Marginal Contribution of Information to Profit in a Zero-sum Game

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1 Introduction

This paper describes how those who are less informed play a zero sum game with those who are better informed. Apparently more informed players can always earn more than a less informed players. In the real world, in the stock and future markets in particular, however, modestly-informed traders seem to miss their guess more frequently than no-informed or random traders do. In this paper we shall show why and how such paradoxical phenomenon can be observed with mathematical analysis, computer simulations and experiments with human subjects.

In the next section we shall show the model of a future market presented by Huber and Kirchler (2004) at 9th Workshop on Economics and Heterogeneous Interacting Agents.

In Section 4 we shall show the results of simulations. The paradoxical income distribution can be realised by the following mechanism: fully-informed traders, who can correctly see the true values of commodities, can earn profits in future markets; no-informed traders, who cannot access any information about the values, buy or sell commodities arbitrarily so that they can expect zero profit in the long run; partly-informed traders can be misled by biased information so that they suffer from losses. We shall also see that less-informed players may increase their profits if they change their strategies according to the strategy of their more informed rivals.

In Section 5 we shall explain our experiments with human subjects and computer agents. The results show that human subjects successfully change their strategies if their competitors (more informed computer agents) change their strategies.

In Section 6 we shall mention game theoretical analysis. The analysis of the three-person game, which is the minimum example of our model, suggests that

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cooperation among less informed players can decrease their losses, though it can be lost by individual profit-maximising behaviour.

In the last section we conclude our analysis with some remarks.

2 The game

The game is played by $2M + 1$ players.[1]

1. At the beginning of each round Player i ($0 \leq i \leq 2M$) shows his reservation price R_i, by which he makes the following contract: he buys (sells) a unit of a future product if R_i is higher (lower) than its price P.

2. The auctioneer gathers all the reservation prices and declares the median of them as P so that demand equals supply in the future market.

3. After the future market is closed, the true value of the commodity V is revealed. It is determined exogenously as the sum of $2M$ stochastic variables: X_1, X_2, ..., and X_{2M}, which are determined to be 0 or 1 independently with one another with the same probability 0.5 for each round.

4. Those who bought (sold) the commodity in the future market must sell (buy) it in the spot market at the true value V to close their accounts. Hence each player's profit is determined as soon as V is revealed. Those players with $P < R_i$ buy the commodity at P in the future market and sell it at V in the spot market so that they each earn $V - P$ of profit, while those players with $R_i < P$ sell the commodity at P in the future market and buy it at V in the spot market so that they each earn $P - V$ of profit. Needless to say, profit is loss if it is negative and the sum of all profit is always zero: $\sum_{i:V<R_i}(P - V) + \sum_{i:R_i<V}(V - P) = 0$.

We assume that before he determines R_1, Player i can correctly see the first i values of X_1, X_2, ..., and X_i (Player 0 cannot predict any of them). Apparently Player i has an advantage over Player j ($j < i$), for the former knows what the latter knows (X_1, X_2, ..., and X_j) as well as what the latter does not (X_{j+1}, X_{j+1}, ..., and X_i). Hence, we will refer to i as "information level".

3 Questions

Player $2M$, the most informed player who can see every X_i, most probably earns positive profit in the long run. In fact he can choose $\sum_{i=1}^{2M} X_i$ as his reservation price R_{2M}; then, as is readily checked, he earns zero profit if P happens to be equal to V or positive profit in more plausible cases where $P \neq V$.

However, Player 0, the least informed player who knows none of X_i may not suffer negative gain in the long run. He may always continue to choose such a

[1] The game is as the same as the one presented by Huber and Kirchler (2004) except that the number of players is not even but odd.

high (low) R_0 that makes him buy (sell) in the future market for every round or he may choose R_0 randomly for each period. In either case he seems to be able to expect zero profit in the long run, because $P < V$ and $V < P$ seem to be realised with the same probability.

A question emerges in the circumstances. The game is zero-sum. If the most-informed player can expect a positive profit while the least-informed player can expect zero profit, some middle-informed player must suffer negative profit. Yet it seems strange that a player who is better informed earns less profit than a player who is less informed.

4 Simulations

We have run a number of simulations to see whether those who are better informed earn smaller profit or suffer larger loss than those who are less informed do.

Such paradoxical results are certainly observed if all players (agents) follow the expected-value strategy:

$$R_i = (\sum_{k=1}^{i} X_i) + 0.5 \times (2M - i). \tag{1}$$

Here the right-hand side represents the value of V that Player i expects; the first term is the sum of the stochastic variables whose values Player i knows, while the second term stands for the expectation of the sum of X_{i+1}, X_{i+2}, ..., and X_{2M}, whose values he does not know. Figure 1 shows the distribution of average profits (per round) among 101 (namely $M = 50$) agents who all followed the expected-value strategy to play the game for 10000 rounds.

The distribution of profits changes drastically if the players change their strategies. Figure 2 shows the distribution of average profits among 101 agents who all played the game for 10000 rounds with the extreme strategy:

$$R_i = \begin{cases} \sum_{k=1}^{i} x_i & \text{with probability 0.5} \\ (\sum_{k=1}^{i} x_i) + (2M - i) & \text{with probability 0.5.} \end{cases} \tag{2}$$

Here $\sum_{k=1}^{i} x_i$ represents the minimum value of V that Player i expects while $(\sum_{k=1}^{i} x_i) + (2M - i)$ stands for its maximum value that Player i expects. The marginal contribution of information to profit, or the increment of profit by knowing the value of an additional variable X_i does not increase significantly until i reaches about 80.

