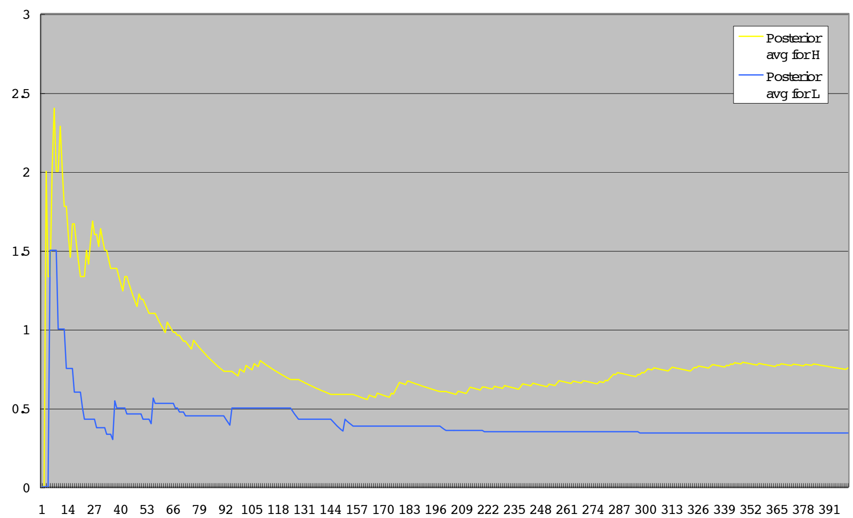

Figure 9: Subject 5 in Problem 3: This shows her posterior average of H at the 300 round exceeds the one of L

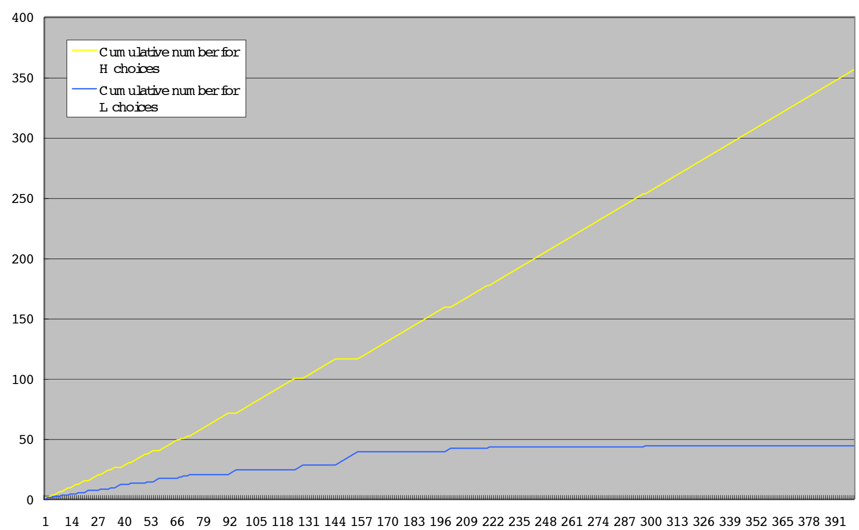

Figure 10: Subject 5 in Problem 3: This shows she often chooses H for the last 100 rounds

It is described here that a subject's decision toward each alternative is not dependent upon only the characteristics of the utility function. For further discussion, a subject's tendency toward each alternative is considered from the following two assumptions.

First, a subject makes her decision depending upon attitudes toward uncertainty: "risk lover" or "risk averter." For example, subject 3 in Problem 2 chooses L most of the rounds although her posterior average of H choices is greater than one of L choices. It is safe to consider that she is risk averter because despite that her posterior average for H choices is greater than that for L, she prefers the alternative which yields 3 points for sure to any risky alternative with expected value 3.2.

Second, subjects make their decision depending upon the posterior average of the button chosen. For example, subject 4 in Problem 4 chose L 389 times because her posterior average of L choices is greater than one of H choices. However, one does not guarantee that she is risk averter with a concave utility function. It is rational that subject 4 continues choosing L to maximize her payoffs.

