2次元QEDと弦理論

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based on arXiv:1812.10064 with Adi Armoni

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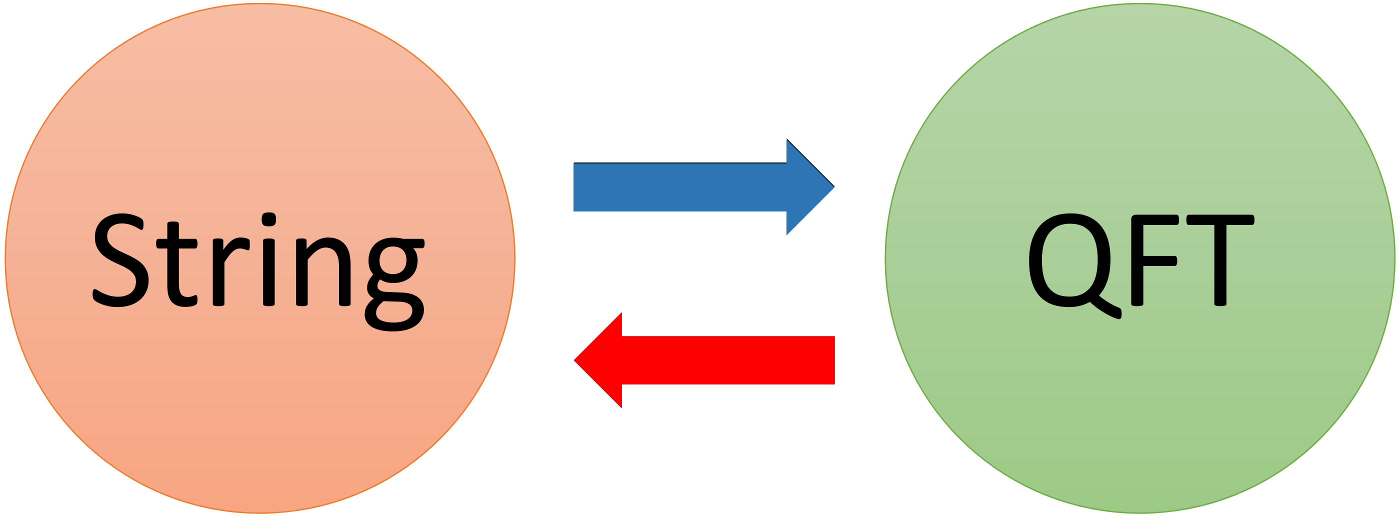
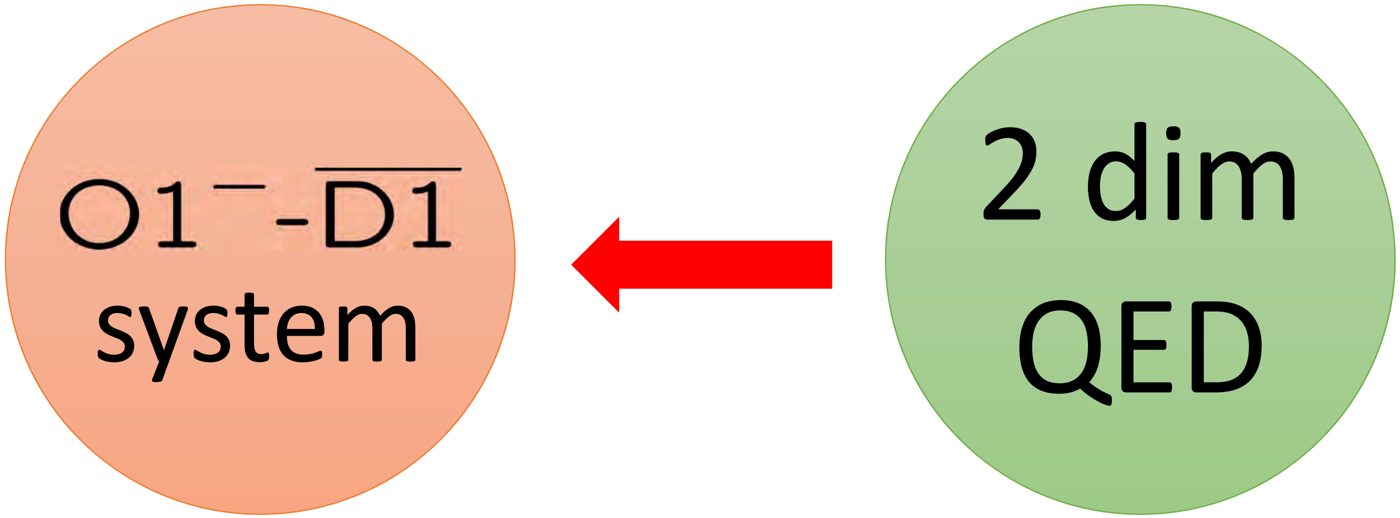
1 Introduction