Figures 1 and 2 may suggest that less informed players may follow the extreme strategy to share relatively small losses. Nevertheless if one player adopts the expected-value strategy while the other follow the extreme strategy, he can do better; see Figure 3.

Yet it applies to the reverse case too. If one player adopts the extreme strategy while the other follow the expected-value strategy, he can do better; see Figure 4.

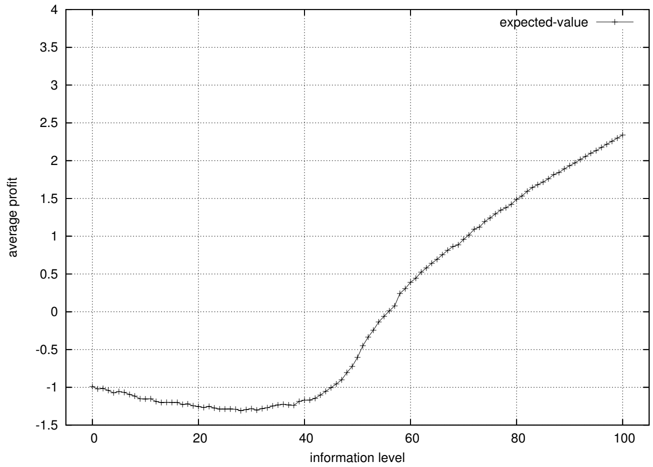

Figure 1: Distribution of average profits among 101 average-strategy agents (10000 rounds). The horizontal axis corresponds to the information level of agents, and the vertical axis corresponds to the average profit per round.

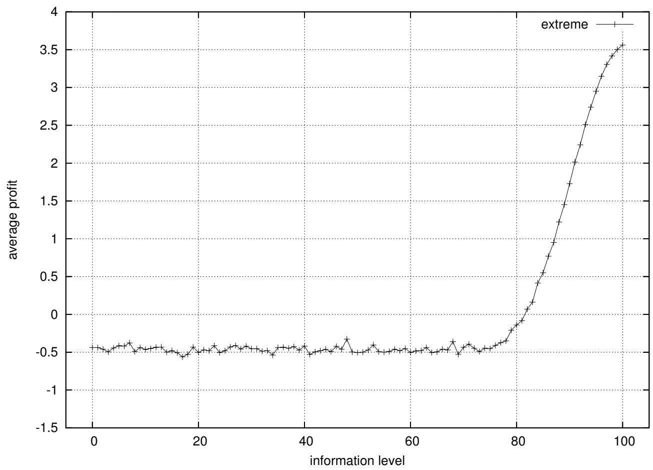

Figure 2: Distribution of average profits among 101 extreme-strategy agents (10000 rounds). The horizontal axis corresponds to the information level of agents, and the vertical axis corresponds to the average profit per round.

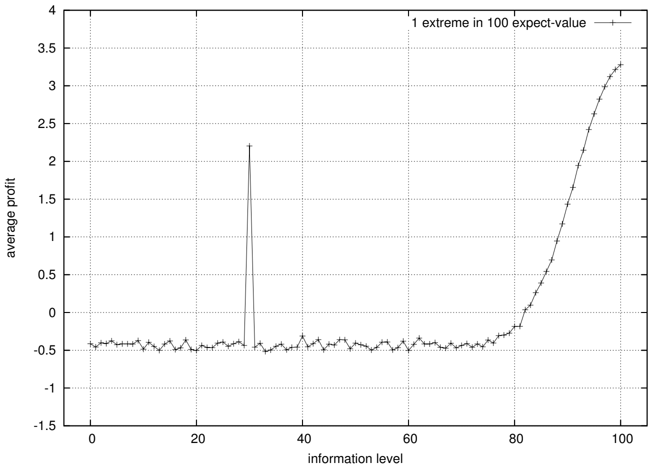

Figure 3: Distribution of profits among an extreme-strategy agent (Player 30) and 100 average-strategy agents (10000 rounds)

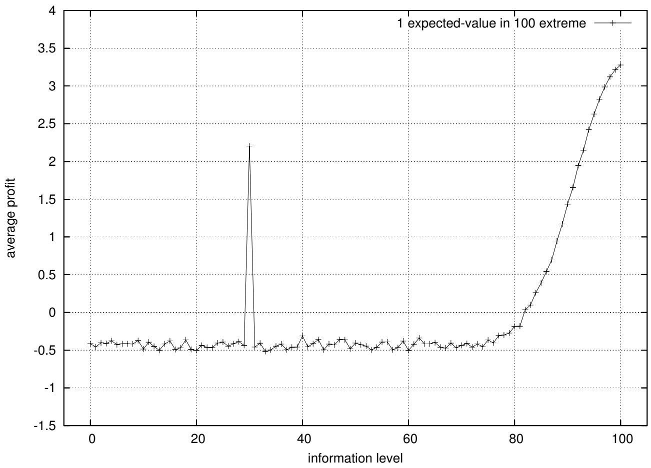

Figure 4: Distribution of profits among an average-strategy agent (Player 30) and 100 extreme-strategy agents (10000 rounds)

Huber and Kirchel (2004) formulated and examined this model with 10 computer agents. They discovered that the distribution of profit among agents is U-shaped if the non-informed agent randomly choose 0 or 10 (the maximum value of the commodity for the ten-agent model) while all the other agents follow the expected-value strategy. In the anticipation that modestly-informed agents could be better off if they do not utilise their private information at all in the circumstances, they increased the number of agents which randomly choose 0 or 10 one by one according to the level of information.