We see from Figure 6 that subject 1, for example, continues choosing H for the last 100 rounds because the posterior average of H choices at the 300 round is greater than the one of L choices in Problem 2. Also, she continues choosing L for the last 100 rounds because the posterior average of L at the 300 round is greater than the one of H. Assuming that it is rational for people to choose the alternative with high expected value, 25 examples shown in the second and fourth columns are regarded as an irrational case. Next, the situation, where the subject's clue of decision is far apart from their actual choice is considered. One considers two cases: (1) a case, where a subject continues choosing H although the posterior average payoff of H choices is less than the one of L and (2) a case, where a subject continues choosing L although the posterior average payoff of H choices is greater than the one of L. Provided that the two cases described above are considered irrational, 14 examples shown in the second and third columns apply roughly reducing by half.

As mentioned above, for some subjects, the posterior average for H choices has become less than the one for L choices. Therefore, they do regard the alternative with high expected value as the one with low expected value and vice versa. Such inconsistency is caused by insufficient search with a small number of rounds for judging which button has high expected value.

5 Experiment 2

5.1 Results

Table 1 indicates the average proportion of H choices in 400 rounds. Table 1 shows that apart from Problem 1, subjects often choose both H and L in each problem. Shown in Table 7 is the proportion of H choices in each subject. For example, the data indicates that subject 3 chose H 248 times in Problem 2, and that

	Problem 1	Problem 2	Problem 3	Problem 4
Subject 1	1	1	0.8275	0.5025
Subject 2	1	1	1	1
Subject 3	0.9975	0.62	0.515	0.7525
Subject 4	1	1	0.8975	0.925
Subject 5	1	0.915	0.665	0.1925
Subject 6	1	0.32	0.5675	0.1875
Subject 7	1	1	0.5575	0.5225
Subject 8	1	0.4875	0.5175	0.07
Subject 9	1	0	0.9975	0.625
Subject 10	1	0.79	0.6275	0.225
Subject 11	1	1	1	1
Subject 12	1	0.4375	0.5625	0.11
Subject 13	1	1	0.8675	0.365
Subject 14	1	0.9425	0.155	0.1725
Subject 15	1	0.97	0.275	0.25
Subject 16	1	0.54	0.295	0.2725
Average	0.99984375	0.75140625	0.64546875	0.44828125
STD	0.000625	0.312432618	0.267918238	0.322727592
Max	1	1	1	1
Min	0.9975	0	0.155	0.07

Table 7: Proportion of H choices

subject 9 only chose L once in Problem 3. It is found from the results that many subjects in Experiment 2 often chose both H and L in each problem except Problem 1.

5.2 Analysis

Each subject in Experiment 2 chooses H or L at the k-th round with the knowledge of the past outcome. This does not serve her knowledge. She knows probabilities from the beginning. Although this might have "asset effect" , we analyze in this section on the presumption that subjects perform repeatedly 400 rounds without being informed the outcome of their past choice, or they are asked how many times they choose H or L once for all.

As a passing reference to the above, the author questionnaires the seven of all subjects.

> Question: How many times you are going to select an alternative H in 400 rounds for each Problem?

	Problem 2	Problem 3	Problem 4
Questionaires	0.47	0.78	0.34
Experiment	0.64	0.88	0.42

Table 8: Proportion for H choices in questionnaires and the experiment.

Although Table 8 shows that the proportion of H choices becomes greater in the experiment than in questionnaires, these observations are insignificant at the .05 level.

5.2.1 Expected Utility Theory

The results of Experiment 2 can be analyzed within the framework of expected utility theory since subjects are informed of the payoff structure. As mentioned above, we analyze the results on the presumption that subjects are asked how many times of 400 rounds they choose H or L once for all.

Let $V_2(m)$, $V_3(m)$ and $V_4(m)$ be the expected utility one acquires when choosing H m times in Problem 2, 3 and 4 respectively.

$$V_2(m) = \sum_{k=0}^{m} \left[{}_mC_k\,(.8)^k\,(.2)^{m-k}\,u(1200 - 3m + 4k) \right] \quad (5)$$

$$V_3(m) = \sum_{k=0}^{m} \left[{}_mC_k \sum_{l=0}^{400-m} \left\{ {}_{400-m}C_l\,(.2)^k\,(.8)^{m-k}\,(.25)^l\,(.75)^{400-m-l}\,u(4k + 3l) \right\} \right] \quad (6)$$

$$V_4(m) = \sum_{k=0}^{m} \left[{}_mC_k\,(.1)^k\,(.9)^{m-k}\,u(1200 - 3m + 32k) \right], \quad (7)$$

where k is the number for the highest payoff of each problem.

