Competitive Burnout in the Laboratory:

Equilibrium Selection in a Two-Stage Sequential Elimination Game*

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Abstract

We examine experimentally equilibrium selection in a two-stage sequential elimination contest in which a group of contestants competes to win a single prize. Only a subset of the participants survives the first stage. In the second stage, the survivors compete once more, with the winner taking home the prize. This game has a continuum of equilibria, many of which are Pareto-rankable, but only one of these equilibria satisfies the Coalition-Proof Nash Equilibrium (CPNE) refinement. That equilibrium involves "burning out" by using all of one's resources in the first stage. It is Pareto-dominated by many other equilibria. We show that CPNE is not a good predictor of behavior when four people compete for two second-stage spots, but that it does predict well when eight people compete for the two available spots. Announcing the successful bids at the end of each stage has little impact on equilibrium selection.

Keywords: all-pay auction, burning out, coalition-proof Nash equilibrium, contests, experiment.

1. Introduction

Contests are an important fact and pervasive aspect of economic life. A contest is a game in which players compete over a prize by making irreversible outlays. Election campaigns, rent-seeking games, R & D races, competition for monopolies, litigation, wars, and sports are all examples of contests.

A common feature of contests is that they involve multiple stages where the set of contestants is narrowed in successive stages of the contest until a winner is finally chosen. Another feature of contests is that the players may be constrained in terms of how much effort or outlay they can expend (e.g., Che and Gale, 1997, 1998; Gravious et al., 2002). In a sequential elimination contest with such a constraint, it may be rational for contestants to expend *all* their efforts in earlier stages, thus burning out and having nothing left to offer in subsequent stages. Amegashie (2004) shows that under certain conditions burning out in this manner may be equilibrium-consistent rational behavior even though the ultimate prize is won only if a contestant is successful in all stages including the final one.

However, in this setting the burning-out equilibrium is not the only equilibrium. There are also equilibria in which the players do not burn out. Indeed, there is a continuum of equilibria, many of which are Pareto-rankable. The presence of multiple Pareto-rankable equilibria suggests that it is desirable for the players to coordinate on Pareto-dominant equilibria. Since the burning-out equilibrium is always Pareto-dominated by many other equilibria, it is never Pareto optimal to burn out.

Similar kinds of coordination problems are common in many economic contexts. A frequently-cited example is the case of team production. If low effort on the part of one worker reduces the marginal products of other team members, it may not be optimal for a particular worker to exert high effort when the efforts of another are low. In this case, the team may be stuck in a low-effort equilibrium even though all team members would be better off in a high-effort equilibrium. An interesting aspect of this kind of coordination problem is that while both low-effort and high-effort outcomes are Nash equilibria, the latter Pareto-dominate the former. Indeed, there may be a continuum of Pareto-rankable equilibria.

Economists and game theorists have proposed solutions to equilibrium selection in such games. Some of these include focal points (Schelling, 1960), belief-learning (Camerer and Ho, 1999), and Pareto dominance (Harsanyi and Selten, 1988). A growing area of research examines coordination games experimentally in order to shed light on the issue of equilibrium selection (e.g., Van Huyck et al., 1990; Van Huyck et al., 1991; Camerer and Knez, 1994; Van Huyck et al., 2001; Anderson et al., 2001; Berninghaus et al., 2002).[1] Generally, this literature finds that smaller groups reach more efficient equilibria than larger groups, especially when play is repeated with a fixed group of participants.

This paper contributes to this line of research by examining equilibrium selection in a two-stage sequential elimination contest in which a group of contestants competes to win a single prize. Only a subset of the participants survives the first stage. In the second stage, the survivors compete once more, with the winner taking home the prize. Like the weak-link team-production coordination game described above, the sequential elimination game has a continuum of Nash equilibria. In contrast to the weak-link coordination game, which has a continuum of Pareto-rankable equilibria, many but not necessarily all of the equilibria in the sequential elimination game are Pareto-rankable. A more significant contrast between the two games is that the main point of a sequential elimination contest is not cooperation to produce a high return for the group, but competition to win a single valuable prize. Thus, in the sequential elimination game, the

[1] Chapter 7 of Camerer (2003) provides an excellent summary of this literature.

equilibrium selected through some process of coordination by group members affects the earnings of the group as a whole even as its members compete for the ultimate prize. Is cooperation to maximize group welfare possible in such a competitive context?

A refinement of Nash equilibrium, in particular the Coaliton-Proof Nash Equilibrium (henceforth CPNE) concept (Bernheim et al., 1987), suggests that the answer to this question is no. The unique CPNE involves the exertion of maximum effort to the point of complete competitive burnout during the first stage of the game, leaving no resources to utilize during the second stage. From the perspective of the competing participants, this burning-out CPNE is Pareto-dominated by many other equilibria in the game. Since the CPNE refinement produces strikingly different outcomes than Pareto-dominance, this is a challenging context in which to assess the predictive power of the refinement.

In the next section, we describe and analyze the two-stage sequential elimination game. Section 3 presents the experimental design and section 4 discusses the results. Section 5 concludes the paper.

2. A Two-Stage Sequential Elimination Game

In Amegashie (2004), the following game is presented. Consider $N \geq 3$ risk-neutral agents contesting for a prize with valuations commonly known to be $V_1 \geq V_2 \geq \dots V_{N-1} \geq V_N > 0$, where V_i is the valuation of the i-th contestant, $i = 1, 2, \dots, N-1, N$. The contest is divided into two stages. In the first stage, the F contestants with the highest bids or effort levels are chosen to compete in a second stage from which the ultimate winner is chosen, where $2 \leq F < N$. Ties are broken randomly in each stage. Formally, the contest success function in stage one is:

$$P_{1i} = \begin{cases} 1 \quad \text{if } e_i \text{ is one of the top F effort levels and is not tied with anyone,} \\ (F-g)/(r+1) \quad \text{if g contestents bid higher than the i-th contestant and this contestant ties with r} \\ \text{other contestants where } g+r+1 > F \text{ and } 0 \le g \le F, \end{cases}$$

where P_{1i} = the probability of advancing from stage one to stage two and e_i = the effort level of player i. In stage two, the contestant with the highest bid wins. Note that the contest in each stage is an all-pay auction.[2]

Following Che and Gale (1997, 1998) and Gravious et al. (2002), suppose all contestants face a common budgetary or effort constraint or cap, B > 0. These papers give examples of caps in contests: caps on campaign contributions, salary caps in US professional sports[3], and caps on how fast Formula 1 racing cars can move. Also, a cap on effort arises because human beings naturally have a limit on how much effort they can expend.

Suppose B can be allocated between the two stages. Let e_i and x_i be the bid or effort levels of the i-th contestant in stages 1 and 2 respectively, where $e_i + x_i \le B$. We assume that e_i and x_i also represent the cost of expending effort, i.e. the cost function of effort is linear. In each stage, the contestants move simultaneously.

Let $P_{1i}(\tilde{e}) = P_{1i}(e_1, e_2, ..., e_N)$ and $P_{2i}(\tilde{x}) = P_{2i}(x_1, x_2, ..., x_F)$ be the success probabilities of the i-th contestant in stages 1 and 2 respectively. Denote the equilibrium success probabilities by $P_{1i}^*(\tilde{e}^*)$ and $P_{2i}^*(\tilde{x}^*)$ for the i-th contestant.

In stage two, the equilibrium expected payoff of the i-th contestant, conditional on making it to that stage, is $\Pi_{2i}^* = P_{2i}^*(\tilde{x}^*)V_i - x_i^*$. Focusing on a subgame perfect Nash equilibrium and applying backward induction, the equilibrium payoff to the i-th contestant in stage one is $\Pi_{1i}^* = P_{1i}^*(\tilde{e}^*)\Pi_{2i}^* - e_i^*$.

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[2] See Baye et. al. (1996); Hillman and Riley (1989); and Clark and Riis (1998) for analyses of all-pay auctions.

[3] As noted by Gravious et al. (2002), in the year 2000, NFL teams faced a salary cap of $62,172,000. This was a cap on the aggregate amount they could spend on their top 51 salaried players.