The results of their simulations show that the negative gain from additional information, or the paradoxical situation where a more-informed agent suffers from a smaller profit or larger loss than a less-informed agent, does not disappear until the number of agents which randomly choose 0 or 10 reaches to 5; see the left column of Figure 5, where how the distribution curve of profit changes as the number of those agents which randomly choose 0 or 10 changes from two (Players 0 and 1 randomly choose 0 or 10 in the top figure) to six (Players 0, 1, ..., and 5 randomly choose 0 or 10 in the bottom figure). Huber and Kirchel claim, from this observation, that the case where five players randomly choose 0 or 10 can be considered an equilibrium if there are only two strategies: the expected-value strategy and the zero-or-ten strategy.

The situation is parallel if the zero-or-ten strategy is substituted with our extreme strategy. The right column of Figure 5 shows how the distribution curve of profit changes as the number of agents which follow the extreme strategy increases from two to six. However, the difference of the profit distribution curve between the two columns increases as the number of those agents which follow the random strategy, or the zero-or-ten or the extreme strategy. In particular the difference is rather large if the number of those agents which follow the random strategy exceeds five; compare the bottom pair of Figure 5.

Let us examine the effect of the difference of the random strategies on the profit distribution curve in the game played by 101 players. The results are quite similar to the above-mentioned ones. See Figure 6, which shows how the distribution curve of profits changes as the number of agents which follow the zero-or-hundred strategy (in the left column) or our extreme strategy (in the right column) increases from 20 (the top pair) to 80 (the bottom pair). In either column the distribution curve is U-shaped until the number of agents which follow the zero-or-hundred strategy or the extreme strategy reaches 80. This value is relatively much larger than it is in the model where there are only 10 agents (80 percent vs. 50 percent). Another visible difference is, though the profit distribution curves are nearly identical until the number of agents which follow either extreme strategy reaches 80, they are very different if the number is greater.

5 Experiments with human subjects

We have done experiments with computer agents and human subjects at Kyoto Experimental Economics Laboratory (KEEL), Kyoto Sangyo University (KSU)

