Experimental Evidence on the Multibidding Mechanism*

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Abstract

Pérez-Castrillo and Wettstein (2002) and Veszteg (2004) propose the use of a multibidding mechanism for situations where agents have to choose a common project. Examples are decisions involving public goods (or public "bads"). We report experimental results to test the practical tractability and effectiveness of the multibidding mechanisms in environments where agents hold private information concerning their valuation of the projects. The mechanism performed quite well in the laboratory: it provided the ex post efficient outcome in roughly three quarters of the cases across the treatments; moreover, the largest part of the subject pool formed their bids according to the theoretical bidding behavior.

Keywords: experiments, mechanisms, uncertainty.

JEL Classification Numbers: C91, C72.

1 Introduction

Economic agents often have to make a common decision, or choose a joint project, in situations where their preferences may be very different from one another. Decisions involving public goods (or public "bads") belong to this class of situations. We can

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consider the examples of several municipalities deciding on the location and quality of a common hospital, several states deciding on the location of a nuclear reactor, or several countries choosing the identity of the leader for an international organization. In these situations the natural tendency for the agents is to try to free ride on the others by exaggerating the benefits and/or losses of a particular decision while, at the same time, minimizing their willingness to pay.

Pérez-Castrillo and Wettstein (2002) address the problem of making this type of decision in environments where the agents have *symmetric information* about everybody's preferences. They propose a simple one-stage *multibidding mechanism* in which each agent submits a bid for each project with the restriction that bids must sum up to zero for each participant. Hence, agents are asked to report on their relative valuations among the projects. The mechanism determines both the project that will be implemented (the one that most bids receives) and a system of (budget-balanced) transfer payments to possibly compensate those agents who are not pleased with the chosen project. Pérez-Castrillo and Wettstein (2002) show that the multibidding mechanism always generates an efficient decision in Nash (and strong Nash) equilibrium.

Veszteg (2004) analyzes the working of the multibidding mechanism in environments where agents hold *private information* regarding their valuation of the projects. He characterizes the symmetric Bayes-Nash equilibrium strategies for the agents, when they have to choose between two projects. He shows that the equilibrium outcomes are always individually rational (i.e. agents have incentives to participate in the mechanism). Veszteg (2004) further proves that, when the decision only concerns two agents, the project chosen at equilibrium is always efficient. Moreover, the number of inefficient decisions diminishes and it approaches zero as the number of agents or uncertainty gets large.

The multibidding mechanism is very simple: its rules are easy to explain, the action that each agent must take is simple, and the outcome is a straightforward function of the actions taken by the agents. Moreover, as we have pointed out, it induces the agents to make, at equilibrium, efficient decisions in a variety of environments. Therefore, we could advocate its use in real economic situations.[1] In this paper, we want to further

[1]In environments with private information, we could also use the (more complex) mechanism proposed by d'Aspremont and Gérard-Varet (1979), that is inspired by the Vickrey-Clarke-Groves schemes. The Bayesian equilibrium outcomes of their mechanism are budget-balanced and efficient. However, it is not necessarily individually rational, some agents may prefer to stay outside of the game. Moreover, if we were to design a mechanism that is Bayesian incentive compatible, (ex post) efficient and balanced we would end up with a mechanism of the d'Aspremont and Gérard-Varet type that is not individually rational in general. For more on this literature check the survey by Jackson (2001). It is worth noting that in this paper we use voluntary participation conditional on the impossibility of avoiding external effects. Even with this definition, it is easy to show that some agents may prefer not to participate in the d'Aspremont and Gérard-Varet mechanisms.

support the use of the multibidding mechanism by providing and analyzing evidence of its functioning in laboratory experiments.[2]

We report the results of two sessions of experiments which have been designed to test the practical tractability and effectiveness of the multibidding mechanism in environments where agents hold private information concerning their valuation of the projects. We implemented two treatments in each session. The first treatment involved decisions by groups of two agents, while we arranged the agents in larger groups for the second treatment. In both treatments, the agents had to choose between two projects. We test the theoretical predictions of the paper by Veszteg (2004).

The first property that we check is to what extend the agents' bids reflect their relative valuations of the projects. According to the rules of the multibidding mechanism, agents are asked to report their relative valuations, and any agent's Bayesian equilibrium bids do indeed only depend on the difference between her valuation for the first and second project. The bids submitted by the individuals in the experiments also follow this pattern. Hence, the mechanism does a good job at extracting the information concerning agents' relative valuation.

Second, the analysis of the joint results also indicates that agents' bidding behavior is close to the theoretical equilibrium prediction. The individual analysis of bidding allows however to identify four types of players. More than half of the individuals were bidding according to the equilibrium. Also, another 20% of them bid in a similar manner, although in a less aggressive way. A third group of individuals (identified in one of the sessions, it accounts for 15% of the people in this session) followed a very safe strategy, by bidding according to maximin strategies. Finally, we could not explain the bidding of around 20% of the individuals participating in the experiments.

In terms of efficiency, the multibidding game picked out the ex post efficient project in roughly three quarter of the cases across the four experimental treatments. In line with the theoretical predictions, efficiency was larger when the individuals were paired than when they formed groups of larger size.

Our work follows the line of research that includes papers as, for example, Smith (1979 and 1980), and Falkinger *et al.* (2000), that advocate for the use of experiments to provide evidence on the empirical properties of mechanisms in public good environments. The characteristics of the multibidding mechanism, and the fact that the experiments were conducted in an environment where individuals hold private information, place our

[2] As Ledyard (1995) points out when he discusses the behavior of individuals in public goods environments: "We need not rely on voluntary contribution approaches but can instead use new organizations... Experiments will provide the basic empirical description of behavior which must be understood by the mechanism designer, and experiments will provide the test-bed in which the new organizations will be tested before implementation."

paper in close relationship with the extensive literature about experiments in auctions; in particular, with experimental papers on independent private-values auctions (see, for example, the early work of Coppinger *et al.*, 1980, and Cox *et al.*, 1982). This literature shows that equilibrium bidding theory correctly predicts the directional relationships between bids and valuations (see Kagel, 1995). Our results show that when a (multi)bidding mechanism is used to make a joint decision (and not to sell an object), theory is still a good predictor of the individuals' behavior.

The paper proceeds as follows: Section 2 introduces the environment and the mechanism to be studied empirically. Section 3 presents the experimental design and Section 4 the empirical results. Finally, Section 5 concludes and offers further directions of research.