We can use duality (S-duality, M-theory lift, holography etc.) in string theory to study QFT

Analysis in QFT can be applied to understand non-perturbative phenomenon in string theory ← This talk

Today, we consider 2 dim QED to study non-perturbative brane dynamics



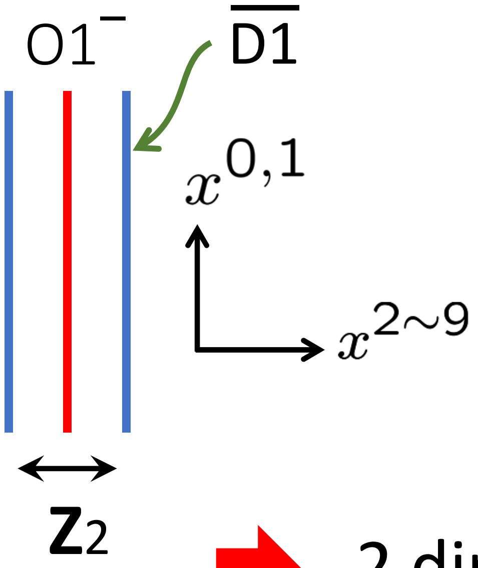
2 dim QED

- massless case is exactly solvable [Schwinger 1962, …]
- ∃ various techniques to analyze
- non-trivial and interesting strongly coupled QFT

➡ It will be interesting to apply it to string theory

$O1^{-}$-$\overline{D1}$ system

(Wait for more explanation)



		gauge	Lorentz 0,1	Rotation 2〜9
		$U(1)$ charge	$SO(1,1)$	$SO(8)$
gauge	a_μ	0	2	1
scalar	ϕ_I	0	1	8_v
fermion	ψ^i_+	2	1_+	8_+
fermion	ψ^i_-	2	1_-	8_-

➡ 2 dim QED with 8 Dirac fermions of charge 2
(coupled with 8 scalar fields)

This motivated us to consider

2 dim QED with N_f fermions of charge k

The k dependence turns out to be very important!

Consider charge *k* fermions x N_f $(k \in \mathbf{Z},\ k > 0)$

$$S = \int d^2x \left(-\frac{1}{4e^2} F_{\mu\nu}^2 + i\overline{\psi}_i \gamma^\mu \left(\partial_\mu + ikA_\mu\right) \psi^i \right) \qquad i = 1, \cdots, N_f$$

- *k* dependence cannot be eliminated by the rescaling $k A_\mu \to A_\mu$

  because of the flux quantization condition $\frac{1}{2\pi} \int F \in \mathbf{Z}$

- *k* dependence appears in the global symmetry

  - $\mathbf{Z}_{kN_f}$ axial sym: $\psi_\pm \to e^{\pm i\alpha} \psi_\pm \qquad \psi^i = \begin{pmatrix} \psi^i_+ \\ \psi^i_- \end{pmatrix}$

    anomaly $\Rightarrow$ $\alpha = \frac{\pi \ell}{k N_f}, \quad \ell = 1, 2, \cdots, k N_f$

  - $\mathbf{Z}_k$ 1-form sym : $A_1 \to A_1 + \frac{1}{kR}, \quad \psi^i \to e^{-ix^1/R} \psi^i$

    (Here, the x^1 direction is compactified to S^1 of radius R)

  It acts on the Wilson loop op. as $W \equiv e^{i \int dx^1 A_1} \to e^{2\pi i/k} W$

Main Results

- **New results in 2 dim QED** *cf* [Anber-Poppitz 2018]

  2 dim QED with N_f massless fermions of charge k

  $$\langle \det(\psi^\dagger_{+j}\psi^i_-)\rangle \neq 0 \qquad \left(\text{although} \quad \langle\psi^\dagger_{+j}\psi^i_-\rangle = 0 \quad (\text{for } N_f > 1)\right)$$

  ⇒ Spontaneous Sym Breaking $\mathbf{Z}_{kN_f} \to \mathbf{Z}_{N_f}$

  ⇒ ∃ k degenerate vacua

- **Non-perturbative calculations in string theory**

  T_Q := tension of (Q,-1)-string (= bound state of Q F1 and 1 $\overline{\text{D1}}$)

  When (Q,-1)-string is placed near O1$^-$-plane with distance Y,

  $$T_Q = \text{const.} - C_Q\, g_s^{1/9} \alpha'^{-17/9} Y^{16/9} \qquad (Y^2 \ll g_s\alpha')$$

  $$\left(C_Q = \frac{18}{\pi^3}\left(\frac{e^\gamma}{2}\right)^{16/9} \times \begin{cases} 1 & (Q = \text{even}) \\ \cos^{16/9}(\pi/8) & (Q = \text{odd}) \end{cases} \right)$$