The solution to this game is summarized in the following proposition:

Proposition 1: *Consider a two-stage contest where the contest in each stage is an all-pay auction and the contestants have valuations commonly known to be* $V_1 \geq V_2 \geq \dots V_{N-1} \geq V_N$. *If* $F \geq 2$ *contestants are chosen in the first stage to compete in the second stage and all the contestants face a common budget (effort) constraint, B, which can be allocated between the two stages, then a given equilibrium effort allocation (e*, B-e*) between the two stages induces a corresponding equilibrium number of active contestants, K, such that* $\Pi_i^* = (F/K)[(1/F)V_i - (B-e^*)] - e^* \geq 0$ *for* $e^* \in [0, B]$, $i = 1, 2, \dots, K-1, K$ *and* $\Pi_i^* = (F/(K+1))[(1/F)V_i - (B-e^*)] - e^* < 0$ *for* $e^* \in [0, B]$, $i = K+1, K+2, \dots, N-1, N$ *and* $F < K \leq N$. *In any equilibrium, the active contestants i = 1, 2, …, K-1, K bid e* in stage one and B-e* in stage two and the rest bid zero in each stage.*[4]

Proof: In any equilibrium the expected payoff for the i-th active player is $\Pi_i^* = (F/K)[(1/F)V_i - (B-e^*)] - e^* \geq 0$, $i = 1, 2, \dots, K-1, K$. If $F \geq 3$, a player who deviates from this equilibrium by bidding marginally more than e* in stage one guarantees entry to stage two, but will then lose in stage two with certainty since he will be joined by, at least two players who, having bid e* in stage one, have bigger caps in stage 2. There exists a pure-strategy equilibrium in the stage-two subgame in which the players with the bigger cap in stage two will bid their cap, yielding an expected payoff lower than the equilibrium expected payoff for the player who deviated. If F = 2, a player who deviates by bidding marginally more than e* in stage one guarantees entry to stage two, but will be joined by a player who bid e* in stage one and hence has a bigger cap in stage two. In this case, there is no equilibrium in pure-strategies in the stage-two subgame. However, in any mixed-strategy equilibrium in stage two, the player with the smaller cap will get a

[4] Equilibria may also exist in which a player with a lower valuation is active (i.e., bids a positive amount in at least one of the stages) while a player with a higher valuation bids nothing in either stage. The existence of such an equilibrium requires that the difference in valuations between these two players be sufficiently small. We do not focus on such equilibria. Note also that we assume that if a player is indifferent between participating in the contest and staying out, he will participate.

zero expected payoff[5] which is less than the expected payoff in the symmetric equilibrium in which everyone bids e* in stage one. Hence, it is not profitable for any player to deviate by bidding more than e* if F = 2. A player who bids less than e* in stage one will lose with certainty in that stage, yielding an expected payoff lower than the equilibrium expected payoff. Hence there is no profitable deviation from the equilibrium stated in the proposition for an active player. The players i = K +1,…, N-1, N, have no incentive to participate if $[F/(K+1)][(1/F)V_i – (B-e^*)] – e^* < 0$ for $e^* \in [0, B]$.**Q.E.D.**

According to proposition 1, a different value of e* may induce a different number of active contestants, K. If K and e* vary simultaneously, a Pareto ranking of the different equilibria is not straightforward. For the sake of exposition, we initially investigate the Pareto ranking of equilibria that share a common number of active participants, K. For a given K, all such equilibria can be ranked by noting that $\delta \Pi_i^* /\delta e^* = F/K – 1 < 0$. Hence the equilibrium with the lowest e* gives the highest expected payoff and the equilibrium with the highest e* gives the lowest expected payoff for i = 1, 2,…, K-1, K. This of course implies that the burning-out equilibrium in which e* = B, the highest possible e*, is Pareto-dominated by all other equilibria with the same number of active participants, K, since each of those equilibria has an e* < B.

As indicated above, a general Pareto ranking of the different equilibria is less straightforward when comparing equilibria with different K's. When equilibria with different K's exist, the burning-out equilibrium may not be Pareto-dominated by all other equilibria. To see this, consider a burning-out equilibrium with K_1 active contestants and $N-K_1$ passive contestants. Then there can be no equilibrium with less than K_1 contestants. The reason is that any contestant in the burning-out equilibrium that has K_1 contestants will want to participate actively in any hypothetical equilibrium with less than K_1 contestants, given our assumption that a player who is indifferent between participating

[5] See appendix A for a proof of this result, adapted with slight modifications from Amegashie (2004).

in the contest and staying out will participate. It follows that only K_1 contestants can sustain a burning-out equilibrium. The remaining equilibria are those with K_1 or more players. Hence the burning-out equilibrium has the lowest number of players. It is possible that some players are better off in the burning-out equilibrium than in some other equilibrium with more active players and hence less chance of winning the prize. We construct an example in Appendix B1 showing that the burning-out equilibrium can weakly Pareto-dominate another equilibrium with a larger number of active participants. However, there will always be many equilibria, including all of those with the same number of participants as the burning-out equilibrium, that will Pareto-dominate burning out.

If we apply the Coalition-Proof Nash Equilibrium (CPNE) refinement,[6] which allows for joint deviations, the burning-out equilibrium, in which $e^* = B$, is the only surviving pure-strategy equilibrium. To see this, consider an equilibrium in which all the contestants in stage one bid $e^* < B$. Suppose a group of M contestants deviate by bidding marginally more than e^* in stage one[7]. If $M = F \geq 2$, then they are all guaranteed entry to stage two. Their payoff will be $\Pi_i^d = (1/F)V_i - B > 0$. It is easy to show that $\Pi_i^d > \Pi_i^*$ as long as $(1/F)V_i - (B-e^*) > 0$ which is true for all active players. Note that such a deviation by the $M = F$ players is immune to further deviations by sub-coalitions of this deviating group, since each coalition member's probability of success in stage one is already at a maximum (i.e., 1). Hence, there exists a profitable joint deviation from any equilibrium where $e^* < B$.[8] Neither a single nor joint deviation is feasible at $e^* = B$. Thus, $e^* = B$ is the unique pure-strategy CPNE. This leads to the following proposition:

[6] See Bernheim et. al (1987) for a discussion of CPNE.

[7] In the experiment, only integer bids were permitted. Thus, in our experimental context, a bid marginally more than e^* may be interpreted as a bid of $e^* + 1$.

[8] Notice that a deviation by $M > F$ players to bid more than e^* is not immune to further deviations by a sub-coalition of F players. This deviation is also not profitable for a deviation by $M < F$ players because they will be joined by, at least, one player who has a bigger cap in stage 2. In any case, to show that any

Proposition 2: *Consider a two-stage contest where the contest in each stage is an all-pay auction and the contestants have valuations commonly known to be $V_1 \geq V_2 \geq \ldots V_{N-1} \geq V_N$. If $F \geq 2$ contestants are chosen in the first stage to compete in the second stage and all the contestants face a common budget (effort) constraint, B, which can be allocated between the two stages, then there exists a continuum of symmetric pure-strategy Nash equilibria in which each active contestant bids $e^* \in [0, B]$ in stage one and $B - e^*$ in stage two but $e^* = B$ is the only coalition-proof Nash equilibrium.*

We experimentally investigate the following issues. First, how does the value of the prize affect the effort or bid level? Given K active contestants bidding $e = e^*$ with $e^* \in [0, B]$, a risk neutral player i should bid e* in stage one and B-e* in stage two if $(F/(K+1))[(1/F)V_i - (B-e^*)] - e^* \geq 0$ and should bid zero in both stages if $(F/(K+1))[(1/F)V_i - (B-e^*)] - e^* < 0$. Actual players need not be risk-neutral. Nonetheless, for each player there should be a critical valuation level consistent with their level of risk aversion that would induce a bid of e* rather than zero.

Second, do we observe the burning out predicted by the CPNE refinement despite the fact that this unique CPNE is Pareto-dominated by other pure-strategy Nash equilibria? The burning-out equilibrium is especially interesting because of its somewhat counter-intuitive prediction that active contestants expend *all* their energies or resources in stage one, get burned-out, and thus have nothing left to offer in stage two. Under what if any circumstances will rational players allocate all their efforts in stage one when there is another stage ahead? Will there be a process of convergence to the burning-out CPNE over the rounds of a finite repeated game? Will the feedback received between rounds make a difference to the convergence process?

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equilibrium with $e^* < B$ is not CPNE, we only need to show that there exists a coalition size which can deviate profitably.

Third, how does the number of players affect the equilibrium. Earlier experimental studies of coordination games have shown that coordination on Pareto-superior outcomes is harder to sustain with more players. For example, Camerer and Knez (1994) argue that coordination on Pareto-superior outcomes in their minimum-effort coordination game was difficult to sustain for more than two players because forming beliefs about the behavior of other players becomes more complex with larger numbers. While two players only have to worry about each other's beliefs, the introduction of additional players forces everyone to think about the beliefs that each player has about the others in order to predict behavior. In our framework the uniqueness of the burning-out CPNE is independent of the number of players. However, the predictive power of the burning-out CPNE may depend on the number of players, since the higher the number of players, the more likely it is that some coalition of $F \geq 2$ players will deviate from a non burning-out equilibrium.

3. Experimental Design

We ran twelve sessions with participants who were undergraduate students at the University of Guelph. They were recruited in the University Centre. A thirteenth session was run using economics professors at the University of Guelph. Participants received a \$3.00 Canadian show-up fee. The rest of their earnings depended on their performance in the game. Average earnings were \$13.20 Canadian, equal to about \$10.00 US, inclusive of the show-up fee. The sessions lasted about one hour.

Upon entering the room, participants were asked to take a seat and were assigned a player number. Written instructions were distributed.[9] The instructions were then read aloud while participants followed along on their own copies. The experiment lasted for eight periods, each of which was divided into two stages. At the beginning of each

[9] A copy of the instructions is attached as Appendix C.

period, each participant was asked to draw an envelope containing an information slip from a box held by the experimenter. The randomly selected information slip told each participant his/her potential prize value. There were four different prize values. Participants were also told the prize values assigned to the other players. The potential prize values determined the monetary payoff of each participant if he/she won the prize at the end of stage two. The information slip also indicated that each participant had an endowment of 50 tokens, some or all of which could be used to place bids in stages one and two. Each token was worth two cents Canadian. Any tokens that were not used in either stage could be cashed in at the end of the game.