2 The environment and the mechanism

Consider an economy where a set of agents $\mathcal{N} = \{1, ..., n\}$ has to choose between two public projects, the set of projects is denoted $\mathcal{K} = \{1, 2\}$. The agents are risk neutral and their utilities depend on the alternative carried out. We shall denote by $x_i^q \in X \subset \mathbb{R}$ the utility that player $i \in \mathcal{N}$ enjoys when project $q \in \mathcal{K}$ is chosen. These values are private information and are treated as random draws from some underlying common distribution. The latter, that characterizes uncertainty, is common knowledge.

The socially desirable outcome is the one that maximizes social welfare. We shall call project q *efficient* if:

$$\sum_{i \in \mathcal{N}} x_i^q = \max \left\{ \sum_{i \in \mathcal{N}} x_i^1, \sum_{i \in \mathcal{N}} x_i^2 \right\}.$$

The presence of external effects in the economy makes the market mechanism unreliable for taking the public decision efficiently. For these environments, Veszteg (2004) proposes the use of a multibidding mechanism, previously suggested by Pérez-Castrillo and Wettstein (2002), to provide a simple incentive scheme for the agents to reveal their private information. The *multibidding mechanism* is a one-stage game and it can be formally defined as follows:

Each agent $i \in \mathcal{N}$ submits a vector of two real numbers that sum up to zero.[3] That is, agent i announces B_i^1 and B_i^2, such that $B_i^2 = -B_i^1$. Agents submit their bids simultaneously.

The project with the highest aggregated bid will be carried out, where the aggregated bid $B_{\mathcal{N}}^q$ for project q is defined as:

$$B_{\mathcal{N}}^q = \sum_{i \in \mathcal{N}} B_i^q.$$

[3]Two is the number of available projects in the experiment.

In case of a tie, the winning project is randomly chosen from the available ones in the tie.

Once the winning project is determined, players enjoy the utility that it delivers, they pay the bids submitted for this project, and they are returned the aggregated winning bid in equal shares.[4] That is, if project q has obtained the largest aggregated bid, then player i receives the payoff V_i^q, where

$$V_i^q = x_i^q - B_i^q + \frac{1}{n} B_{\mathcal{N}}^q.$$

A key property of the multibidding mechanism is that it can be operated without any positive or negative amount of money by the social planner, i.e., it is safe for the central government or for the authority entitled to carry out a social project. *Budget-balance* is achieved by construction since funds raised through the bidding process are entirely given back to participants in equal shares.

Moreover, the multibidding mechanism is safe for bidders, too. Once we suppose that members of the economy may abstain from participating in the bidding, but cannot avoid external effects, the mechanism assures that agents cannot do better by staying out of the decision-making process. Bidding exactly half of the difference between her private valuations, any agent can secure for herself a final payoff that is never less than the average of the private valuations. That is, if agent i takes the decisions of bidding:

$$B_i^1 = \frac{x_i^2 - x_i^1}{2} \text{ and } B_i^2 = -B_i^1,$$

then her payoff V_i is at least

$$\frac{x_i^2 + x_i^1}{2},$$

independently on whether project 1 or 2 is chosen. We shall refer to this bidding behavior as bidding according to *maximin strategies*.

Maximin strategies are not equilibrium strategies, an agent can typically obtain a higher expected payoff if she follows a different strategy. Hence, it is more interesting to consider the *Bayes-Nash equilibria* of the multibidding game. In particular, we concentrate on *symmetric Bayes-Nash equilibria*. The bidding behavior in these equilibria is substantially different for different types/degrees of uncertainty that individuals face in their decision making. That means that the optimal bidding function depends both on the underlying probability distribution and the number of agents in the economy. We offer a brief summary of the theoretical results related to the empirical problem studied in the experimental sessions. For more general results and formal proofs, we refer to Veszteg (2004).

[4]The aggregate bid for the winning project is always non-negative, since bids of each agent sum up to zero.

We denote by $B_i(x_i^1, x_i^2)$ the equilibrium bid by agent i on project 1, when the utility levels that this agent enjoys for the two projects are x_i^1 and x_i^2. Given the restriction on the bids, agent i shall bid $-B_i(x_i^1, x_i^2)$ on project 2.

In the multibidding game, players must submit bids that add up to zero, that is, they are asked to report their relative preferences over the alternative projects. The first important result we highlight is that the optimal bidding behavior also reflects only relative preferences:

Proposition 1 *The bidding function in symmetric Bayes-Nash equilibria depends only on the difference between the true private valuations. That is,* $B_i(x_i^1, x_i^2) = B_i(\widehat{x}_i^1, \widehat{x}_i^2)$ *whenever* $x_i^1 - x_i^2 = \widehat{x}_i^1 - \widehat{x}_i^2$.

This result allows for an important simplification in the notation and in the numerical analysis of the problem. Let the difference between player i's private valuations be d_i with the following definition: $d_i = x_i^1 - x_i^2$. We denote by $f(d)$ the density and by $F(d)$ the cumulative distribution functions of the difference d for both agents. Also, we denote d_M the median of the distribution. The next proposition states the optimal bidding function when there are two agents:

Proposition 2 *In the case of two agents and symmetric distributions, the symmetric Bayes-Nash bidding function is given by the following expression:*

$$B_i(d_i) = \begin{Bmatrix} \frac{1}{2}d_i + \frac{1}{2}\left[1 - 2F(d_i)\right]^{-2} \cdot \int_{d_i}^{d_M} \left[1 - 2F(t)\right]^2 dt & \text{if } d_i < d_M \\ \frac{1}{2}d_i & \text{if } d_i = d_M \\ \frac{1}{2}d_i - \frac{1}{2}\left[1 - 2F(d_i)\right]^{-2} \cdot \int_{d_M}^{d_i} \left[1 - 2F(t)\right]^2 dt & \text{if } d_i > d_M \end{Bmatrix} \tag{1}$$

The optimal bidding behavior coincides with the maximin strategy at the median, d_M. Due to the strategic behavior that takes into account the distribution of valuations in the economy, below this value agents submit higher bids, while under the median they bid less aggressively.

In the experiments, we used the uniform distribution from the interval $[0; 300]$ to assign private valuations to each subject and for each project. With this choice, the variable of the difference between private valuations follows a symmetric triangular distribution over the interval $[-300; 300]$. By the continuity of the underlying distribution and the rules of the multibidding game, the optimal bidding function is continuous and strictly increasing in d_i. Graph 1 plots the optimal bidding function according to Bayes-Nash (thick line) and to maximin strategies (thin line) for the triangular distribution over the interval $[-300; 300]$. Calculations are to be found in Appendix A.