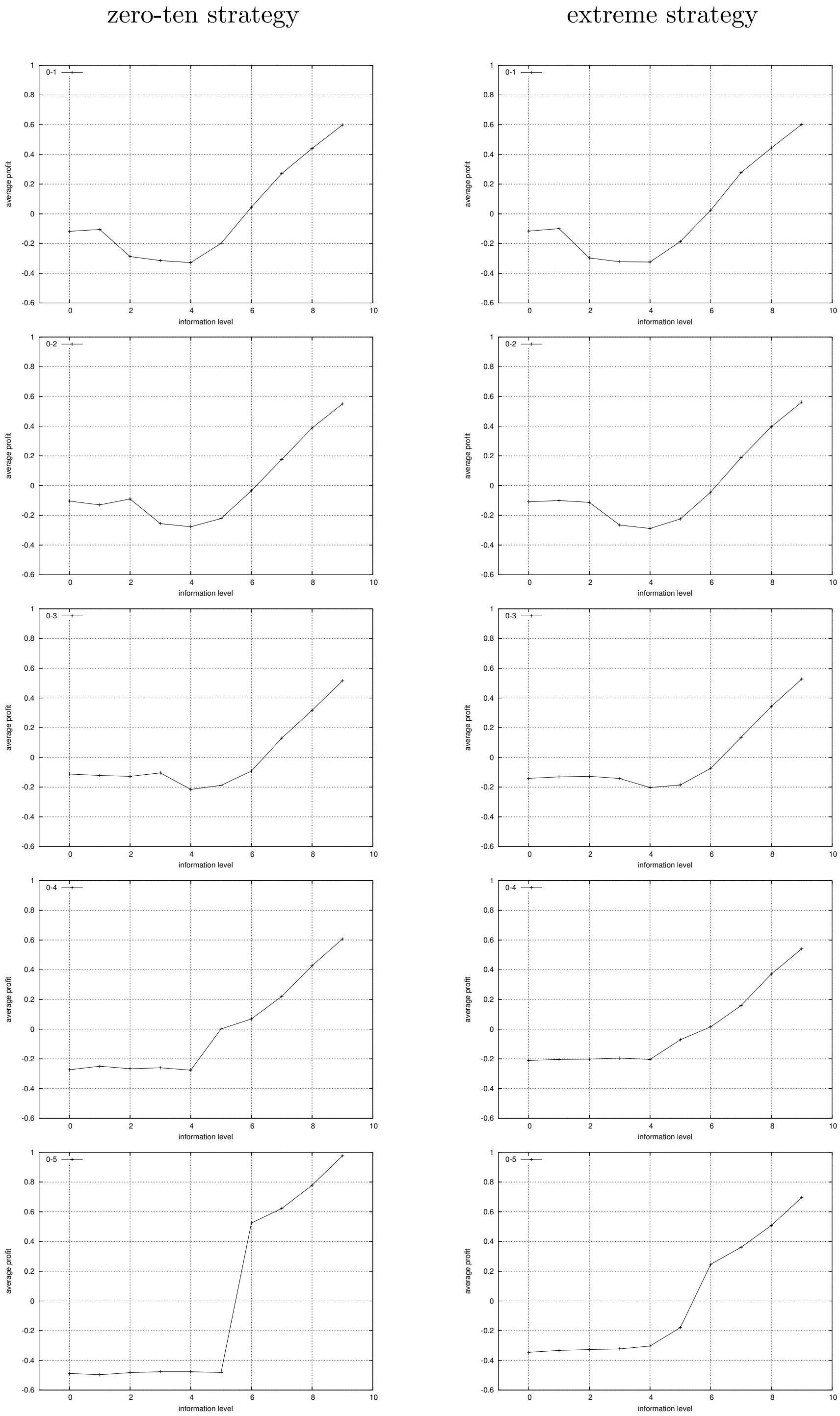

Figure 5: Distributions of profits among 10 agents

zero-ten strategy extreme strategy

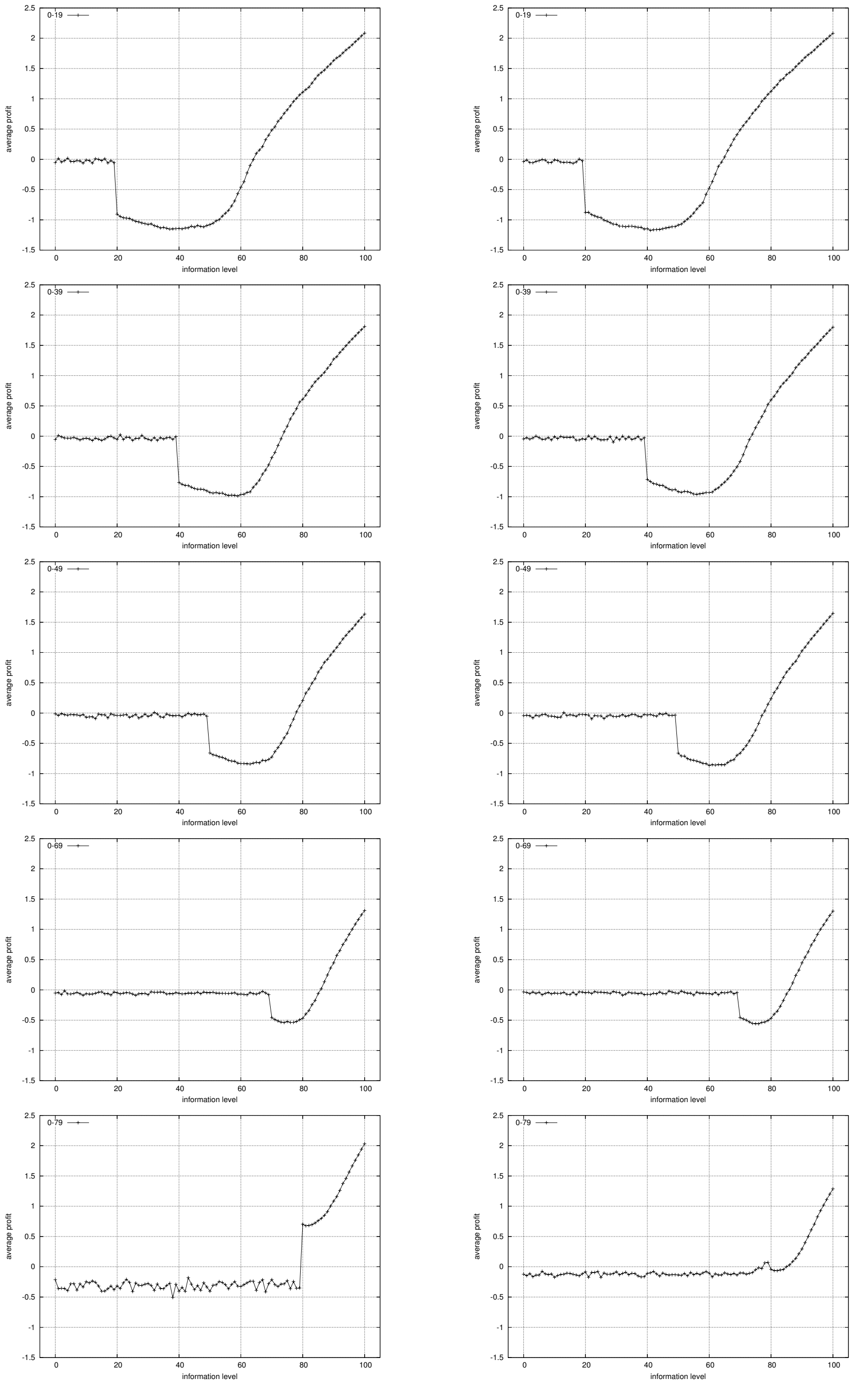

Figure 6: Distributions of profits among 100 agents

on November 13 and 16, 2004. In total 43 undergraduates of KSU played the game mentioned in the previous section as a unique human player with 100 computer agents. To put it concretely, each subject was asked to play the game as Player 100 with 100 computer agents following the expected-value strategy for 100 rounds (Session A100), as Player 30 with 100 computer agents following the expected-value strategy for 100 rounds (Session A30) and as Player 30 with 100 computer agents following the extreme-value strategy for 100 rounds (Session B30). Twenty one subjects played the three sessions in the above-mentioned order, while the other subjects played B30 as the second session and A30 as the third session. Since there was no significant difference in subjects' performances between the two groups, we shall refer hereinafter to Session A30 or B30 without mentioning whether it was the second session or the third session for the subject.