In stage one, participants were given the opportunity to bid any integer amount of tokens between zero and their budgetary caps of 50. After writing their bids in the designated space on their information slips, participants raised their hands and the experimenter collected the slips. Participants understood that once bids were placed, the amount bid would not be returned, regardless of whether or not they won the prize. The two participants with the highest bids were then privately informed that they would move on to stage two. Ties were broken randomly by a draw. Other participants were informed privately that they would not be moving on. Their earnings for the period were 50 tokens minus their stage-one bids.

The two participants who reached stage two were then given the opportunity to bid any amount of tokens from zero up to whatever number of tokens remained after their stage-one bids by writing the desired amount in the designated space on their information slips. The participants who had not reached stage two were asked to write zero in the designated space so that it would not be obvious which two players were still in the game. The person who placed the highest stage-two bid was then privately informed that he/she had won the prize, which was worth the amount that had been indicated on his/her information slip. As in stage one, a random draw was used to determine the final winner if both participants bid the same amount.

At the end of each period, the information slips were returned to each participant, indicating his/her earnings for the period. Earnings were equal to the 50-token endowment plus the payoff from playing the game. Thus, the earnings of the final winner consisted of the 50-token endowment, minus the tokens bid in stages one and two, plus the prize value drawn at the beginning of the period. The earnings of the other participants consisted of the 50-token endowment, minus the bid or bids placed during the period.

At the beginning of a new period, each participant drew a new information slip at random containing a new prize value. Tokens from earlier periods could not be used in the new period. Each participant began each period with exactly 50 tokens.

We ran four treatments, which are summarized in Table 1.

Treatment 1 - Four persons, no announcement of winning bids: In the first treatment, four persons participated in the game. Participants were informed at the end of stage one whether or not they would advance to stage two. However, they were not given any information about the level of the successful bids. Similarly, at the end of stage two, continuing participants were informed whether or not they had won the prize. However, they were not told the level of the winning bid.

Treatment 2 - Four persons, announcement of winning bids: Once again in treatment 2, four persons participated in the game. However, in this treatment, the two stage-one bids of those moving on to stage two were publicly announced after stage one and the stage-two bid of the final winner was publicly announced after stage two.

Treatment 3 - Eight persons, no announcement of winning bids: In treatment 3, eight persons participated in the game. As in treatment 1, successful wins were not announced.

Treatment 4 - Eight persons, announcement of winning bids: In treatment 4, eight persons participated in the game. As in treatment 2, successful bids were announced.

In treatments 1 and 2, the prize values, which were assigned randomly to the four participants, were set at 100, 170, 230 and 300 tokens. Consider a risk-neutral participant who believed the other three participants would also behave as if they were risk-neutral. If such a participant drew the possibility of winning the 100-token prize, proposition 1 indicates that he/she would bid zero in both stages for all non-zero equilibria since $(F/K+1)[(1/F)V_i - (B\text{-}e^*)] - e^* < 0$, for $e^* \in (0, B]$ in this case. If $e^* = 0$, then $(F/K+1)[(1/F)V_i - (B\text{-}e^*)] - e^* = 0$. Given the assumption that a player who is indifferent between participating in the contest and staying out will participate, the risk-neutral player with a valuation of 100 tokens would bid zero in stage one and B = 50 in stage two. However, if such a risk-neutral participant drew the possibility of winning one of the other three prizes, proposition 1 indicates that he/she would bid $0 \leq e^* \leq B$ in equilibrium in stage one and $x^* = B - e^*$ in stage two since $(F/K+1)[(1/F)V_i - (B\text{-}e^*)] - e^* > 0$ in these cases. In appendix B2 we show that the burning-out CPNE in which K = 3 and $e^* = B = 50$ is the worst equilibrium in the sense that it is Pareto-dominated by all of the other Nash equilibria in the four-player case. We also demonstrate that $e^* = 1$ and K = 3 is the Pareto-optimal equilibrium in this case.

In treatments 3 and 4, the prize values were doubled relative to treatments 1 and 2 in order to hold expected earnings constant across the four- and eight-person treatments. The prize values were accordingly set at 200, 340, 460 and 600 tokens. Each of these prize values was randomly assigned to two of the eight participants. Employing the same reasoning as above, risk neutrality implies a bid of zero for those drawing the 200-token prize in stages one and two when $20 < e^* \leq B$. When $16.667 < e^* \leq 20$, the equilibrium calls for one of the players with the 200-token valuation to bid e^* in stage one and $B - e^*$ in stage two, while the other bids zero in both stages. Both of the players with the 200-token valuations will bid e^* in stage one and $B - e^*$ in stage two in any equilibrium in which $0 \leq e^* \leq 16.667$. Those drawing any of the other prize values will place a bid of $0 \leq e^* \leq B$ in equilibrium in stage one and $x^* = B - e^*$ in stage two.

In appendix B3, we show that the burning-out CPNE in which K = 6 and e* = B = 50 is the worst equilibrium in the sense that it is dominated by all other equilibria in the eight-player case. In appendix B4, we demonstrate that there are two Pareto-optimal Nash equilibria that are not themselves Pareto-rankable in the eight-person case: K = 8, e* = 0 and K = 6, e* = 21.

Three sessions of each treatment were run using undergraduate student participants and were analyzed in a two-by-two factorial design framework. One session of treatment 2 was run using economics professors. We hypothesized that both announcements of the winning bids and larger numbers of players might facilitate convergence to the burning-out CPNE. In the case of announcements, we guessed that if everyone learned how much those moving on to stage two had bid in stage one, it might encourage attempts to bid even higher. In the case of eight-person versus four-person competitions, we reasoned that more competitors would increase the likelihood of coalition formation and defection, pushing bids higher.

4. Results

We focus our analysis on the stage-one bids. The Coalition Proof Nash Equilibrium (CPNE) refinement calls for all participants for whom the prize value is sufficiently large to burn out by bidding their entire 50-token endowment in the first stage. Participants for whom the prize value is not large enough to justify bidding withdraw from the contest by bidding zero. Of course, any outcome in which all active participants bid a common amount in stage one is consistent with a Nash equilibrium. The CPNE is Pareto inferior to all of the other pure-strategy Nash equilibria in both the four- and eight-person treatments.

Figures 1 to 5 present representative results from five of the 13 experimental sessions, one from each of the student treatments as well as the one session with economics professors as participants. The bars in the figures indicate the bids placed by the individual participants in the first stage of each period. The bars are ordered by participant number identically in each period. Asterisks indicate bids of zero.

As the figures indicate, the Pareto-optimal equilibria (e* = 1, K= 3 in the four-person treatments; e* = 0, K = 8 or e* = 21, K = 6 in the eight-person treatments) were not achieved in any of the experimental sessions. The economics professors playing the four-person announcement treatment, illustrated in Figure 3, came closest, converging to a bid of about e* = 20, K=3, which was nonetheless still a whopping 19 tokens above the Pareto-optimal equilibrium bid for the four-person case. Eight-person sessions converged to a bid very close to the CPNE, while four-person sessions did not.

The figures also indicate that some participants placed a bid of zero. However, only in the case of the economics professors did the bidding behavior suggest reasonably consistent risk neutrality. In every period with the exception of period 2, the economics professor who drew the lowest prize value of 100 bid zero. In both periods 2 and 8, the professor who had drawn the second-lowest prize value of 170 also placed a zero bid, showing some risk aversion. In the student sessions, some participants who drew low prize values bid positive amounts, while some who drew higher prize values bid zero. Thus, there is evidence of both risk-averse and risk-loving behavior.

If participants had different attitudes toward risk, the prize value required to produce a level of expected earnings high enough to warrant a positive bid at a given e* would differ from person to person. However, one would nonetheless expect the overall probability of a positive bid to be higher, the higher the prize value drawn. In fact, those drawing the lowest prize bid zero 30% of the time, those drawing the second lowest prize bid zero 15% of the time, those drawing the second highest prize bid zero 6% of the time, and those drawing the highest prize bid zero just 4% of the time. These observations indeed suggest a positive relationship between the probability of a positive bid and the prize value drawn. To examine this issue more formally, we employ a three-level hierarchical logit model and estimate it using the data from the twelve student sessions.[10] The binary dependent variable is equal to one if a positive bid is placed and zero if a zero bid is placed. We hypothesize that the probability of a positive bid will be positively related to the prize value drawn, while controlling for the period of play, possible treatment effects, and random effects related to the actions of individual participants over time and to particular sessions across individuals.

Level 1 is a logit model, defined for each individual participant 'i' in every session 's'over the eight periods of play 't':

$$\log[P_{tis}/(1-P_{tis})] = \pi_{0is} + \pi_{1is}(PER_t) + \pi_{2is}(NV_{tis}), \quad (1)$$

where P_{tis} is the probability of a positive bid in period 't' for individual 'i' in session 's', PER_t is the period number minus eight in period 't', NV_{tis} is the normalized prize value in period 't' for individual 'i' in session 's', and the π's are individual–level coefficients. Subtracting eight from the period number allows the effect of treatment variables that may interact with the period of play to be tested during the last period of the game when convergence to an equilibrium is most likely to have occurred. The prize value is normalized to correspond with the expected earnings it represents by dividing prize values by the number of participants in the session, either four or eight.