The explicit formula of the optimal bidding function for economies with more than two players is not available. Veszteg (2004) shows that it can be approximated with a proportional function, the slope of which depends on the number of bidders, n:

Proposition 3 *If the number of agents is large, the symmetric Bayes-Nash bidding function is close to a proportional function:*

$$B_i(d_i) \approx \frac{n}{4n-2} d_i. \tag{2}$$

According to the rules of the multibidding game, the project to be carried out is the one that receives the highest aggregate bid. Taking into account our experimental setup, i.e., uncertainty is characterized by a symmetric triangular distribution, theory predicts ex post efficient public decisions in case of two bidders. If there are more than two participants, inefficiencies may appear. Although we do not have analytical results for the latter case, simulation shows that one can expect around 99% of the public decisions taken to be ex post efficient when the number of agents is larger than 5.

3 Experimental design

To investigate the empirical properties of the multibidding game, computerized sessions were conducted at the Universitat Jaume I in Castellón and at the Universitat Pompeu Fabra in Barcelona. We have invited 20 and 16 participants, respectively, to take part in the experiment. Sessions lasted less than two hours and the average net pay, including EUR 3 show-up fee, was about EUR 20 per subject and session.

The experiment was programmed and conducted with the software z-Tree (Fischbacher, 1999). We implemented two treatments in each session. At the beginning of each treatment, printed instructions were given to subjects and were read aloud to the entire room. Instructions explained all rules to determine the resulting payoff for each participant. They were written is Spanish, contained a numerical example to illustrate how the program works, and presented pictures of each screen to show up. The English translation of the instructions, without pictures, can be found in Appendix C.

At the start of each round the computer randomly assigned subjects to groups. We applied stranger treatment, that is participants were not informed about who the other members of their group were. Also, the assignment was done every period, hence participants knew that the groups were typically different from period to period. Subjects were not allowed to communicate among themselves, the only information given to them in this respect was the size of the group. In the first treatment of each session groups of two were formed, while in the second treatment groups of ten (in Castellón) and eight (in Barcelona) were constituted.

Private valuations for the two projects at each round were assigned to subjects by the computer in a random manner. We used the built-in function of z-Tree to generate random draws from the $U[0; 300]$ uniform distribution. For this reason, valuations for the alternatives were typically different in each round and for each subject. Treatments consisted in 3 practice and 20 paying rounds. Table 1 summarizes the features of the four treatments.

For computational convenience, numbers (valuations, bids, and gains) used in the experiment were rounded to integers. Since our objective had been to verify theoretical results on the multibidding game in an environment where agents hold private information and common prior beliefs, we dedicated a paragraph in the instructions to explain the nature of the uniform distribution.[5]

In each round, participants were asked to enter their bids over the two projects. Taking into account the rules of the multibidding game, the winning project was determined and payoffs were calculated automatically by the computer.[6] At the end of each round, subjects received on-screen information about the aggregated bid of other players in the same group; and also detailed information about the determining components of the personal final payoff. The history of personal earnings was always visible on screen during the experiment.

At the end of each session participants were paid individually and privately. Final profits were computed according to a simple conversion rule, based on the personal gains in experimental monetary units during the whole session.

4 Results

4.1 Efficiency

The multibidding game achieved efficiency in the large majority of decision problems, as it picked out the ex post efficient public projects roughly in 3/4 of the cases across the four experimental treatments. Table 2 contains detailed information on efficiency for each treatment and also presents the 90% confidence intervals around the data. Theoretical result on the multibidding game refer to the number (or proportion) of ex post efficient public decisions when talking about efficiency. Nevertheless, a different measure can be constructed to capture the empirical efficiency of the mechanism that also takes into account the magnitude of the efficiency loss when an inefficient project is chosen. We call

[5]Although theoretical results are provided for a wide range of probability distributions, we had chosen the uniform. We thought that this one would be the most intuitive and simplest to explain to subjects who are not familiar with probability theory. We followed the example of Binmore *et al.* (2002) in the instructions.

[6]In case of a tie, the program breaks the tie choosing the project randomly assigning equal probability to the alternatives.

it realized efficiency (RE) and define it as

$$RE = \frac{\sum_{i\in\mathcal{N}} x_i^{winning\ project}}{\max\left\{\sum_{i\in\mathcal{N}} x_i^1, \sum_{i\in\mathcal{N}} x_i^2\right\}} \cdot 100 \text{ percent.}$$

Table 2c reports the realized efficiency with the 90% confidence interval. The point estimates are above 90% in all of our treatments.

In order to extend the efficiency analysis we estimated a Logit model for each treatment, trying to establish some empirical relation between the probability of an efficient decision and the absolute difference between the two projects. Table 2d contains the estimation results, and shows that the larger the difference between the projects, the higher the probability of an efficient decisions in treatments B1 and C1. That is, observed inefficiencies tended to occur in cases in which the projects were similar, causing a relatively small drop in realized efficiency. In the remaining two treatments we can not identify any significant relationship of the above type.

Due to the small number of experimental sessions, we can not establish the empirical ranking of group sizes according to efficiency. Nevertheless, it is worth to note that our treatments with groups of two do not give statistically different results on efficiency. Operating with groups of eight, in treatment B2, the multibidding game performed significantly better than with groups of ten in C2. Simulation results suggest that the efficiency change caused by an increase of group size from eight to ten is minimal in theory. And according to theory we would expect this minimal change to be a gain, in spite of the loss we observed in the experiments. We explain this observed feature with differences in the subject pool and attribute it to different individual bidding behavior from one session to the other. Before starting with the empirical analysis of the bidding function, let us point out that we could not identify any significant linear time trend in the evolution of efficiency over the 20 paying periods (see Table 3).

4.2 Bidding behavior

We have used experimental data from two sessions and a simple linear model to estimate the empirical bidding function. The linear approximation is the strongest available theoretical result for the bidding behavior with decision problems that involve more than two agents. In case of groups of two, the optimal bidding function shows pronounced curvature at the extremes of the support. Since we generated private valuation according to a uniform distribution in the experiments, i.e., the theoretical distribution of the difference was triangular, we do not have many observations on the positive and negative ends of the support and could not give significant estimates for the curvature. Moreover, the linear specification allows for a single expression that approximates optimal bids as a function of group size and the difference between private valuations. In Appendix A we show that the first-order Taylor-approximation - around zero - of the optimal bidding function has slope 1/3. For this reason, in this analysis we shall treat Expression (2) as the theoretical optimal bidding function in all of our treatments. The maximin bidding behavior can also be characterized by a linear function in the multibidding game. That function has slope 1/2 independently from the group size.