Figure 7 shows the distribution of average profits per round of the session A100, namely where Player 100 is a human subject and the other 100 players are computer agents following the expected-value strategy. We see from Figure 7 that the human subject plays worse as Player 100 than the agent. Because the subjects can see the all stochastic variable X_is, they know the value of V. If the subject chooses V as his reservation price R_{100}, he can obtain the higher profit as shown in Figure 1. It should be noted that choosing V as his reservation price R_{100} is the weakly dominant strategy for Player 100.

Figure 8 shows the distributions of average profits among players of A30 (above) and of B30 (below). In these session, Player 30 is a human subject, and the other 100 players are computer agents following the expected-value strategy in the figure above (A30) and the extreme strategy in the figure below (B30), respectively. These figures show that the human subject played so well as Player

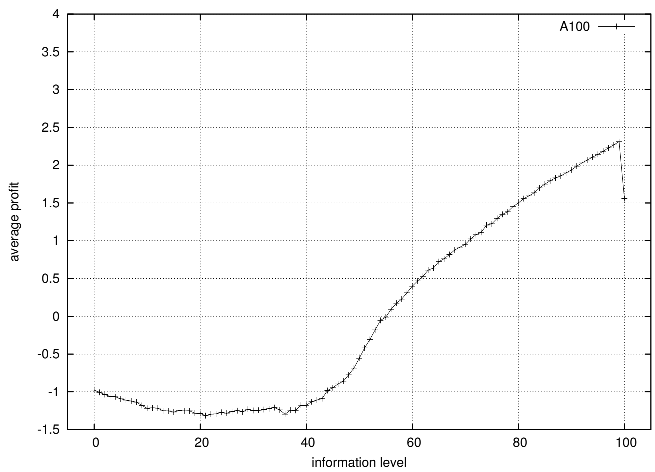

Figure 7: Distribution of average profits among a human subject (Player 100) and 100 agents with expected-value strategy (100 rounds).

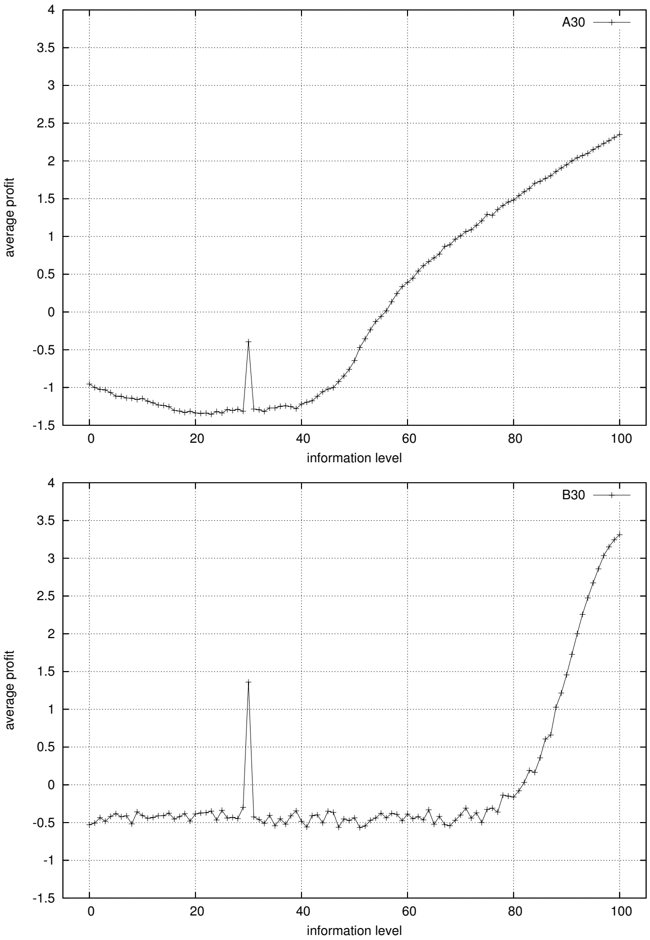

Figure 8: Distribution of average profits among a human subject (Player 30) and 100 expected-value strategy agents in the figure above; 100 extreme strategy agents in the figure below (100 rounds)

30 and they earned the better profit than the agents of the same informed level in both circumstances.

As noted above, the extreme strategy is better strategy among the expected-value strategy, and the expected-value strategy is better strategy among the extreme strategy. We therefore focused on the deviation from the expected value of Player 30 (see Eq. (1)) for characterizing the subjects behavior. Obviously, if the subject plays as the expected-value strategy, the deviation from the expected value is zero; if the subject plays as the extreme strategy, the deviation is $\frac{100-i}{2}$ for Player i. Figure 9 shows the correlation between the average profit of Player 30 and the deviation from the expected-value of his strategy. The horizontal axis corresponds to the deviation from the expected-value, and the vertical axis corresponds to the average profit. The solid line indicates the average profit of Player 30 among the expected-value strategies, while the broken line indicates the average profit among the extreme strategies.

The average deviations from the expected value of the subjects as Player 30 are shown in the Table 1. It is shown that the deviation in the session with the agents of extreme strategy is smaller than in the session with the agents of expected-value strategy. In other words, most of the subjects change their strategy in the right way.