The level-2 model takes account of possible individual-specific random effects on the level-1 coefficients:

$$\pi_{0is} = \beta_{00s} + \eta_{0is}$$
$$\pi_{1is} = \beta_{10s} + \eta_{1is} \quad (2)$$
$$\pi_{2is} = \beta_{20s} + \eta_{2is},$$

where the β's are session-level coefficients and the η's represent individual-specific random effects.

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[10] Raudenbush and Bryk (2002), and Snijders and Bosker (1999) both provide excellent discussions of hierarchical linear and logit models (also called mixed models or random-effects models) incorporating both fixed and random effects.

The level-3 model takes account of possible session-specific treatment and random effects on the level-2 coefficients:

$$\beta_{00s} = \gamma_{000} + \gamma_{001}(NA_s) + \gamma_{002}(8P_s) + \mu_{00s}$$
$$\beta_{10s} = \gamma_{100} + \gamma_{101}(NA_s) + \gamma_{102}(8P_s) + \mu_{10s} \qquad (3)$$
$$\beta_{20s} = \gamma_{200} + \gamma_{201}(NA_s) + \gamma_{202}(8P_s) + \mu_{20s},$$

where the γ's are level-3 coefficients and the μ's represent possible session-specific random effects. The treatment dummy variable NA_s is equal to 0 for sessions in which the winning bids are announced and 1 if they are not announced. The treatment dummy variable $8P_s$ is equal to 0 for the four-person treatments and 1 for the eight-person treatments. Combining the three sets of equations, we estimate:

$$\log[P_{tis}/(1-P_{tis})] = \gamma_{000} + \gamma_{001}(NA_s) + \gamma_{002}(8P_s) + \gamma_{100}(PER_t) + \gamma_{101}(PER_t \times NA_s) + \gamma_{102}(PER_t \times 8P_s) + \gamma_{200}(NV_{tis}) + \gamma_{201}(NV_{tis} \times NA_s) + \gamma_{202}(NV_{tis} \times 8P_s) + \eta_{0is} + \eta_{1is}(PER_t) + \eta_{2is}(NV_{tis}) + \mu_{00s} + \mu_{10s}(PER_t) + \mu_{20s}(NV_{tis}). \qquad (4)$$

Table 2 reports the results. The prize value is positively related to the probability of a positive bid as hypothesized, rejecting the null hypothesis with a two-tailed p-value of 0.076, which corresponds to a one-tailed p-value of 0.038. We can thus reject the null in the direction of the hypothesized positive relationship. Neither the period variable nor either of the treatment variables or their interactions is significantly related to the probability of a positive bid. Thus, the positive relationship between prize value and the probability of a positive bid appears to be invariant to both the period in which the prize is drawn and the four treatments. If we drop all of the insignificant variables, maintaining only NV and the individual-specific and session-specific random effects, the two-tailed p-value on NV falls to 0.001, strongly supporting the hypothesized relationship.[11]

[11] If the data from the professor treatment is added to the estimation of equation 4, the two-tailed p-value becomes 0.019 and all the other variables remain insignificant. When the insignificant variables are dropped the two-tailed p-value becomes 0.000.

We are primarily interested in how close participants came to the burning-out CPNE in the various treatments. The CPNE is consistent with some participants bidding zero in stage one if they determine that the expected gains from bidding are not worth the cost. Of course, if everyone bid zero in stage one, they would be playing a different Nash equilibrium. Nothing close to this ever happened in any period of any session. In the CPNE, while some participants may bid zero, many others burn out by bidding their entire 50-token endowment in stage one of the game. Since a bid of either zero or 50 is consistent with the burning-out CPNE, we define EQDIST = Min(50-Bid, Bid-0) as the dependent variable in a three-level hierarchical linear model.

The level-1 model is defined over time 't' for each individual participant 'i' in each session 's' to account for convergence over the course of the game as:

$$EQDIST_{tis} = \pi_{0is} + \pi_{1is}(PER_t) + \varepsilon_{tis}, \tag{5}$$

where ε_{tis} is an observation-specific disturbance term. The level-2 model takes into account the possibility of individual-level random effects:

$$\pi_{0is} = \beta_{00s} + \eta_{0is}$$

$$\pi_{1is} = \beta_{10s} + \eta_{1is}, \tag{6}$$

The level-3 model introduces the session-specific treatment effects, which are now our primary focus of interest, as well as session-specific random effects:

$$\beta_{00s} = \gamma_{000} + \gamma_{001}(NA_s) + \gamma_{002}(8P_s) + \mu_{00s}$$

$$\beta_{10s} = \gamma_{100} + \gamma_{101}(NA_s) + \gamma_{102}(8P_s) + \mu_{10s}. \tag{7}$$

Initially, we included interaction effects between NA, the no-announcement dummy, and 8P, the eight-person dummy at level 3. These effects were very far from significance and therefore dropped from the model. Combining equations (5), (6), and (7), we estimate:

$$EQDIST_{tis} = \gamma_{000} + \gamma_{001}(NA_s) + \gamma_{002}(8P_s) + \gamma_{100}(PER_t) + \gamma_{101}(PER_t \times NA_s) +$$

$$\gamma_{102}(PER_t \times 8P_s) + \eta_{0is} + \eta_{1is}(PER_t) + \mu_{00s} + \mu_{10s}(PER_t) + \varepsilon_{tis}. \tag{8}$$

Table 3 outlines the results. It is important to remember that there are eight periods in the game and that PER is defined as the period number minus eight. Thus, the estimated intercept and coefficients on both NA and 8P are calculated with respect to the last period. The intercept is equal to about 14.5 and highly significant ($p = 0.000$), indicating that in the last period of the four-person sessions with announcements, bids were about 14.5 tokens away from the burning-out CPNE. NA is insignificant, implying that whether or not there was an announcement made no difference to the distance from the burning-out CPNE in the last period. The insignificance of the interaction between PER and NA indicates that whether or not there was an announcement did not affect the speed of convergence to the CPNE either.

In contrast, 8P is negative and highly significant ($p = 0.000$), indicating that more players push participants significantly closer to the CPNE. The sum of $\gamma_{000} + \gamma_{002}$, which represents an estimate of the distance from the CPNE in the last period of the eight-person sessions, is insignificant, indicating that bids were very close to the burning-out CPNE in the eight-person case.

The coefficient on PER is not significant, implying that in the four-person games, there is no significant movement towards or away from the CPNE. However, the interaction between PER and 8P is negative and highly significant ($p=0.009$), indicating that in the eight-person sessions the period-to-period movement towards the CPNE was significantly higher than in the four-person case. The sum of $\gamma_{100} + \gamma_{102}$, which represents that movement, is significant ($p = 0.001$) and equal to about –1.41, indicating that from period to period, bids moved about 1.41 tokens closer to the burning-out CPNE in the eight-person case.

How did participants behave in stage two? Table 4 summarizes stage-two bids in the student sessions. In all of the pure-strategy Nash equilibria, both participants who reach stage two after bidding identical amounts as required by all the pure-strategy equilibria in stage one, should bid all of their remaining endowments in the second stage. In 16 out of the 17 cases in which the announcement indicated that the two players entering stage two were tied in stage one, both players did in fact bid all of their remaining endowments in stage two as predicted. The professors did so in four out of four tied cases.

There were cases, however, in which the announcement revealed that the two participants entering stage two bid different amounts in stage one, despite the fact that such behavior is not part of a pure-strategy Nash equilibrium. If the highest stage-one bid was just one token higher than the second-highest bid, the player who had bid less in stage one could win for sure by using all of his/her remaining resources in stage two. Furthermore, if the highest stage-one bid was more than one token higher than the second-highest bid, the player who had bid less in stage one could win for sure even without using all of his/her remaining resources in stage two. In either case, the player who had bid more in stage one might thus give up and bid zero in stage two. If the player who had bid less in stage one knew this might happen, he/she might try to get away with bidding a low amount. On the other hand, if the player who had bid more anticipated this, he/she would not give up after all. Since in these cases the participants in the stage-two subgame have unequal caps, there is no pure-strategy equilibrium for the subgame, but only an equilibrium in mixed strategies (Che and Gale, 1997). Out of the 15 instances in which the announced winning stage-one bids differed by one token, both players bid the rest of their endowments nine out of 15 times, one player bid the rest of his/her endowment five out of 15 times, and neither player bid the rest of his/her endowment one out of 15 times. In the professor session, one player bid the rest of his/her endowment three out of three times in this case. Out of the 16 instances in which the announced winning stage-one bids differed by more than one token, both players bid the rest of their endowments five out of 16 times, one player bid the rest of his/her endowment eight out of 16 times, and neither player bid the rest of his/her endowment three out of 16 times. In the one professor case, neither player bid the rest of her/her endowment.