It is found from Table 1 that subjects make their decision by choosing both alternatives in Experiment 2, where they are further informed of payoff structure. Employed for explaining this phenomena is the utility function $u(x) = -\frac{1}{a}e^{-ax} + \frac{1}{a}$ with an Arrow-Pratt measure of absolute risk aversion equal to the constant a at all x. Note that given a twice-differentiable utility function $u(\cdot)$ for money, defined as $r_A(x) = -u''(x)/u'(x)$ is the Arrow-Pratt coefficient of absolute risk aversion at x.

The greater the value of a is, the larger the degree of risk aversion. In expected utility theory, risk aversion is equivalent to the concavity of the utility function. The prevalence of risk aversion is perhaps the best known generalization regarding risky choices. Indeed, economists have a simple and elegant explanation for risk aversion: It derives from expected utility maximization of a concave utility-of-wealth function. This rationale is used ubiquitously in theoretical and empirical economic research.

Figure 11 shows the utility function $u(x)$ is linearly increasing for $a \doteqdot 0$ since $u(x) = x$. Figure 14 shows the function $V_2(m)$ is also linearly increasing, where $u(x) = x$. The function $V_2(m)$ has its maximum at $m = 400$ for $0 \leq m \leq 400$. Hence, a subject with the utility function $u(x) = x$ maximizes her utility by choosing H at all times. Figure 12 indicates the utility function $u(x)$ for $a = .005$. It is found from Figure 18 that the function $V_3(m)$ has its maximum at $m = 203$. Hence, a subject maximizes her utility by choosing H 203 and L 197 times respectively. Figure 13 indicates the utility function $u(x)$ for $a = 10^{-2}$. We see from Figure 22 that the function $V_4(m)$ has its maximum at $m = 0$. Hence, a subject maximizes her utility by choosing L at all times.

It does not guarantee that the above holds in all choice problems considered in Experiment 2. One possibility is to assume that there are other types of utility functions which capture all choice problems well than the one with Arrow-Pratt curvature index equal to a specific value. Another possibility is that the function $V_2(m)$, $V_3(m)$ and $V_4(m)$ seem to have no peak in the range of $0 \leq m \leq 400$.

The results of Problem 2 seem to reveal a reversed certainty effect which is referred by prospect theory. While the mean proportion of H choices is .75 when L provided 3 points with certainty (in Problem 2), it decreases to .65 when payoff probabilities are divided by four (in Problem 3) [2]. Thus, subjects deviated from

[2] These observations are insignificant at the .05 level.