Plan

1. Introduction ✓
2. Symmetry of 2 dim QED
3. Bosonized description
4. Application to string theory
5. Conclusion

2 Symmetry of 2 dim QED

$$S = \int d^2x \left(-\frac{1}{4e^2} F_{\mu\nu}^2 + i\overline{\psi}_i \gamma^\mu (\partial_\mu + ikA_\mu) \psi^i \right) \qquad i = 1, \cdots, N_f$$

- (classical) global symmetry

$$G_{\text{classical}} = \frac{SU(N_f)_- \times SU(N_f)_+ \times U(1)_A / \mathbf{Z}_2}{(\mathbf{Z}_{N_f})_- \times (\mathbf{Z}_{N_f})_+}$$

  - $\psi_\pm \to g_\pm \psi_\pm \qquad g_\pm \in SU(N_f)_\pm \qquad \psi^i = \begin{pmatrix} \psi^i_+ \\ \psi^i_- \end{pmatrix}$
  - $\psi_\pm \to e^{i\pm\alpha} \psi_\pm \qquad e^{i\alpha} \in U(1)_A \qquad$ ($e^{i\alpha} = \pm 1$ is a part of U(1)$^{\text{gauge}}$)

  ($g_\pm = \omega_\pm \mathbf{1}$, $e^{i\alpha} = \omega_-^{1/2} \omega_+^{-1/2}$ with $(\omega_\pm)^{N_f} = 1$ is a part of U(1)$^{\text{gauge}}$)

- U(1)A anomaly : $\mathcal{D}\psi \to \mathcal{D}\psi \, \exp\left(-i\alpha \frac{kN_f}{\pi} \int F \right) \qquad e^{i\alpha} \in U(1)_A$

$$\frac{1}{2\pi} \int F \in \mathbf{Z} \quad \Rightarrow \quad \alpha = \frac{\pi \ell}{kN_f} \, , \quad \ell = 1, 2, \cdots, kN_f$$

➡ $U(1)_A/\mathbf{Z}_2$ is broken to $\mathbf{Z}_{kN_f}$

1-form symmetry

$$S = \int d^2x \left(-\frac{1}{4e^2} F_{\mu\nu}^2 + i\bar{\psi}_i \gamma^\mu (\partial_\mu + ikA_\mu)\psi^i \right)$$ is invariant under

Z$_k$ 1-form sym : $A_1 \to A_1 + \frac{1}{kR}\,, \quad \psi^i \to e^{-ix^1/R}\psi^i$

(Here, the x^1 direction is compactified to S^1 of radius R)

($A_1 \to A_1 + \frac{1}{R}\,, \quad \psi^i \to (e^{-ix^1/R})^k \psi^i$ is a large gauge transformation)

It acts on the Wilson loop op. as $W \equiv e^{i\int dx^1 A_1} \to e^{2\pi i/k} W$

Mixed 't Hooft anomaly

[Anber-Poppitz 2018]

This **Z**$_k$ 1-form sym is a global symmetry, but if we gauge it, the **Z**$_{kN_f}$ axial symmetry is broken to **Z**$_{N_f}$.

gauging **Z**$_k$ 1-form sym ⇔ introducing flux with $\frac{1}{2\pi}\int F \in \frac{1}{k}\mathbf{Z}$ (see next)

➡ $\mathcal{D}\psi \to \mathcal{D}\psi \exp\left(-i\alpha \frac{kN_f}{\pi} \int F \right)$ is invariant when $\alpha = \frac{\pi \ell}{kN_f}\,, \quad \ell = k, 2k, \cdots, kN_f$

gauging $\mathbf{Z}_k$ 1-form sym ⇔ introducing flux with $\frac{1}{2\pi}\int F \in \frac{1}{k}\mathbf{Z}$

cf) gauging $\mathbf{Z}_k$ 0-form symmetry

- 0-form gauge field: scalar field φ with gauge sym $\varphi \to \varphi + 2\pi$
  (phase of an ordinary scalar field: $\Phi = |\Phi| e^{i\varphi}$)

- U(1) 0-form global sym: $\varphi \to \varphi + \eta$ with $d\eta = 0$

- gauging: (1) promote η to be a 0-form gauge field
  (2) introduce a 1-form gauge field A with $A \to A + d\eta$
  and replace $d\varphi$ with $d\varphi - A$

- breaking to $\mathbf{Z}_k$: introduce another 0-form gauge field φ'
  with $\varphi' \to \varphi' + k\eta$ and impose $d\varphi' - kA = 0$

➡ $\frac{1}{2\pi}\int d\varphi \in \mathbf{Z}$ is replaced with $\frac{1}{2\pi}\int \left(d\varphi - \frac{1}{k} d\varphi'\right) \in \frac{1}{k}\mathbf{Z}$

gauging $\mathbf{Z}_k$ 1-form sym ⇔ introducing flux with $\frac{1}{2\pi}\int F \in \frac{1}{k}\mathbf{Z}$

gauging $\mathbf{Z}_k$ 1-form symmetry

[Gaiotto-Kapustin-Komargodski-Seiberg 2017]