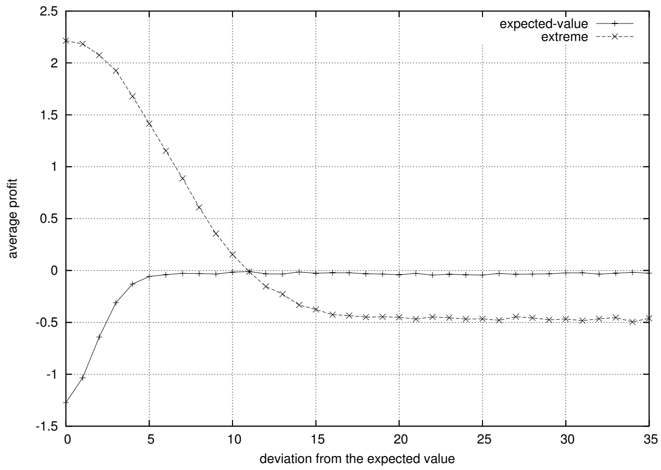

Figure 9: The correlation between the deviation from the expected-value and the average profit

6 Mathematical analysis

The results of simulation suggest that less-informed players could follow some 'cooperative' strategy to distribute small losses among them. In fact the non-perfectly informed players, or Players 0, 1, ..., and $2M-1$, may follow the

Table 1: Deviation from the expected-value

A30 (with 100 expected-value strategies)	B30 (with 100 extreme-strategies)
32.21	34.20
2.75	2.43
5.38	2.17
1.92	1.04
13.34	9.45
10.74	6.09
3.95	2.44
4.35	3.49
7.96	6.95
7.06	4.04
3.48	2.70
3.44	5.20
0.65	1.08
5.62	4.57
4.91	3.45
3.83	1.93
2.14	2.34
7.40	3.11
7.96	1.57
17.76	21.34
8.49	4.83
3.20	0.01
6.75	6.19
3.54	3.30
2.82	2.58
9.04	0.42
2.50	4.55
4.03	0.58
5.72	6.64
3.13	0.99
6.15	0.32
9.48	4.35
3.82	2.79
8.75	5.00
30.55	23.66
26.87	14.63
12.71	24.20
6.63	6.84
8.06	8.39
27.79	34.13
5.54	4.36
4.25	3.83
9.59	6.05

following strategies as an example: that a half of them choose 0 as their reservation prices while the other half of them select $2M$ as their reservation prices. If the perfectly informed Player $2M$ follows his weakly dominant strategy, he declares the true value of the commodity as his reservation price; then, as is readily checked, every players gets zero profit. Yet it is most probably broken by relatively well informed players, who can choose a value nearer to the true value as their reservation prices to increase their profits.

Although the mathematical analysis of the general $2M$ person game is not practical, we can examine the 3 person model even with small generalisation to examine what is mentioned above:

$$N = 3 \tag{3}$$
$$V_k \in \{0, 1\} \qquad \text{for } k = 1, 2 \tag{4}$$
$$0 < \theta = \mathrm{Prob}(V_k = 0) < 1 \qquad \text{for } k = 1, 2 \tag{5}$$
$$P_i \in \{0, 1, 2\} \qquad \text{for } i = 0, 1, 2 \tag{6}$$

We assume that no players choose weakly dominated strategies: Player 0 chooses j ($j = 0, 1, 2$) as P_0 with probability x_j; seeing $V_0 = i$ ($i = 0, 1$), Player 1 chooses j ($j = i, i + 1$) as P_1 with probability y_{ij}; Player 2 always chooses $V_1 + V_2$ as P_2. Here, by definition,

$$x_0 + x_1 + x_2 = y_{00} + y_{01} = y_{11} + y_{12} = 1. \tag{7}$$

Then there are $24(= 2\times2\times3\times2\times1)$ possible combinations of the values of V_1, V_2, P_0, P_1 and P_2. Table 2 shows the three players' profits π_0, π_1 and π_2 for each case with which the probability with which each case will occur.

We can see each player's expected profit:

$$\pi_0 = -\frac{\theta^2 x_1 y_{01} + \theta(1-\theta)(x_0 y_{00} + x_2 y_{12}) + (1-\theta)^2 x_1 y_{11}}{2} - \theta^2 x_2 y_{01} - (1-\theta)^2 x_0 y_{11} \tag{8}$$
$$\pi_1 = -\frac{\theta^2 x_1 y_{01} + \theta(1-\theta)(x_0 y_{00} + x_2 y_{12}) + (1-\theta)^2 x_1 y_{11}}{2} \tag{9}$$
$$\pi_2 = \theta^2 (x_1 + x_2) y_{01} + \theta(1-\theta)(x_0 y_{00} + x_2 y_{12}) + (1-\theta)^2 (x_0 + x_1) y_{11} \tag{10}$$

or equivalently

$$\pi_0 = A_0 x_0 + A_2 x_2 - \frac{\theta^2 (1 - y_{00}) + (1-\theta)^2 (1 - y_{12})}{2} \tag{11}$$
$$\pi_1 = B_0 y_{00} + B_2 y_{12} - \frac{(\theta^2 + (1-\theta)^2)(1 - x_0 - x_2)}{2} \tag{12}$$
$$\pi_2 = \theta^2 (1 - x_0 - y_{00}) + (1-\theta)^2 (1 - x_2 - y_{12}) + \theta x_0 y_{00} + (1-\theta) x_2 y_{12} \tag{13}$$