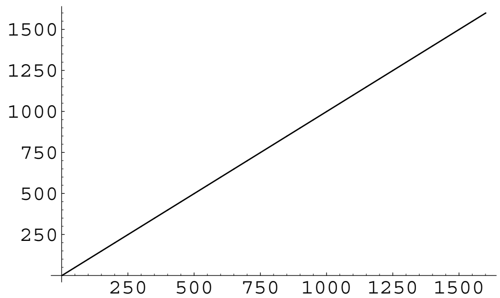

Figure 11: $u(x)$ for $a = 10^{-6}$

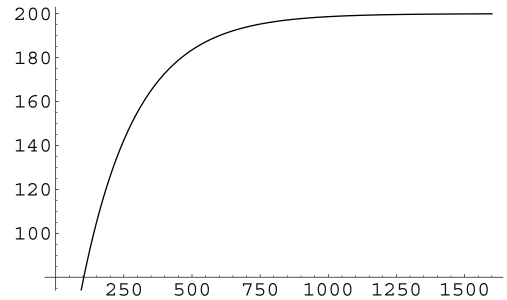

Figure 12: $u(x)$ for $a = .005$

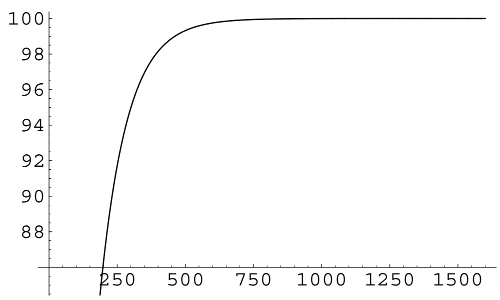

Figure 13: $u(x)$ for $a = 10^{-2}$

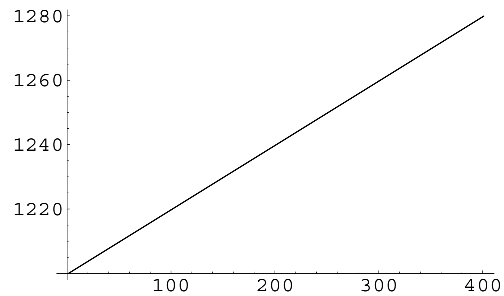

Figure 14: $V_2(m)$ for $a = 10^{-6}$

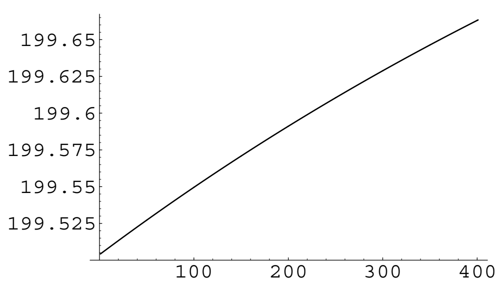

Figure 15: $V_2(m)$ for $a = .005$

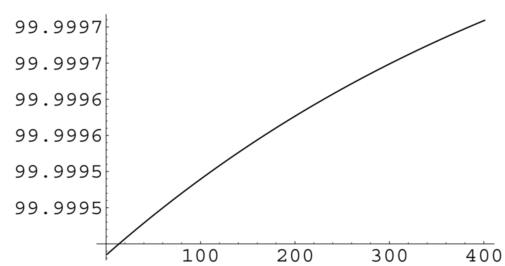

Figure 16: $V_2(m)$ for $a = 10^{-2}$

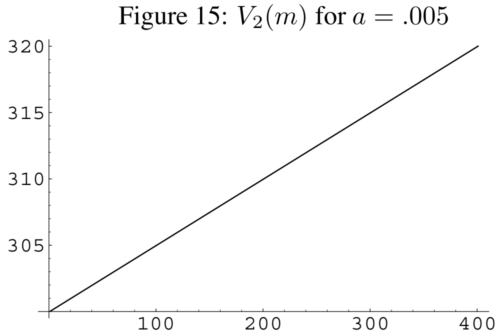

Figure 17: $V_3(m)$ for $a = 10^{-6}$

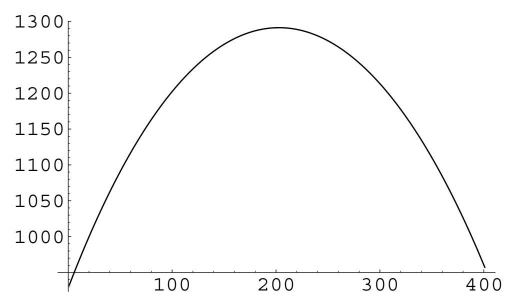

Figure 18: $V_3(m)$ for $a = .005$

the predictions of identical choice proportion in the two problems in the opposite direction of the deviation observed in description-based decisions.

So far, we have seen that the optimal value of m exists in each problem. Hence, the subject in each problem can maximize her utility by choosing H m times and L $(400 - m)$ times. However, the above holds true under the utility function with considerably large degree of risk aversion. Although the function $V_2(m)$ and $V_4(m)$ seem to have a single peak, they do not have such a peak for $10^{-6} \leq a \leq .005$.

6 Comparison of both experiments

Figure 23 shows that the difference between the number for H choices and the one for L in each 100 rounds on the average. It means that the greater the difference is, the more frequently H is chosen. Assuming that a subject chooses H 50 times and L 50 times, the difference is equal to 0.