- U(1) 1-form global sym: $A \to A + \eta_1$ with $d\eta_1 = 0$

- gauging: (1) promote η_1 to be a 1-form gauge field
  (2) introduce a 2-form gauge field *B* with $B \to B + d\eta_1$
  and replace dA with $dA - B$

- breaking to $\mathbf{Z}_k$: introduce another 1-form gauge field A'
  with $A' \to A' + k\eta_1$ and impose $dA' - kB = 0$

➡ $\frac{1}{2\pi}\int dA \in \mathbf{Z}$ is replaced with $\frac{1}{2\pi}\int \left(dA - \frac{1}{k}dA'\right) \in \frac{1}{k}\mathbf{Z}$ ■

3 Bosonized description

Bosonization

- non-Abelian bosonization [Witten 1984]

$$\psi^i = (\psi^i_+, \psi^i_-)^T \longleftrightarrow u = ({u^i}_j) \in U(N_f)$$

N_f Dirac fermions — $U(N_f)$ valued scalar field

$$\psi^\dagger_+ \psi_- \sim c\, u \,, \quad \psi^\dagger_- \psi_- \sim \frac{i}{2\pi} u \partial_- u^{-1} \,, \quad \psi^\dagger_+ \psi_+ \sim \frac{i}{2\pi} u^{-1} \partial_+ u \,,$$

- $u = e^{i\varphi} g \qquad (e^{i\varphi}, g) \in U(1) \times SU(N_f)$

Identification $\quad \varphi \to \varphi - \frac{2\pi}{N_f} \,, \quad g \to e^{\frac{2\pi i}{N_f}} g$

- Action for the bosonized description

level 1 WZW action

$$S = \int d^2x \left(-\frac{1}{4e^2} F^2_{\mu\nu} + \frac{N_f}{8\pi} \partial_\mu \varphi \partial^\mu \varphi + \frac{k N_f}{2\pi} \varphi F_{01} + S_{\mathrm{WZW}}(g) \right)$$

$$S = \int d^2x \left(-\frac{1}{4e^2} F_{\mu\nu}^2 + \frac{N_f}{8\pi} \partial_\mu \varphi \partial^\mu \varphi + \frac{k N_f}{2\pi} \varphi F_{01} + S_{\rm WZW}(g) \right)$$

- Identification $\varphi \to \varphi - \frac{2\pi}{N_f}$, $g \to e^{\frac{2\pi i}{N_f}} g$: gauged **Z**$_{N_f}$ sym
- **Z**$_{kN_f}$ axial sym $\varphi \to \varphi - \frac{2\pi}{kN_f}$
- **Z**$_k$ 1-form sym $A_1 \to A_1 + \frac{1}{kR}$

- Canonical momenta conjugate to A_1, φ ($A_0 = 0$ gauge)

$$\Pi_A \equiv \frac{1}{2e^2} \partial_0 A_1 + \frac{k N_f}{2\pi} \varphi \ , \qquad \Pi_\varphi \equiv \frac{N_f}{4\pi} \partial_0 \varphi$$

$$\mathbf{Z}_{kN_f} \ : \ \hat{V} \equiv \exp \left(-\frac{2\pi i}{k N_f} \int dx^1 \Pi_\varphi + i \int dx^1 A_1 \right)$$

$$\mathbf{Z}_k \ : \ \hat{U} \equiv \exp \left(\frac{2\pi i}{k} \Pi_A \right)$$

Mixed 't Hooft anomaly

cf [Anber-Poppitz 2018]

$$\mathbf{Z}_{kN_f} \;:\; \hat{V} \equiv \exp\left(-\frac{2\pi i}{kN_f}\int dx^1 \Pi_\varphi + i\int dx^1 A_1\right)$$

$$\mathbf{Z}_k \;:\; \hat{U} \equiv \exp\left(\frac{2\pi i}{k}\Pi_A\right)$$

$$\rightarrow \quad \hat{U}\hat{V} = \hat{V}\hat{U}e^{\frac{2\pi i}{k}}$$

If $\mathbf{Z}_k$ is gauged, $\mathbf{Z}_{kN_f}$ is no longer well-defined

⇒ ∃ mixed 't Hooft anomaly

(reproducing the previous discussion)

⇒ The vacuum cannot be trivial

The vacuum states have to be consistent with the above algebra

Vacuum structure

- $N_f = 1$ case $\langle \overline{\psi}\psi \rangle \neq 0$ [Anber-Poppitz 2018]