Table 2: All Cases

Case	V_1	V_2	P_0	P_1	P_2	π_0	π_1	π_2	Prob.
1	0	0	0	0	0	0	0	0	$\theta^2 x_0 y_{00}$
2	0	0	1	0	0	0	0	0	$\theta^2 x_1 y_{00}$
3	0	0	2	0	0	0	0	0	$\theta^2 x_2 y_{00}$
4	0	0	0	1	0	0	0	0	$\theta^2 x_0 y_{01}$
5	0	0	1	1	0	-0.5	-0.5	1	$\theta^2 x_1 y_{01}$
6	0	0	2	1	0	-1	0	1	$\theta^2 x_2 y_{01}$
7	0	1	0	0	1	-0.5	-0.5	1	$\theta(1-\theta) x_0 y_{00}$
8	0	1	1	0	1	0	0	0	$\theta(1-\theta) x_1 y_{00}$
9	0	1	2	0	1	0	0	0	$\theta(1-\theta) x_2 y_{00}$
10	0	1	0	1	1	0	0	0	$\theta(1-\theta) x_0 y_{01}$
11	0	1	1	1	1	0	0	0	$\theta(1-\theta) x_1 y_{01}$
12	0	1	2	1	1	0	0	0	$\theta(1-\theta) x_2 y_{01}$
13	1	0	0	1	1	0	0	0	$(1-\theta)\theta x_0 y_{11}$
14	1	0	1	1	1	0	0	0	$(1-\theta)\theta x_1 y_{11}$
15	1	0	2	1	1	0	0	0	$(1-\theta)\theta x_2 y_{11}$
16	1	0	0	2	1	0	0	0	$(1-\theta)\theta x_0 y_{12}$
17	1	0	1	2	1	0	0	0	$(1-\theta)\theta x_1 y_{12}$
18	1	0	2	2	1	-0.5	-0.5	1	$(1-\theta)\theta x_2 y_{12}$
19	1	1	0	1	2	-1	0	1	$(1-\theta)^2 x_0 y_{11}$
20	1	1	1	1	2	-0.5	-0.5	1	$(1-\theta)^2 x_1 y_{11}$
21	1	1	2	1	2	0	0	0	$(1-\theta)^2 x_2 y_{11}$
22	1	1	0	2	2	0	0	0	$(1-\theta)^2 x_0 y_{12}$
23	1	1	1	2	2	0	0	0	$(1-\theta)^2 x_1 y_{12}$
24	1	1	2	2	2	0	0	0	$(1-\theta)^2 x_2 y_{12}$

where

$$A_0 = \frac{2\theta - 1 - \theta y_{00} + (1-\theta)^2 y_{12}}{2} \qquad = -\theta(y_{00}+\alpha) + (1-\theta)^2(y_{12}-\alpha) \tag{14}$$

$$A_2 = \frac{1 - 2\theta + \theta^2 y_{00} - (1-\theta) y_{12}}{2} \tag{15}$$

$$B_0 = \frac{\theta(\theta - x_0 - \theta x_2)}{2} \tag{16}$$

$$B_2 = \frac{(1-\theta)(1-\theta-(1-\theta)x_0 - x_2)}{2} \tag{17}$$

Nash equilibria

A Nash equilibrium exists with $0 = A_0 = A_2$ only if $\theta = 0.5$. [Proof] Suppose

$0 = A_0 = A_2$. Then

$$y_{00} = -\alpha \qquad \text{and} \qquad y_{12} = \alpha \qquad \text{where} \qquad \alpha = \frac{1-2\theta}{1-\theta+\theta^2}. \tag{18}$$

They are consistent with $0 \le y_{00}$ and $0 \le y_{12}$ only if $\alpha = 0$, or $\theta = 0.5$. ■ The equilibrium with $0 = A_0 = A_2$ and $\theta = 0.5$ is as follows:

$$\{(x_0, x_1)|\ 1 \le 2x_0 + x_2;\ 1 \le x_0 + 2x_2;\ x_0 + x_1 \le 1\} \qquad \text{and} \qquad y_{00} = y_{12} = 0. \tag{19}$$

Here $\le 2x_0 + x_2$ and $1 \le x_0 + 2x_2$ are necessary for $B_0 \le 0$ and $B_2 \le 0$, which makes $y_{00} = y_{12} = 0$ be the best response of Player 1.

A Nash equilibrium exists neither with $0 \le A_0$ and $0 < A_2$ nor with $0 < A_0$ and $0 \le A_2$. In fact no feasible combination of x_0, x_2, y_{01} and y_{12} in either case. [Proof] Suppose $0 \le A_0$ and $0 < A_0$. Then $\theta(y_{00} + \alpha) \le (1-\theta)^2(y_{12} - \alpha)$ and $(1-\theta)(y_{12} - \alpha) < \theta^2(y_{00} + \alpha)$. By combining these inequalities, $\theta(y_{00} + \alpha) \le (1-\theta)^2(y_{12} - \alpha) < \theta^2(1-\theta)(y_{00} + \alpha) \Rightarrow y_{00} + \alpha < 0$; $(1-\theta)(y_{12} - \alpha) < \theta^2(y_{00} + \alpha) \le (1-\theta)^2\theta(y_{12} - \alpha) \Rightarrow y_{02} - \alpha < 0$. Hence $y_{00} + y_{12} = (y_{00} + \alpha) + (y_{12} - \alpha) < 0$, which is inconsistent with $0 \le y_{00} \le 1$ and $0 \le y_{12} \le 1$. Similarly it is checked that $0 < A_0$ and $0 \le A_2$ are inconsistent with $0 \le y_{00} \le 1$ and $0 \le y_{12} \le 1$. ■