We see from Figure 23 that the more rounds subjects perform in Experiment 1, the greater the proportion of H is. The proportion for H choices in each problem in Experiment 2 is greater than the one in Experiment 1[3]. At the beginning of each problem in Experiment 1, where subjects have no information as to payoff structure, they are asked to try out frequently both buttons to update their judged subjective probability discussed in chapter three with a mental simulation process.

Figure 23 shows that subjects in Problem 2, 3 and 4, on the average, search payoff structure by trying out both H and L over and over at the first 100 rounds, and they often choose H in the last 100 rounds in Problem 2 because they search that H has higher expected value than L in their mental simulation process. ¿From what has been discussed above, it seems reasonable to conclude that the difference between the number of H and the one of L in each problem reflects the effect of the adjustment process in ambiguity model. This examination, however, remains as a matter to be investigated further.

This study explores attitudes toward uncertainty within the framework of the search and ambiguity models. In considering the role of ambiguity and uncertainty in inferential judgments, a further direction of this study will be to develop a quantitative model that accounts for much existing data as well as experimental findings proposed by the authors.

7 Conclusion and discussion

7.1 Experiment 1 (search under uncertainty)

The search model tells us that the probability that subjects misestimate the payoff structure can be rather large if they choose an alternative only 400 times. As a result, the subjects' posterior average of L exceeds that of H in 17 out of the 32 sessions for Problem 3 and 4. Problem 2 and 3 are quite different search problems. Problem 3 is created by only dividing the probability of winning in Problem 2 by four. However, the probability of misunderstanding of payoff structure is much larger in Problem 3 than in Problem 2. The results show that the posterior average of L exceeds the one of H for only one subject in Problem 2, while for 7 of 16 subjects in Problem 3. Therefore, contrary to "description-based decisions" , in "feedback-based decisions" Problem 2 and 3 should be dealed with as distinct search problems.

Subjects tend to choose the alternative more frequently that has produced higher posterior average points. We see from the results that each subject makes her decision for the last 100 rounds to follow her posterior average at the 300 round. These subjects, whose posterior average is higher for H (L) at the 300 round, usually choose it more frequently for the following 100 rounds.

"*Apparent reversed certainty/Allais effect*" is not always "real" one. Subjects may simply choose the alternative more frequently that has higher posterior average, which has in reality a lower expected value.

7.2 Experiment 2 (choice under uncertainty)

Most subjects choose both H and L in each session except for the session for Problem 1. The results show that out of 48 sessions, only 11 are sessions, where either H or L are chosen. However, the results show that subjects choose H 300, 260 and 180 times in Problem 2, 3 and 4 respectively.

[3]Each of these differences is significant at the .05 level except Problem 3.