  ⇒ the axial $\mathbf{Z}_k$ is spontaneously broken

  ⇒ ∃ k degenerate vacua

(*cf* $\mathcal{N} = 1$ SU(N) SYM in 4 dim

$$U(1)_R \xrightarrow{\text{anomaly}} \mathbf{Z}_{2N} \xrightarrow[\langle \lambda\lambda \rangle \neq 0]{\text{SSB}} \mathbf{Z}_2 \quad \Rightarrow \exists\ N \text{ vacua}$$
)

- $N_f > 1$ case $\langle \overline{\psi}_i \psi^j \rangle = 0$

massive scalar

$$\text{But,} \quad \langle \det(\psi^\dagger_{+i}\psi^j_-) \rangle \overset{\text{bosonize}}{\sim} \langle \det u \rangle = \langle e^{iN_f\varphi} \rangle \neq 0$$

⇒ the axial $\mathbf{Z}_{kN_f}$ $\left(\varphi \to \varphi - \frac{2\pi}{kN_f}\right)$ is spontaneously broken to $\mathbf{Z}_{N_f}$

⇒ ∃ k degenerate vacua

Explicit construction of the *k* vacua

- Pick a vacuum, which is an eigenstate of $\hat{U} = \exp\left(\frac{2\pi i}{k}\Pi_A\right)$

$$\hat{U}|\theta\rangle = e^{i\theta/k}|\theta\rangle$$

comments

- $\hat{U}$ commutes with the gauge inv. local operators $F_{\mu\nu}, \varphi, \partial_\mu\varphi, \cdots$

  ⇒ Superselection sectors are characterized by the eigenvalue of $\hat{U}$

- θ is the θ parameter and $|\theta>$ is the θ vacuum

- The other vacua can be obtained by acting $\hat{V} = e^{i\int dx^1\left(-\frac{2\pi}{kN_f}\Pi_\varphi + A_1\right)}$

$$\{\text{vacuum}\} = \left\{|\theta\rangle, \hat{V}|\theta\rangle, \cdots, \hat{V}^{k-1}|\theta\rangle\right\} \quad \left(\hat{V}^n|\theta\rangle = |\theta + 2\pi n\rangle\right)$$

$\hat{V}^k|\theta\rangle = |\theta + 2\pi k\rangle$ and $|\theta\rangle$ are identified by the gauged $\mathbf{Z}_{N_f}$ sym

⇒ *k* dim representation of the algebra $\hat{U}\hat{V} = \hat{V}\hat{U}e^{\frac{2\pi i}{k}}$

Mass deformation

- Consider adding a fermion mass term $\sim M_0 \overline{\psi}\psi$

  $\mathbf{Z}_{kN_f}$ sym is explicitly broken ⇒ The degeneracy of the k vacua is lifted

- For $M_0 \ll e$,

[Smilga 1992,
Hetrick-Hosotani-Iso 1995,
Rodriguez-Hosotani 1996]

$$\mathcal{E}(\theta) \propto M_0 \langle \theta | \overline{\psi}_j \psi^j | \theta \rangle = f(\theta)\, m_h^{\frac{2}{N_f+1}} M_0^{\frac{2N_f}{N_f+1}}$$

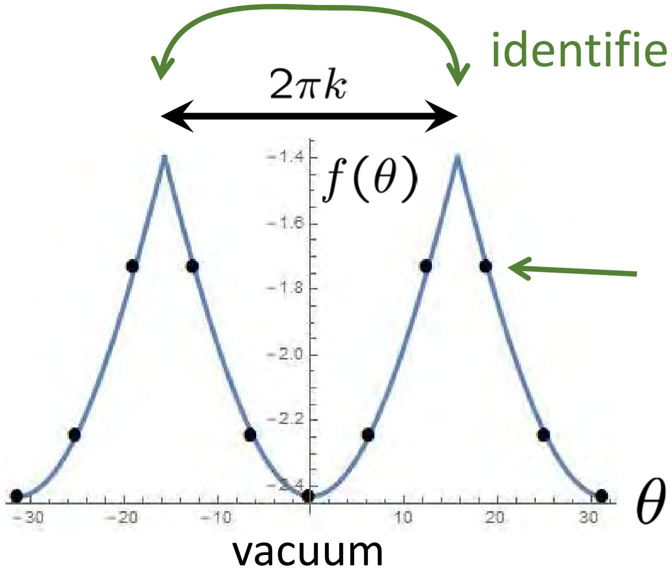
energy density of $|\theta>$

$$m_h^2 \equiv \frac{e^2 k^2 N_f}{\pi}$$

$$f(\theta) \equiv -\frac{N_f}{4\pi}\left(2\exp(\gamma)\cos\left(\frac{1}{N_f}\overline{(\theta/k)}\right)\right)^{\frac{2N_f}{N_f+1}}$$