Nash equilibria exist with $A_0 < 0 < A_2$:

$$x_0 = 0, \qquad x_2 = 1, \qquad \max(0, \frac{2\theta - 1}{\theta^2}) < y_{00} \le 1 \qquad \text{and} \qquad y_{12} = 0. \tag{20}$$

Nash equilibria exist with $A_2 < 0 < A_0$:

$$x_0 = 1, \qquad x_2 = 0, \qquad y_{00} = 0 \qquad \text{and} \qquad \max(0, \frac{1-2\theta}{(1-\theta)^2}) < y_{12} \le 1. \tag{21}$$

A Nash equilibrium exists with $A_0 < 0$ and $A_2 < 0$:

$$x_0 = 0, \qquad x_2 = 0, \qquad y_{00} = 1 \qquad \text{and} \qquad y_{12} = 1. \tag{22}$$

A Nash equilibria exists with $A_0 < 0$ and $0 = A_2$ for all θ $(0 < \theta < 1)$:

$$x_0 = 0, \qquad x_2 = 1 - \theta, \qquad y_{00} = 1 \qquad \text{and} \qquad y_{12} = 1 - \theta. \tag{23}$$

Another Nash equilibria exists with $A_0 < 0$ and $0 = A_2$ if $0.5 < \theta < 1$:

$$x_0 = 0, \qquad x_2 = 1, \qquad y_{00} = \frac{2\theta - 1}{\theta^2} \qquad \text{and} \qquad y_{12} = 0. \tag{24}$$

A Nash equilibria exists with $A_0 = 0$ and $A_2 < 0$ for all θ $(0 < \theta < 1)$:

$$x_0 = \theta, \qquad x_2 = 0, \qquad y_{00} = \theta \qquad \text{and} \qquad y_{12} = 1. \tag{25}$$

Another Nash equilibria exists with $A_0 = 0$ and $A_2 < 0$ if $0 < \theta < 0.5$:

$$x_0 = 1, \qquad x_2 = 0, \qquad y_{00} = 0 \qquad \text{and} \qquad y_{12} = \frac{1-2\theta}{(1-\theta)^2}. \tag{26}$$

Table 3: Nash equilibria

Case	x_0	x_1	x_2	y_{00}	y_{01}	y_{11}	y_{12}	π_0	π_1	π_2
1	0	1	0	1	0	0	1	0	0	0
2	0	θ	$1-\theta$	1	0	θ	$1-\theta$	$-\frac{\theta(1-\theta)^2}{2}$	$-\frac{\theta(1-\theta)^2}{2}$	$\theta(1-\theta)^2$
3	θ	$1-\theta$	0	θ	$1-\theta$	0	1	$-\frac{\theta^2(1-\theta)^2}{2}$	$-\frac{\theta^2(1-\theta)^2}{2}$	$\theta^2(1-\theta)^2$
4	0	0	1	$0 < y_{00} \leq 1$ $\frac{2\theta-1}{\theta^2} < y_{00}$ $y_{01} = 1 - y_{00}$		1	0	$-\theta^2 y_{01}$	0	$\theta^2 y_{01}$
5	1	0	0	0	1	$y_{11} = 1 - y_{12}$ $\frac{1-2\theta}{(1-\theta)^2} < y_{12}$ $0 < y_{12} \leq 1$		$-\theta^2 y_{12}$	0	$\theta^2 y_{12}$
$6_{\theta<0.5}$	1	0	0	0	1	$\frac{\theta^2}{(1-\theta)^2}$	$\frac{1-2\theta}{(1-\theta)^2}$	$-\theta^2$	0	θ^2
$6_{\theta=0.5}$	$1 \leq 2x_0 + x_2$ $1 \leq x_0 + 2x_2$ $x_0 + x_2 \leq 1$			0	1	1	0	$-\frac{1}{4}$	$-\frac{xpla_1}{4}$	$\frac{1+x_1}{4}$
$6_{0.5<\theta}$	0	0	1	$\frac{2\theta-1}{\theta^2}$	$\frac{(1-\theta)^2}{\theta^2}$	1	0	$-(1-\theta)^2$	0	$(1-\theta)^2$

7 Concluding Remarks

The results of our simulations may suggest that human players can think and/or learn so flexibly that they can change their strategies according to changes in their circumstances. In our experiments each game was played by 100 computer agents and a human subject. Most human subjects outwitted computer agents in the uncertain circumstances. This performance is even more impressive if we take it into account that not a few subjects failed to find out the best strategy when has the perfect information.

However, the good performance of our subjects may be benefited largely from the fact that their rivals are all such simple computer agents that cannot change their strategies. If they also could change their strategies according to their experience, the dynamics of the game would be so complicated that they could not be outwitted by human subjects.

The model invented by Huber and Kirchel (2004) gives a very simple description of competition/cooperation among traders with different private informations. It could be a good test bed for the theory and simulation in the field of study. We hope that ours could be a first attempt in the direction.

References

[1] 27-29 May 2004: Jüergen Huber and Michael Kirchler "The Value of Information in Markets with Heterogeneously Informed Traders - and Experimental and a Simulation Approach", presented at 9th Workshop on Economics and Heterogeneous Interacting Agents(WEHIA2004), Kyoto University, Kyoto, Japan