We have estimated two linear specifications of the bidding function:

$$\widehat{B}_i = \widehat{\alpha}_1 + \widehat{\beta}_1 x_i^1 + \widehat{\beta}_2 x_i^2, \qquad (3)$$

$$\widehat{B}_i = \widehat{\alpha}_2 + \widehat{\beta} d_i. \qquad (4)$$

Equation (3) represents a linear bidding function that does not force bids to depend solely on the difference between private valuations, while equation (4) does. The dependent variable in both specification is the bid submitted for project 2. Recall that in the original theoretical model the function B stands for bids for project 1. This switch is due to the following: theoretical models deal with positive bids as amounts that agents are willing to pay; nevertheless in the experiments we asked subjects to type in a negative number in case they were willing to pay for a given project and a positive one in the opposite case. Since the multibidding game operates both with positive and negative bids we thought that in this way concepts might be more intuitive for people participating in the sessions.

Tables 4a through 8a contain the OLS estimates (with indexes for significance of the results) of the empirical bidding functions both individually for each subject and jointly for the subject pool across different treatments.

When considering treatments globally, at 5% significance level we can not reject the hypothesis that subjects decide their bids taking into consideration only the difference between their private valuations for the two public projects. That is, the empirical results are in accordance with Proposition 1, in the sense that individuals seem to "report" (through the bids) their relative valuations of the two projects. Moreover, this is a robust result, since it holds in each of the four treatments that we implemented.

We now turn our attention to the fit between the experimental data and propositions 2 and 3, which state the expressions for the equilibrium bids as function of the distance between the valuations. Table 4a provides the estimated bidding functions. For treatments C1 and C2, these functions are not different statistically from the theoretical ones, i.e. they are proportional with slopes (statistically) equal to 1/3 and 0.26 respectively. Estimates for the other two treatments, B1 and B2, are more precise in that we obtained a better fit with smaller variance of the estimates. For B2 the constant term turns out to be significantly different from zero at 5%, but its estimated absolute value, 1.45, is very small compared to the magnitude of the private valuations, $[0; 300]$, used in the experiment. In the latter two treatments, subjects seem to have bid according to a linear function, though more conservatively than predicted by theory: the small variance of the estimates confirms that bidding behavior can be approximated by a simple linear function with slopes 0.22 for B1 and 0.20 for B2, significantly less than 1/3 and 0.27, respectively.

The individual analysis of bidding offers a deeper insight into the above pooled results and their consequences on the number of ex post efficient public decisions. We have estimated the two linear models in Equation (3) and (4) for each subject separately, and performed the same tests that we have done for the subject pools. Detailed estimation results are to be found in Appendix B. In order to have a structured summary of the subject pool we have grouped agents into three groups based on the estimated slope coefficient of the empirical bidding function. Table 9 shows that the largest part of our subjects falls into the two strategy groups studied by theory, i.e. maximin with slope 1/2 and Bayes-Nash with slope either 1/3 or $n/(4n-2)$.

Bidding in treatment B1, in spite of being the most efficient among the four, seems to be difficult to explain at first sight with the latter two types of strategies. As mentioned before, in B1 subject formed their bids linearly, but less aggressively than predicted by theory in Bayes-Nash equilibrium. This is why we split the residual category of Other in Table 9 into two: linear bidding behavior based on the difference between private valuations and other kind of behavior we can not account for.[7] The distinction clearly improves statistics presented in Table 10a for our Barcelona session, and leaves at most 30% of the subjects as *irrational*.

Different reasons may explain why a sensible share of the subjects bid less aggressively than predicted by the bayes-Nash equilibrium. Agents, for example, may form their bids according to the symmetric Bayes-Nash equilibria of the multibidding game, but perceive uncertainty in a biased way. Therefore, the bidding function $B(d_i) = 0.2 \cdot d_i$ may be optimal. It turns out that this is the case under uncertainty characterized by the distribution function $\widetilde{F}(d_i) = \frac{1}{2} \cdot \left[1 + \left(\frac{d_i}{300}\right)^{\frac{1}{3}}\right]$ over the interval $[-300; 300]$.[8] The comparison of this and the underlying true triangular distribution, presented numerically in Table 12, shows that participants possibly overweighted high-probability events and underweighted the low-probability ones. The distribution defined by $\widetilde{F}(d_i)$ is symmetric around zero, has a smaller standard deviation than the triangular and it is more peaked

[7] The latter category includes some subject that handed in their bids independently from the difference between their private valuations, and some that we have estimated negative slope coefficient for. Table 10 has been built at 1% significance level, but results do not change in the Other category if we move to 5%, either.

[8] We do not provide the proof of this result here, but it is available upon request.

around zero.[9]

Interestingly, these subjects who bid in a linear way, but did not follow the Bayes-Nash strategy, did very well in terms of (ex post) profits in every treatment. Table 10c shows the mean payments in four bidding categories. Subjects in the Other category were the ones who gained less, even though the difference between the first three and the fourth category is not significant statistically in our Barcelona treatments, at any usual significance level.

Bidding less aggressively is also self-consistent in the following sense: if players, in groups of two, are applying linear bidding functions and believe that their opponents bid according to $B(d_i) = 0.2 \cdot d_i$, they maximize their expected payoff by bidding slightly more for any given d_i. The best response in this example is $B(d_i) \approx 0.222 \cdot d_i$ and very well may explain the observed behavior.[10] In order to understand and provide further support for a more conservative empirical bidding function in B1 and B2, we have also studied whether subjects could have been better off applying the Bayes-Nash bidding function against the others' observed behavior. Taking into account those who bid in a linear way, but significantly different from the predicted one by theory, we get that the Bayes-Nash bidding function (ceteris paribus) could have improved their gains only moderately: by 1.23% in B1 and 2.56% in B2. That is, facing the others' bids these participants did not have enough incentives to abandon their bidding function and play Bayes-Nash instead.

It is important to point out that in Castellón we encountered subjects bidding safe according to maximin strategies. This feature of the observed behavior, along with more conservative bidding in B1 and B2, gives partial explanation for the reported efficiency rates, too. The available theoretical results deal with symmetric equilibria. An important part of the ex post efficiency of the multibidding game is due to the fact that agents bids according to the same theoretical function. The heterogeneity of the subject pool in C1 and C2 appears also in the observed efficiency loss. In treatments B1 and B2, even though subjects do not play strictly Bayes-Nash, the number of ex post efficient decisions is larger because the subject pool was more homogeneous.[11]

We have discussed above that the available data set is not large enough to deliver

[9] This finding is in line with those presented by Harbaugh *et al.* (2002) who examine how risk attitudes change with age. The ages of participant in their experiments range from 5 to 64. They observe that young people's choices are consistent with the underweighting of low-probability events and the overweighting of high-probability ones, and that this tendency diminishes with age. Participants in our sessions were university student with approximately 20 years of age.