$\overline{(x)} = x$ for $-\pi < x < \pi$

and $\overline{(x+2\pi)} = \overline{(x)}$



$f(\theta)$ for $N_f = 4$, $k = 5$

Dots:
energy of the k states

$$\widehat{V}^n|0\rangle = |2\pi n\rangle\ , \quad n = 0, 1, \cdots, k-1$$

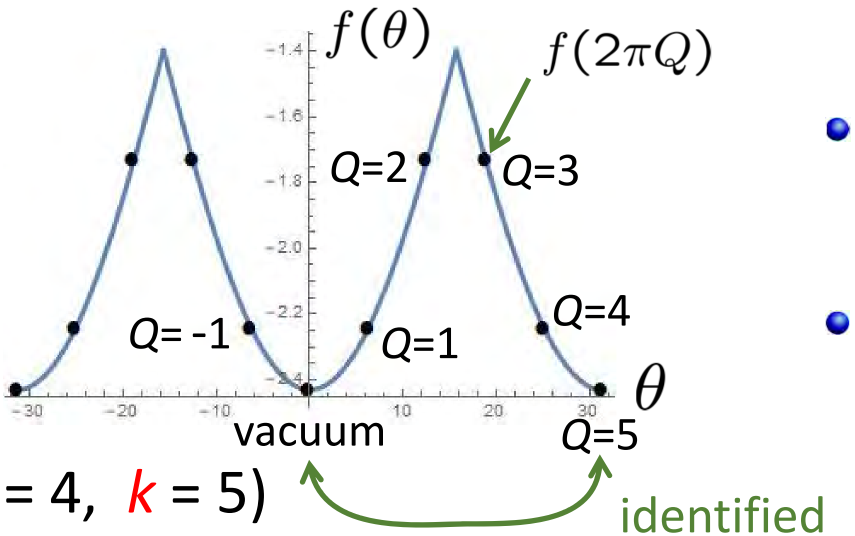
(vacuum for M_0=0 and θ = 0)

Q-string tension

$$S_{\rm int} = Q\int dt A_0\Big|_{x^1=-L} - Q\int dt A_0\Big|_{x^1=+L} = Q\int F \quad \Leftrightarrow \quad \Delta\theta = 2\pi Q$$

➡ Q-string tension (= energy density of Q-flux) (for $\theta = 0$)

$$\sigma(Q) = \mathcal{E}(2\pi Q) - \mathcal{E}(0) \propto (f(2\pi Q) - f(0))\, m_h^{\frac{2}{N_f+1}} M_0^{\frac{2N_f}{N_f+1}}$$



($N_f = 4$, $k = 5$)

- $\sigma(Q + k) = \sigma(Q)$
  screening by charge k fermions
- $\sigma(Q) \to 0$ as $M_0 \to 0$
  Q-string state becomes $Q^{\rm th}$ vacuum

④ Application to String Theory

D-brane & orientifold plane

- D*p*-brane: (*p*+1) dim plane on which open strings can end

➡ (p+1) dim U(N) gauge theory

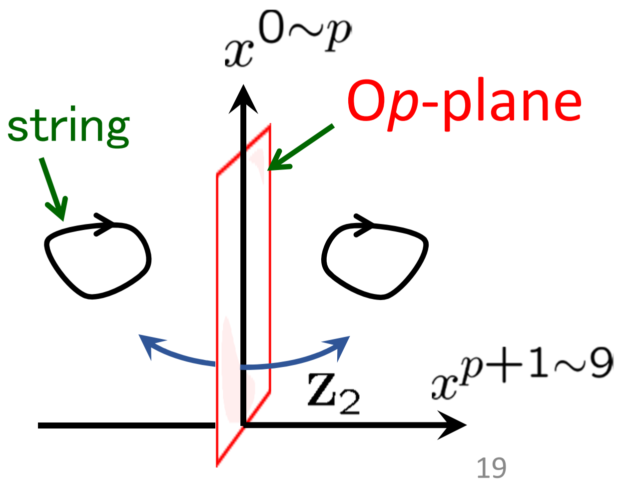
- O*p*-plane:

(*p*+1) dim fixed plane of $\mathbf{Z}_2$ $\begin{cases} x^{p+1\sim 9} \to -x^{p+1\sim 9} \\ \text{and flip orientation of strings} \end{cases}$