In the treatments where the successful bids were not announced, a participant moving on to stage two would only know his own stage-one bid and whether there had been zero, one, or two random draws. Since such draws were used only in the event of a tie for one or both of the two winning positions, the following inferences could be drawn. If there were two draws, three or more players must have been tied, requiring two draws to choose the two players who would advance to stage two. Thus, in this case, the two advancing players could determine that they must have bid identical amounts in stage one and thus have identical caps in stage two. This is of course consistent with all of the pure-strategy equilibria of the game, each of which requires the advancing players to bid the rest of their endowments in stage two. This actually occurred in six out of the seven no-announcement cases in which there were two draws. If there was only one draw, the two players advancing to stage two could determine that they had bid different amounts in stage one. The one with the highest bid had advanced to stage two without the need for a draw, while the draw was used to break a tie for the second advancing position. However, neither player would have any way of knowing which one of them had placed the higher bid. In this instance, in five out of six cases, both players used up all of their remaining resources. Having no draw was consistent with either a tie or no tie between the bids of the two advancing players. Thus, no inference could be drawn about the relative size of the caps in stage two. Out of the 34 cases of this type, both players bid all of their remaining resources 23 times, one player bid all of his/her resources nine times and neither player bid all of his/her resources twice. In general, when the two advancing participants knew that they faced identical caps in stage two, they selected the strategy associated with a pure-strategy equilibrium 22 out of 24 times. In other cases, more mixed strategies were selected.

5. Conclusion

We have examined equilibrium selection in a two-stage sequential elimination game with a continuum of equilibria in the first stage. Many of these equilibria can be ranked according to the Pareto criterion. A set of such rankable equilibria resembles the continuum of Pareto-rankable equilibria in the weak-link coordination game. In that game, groups of two and to a lesser extent three are better able than larger groups to maintain a Pareto-dominant equilibrium over a series of periods in which the game is repeated in a partner protocol. Our game differs from such weak-link games in that the main point is not to cooperate, but to win the prize. In addition, in our game but not in weak-link games, the Coalition-Proof Nash Equilibrium refinement rules out all equilibria but the one in which everyone who chooses to bid burns out by bidding all of their resources in stage one. In all of our treatments, this is the least efficient pure-strategy equilibrium in the sense that it is Pareto-dominated by all of the other equilibria.

Our first finding is that some players withdraw from the game by bidding zero, while others bid substantial amounts. This is reminiscent of a laboratory result that emerged unexpectedly in Muller and Schotter's (2003) recent experimental examination of a model developed by Moldovanu and Sela (2001) in which players had different costs of effort. Although Moldovanu and Sela's theoretical model predicted that the amount of effort exerted should be a continuous inverse function of cost, the laboratory results indicated a discontinuity: higher-cost players generally gave up, expending little effort, while lower-cost players generally tried hard, exerting a lot of effort. In the Amegashie (2004) model, the cost of effort is identical for all players, but prize valuations can differ.[12] The specific version of the Amegashie model adopted in this paper predicts that players with lower valuations will withdraw from the contest by bidding zero, while players with higher valuations will compete for the prize by bidding substantial amounts in the first stage of a two-stage game, a prediction that is corroborated by the data.

When augmented by the CPNE refinement, the Amegashie model goes further, predicting that active players will use up all of their resources to place the highest

[12] However, as shown by Baye et. al (1996) and Clark and Riis (1998), a contest where the players have different valuations but a common cost of effort is analytically equivalent to a contest where the players have common valuations but different costs of effort.

possible stage-one bids. Our second finding is that while CPNE predicts quite well for eight-person groups, it does not predict well for four-person groups. However, even in the case of the smaller groups, the Pareto-optimal equilibrium has no predictive power at all. Rather small groups seem to coordinate on an equilibrium in between that predicted by the Pareto criterion and that predicted by CPNE. The likelihood of stage-one burnout amongst the active bidders seems to depend on the number of people competing for entry into the second stage where the possibility exists of winning the prize. When eight people compete for two chances to win one prize, we frequently observe burnout or near burnout in the first stage of the two-stage game. Thus, the stage-one burnout predicted by the Coalition-Proof Nash Equilibrium refinement arises consistently only when there is sufficient competition: early burnout is indeed competitive burnout in the laboratory.

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Table 1

Summary of Treatments

	4-person group	8-person group
Without announcement (eight periods)	3 sessions with students	3 sessions with students
With announcement (eight periods)	3 sessions with students 1 session with economics professors (excluded from statistical analysis)	3 sessions with students

Table 2
Positive versus Zero Bid Results

Repeated Measures Three-level Hierarchical Logit Model with Random Effects on Intercept, Period and Normalized Valuation, using Full PQL (Penalized Quasi-Likelihood) Estimation.

Equation estimated: $\log[P_{tis}/(1\text{-}P_{tis})] = \gamma_{000} + \gamma_{001}(NA_s) + \gamma_{002}(8P_s) + \gamma_{100}(PER_t) + \gamma_{101}(PER_t \times NA_s) + \gamma_{102}(PER_t \times 8P_s) + \gamma_{200}(NV_{tis}) + \gamma_{201}(NV_{tis} \times NA_s) + \gamma_{202}(NV_{tis} \times 8P_s) + \eta_{0is} + \eta_{1is}(PER_t) + \eta_{2is}(NV_{tis}) + \mu_{00s} + \mu_{10s}(PER_t) + \mu_{20s}(NV_{tis})$

Independent Variables	*Estimate*	*t value*	*Pr > |t|*
Intercept	0.004332	0.004	0.997
No Announcement (NA)	-0.635910	-0.516	0.618
8 Participants (8P)	-0.980644	-0.762	0.465
Adjusted Period (PER)	0.008298	0.059	0.954
NA × PER	0.045866	0.331	0.748
8P × PER	-0.191543	-1.272	0.236
Normalized Valuation (NV)	0.052823	1.999	0.076
NA × NV	0.037350	1.262	0.239
8P × NV	-0.009759	-0.320	0.756

Table 3
Distance from Burning-out CPNE Results

Repeated Measures Three-level Hierarchical Linear Model with Random Effect on Intercept and Adjusted Period using Full Maximum Likelihood.

Equation estimated: $EQDIST_{tis} = \gamma_{000} + \gamma_{001}(NA_s) + \gamma_{002}(8P_s) + \gamma_{100}(PER_t) + \gamma_{101}(PER_t \times NA_s) + \gamma_{102}(PER_t \times 8P_s) + \eta_{0is} + \eta_{1is}(PER_t) + \mu_{00s} + \mu_{10s}(PER_t) + \varepsilon_{tis}$.

Independent Variables	*Estimate*	*t value*	*Pr > |t|*
Intercept [γ_{000}]	14.544341	6.053	0.000
No Announcement (NA) [γ_{001}]	0.425206	0.155	0.881
8 Participants (8P) [γ_{002}]	-15.001736	-5.464	0.000
Adjusted Period (PER) [γ_{100}]	-0.159603	-0.474	0.646
PER×NA [γ_{101}]	-0.227421	-0.615	0.553
PER×8P [γ_{102}]	-1.254216	-3.347	0.009
Other Hypothesis Tests			
$\gamma_{000} + \gamma_{002}$	-0.457395	-0.195	0.850
$\gamma_{100} + \gamma_{102}$	-1.413819	-4.577	0.001

Table 4
Summary of Stage-Two Behavior in Student Sessions

Announce-ment	*Stage One Winning Bids*	*Both spend rest of Endowment*	*One spends rest of Endowment*	*None spend rest of Endowment*	*Total*
Yes	Tie	16	1	0	17
Yes	Difference = 1	9	5	1	15
Yes	Difference >1	5	8	3	16
No	Two chosen randomly	6	1	1	8
No	One chosen randomly	5	1	0	6
No	No random draw	23	9	2	34