[10] This numerical result follows directly from the expected utility maximization problem with the triangular distribution.

[11] A measure for homogeneity could be the (length of the) range of our estimates for the slope coefficient of the bidding function according to Equation 4: C1 - $[-0.43; 1.82]$; C2 - $[-0.85; 1.18]$; B1 - $[0.05; 0.036]$; B2 - $[0.02; 0.35]$.

empirical evidence for the curvature of the bidding function. This curvature is responsible, as theory predicts, for the occurrence of ex post inefficient decisions once the group size is larger than two. Unfortunately we can not present empirical proofs for this feature, nevertheless we can explore statistically how bidding behavior changes when the group size (and with it uncertainty) increases. As a response to this, according to theory, the slope of the Bayes-Nash bidding function should decrease. We can verify a change in this direction looking at the estimated bidding function for the whole subject pool both in Barcelona and Castellón. This drop is significant at 5% in Castellón, while it is not in Barcelona.

Table 11 offers a summary of the individual data in this respect. The estimated slope coefficient of the individual bidding function decreases in 45% of the cases in Castellón, and in 63% in Barcelona. Though, the vast majority of these estimated changes is not significant individually at 5% or 10%.[12]

Subjects were asked to decide over public projects and their alternative in 20 paying rounds. Even though private valuations were different from round to round, according to the underlying uniform probability distribution, one might expect that participants get trained and gain experience in each treatment. In order to show possible learning effects we have split every data set into two,[13] and estimated the individual bidding function (according to Equation 4) separately for the subsamples. Tables 4b through 8b offer the estimation results. For a quick view consider Table 10b in which we repeated the categorization of bidding behavior taking into account four groups. Unexpected bidding behavior, i.e., frequencies in the Other category, barely or do not change from the first to the last 10 playing rounds. In treatment C1 three subjects, while in treatments C2 and B1 one and one subject seem to adjust their bidding behavior to the one predicted by theory.

5 Conclusion

In this paper we have studied the empirical properties of the multibidding game under uncertainty described by Veszteg (2004). The results of our four treatments, with two projects to choose between, show that the mechanism performs well in the laboratory. We find that the one-shot multibidding game with its simple rules succeeds in extracting private information from agents, as the observed bids were formed taking into account relative private valuations between two projects.

[12]When fixing the significance level at 15% the only change in Table 11 is that a difference into the unexpected direction becomes significant for a subject in Castellón.

[13]We wanted to form two independent data set for each subject and treatment. Taking into account our relatively small sample size we decided no to eliminate any observation from the analysis.

Though not all participants followed the Bayes-Nash equilibrium predicted by theory, the mechanism gave rise to ex post efficient outcomes in almost 3/4 of the cases across the treatments. Apart from the expected utility maximizing Bayes-Nash behavior we could identify bidding behavior according to the safe maximin strategies in one of our sessions. Unfortunately our sample size, due to feasibility constraints in the laboratory, does not allow for verifying theoretical predictions for large groups in a significant way. More subjects and more repetitions are needed to possibly reduce the observed variance of the data and study those effects.

Our data set does not contain any significant linear trend in time. Neither if we consider global efficiency or in the case of individual bidding behavior. A longer time series would also be able to show whether the rules of the multibidding game are simple enough to understand, or learning indeed plays an important role in the performance of the mechanism.

It is important to point out that a considerable fraction of participants (especially in the Barcelona treatments) applied linear bidding function based on their relative valuations, though they bid less aggressively than expected in theory. Since they did well in monetary term among all the participants and did not harm ex post efficiency, we suggest to obtain theoretical results for economies in which there are several groups (types) of agents: some play maximin strategies, some Bayes-Nash. Beside the expansion of theoretical work on the multibidding game, undoubtedly also more empirical research is needed to explore its empirical performance. We think that further experiments can help to identify features that allow for designing successful practical mechanisms.

6 Appendix A. Optimal bidding behavior

The triangular distribution over the interval $[-300; 300]$ of our experimental design can be characterized by the following density function:

$$f(x) = \left\{ \begin{array}{cc} 0 & x \notin [-300; 300] \\ \frac{1}{90000}x + \frac{1}{300} & x \in [-300; 0] \\ -\frac{1}{90000}x + \frac{1}{300} & x \in [0; 300] \end{array} \right\},$$

and cumulative density function:

$$F(x) = \left\{ \begin{array}{cc} 0 & x \in (-\infty; -300) \\ \frac{1}{180000}x^2 + \frac{1}{300}x + \frac{1}{2} & x \in [-300; 0] \\ -\frac{1}{180000}x^2 + \frac{1}{300}x + \frac{1}{2} & x \in [0; 300] \\ 1 & x \in (300; \infty) \end{array} \right\}.$$

It is symmetric to the origin and for this reason its median is zero. By substituting the above function into equation (1), we have that the optimal bidding function in our example can be written as:

$$B_i(d_i) = \left\{ \begin{array}{cc} \frac{1}{2}d_i + \frac{-600\,000d_i - 1500d_i^2 - d_i^3}{12\,000d_i + 10d_i^2 + 3600\,000} & \text{if } d_i < 0 \\ 0 & \text{if } d_i = 0 \\ \frac{1}{2}d_i + \frac{-600\,000d_i + 1500d_i^2 - d_i^3}{10d_i^2 - 12\,000d_i + 3600\,000} & \text{if } d_i > 0 \end{array} \right\}.$$

If we consider the first-order Taylor-approximation of this resulting bidding function around zero, we have $B_T(d_i) = \frac{1}{3}d_i$.

7 Appendix B. Results

Treatment	Number of groups	Group size	Uncertainty	Practice periods	Paying periods
C1	10	2	$U[0;300]$	3	20
C2	2	10	$U[0;300]$	3	20
B1	8	2	$U[0;300]$	3	20
B2	2	8	$U[0;300]$	3	20

Table 1. Treatment summary.

Treatment	C1	C2	B1	B2
Efficient decisions	72%	58%	81%	70%
Upper bound	77%	71%	86%	82%
Lower bound	67%	44%	76%	58%

Table 2a. Proportion of efficient decisions with 90% confidence interval.

Treatment	C1	C2	B1	B2
First 10 - Efficient decisions	73%	45%	84%	75%
First 10 - Upper bound	80%	64%	91%	91%
First 10 - Lower bound	66%	26%	77%	59%
Last 10 - Efficient decisions	71%	70%	79%	65%
Last 10 - Upper bound	79%	87%	86%	83%
Last 10 - Lower bound	63%	53%	71%	47%

Table 2b. Proportion of efficient decisions with 90% confidence interval

for the first and last 10 rounds.