- Two basic types: O*p*$^-$ & O*p*$^+$

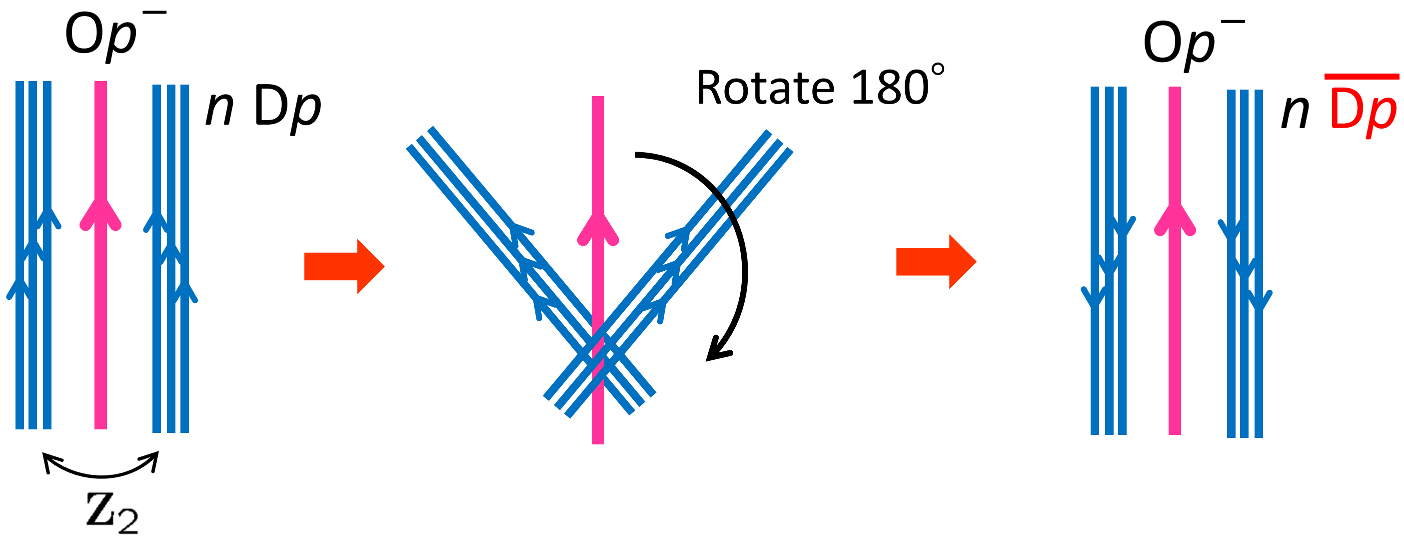
O*p*$^-$ + *n* D*p* ➡ SO(2*n*) gauge theory

O*p*$^+$ + *n* D*p* ➡ USp(2*n*) gauge theory

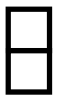


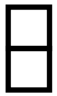
$\mathrm{O}p^{-}$-$\overline{\mathrm{D}p}$ system

[SS 1999]

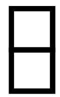


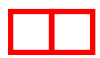
(p+1) dim maximally SUSY
SO(2n) gauge theory

A_μ, Φ_I (gauge, scalar)  of SO(2n)

Ψ^i (fermion)  of SO(2n)

(p+1) dim non-SUSY
SO(2n) gauge theory

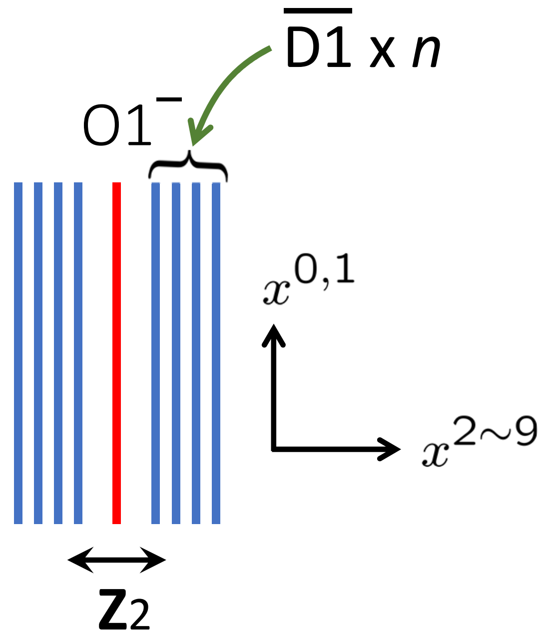
A_μ, Φ_I (gauge, scalar)  of SO(2n)

Ψ^i (fermion)  of SO(2n)

O1$^-$- $\overline{\text{D1}}$ system

- O1$^-$-plane + $\overline{\text{D1}}$-brane x *n*

		gauge $SO(2n)$	Lorentz $SO(1,1)$	$SO(8)$
gauge	A_μ	adj	2	1
scalar	Φ_I	adj	1	8_v
fermion	Ψ^i_+	sym	1_+	8_+
fermion	Ψ^i_-	sym	1_-	8_-



- *n* = 1 case

		gauge $U(1)$ charge	Lorentz $SO(1,1)$	$SO(8)$
gauge	a_μ	0	2	1
scalar	ϕ_I	0	1	8_v
fermion	ψ^i_+	2	1_+	8_+
fermion	ψ^i_-	2	1_-	8_-

(+ neutral fermions)