Figure 1
Four Persons Without Announcement

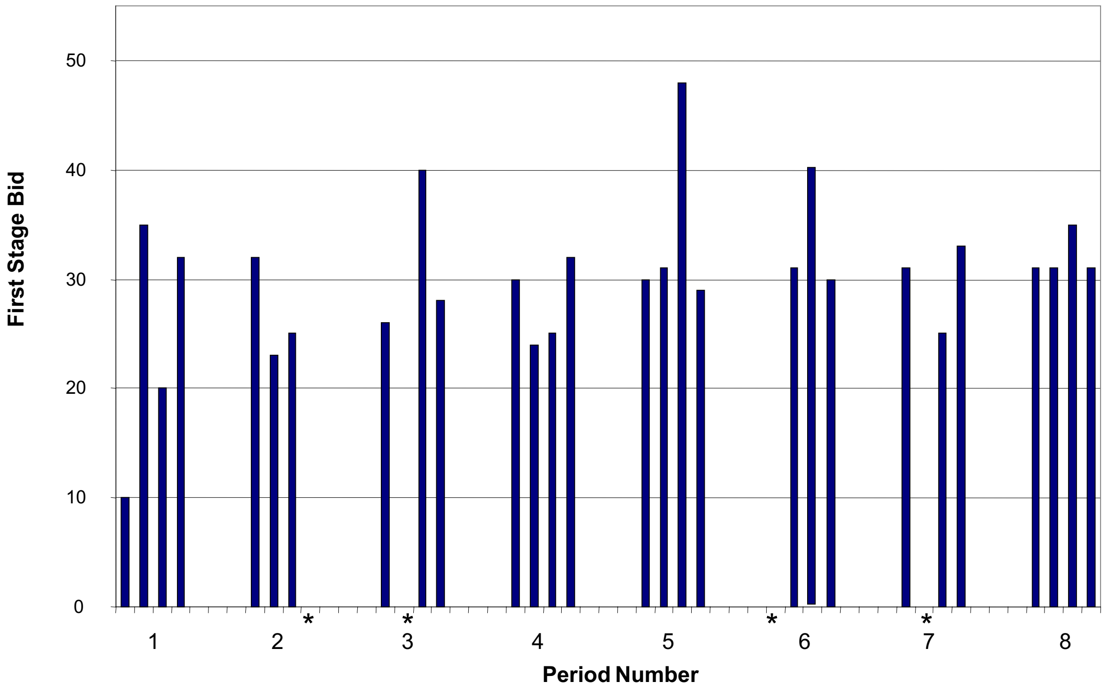


Figure 2
Four Persons With Announcement

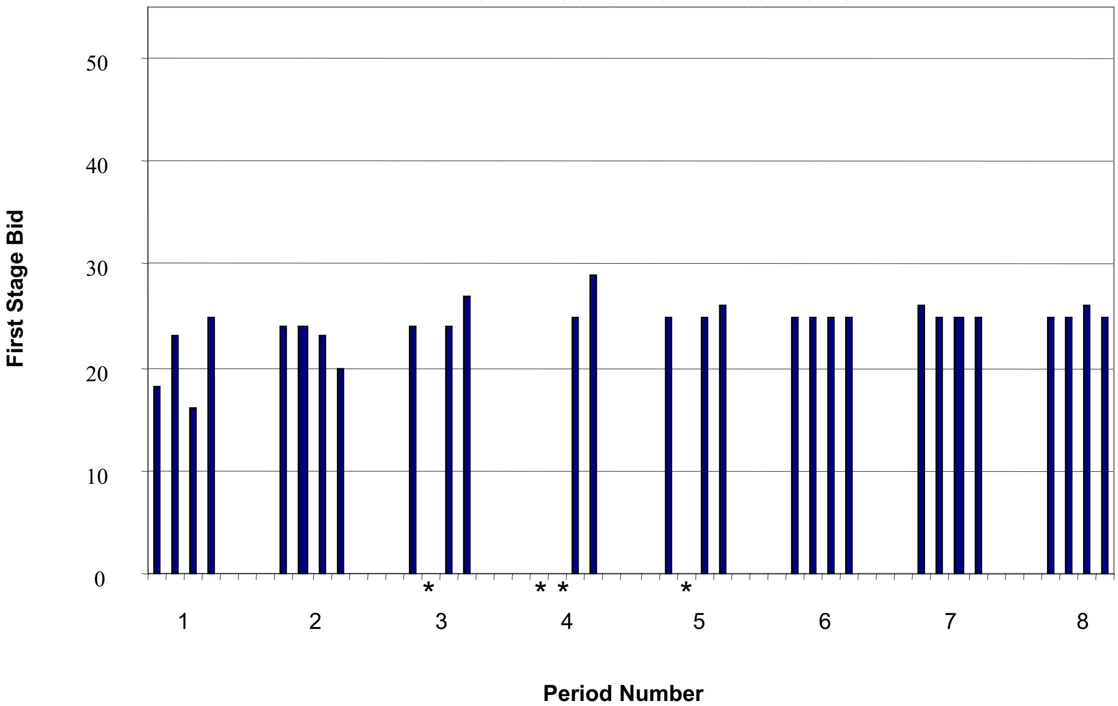


Figure 3
Four Professors With Announcement

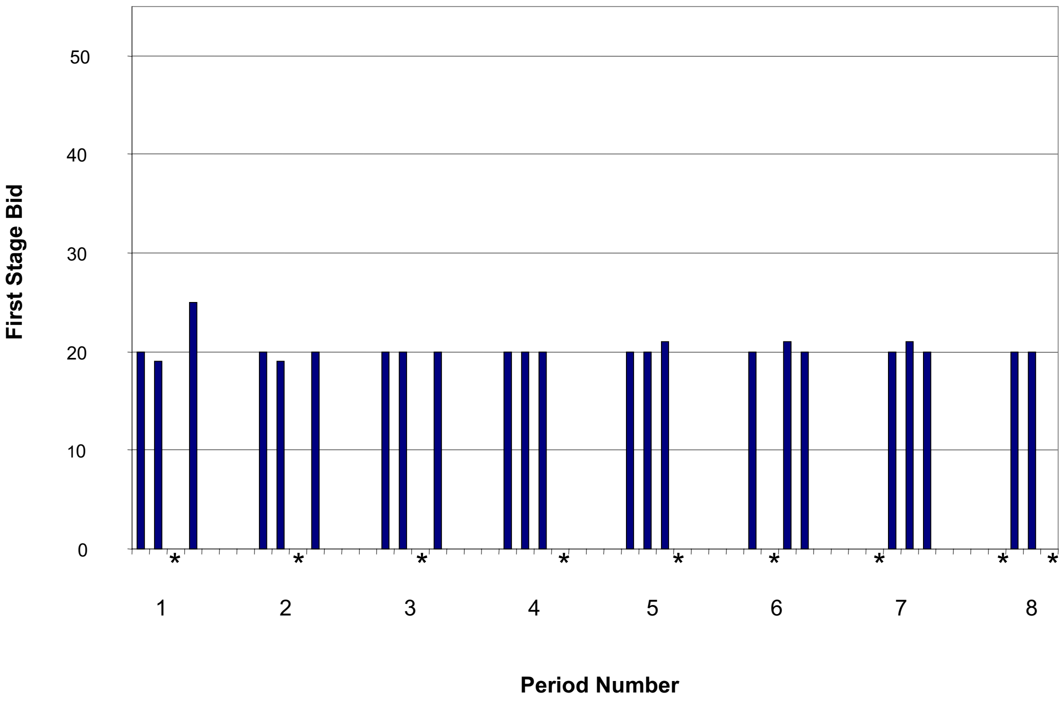


Figure 4
Eight Persons Without Announcement

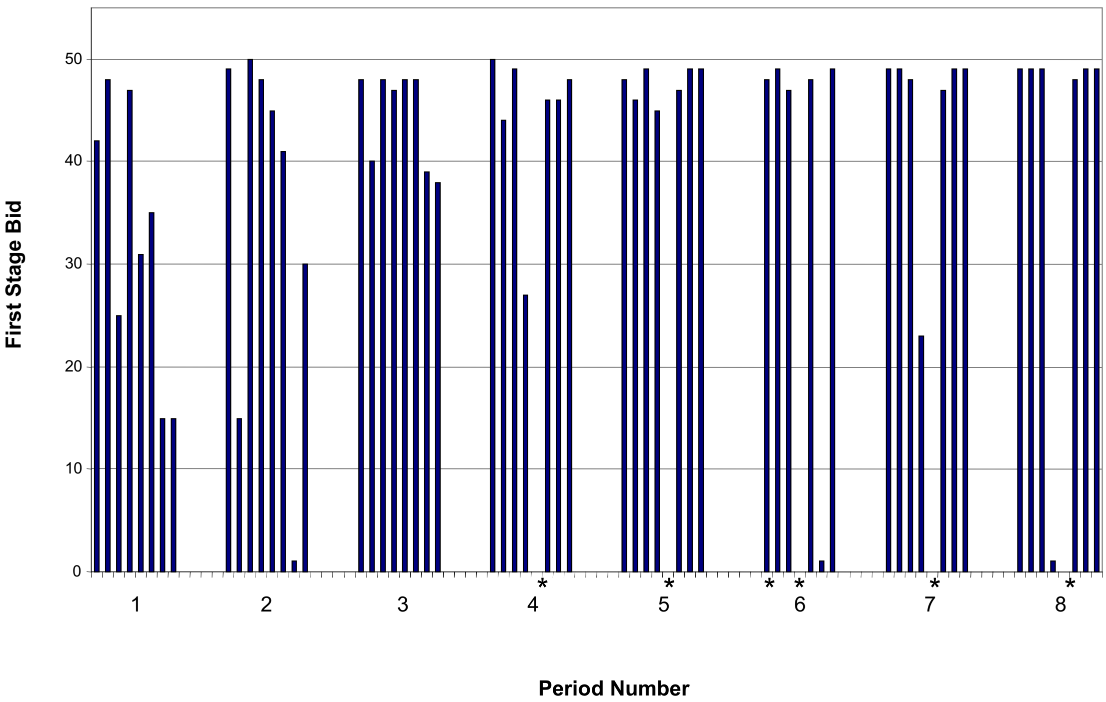
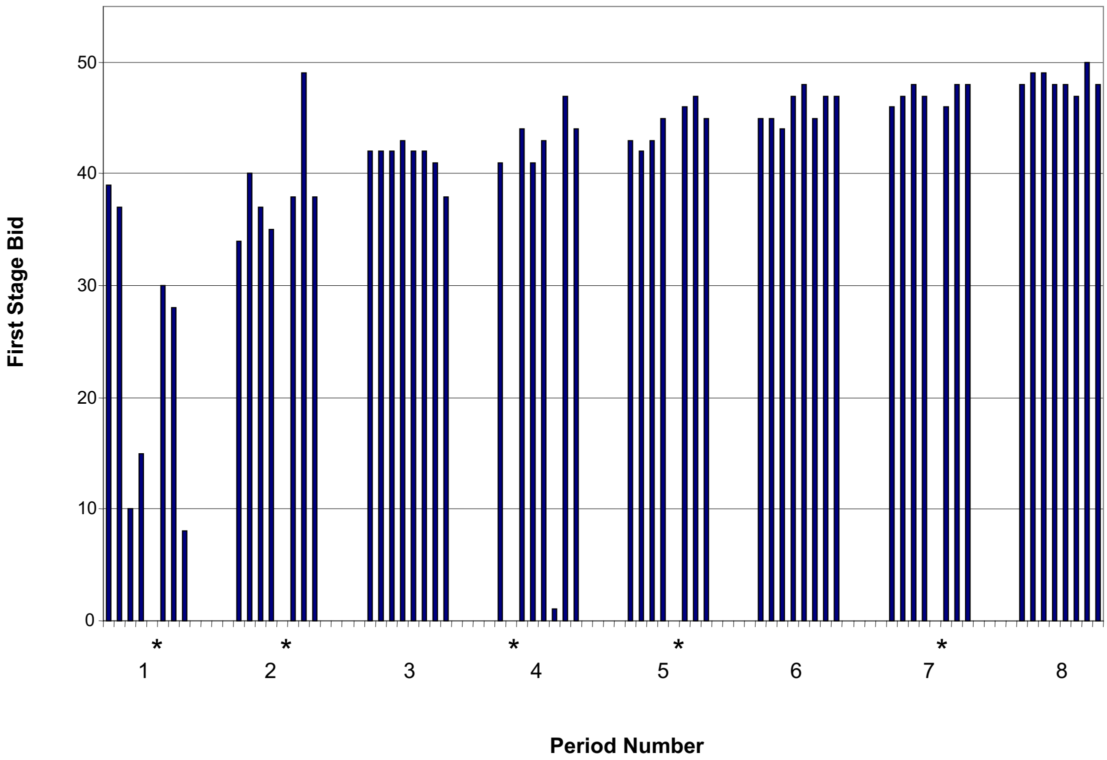