Treatment	C1	C2	B1	B2
Realized efficiency	91%	94%	96%	95%
Upper bound	94%	100%	98%	100%
Lower bound	88%	87%	93%	89%

Table 2c. Realized efficiency with 90% confidence interval.

Treatment	C1	C2	B1	B2
Constant term	0.30	0.57^*	0.23	0.99^{**}
$\|d_i\|$	0.005^*	0.00	0.01^*	0.00

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

Table 2d. Estimated coefficients of the impact of the absolute difference between private valuation on the probability of an efficient decision (Logit)

	Treatment			
Round	C1	C2	B1	B2
1	60%	50%	88%	100%
2	60%	50%	100%	0%
3	70%	50%	75%	100%
4	80%	50%	88%	100%
5	80%	100%	63%	50%
6	80%	0%	75%	100%
7	70%	50%	75%	50%
8	70%	100%	100%	100%
9	70%	0%	88%	50%
10	90%	0%	88%	100%
11	60%	50%	88%	0%
12	70%	100%	88%	0%
13	90%	50%	75%	50%
14	60%	100%	63%	100%
15	50%	0%	88%	100%
16	70%	50%	75%	100%
17	70%	100%	63%	50%
18	80%	50%	75%	100%
19	80%	100%	100%	50%
20	80%	100%	75%	100%

Table 3. Proportion of efficient decisions per round.

Treatment†	Constant term	Slope coefficient		Constant term	Slope coefficient
		Project 1	Project 2		
C1	-2.22	0.37*	-0.36*	-0.84	0.37*
C2**	-22.80*	0.30*	-0.18*	-5.95***	0.24*
B1	-0.32	0.21*	-0.22*	-0.80	0.22*
B2	1.12	0.20*	-0.20	1.45*	0.20*

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

†Diff. between the absolute value of slope coefficients for the two projects stat. significant.

Table 4a. Bidding functions (OLS).

	First 10	First 10	Last 10	Last 10
Treatment	Constant term	Slope coefficient	Constant term	Slope coefficient
C1	-7.94	0.37*	6.29	0.36*
C2	-7.63	0.22*	-4.11	0.27*
B1	0.18	0.22*	-1.79	0.21*
B2	-0.15	0.19*	3.31**	0.21*

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

Table 4b. Bidding functions for the first and last 10 rounds (OLS).

Treatment C1					
Subject†	Constant term	Slope coefficient		Constant term	Slope coefficient
		Project 1	Project 2		
1	-5.53	0.08	-0.28	-32.68**	0.22
2***	-48.14***	0.57*	-0.27*	-3.37	0.40*
3	15.72	0.03	-0.22	-9.97	0.12
4	21.05	0.05	-0.21***	-1.39	0.14***
5	-6.85	0.62*	-0.54*	1.79	0.57*
6	-16.78	-0.39*	0.48*	-3.25	-0.43*
7	37.67	0.62*	-0.66*	31.57*	0.64*
8	-35.39	0.47*	-0.20***	3.71	0.31*
9	34.36	1.19*	-1.31*	15.39	1.26*
10	9.10	-0.06	-0.01	-0.37	-0.02
11	12.00	-0.41	0.19	-17.47	-0.31**
12	-7.50	0.38*	-0.39*	-8.58	0.39*
13**	-92.31**	0.13	0.40*	-7.14	-0.20
14	4.98	0.16*	-0.22*	-4.79	0.19*
15	127.87	1.41*	-2.24*	-10.73	1.82*
16	4.72	0.13**	-0.15**	1.50	0.14*
17	-35.71	0.50*	-0.05	31.22***	0.28***
18	-13.7	0.25***	-0.24***	-11.29	0.24*
19	10.84	0.19***	-0.24**	3.28	0.22*
20	-6.14	0.17	-0.08	5.87	0.12

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

†Diff. between the absolute value of slope coefficients for the two projects stat. significant.

Table 5a. Individual bidding functions (OLS).

Treatment C1				
	First 10	First 10	Last 10	Last 10
Subject	Constant term	Slope coefficient	Constant term	Slope coefficient
1	-13.12	0.00	-29.26	0.48**
2	-16.96	0.43*	14.37	0.33*
3	-22.88	0.04	3.06	0.24**
4	8.60	0.08	-18.98	0.30**
5	4.80	0.60*	2.44	0.55*
6	6.85	-0.55*	-10.25	-0.40*
7	43.76***	0.80*	31.56***	0.53*
8	-0.97	0.32***	8.55	0.31*
9	-9.98	1.18*	42.04	1.23*
10	0.22	-0.21**	0.67	0.21**
11	-23.09	-0.36***	-13.93	-0.26
12	-11.83	0.42*	0.26	0.25*
13	19.45	-0.06	-36.91	-0.28
14	-6.20	0.21*	-3.36	0.15***
15	40.11	1.78*	-66.13	2.15*
16	9.75	0.10	-5.41	0.15*
17	-0.46	0.21*	52.00	0.57
18	-46.61*	-0.06	26.98**	0.46*
19	-13.69	0.16	19.63	0.25**
20	30.48***	0.52*	-0.25	0.03

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

Table 5b. Individual bidding functions for the first and last 10 rounds (OLS).

Treatment C2					
Subject†	Constant term	Slope coefficient		Constant term	Slope coefficient
		Project 1	Project 2		
1	20.01	0.07	-0.21	-1.40	0.12
2*	-43.94**	0.71*	-0.38*	5.30	0.55*
3***	34.48	0.43*	-0.72*	-8.46	0.60*
4	-29.39	-0.68**	0.98*	12.95	-0.85*
5	24.00	0.40*	-0.67*	-20.14***	0.52*
6	-11.13	0.19*	-0.04	12.91***	0.12*
7	40.86	0.23*	-0.47*	-0.73	0.36*
8*	-100.79*	0.79*	-0.22***	-1.01	0.52*
9	0.79	0.13**	-0.14**	-0.79	0.14*
10*	36.20***	0.16**	-0.49*	-12.42	0.28*
11*	-39.60	0.38*	-0.13**	-2.50	0.28*
12	2.33	0.18**	-0.17**	4.04	0.18*
13	-22.9**	0.13*	-0.02	-7.46	0.07
14*	-121.01**	1.51*	-0.70*	-5.48	1.18*
15	-84.13**	0.40*	0.01	-30.58***	0.28**
16	-0.14	0.22*	-0.23*	-2.23	0.22*
17*	84.87	-0.19	-0.76*	-51.86*	0.36
18	-30.75	0.45**	-0.26	-1.26	0.35**
19	-67.73	0.03	0.50*	7.87	-0.27*
20	2.00	0.21*	-0.25*	-3.23	0.23*

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

†Diff. between the absolute value of slope coefficients for the two projects stat. significant.