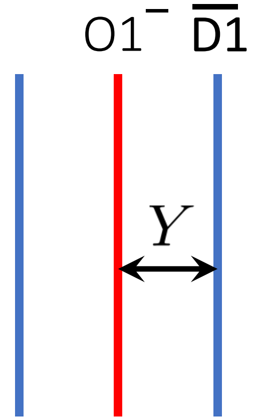
➡ 2 dim QED with k = 2, N_f = 8 coupled with 8 scalar fields

Interpretation

$$Z_{\text{Full}} = \int \mathcal{D}\phi \, e^{i\int d^2x \frac{1}{2}(\partial_\mu \phi_I)^2} Z_{\text{QED}}[\phi_I] \, ,$$

$$Z_{\text{QED}}[\phi_I] \equiv \int \mathcal{D}a\mathcal{D}\psi\mathcal{D}\overline{\psi} \, e^{iS_{\text{QED}}[a_\mu,\psi^i]+iS_{\text{Yukawa}}[\phi_I,\psi^i]}$$

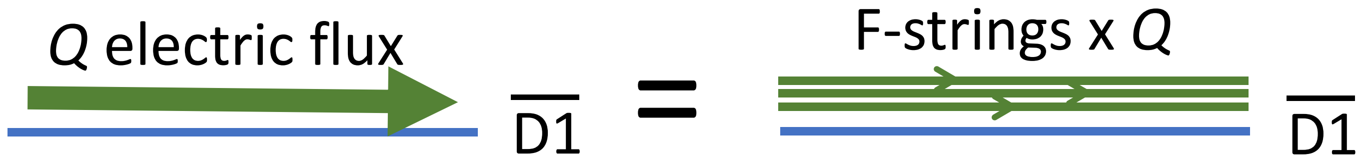
We focus on this QED part.



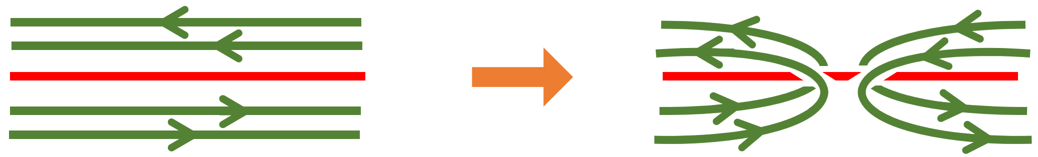
- $|\phi_I| \propto$ distance between O1$^-$ and $\overline{\text{D1}}$ $\equiv Y$

  $\propto$ fermion mass M_0 ⬅ $S_{\text{Yukawa}} \sim \phi\overline{\psi}\psi$

- Q-string = (Q,-1)-string = bound state of $\overline{\text{D1}}$ and F1 x Q



- F1 x 2 can be screened



Prediction

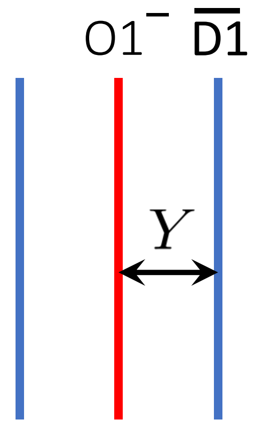
T_Q := tension of (*Q*,-1)-string (= bound state of *Q* F1 and 1 $\overline{\mathrm{D1}}$)

When (*Q*,-1)-string is placed near $\mathrm{O1}^-$-plane with distance *Y*,

$$T_Q = \text{const.} + \mathcal{E}(2\pi Q) \qquad (\text{valid when } Y^2 \ll g_s\alpha')$$

$$= \text{const.} - C_Q\, g_s^{1/9} \alpha'^{-17/9} Y^{16/9}$$

$$\left(C_Q = \frac{18}{\pi^3}\left(\frac{e^\gamma}{2}\right)^{16/9} \times \begin{cases} 1 & (Q = \text{even}) \\ \cos^{16/9}(\pi/8) & (Q = \text{odd}) \end{cases} \right)$$



In particular, $T_1 = T_0$ when $Y = 0$

(*cf* Behavior at large *Y*

$$T_Q = \frac{1}{2\pi\alpha'}\sqrt{Q^2 + \frac{1}{g_s^2}}$$

)

5 Conclusion

Summary

- We found SSB $\mathbf{Z}_{kNf} \rightarrow \mathbf{Z}_{Nf}$ and k vacua in 2 dim QED with Nf fermions of charge k
- Non-perturbative calculations in O1^{-}-$\overline{\text{D1}}$ system

Discussion

- How about 2 dim QCD? 3 dim QED? 4 dim QED?
- Curious relation: $\langle \overline{\psi}_i \psi^i \rangle = \pi\alpha' \dfrac{\partial \mathcal{E}(Y)}{\partial Y}$

chiral condensate — force between O1^{-} and $\overline{\text{D1}}$

VEV in QFT — Brane dynamics

How general can this be?

How much can we learn from such relations?

Thank you !