Figure 5
Eight Persons With Announcement



Appendix A: Proof that in an all-pay auction with two players who have different caps, the player with the smaller cap gets a zero expected surplus if the cap is sufficiently small

Consider stage two of the game (an all-pay auction) where there are two active players with different caps. For the sake of argument, suppose the players are 1 and 2, with valuations V_1 and V_2 and caps B_1 and B_2, where $B_2 < B_1 \leq B$ and $V_1 > V_2 > B_2$. Note that $V_2 > B_2$ since $(1/F)V_i - B > 0$, for all active players. We follow the proof in Che and Gale (1997), although in their model the players have different caps but the same valuations.

If $B_2 = 0$, then the only equilibrium is in pure strategies in which player 2 bids zero and player 1 bids a small but positive amount.

Now suppose $B_2 > 0$. First, there is no equilibrium in pure strategies. The proof is straightforward, so it is omitted. There is an equilibrium in mixed strategies (Che and Gale, 1997). Second, no player has a mass point at any bid $x \in (0, B_2)$ in stage two. Without loss of generality, suppose the contrary that player 1 has a mass point at $x \in (0, B_2)$, say at x_1. Then the probability that player 2 wins rises discontinuously as a function of his bid at x_1. Hence there is some $\varepsilon > 0$ such that player 2 will bid on the interval $[x_1 - \varepsilon, x_1]$ with zero probability. But then player 1 is better off bidding $x_1 - \varepsilon$ instead of x_1 since his probability of winning is the same. This contradicts the hypothesis that putting a mass point at $x \in (0, B_2)$ is an equilibrium strategy. Third, only one player can receive a strictly positive expected surplus. Suppose instead that both players receive positive expected surpluses. Then both players must have the same infimum bid. If not, the player with the strictly lower infimum would lose with probability one when he bids below the other player's infimum bid, so his expected surplus cannot be strictly positive, since every bid in the support of his equilibrium mixed strategy must yield the same expected surplus. If both players have the same infimum bid, $\underline{x} > 0$, then in order for each of them to win with positive probability when bidding $\underline{x}$, they must both have mass points at $\underline{x}$. But this is not possible since no player puts a positive mass at $x \in (0, B_2)$ and B_2 cannot be either player's lowest bid since there is no pure-strategy equilibrium. Hence, only one player can have a strictly positive expected surplus. Finally, the player with the bigger cap (i.e., player 1) gets a positive expected surplus and therefore player 2's expected surplus is zero. To see this, note that player 1 can guarantee himself a positive expected surplus by submitting a bid above B_2. Since there exists a bid that guarantees player 1 a positive expected surplus, this player cannot make a zero expected surplus in a mixed-strategy equilibrium. Hence player 2 (i.e., the player with the smaller cap) gets a zero expected surplus. **QED.**

Appendix B

B1: An example to show that a burning-out equilibrium can weakly Pareto-dominate a non-burning-out equilibrium with higher K.

Consider N players with valuations, $V_1 = 600$, $V_2 = 600$, $V_3 = 460$, $V_4 = 460$, $V_5 = 340$, $V_6 = 340$, $V_i = 100$ for i = 7, 8, …, N. The cap is B = 50 and F = 2.

Then K= 6 and e* = B is an equilibrium because $(1/6)V_i – B > 0$ for i = 1, 2,, …, 6. If players 7 to N bid B = 50 they will each get a negative payoff. However, K = N, e* = 0 is also an equilibrium because $(2/N)(V_i/2 – 50) \geq 0$ for i = 1, 2, …, N. The players indexed 7 to N are neither better off nor worse off in this equilibrium than in the six-player burning-out equilibrium, since expected payoffs equal zero in both cases. For the burning-out equilibrium to Pareto-dominate K = N, e* = 0, we require $(2/N)(V_i/2 – 50) \leq (1/6)V_i – 50$, or equivalently, if $N \geq 6(V_i – 100)/(V_i – 300)$ for i = 1, 2,, …, 6 with strict inequality for at least one i. Given the players' valuations above, this is true if $N \geq 36$. **QED**.

B2: (i) Proof that K = 3 and e* = B = 50 is dominated by all other pure-strategy equililbria in the four-player case, and (ii) Proof that e* = 1 and K = 3 dominates all other pure-strategy equilibra in the four-player case

There are four players with valuations $V_1 = 300$, $V_2 = 230$, $V_3 = 170$, and $V_4 = 100$. The cap is B = 50 and F = 2.

Part (i): First, K = 3 and e* = B = 50 is an equilibrium because $(1/3)V_i - B > 0$ for i = 1, 2, 3. If player 4 bids B = 50 given K = 3, his payoff is negative because $100/4 - 50 < 0$. Given K = 3, we know from the discussion in the text that all other equilibria for which K = 3 (i.e., $0 < e^* < 50$) Pareto-dominate K = 3, e* = B = 50.

Note that there is no equilibrium with K < 3 players since any player who participated in the three-player burning-out equilibrium would also participate in any hypothetical equilibrium having less than three players. Hence we only need to compare the equilibria with K = 4 to the three-player burning-out equilibrium.

For K = 4 to be an equilibrium, we require that $(2/4)[(1/2)V_i - (50\text{-}e^*)] - e^* \geq 0$ for i = 1, 2, 3, 4. This holds so long as $e^* \leq (1/2)V_i - 50$ or, substituting the lowest valuation for V_i, $e^* \leq 0$. Hence, the only equilibrium is e* = 0 given K = 4. Now the equilibrium in which K = 4 and e* = 0 Pareto-dominates the three-player burning-out equilibrium if $(2/4)[(1/2)V_i - 50] \geq (1/3)V_i - 50$, with strict inequality for at least one i, i = 1, 2, 3. This holds if $V_i \leq 300$. Hence players 2 and 3 are better off in the equilibrium with K = 4 and e* = 0 and players 1 and 4 are no worse off. Hence K = 3, e* = B is the worst equilibrium. **QED**.

Part (ii): First, note that player 4 gets a zero expected payoff whether K = 3 or 4. Given K = 3, the equilibrium which gives the highest payoff is the equilibrium with the lowest effort, e*, in stage one. Since we only allow integer bids in our experiments, the lowest such bid in stage one consistent with K=3 is e* = 1. Hence to show that K = 3, e* = 1 is the best equilibrium, we need to compare this equilibrium to K = 4, e* = 0. To do this, we need to show that

$(2/3)[(1/2)V_i - (50 - 1)] - 1 \geq (2/4)[(1/2)V_i - 50]$, for i = 1, 2, 3, with strict inequality for, at least, one i. This holds if $V_i \geq 104$, with strict inequality for at least one i. This is true, given $V_1 = 300$, $V_2 = 230$, and $V_3 = 170$. **QED**.

B3: Proof that K = 6 and e* = B = 50 is Pareto-dominated by all other pure-strategy equilibria in the eight-player case.

There are eight players with valuations, $V_1 = 600$, $V_2 = 600$, $V_3 = 460$, $V_4 = 460$, $V_5 = 340$, $V_6 = 340$, $V_7 = 200$, and $V_8 = 200$. The cap is B = 50 and F = 2.