Table 6a. Individual bidding functions (OLS).

Treatment C2				
	First 10	First 10	Last 10	Last 10
Subject	Constant term	Slope coefficient	Constant term	Slope coefficient
1	-5.74	-0.11	-5.40	0.46
2	3.21	0.37***	21.09**	0.72*
3	-12.47	0.64*	-6.49***	0.57*
4	21.44	-1.07*	-3.05	-0.66*
5	-20.93	0.55*	-20.54	0.50*
6	20.18	0.13	4.51	0.08**
7	-7.63	0.50*	8.55	0.24*
8	0.02	0.52*	-2.12	0.51*
9	3.00	0.17**	-5.56	0.10*
10	-14.11	0.35*	-8.96	0.22***
11	2.17	0.28*	-7.06	0.27*
12	0.26	0.01	1.94	0.28*
13	-7.27	0.14	-9.93	0.02
14	-49.76	0.97*	32.22	1.15*
15	-64.64*	0.37**	3.89	0.39***
16	2.36	0.23*	-7.01	0.21*
17	-104.40*	0.13	-8.45	0.37
18	27.09	0.43**	-37.06	0.42
19	6.91	-0.38**	-0.49	-0.09
20	-2.09	0.22*	-5.38	0.25*

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

Table 6b. Individual bidding functions for the first and last 10 rounds (OLS).

Treatment B1					
Subject†	Constant term	Slope coefficient		Constant term	Slope coefficient
		Project 1	Project 2		
1	1.53	0.11	-0.14**	-3.60	0.13*
2	9.29	0.20*	-0.23*	5.44	0.22*
3	-10.76	0.07	-0.04	-6.25	0.05
4	1.78	0.26*	-0.26*	2.09	0.26*
5	2.79	0.17*	-0.17*	1.74	0.17*
6	6.52	0.22*	-0.25*	1.00	0.23*
7	-1.48	0.25*	-0.22*	0.59	0.23*
8	2.02	0.35*	-0.37*	-1.71	0.36*
9	3.97	0.20*	-0.19*	4.58	0.19*
10***	-24.41	0.36*	-0.22*	-3.57	0.29*
11	-4.82	0.24*	-0.22*	-2.39	0.22*
12	8.46	0.23*	-0.36*	-11.00***	0.31*
13	10.42	0.10	-0.11	8.58	0.10***
14	10.67	0.17**	-0.28*	-3.61	0.23*
15	-14.14	0.31*	-0.31*	-14.82	0.31*
16	3.03	0.07	-0.07	3.18	0.07

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

†Diff. between the absolute value of slope coefficients for the two projects stat. significant.

Table 7a. Individual bidding functions (OLS).

Treatment B1				
	First 10	First 10	Last 10	Last 10
Subject	Constant term	Slope coefficient	Constant term	Slope coefficient
1	-13.66	0.22*	3.45	0.08
2	10.52	0.15*	2.92	0.35*
3	-5.01	0.00	-6.40	0.11***
4	2.80	0.25*	-1.33	0.29*
5	-0.04	0.16*	3.93	0.21*
6	2.52	0.22*	0.47	0.23*
7	7.14	0.22*	-5.16	0.28*
8	2.01	0.38*	-5.58**	0.36*
9	-0.21	0.29*	4.65***	0.13*
10	-5.28	0.31*	-0.26	0.25*
11	-0.56	0.20*	-4.16	0.23*
12	3.95	0.32*	-27.36*	0.36*
13	10.37	0.27**	3.08*	0.02*
14	-0.84	0.23*	-6.57	0.24*
15	-4.16	0.26*	-26.86	0.30***
16	-2.43	0.15*	8.28	0.01

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

Table 7b. Individual bidding functions for the first and last 10 rounds (OLS).

Treatment B2					
Subject†	Constant term	Slope coefficient		Constant term	Slope coefficient
		Project 1	Project 2		
1	3.90	0.21*	-0.20*	6.46	0.21*
2	0.24	0.19*	-0.19*	0.99	0.19*
3	5.98	0.03	-0.05	1.17	0.04***
4	2.07	0.25*	-0.26*	-0.52	0.26*
5	-0.99	0.26*	-0.24*	1.65	0.25*
6	-0.40	0.14*	-0.14*	0.58	0.14*
7	-16.10	0.30*	-0.19*	-0.77	0.25*
8	0.53	0.36*	-0.35*	2.51	0.35*
9	-4.85	0.30*	-0.23*	4.21	0.27*
10	19.06	0.16*	-0.26*	3.87	0.21*
11*	18.53	0.12	-0.35*	-17.90*	0.28*
12	-7.61	0.25*	-0.21*	-1.47	0.23*
13	8.34	0.03***	-0.07*	2.12	0.05*
14	-25.17**	0.33*	-0.19*	-3.41	0.25*
15	-10.25	0.25*	-0.13**	8.70***	0.21*
16***	-9.24	0.05**	0.00	-2.31	0.02

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

†Diff. between the absolute value of slope coefficients for the two projects stat. significant.

Table 8a. Individual bidding functions (OLS).

Treatment B2				
	First 10	First 10	Last 10	Last 10
Subject	Constant term	Slope coefficient	Constant term	Slope coefficient
1	-0.22	0.18**	13.15	0.23*
2	-0.67	0.25*	1.17	0.18*
3	-0.33	0.04***	2.63	0.03
4	7.74	0.30*	-7.53	0.22*
5	2.32	0.24*	-1.29	0.28*
6	-0.50	0.15*	-1.59	0.13*
7	-0.41	0.24*	-0.87	0.26*
8	1.11	0.39*	2.63	0.33*
9	-1.19	0.21*	10.02	0.32*
10	-4.37	0.15*	11.47	0.29*
11	-8.15	0.17	-25.99**	0.38*
12	0.93	0.30*	-2.70	0.19*
13	1.77	0.05*	2.50	0.05**
14	-18.41**	0.28*	10.56***	0.27*
15	17.84**	0.16**	1.07	0.21*
16	-4.79	0.01	1.77	0.06***

*Stat. significant at 5%; **Stat. significant at 10%; ***Stat. significant at 15%.

Table 8b. Individual bidding functions for the first and last 10 rounds (OLS).

	C1	C2	B1	B2
Maximin	10%	20%	0%	0%
Bayes-Nash	50%	55%	50%	63%
Other	40%	25%	50%	37%
Total	100%	100%	100%	100%

Table 9. Observed strategies grouped into theoretical categories at 1% significance level.