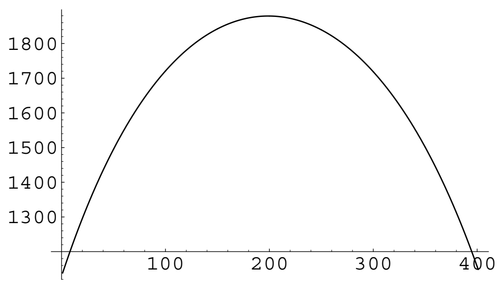

Figure 19: $V_3(m)$ for $a = 10^{-2}$

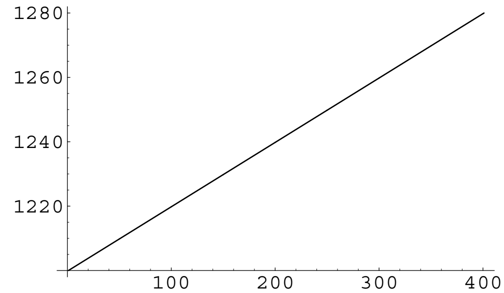

Figure 20: $V_4(m)$ for $a = 10^{-6}$

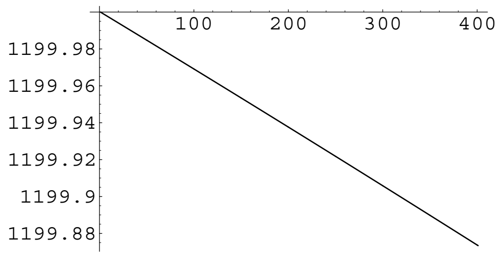

Figure 21: $V_4(m)$ for $a \doteqdot .005$

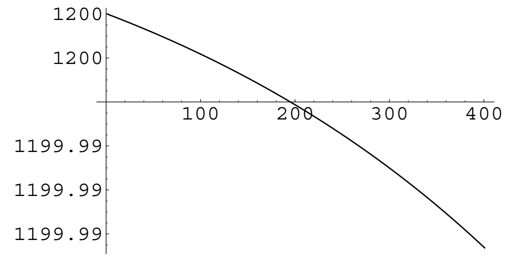

Figure 22: $V_4(m)$ for $a = 10^{-2}$

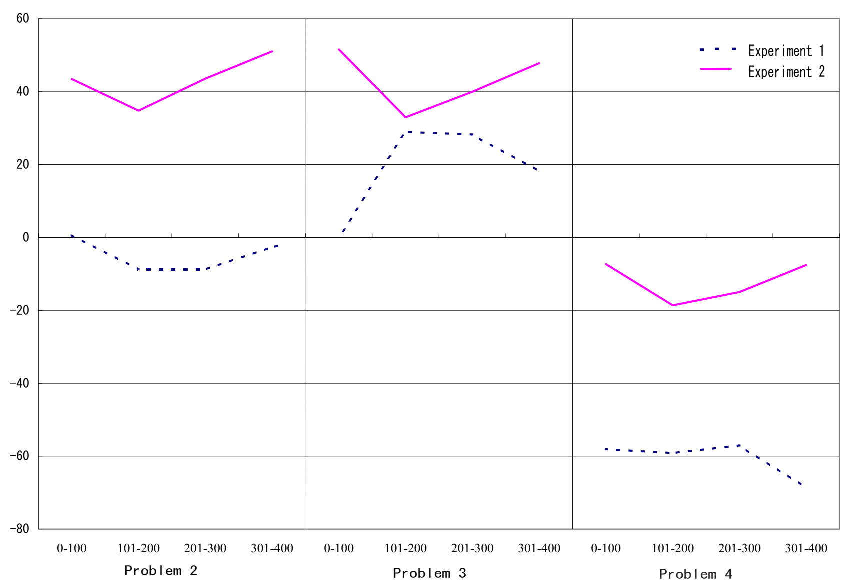

Figure 23:

The optimal behavior for a case, where the same problem is repeatedly asked, is not necessarily to repeat the optimal choice for the problem. Suppose that one chooses an alternative if she is asked to choose only once. This does not necessarily imply that she will choose the alternative 400 times, if she is asked to choose an alternative 400 times.

Suppose that a utility function with the Arrow-Pratt measure is assumed, expected utility theory cannot explain subjects' behavior consistently. However, our calculation shows: within the range of a plausible value of a, to maximize her utility, a subject must choose only H for the session of Problem 2; she must choose only L for the session of Problem 4, and she must choose both H and L for the session of Problem 3.

Although another specification of utility function may explain the above-mentioned subjects' behavior consistently, it should be mentioned that the typical utility function fails to explain the results of our simple experiments.

7.3 Experiment 1 and Experiment 2

Subjects choose H more frequently in Experiment 2 than in Experiment 1. This seems to be explored within the framework of the ambiguity model. Subjects often choose H in the latter part of Problem 2 since they judged that H has higher expected value than L in their mental simulation process. It seems reasonable to conclude that the difference between the proportion of H and that of L reflects the effect of the adjustment process in the ambiguity theory.

As mentioned above, studies of search under uncertainty have been proposed in various fields. The current results under ambiguity are captured by the ambiguity model. However, there are some studies concerning human decision making, such as Bayesian analysis (e.g. Berger [3]) and Two-armed bandit problems (e.g. Sanjeev and Lugosi [32], Aoyagi [27]). A further direction of this study will be to provide evidence from the perspectives of studies above.

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