First, K= 6 and e* = B, is an equilibrium because $(1/6)V_i - B > 0$ for i = 1, 2,, …, 6. If either player 7 or player 8 bids B = 50, he/she will each get a negative expected payoff. Given K = 6, we know from the text that all other equilibria (i.e., $0 \leq e^* < 50$), if they exist, Pareto-dominate K = 6, e* = B = 50.

Note that there is no equilibrium with K < 6 players since any player who participated in the six-player burning-out equilibrium would also participate in any hypothetical equilibrium having less than six players. Hence we only need to compare the equilibria with K = 7 and K = 8 to the six-player burning-out equilibrium.

We now need to show that in any equilibrium with K = 7 or K = 8, players 7 and 8 get an expected payoff greater than or equal to zero and players 1 to 6 get expected payoffs greater than or equal to $(1/6)V_i - B$ with strict inequality for at least one i. Since players 7 and 8 get a zero payoff in the six-player equilibrium and cannot be forced to choose a negative expected payoff in any other possible equilibrium, we focus primarily on players 1 to 6 unless otherwise indicated.

Any equilibrium with K = 7, 8 Pareto dominates the six-player burning-out equilibrium if $(F/K)[(1/F)V_i - (B\text{-}e^*)] - e^* \geq (1/6)V_i - B$, with strict inequality for at least one i, i = 1, 2, 3,…, 6. Solving for e*, gives

$$e^* \leq B + V_i\left(\frac{1 - K/6}{K - F}\right), \qquad (1)$$

$i = 1, 2, \ldots, 5, 6$.

If (1) holds for $V_1 = V_2 = 600$, then it holds for lower V_i with strict inequality. Substituting $K = 8$, $V_i = 600$, $F = 2$, and $B = 50$ into (1) gives $e^* \leq 16.667$ as the required condition. Now for $K = 8$ to be an equilibrium, we require that $(F/K)[(1/F)V_i - (B\text{-}e^*)] - e^* \geq 0$ for $i = 7$ and 8. This will also be true as long as $e^* \leq 16.667$. It follows that when an equilibrium exists for $K = 8$, it satisfies the inequality in (1) and thus Pareto-dominates the six-player burning-out equilibrium.

We now compare equilibria with $K = 7$ to the six-player burning-out equilibrium. Substituting $K = 7$, $V_i = 600$, $F = 2$, and $B = 50$ into (1) gives $e^* \leq 30$ as the required condition for Pareto dominance. For $K = 7$ to be an equilibrium, we require that $(F/K)[(1/F)V_i - (B\text{-}e^*)] - e^* \geq 0$ for either $i = 7$ or 8 and $[F/(K+1)][(1/F)V_i - (B\text{-}e^*)] - e^* < 0$ for either $i = 7$ or 8. Substituting into these two expressions yields $16.667 < e^* \leq 20$. It follows that if players 1 to 7 bid $e^* \in (16.667, 20]$, then player 8 will stay out of the contest.[13] Hence equilibria with $K = 7$ exist for $e^* \in (16.667, 20]$.[14] Since $e^* \in (16.667, 20]$ satisfies $e^* \leq 30$, it follows that when an equilibrium exists for $K = 7$, it satisfies the inequality in (1) and thus Pareto-dominates the six-player burning-out equilibrium.

We have therefore proven that any pure-strategy equilibrium Pareto-dominates the six-player burning-out equilibrium. **QED**.

B4: Proof that ($K = 8$, $e^* = 0$) and ($K = 6$, $e^* = 21$) are the only pure-strategy equilibria that are not Pareto-dominated

Recall that equilibria with $K = 7$ exist for $e^* \in (16.667, 20]$. Hence the best equilibrium when $K = 7$ has $e^* \approx 16.667$. However, since in our experiments, we allow

[13] By symmetry, the roles of players 7 and 8 are interchangeable.

[14] For $K = 7$, there is no equilibrium in which the players with valuations $V_i = 200$, are active players but one of the other players is not. As shown above, for player 7 or 8 to be an active player for $K = 7$, we require that $e^* \leq 20$. Then for one of the other players to be non-active, we require that $(F/(K+1))[(1/F)V_i - (B\text{-}e^*)] - e^* < 0$ or $(2/8)[(V_i/2 - (50 - e^*)] - e^* < 0$. This gives $e^* > 40$ for $V_i = 340$. Since $e^* \leq 20$ and $e^* > 40$ cannot simultaneously hold, it follows that there is no equilibrium with $K = 7$ where the players with $V_i = 340$ are non-active and the players with $V_i = 200$ are active. There is also no such equilibrium with $K < 7$. A similar argument holds for the players with $V_i = 460, 600$.

only integer bids, the best equilibrium for K = 7 is at e* = 17. Call this equilibrium (K =7, e* = 17).

When K = 8, the best equilibrium has e* = 0. Call this (K = 8, e* = 0). To find the best equilibrium for K = 6, we need to find the lowest value of e* for which K = 6 is an equilibrium. When K = 6, then players 7 and 8 will stay out of the contest if $(2/7)[200/2 - (50 - e^*)] - e^* < 0$. This gives e* > 20. Hence the best equilibrium for K = 6 is at e* = 21, given that we allow only integer bids in our experiments. Call this (K = 6, e* = 21).

First, let's compare (K = 8, e* = 0) and (K = 7, e* = 17). The payoff of a player with valuation, V_i, when K = 8 and e* = 0, is $\prod_{8i} = (1/8)V_i - 12.5$. The payoff of a player with valuation, V_i, when K = 7 and e* = 17, is $\prod_{7i} = (1/7)(V_i - 185)$. Therefore, $\prod_{8i} - \prod_{7i} = 13.92857143 - 0.0178571429V_i > 0$ for $V_i \in [200, 600]$. Hence, (K = 8, e* = 0) Pareto-dominates (K = 7, e* = 17).

We now compare (K = 8, e* = 0) and (K = 6, e* = 21). The payoff of a player with valuation, V_i, when K = 6 and e* = 21, is $\prod_{6i} = (1/6)V_i - 92/3$. Therefore, $\prod_{8i} - \prod_{6i} = 18.66667 - 0.04167V_i$. Now $\prod_{8i} - \prod_{6i} > 0$ for $V_i = 340$ but $\prod_{8i} - \prod_{6i} < 0$ for $V_i = 460$ and 600. Hence, (K = 8, e* = 0) and (K = 6, e* = 21) cannot be ranked according to the Pareto criterion. **QED**.

Appendix C: Experimental Instructions

This is an experiment in the economics of decision making. The Social Sciences and Humanities Research Council of Canada has provided funds for this research. The instructions are simple and if you follow them carefully, you may make money in this experiment. This money along with a $3.00 participation fee will be paid to you by cheque at the end of the session.

The session will last for eight periods and each period consists of two stages. You will be playing with three other persons. Your total earnings will depend on your decisions together with the decisions of the other players and your luck during the sessions. You should not communicate with anyone else in the room during the session.

The game uses a fictional currency called tokens. All game transactions are denominated in this fictional currency. Your information slip contains the rate that allows you to convert the tokens that you earn in the experiment into Canadian dollars. The total amount of money you earn in all of the rounds will determine your dollar payoff at the end of the game.

At the beginning of each period, you will be asked to draw an information slip from a box held by the experimenter. On each slip you should enter the date, your assigned player number, and the period number.

At the end of stage 2, one of the four persons will be awarded a monetary prize. The value of the prize for you and for the other members of your group will be specified on your information slip. Your prize value may differ from the prize value for the other members of your group. Each information slip will also indicate that you have 50 tokens that you may either keep or use in order to bid for the prize.

In stage 1 of each period, you will be given the opportunity to bid any amount of money from zero up to 50 tokens. You are not allowed to bid more than 50 tokens. Enter the value of your bid in the designated space on the information slip. Once you make your decision, please raise your hand and your information slip will be collected by the experimenters. The person who places the highest bid and the person who places the second-highest bid will move on to stage 2. The other two players will earn 50 tokens minus their bids in that period. If two players choose the same bid and it is the highest bid, they will both move on to stage 2. If more than two players choose the same bid, and it is the highest bid, a random draw will be used to determine which two will move on to stage 2. Finally, if one player places the highest bid and two or more players place the same bid and it is the second-highest bid, a random draw will be used to determine which one of the latter will move on to stage 2.

If you reach stage 2, you will be given the opportunity to bid any amount of money from zero up to whatever amount of money remains after your stage-1 bid. The person who places the highest bid will receive the prize. Its value will be as specified on that person's information slip. If both players choose the same bid, a random draw will be used to determine which of the two will receive the prize.

If you receive the prize, your total earnings will simply be 50 tokens, minus the tokens you bid in both stages, plus the prize value you drew at the beginning of the game. If you do not receive the prize, your total earnings for each period will just be 50 tokens, minus your bid or bids in the period.

At the end of each period, the amount you have earned in tokens will be indicated by the experimenter on your information slip, which will then be returned to you. Please note that you will have 50 tokens allocated to you at the beginning of each period. You may not use your earnings from an earlier period to make bids in a later period.

At the end of the session, you will be called up one at a time and paid by cheque the total amount that you earned for all periods in the sessions. All slips used in the session should be returned at that time.