	C1	C2	B1	B2
Maximin	10%	20%	0%	0%
Bayes-Nash	50%	55%	50%	63%
Other linear bidding	10%	5%	38%	19%
Other	30%	20%	13%	19%
Total	100%	100%	100%	100%

Table 10a. Observed strategies grouped into four theoretical categories at 1% significance level.

	C1		C2		B1		B2	
	First 10	Last 10	First 10	Last 10	First 10	Last 10	First 10	Last 10
Maximin	25%	25%	30%	35%	0%	0%	0%	0%
Bayes-Nash	35%	50%	50%	35%	69%	56%	69%	69%
Other linear bidding	5%	5%	5%	10%	31%	38%	13%	13%
Other	35%	20%	15%	20%	0%	6%	19%	19%
Total	100%	100%	100%	100%	100%	100%	100%	100%

Table 10b. Observed strategies grouped into four theoretical categories at 1% significance level for the first and last 10 rounds.

	C1	C2	B1	B2
Maximin	8.89	8.93	*	*
Bayes-Nash	8.87	7.99	8.38	8.17
Other linear bidding	10.28	8.73	9.42	9.02
Other	6.92	4.90	7.94	8.02

Table 10c. Mean payment (without show-up fee in EUR) according to the four theoretical equilibrium categories.

	Change in	Direction of significant* change		Not significant*
	expected direction	expected	unexpected	change
Castellón	45%	25%	20%	55%
Barcelona	63%	6%	6%	88%

*Significance at 5% and 10%.

Table 11. Change in the slope of the empirical bidding function due to group size.

Distribution	Mean	St.deviation	Kurtosis	Skewness
$\widetilde{F}$	0	113.39	3.78	0
Triangular $[-300; 300]$	0	212.13	0.27	0

Table 12. Comparison between the triangular distribution and $\widetilde{F}$

Graph 1. Optimal Bayes-Nash and maximin bidding function for groups of two.

8 Appendix C. Instructions

8.1 First treatment

Thank you for participating in the experiment.

This session has 3 practice periods and other 20 that will determine a part of the amount of money that you will receive by the end of the experiment.

In each game groups of two will be formed in a random manner. Your task is to make decisions on your own and for this reason you are not allowed to talk to other participants. Games have a unique stage in which you will have to choose between two projects (project 1 and project 2). The resulting choice will influence the benefit you obtain in each period.

The first screen will inform you about the value each project has for you. The table on the left, in this example, shows that if project 1 is chosen you receive 33 monetary units; while if project 2 is chosen you receive 128 monetary units. These values are integer numbers between 0 and 300, and are assigned randomly in each game, such that every number has the same probability to be picked out. For this reason, these values are typically different for each player and for each project.

The other player in your group receives similar information on the values that each project has for him/her. You do not know the value of the projects for the other player, not even which project he/she prefers. He/she does not know the value of the project for you either. The only information in this aspect is the following:

The value of each project is an integer number between 0 and 300 (including limits) for each player. Each value within the limits occurs with the same probability. A common question is: what does it mean that each value occurs with the same probability? Suppose that we have a roulette wheel with 301 slots of equal size, numbered from 0 to 300. The ball in this case will stop with equal probability at each slot. In the experiment, the four values – for project 1 and project 2 for both players – are assigned using a similar method, with the help of the computer.

In our example, chance has assigned the values 250 and 102 for projects 1 and 2, respectively, for the other player.

The project is chosen through an auction especially designed for this occasion, according to which you have to decide how many monetary units you are willing to pay for project 1, for example, to be chosen. It is also possible that you prefer project 2 and for this reason if project 1 is chosen you wish to receive some amount of compensation. In the auction you have to choose two bids (one for each project) that must sum up to zero. Negative numbers will indicate the amounts you are willing to pay, while positive numbers the amounts that you wish to receive. Suppose that you are willing to pay 10 units if project 1 is chosen and you would like to receive 10 if project 2 is chosen. In this case you have to type the number -10 and 10 in the purple cells of the table on the right hand side; and after that click on the "OK" button to continue.

Let us suppose that the other player decides to bid -25 for project 1 and 25 for project 2. With this project 1 receives a total of -35 bids, while project 2 gets 35.

The project with more negative bids is chosen to carry out. In case of a tie the result is determined randomly. Bids for the chosen project will be paid / received and the aggregated bid will be given back to the members of the group in equal shares. When all of you have chosen your bids, a screen appears with the results.

The right side of the screen with the results informs you about the other player's bids. In our example project 1 has received $(-10) + (-25) = (-35)$ bids, while project 2 has received $10 + 25 = 35$. Project 1 is chosen. Your profit in the game appears on the left part of the results screen. In this case it is computed as follows:

- you receive 33 units, because project 1 has been chosen,

- you have to pay your bid for this project, that is 10 units, and
- you receive half of the aggregated bid, 17.5 units

Summing up: $33 - 10 + 17.5 = 40.5$ monetary units. The other player in the example earns $250 - 25 + 17.5 = 242.5$ units.

If you click on the "OK" button of the results screen, the game ends.

A table down on the left hand side keeps you informed about your profit obtained during the whole session. 400 monetary units are equal to 1 euro. For any computation you might want to perform, you may use the Windows calculator by clicking on its icon next to the "OK" button.

8.2 Second treatment

In this session, we will use the game from the previous session, but with one modification. The groups that form randomly in each game will have 8 members (not 2 as in the first session). Each group of 8 will choose a common project.

The auction to be used is the same. Your task is to make decisions on your own and for this reason you are not allowed to talk to other participants. Your principal task is to choose a project between two alternatives. The value of each project for each player is assigned in a random manner, therefore these values can be equal to any integer number between 0 and 300 (including limits), and each occurs with the same probability.

There will be 3 practice periods and other 20 that will determine a part of the amount of money that you will receive by the end of the experiment

The computer screens you will see are identical to the ones you have seen before except for one detail. On the results screen the aggregated bid of the other players in your group will appear.

The table on the left informs you about bids in the auction. Following the example in the instructions, let us suppose that you are willing to pay 10 monetary units if project 1 is chosen, and wish to receive 10 units if project 2 is chosen. The column of the other players' bid in this example indicates that the bids of the other 7 members of your group or project 1 sum up to -135 monetary units. The seven bids for project 2 sum up to 135.

Taking into account your bids, the aggregated bid for the projects are -145 and 145, respectively. For this reason, project 1 is chosen and you earn 37.5 monetary units: 33 (the value of project 1 for you) -10 (your bid for project 1) $+14.5$ (your share from the aggregated bid).

400 monetary units are equal to 1 euro. For any computation you might want to perform you may use the Windows calculator by clicking on its icon next to the "OK" button.

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