弱い重力予想と現象論

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based on arXiv:1802.04287

w/Stefano Andriolo, Daniel Junghans, Gary Shiu

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主なメッセージ

- 弱い重力予想 (Weak Gravity Conjecture) ← レビュー

量子重力理論には質量比電荷や axion の崩壊係数に上下限値

$$\frac{q}{m} > 1 \qquad\qquad f > M_{\mathrm{Pl}}$$

→ インフレーションや暗黒物質の模型への示唆

- Tower Weak Gravity Conjecture [Andriolo-Junghans-TN-Shiu '18]

無限個の荷電粒子がタワー状に存在（cf. KK tower, string spectrum）

two key words: Landscape and Swampland（沼地）

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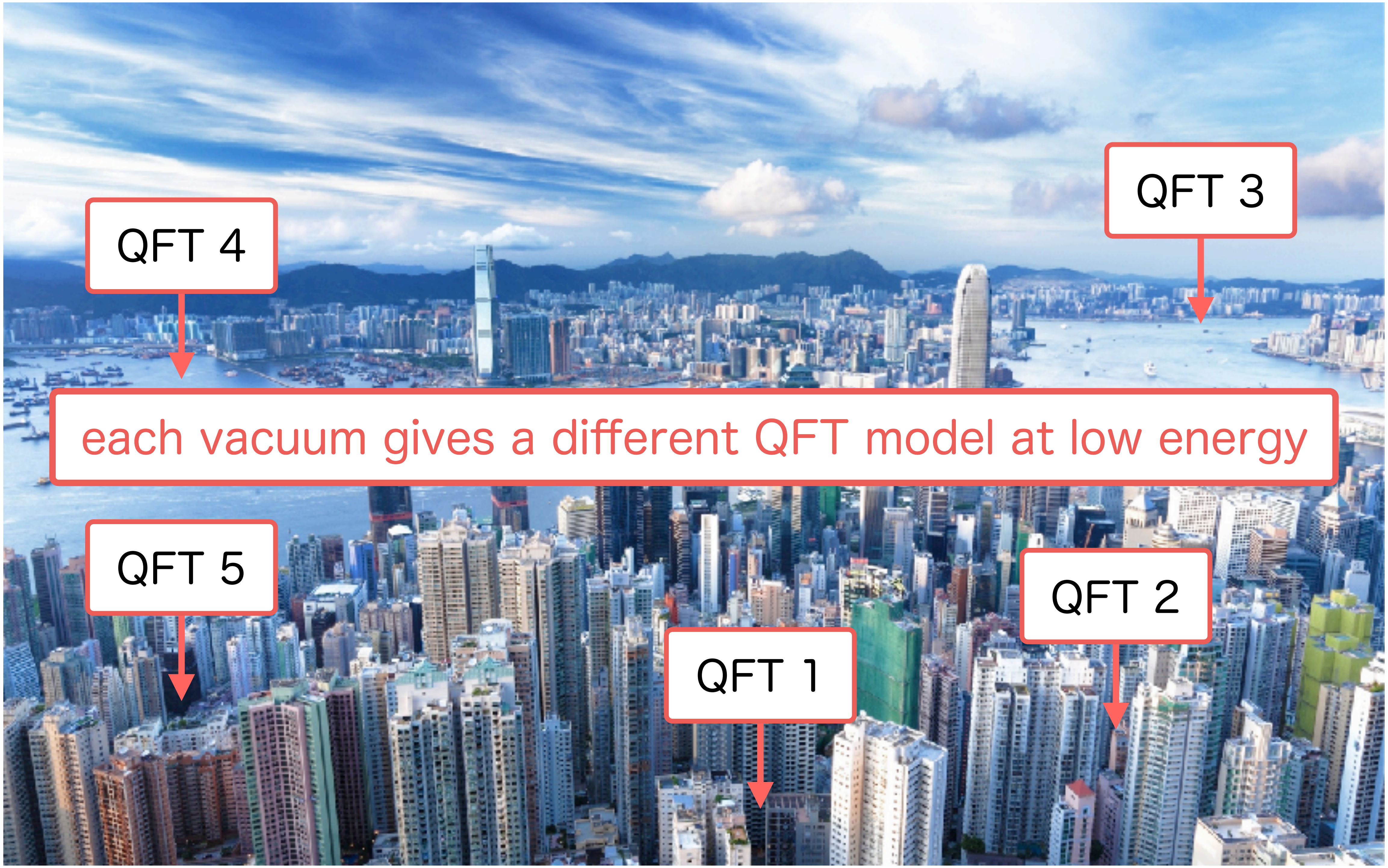
probably, you have heard of

the word “String Theory Landscape”

there seem to exist almost infinite vacua in string theory

- how to compactify the extra dimensions
- how to put D-branes, ⋯



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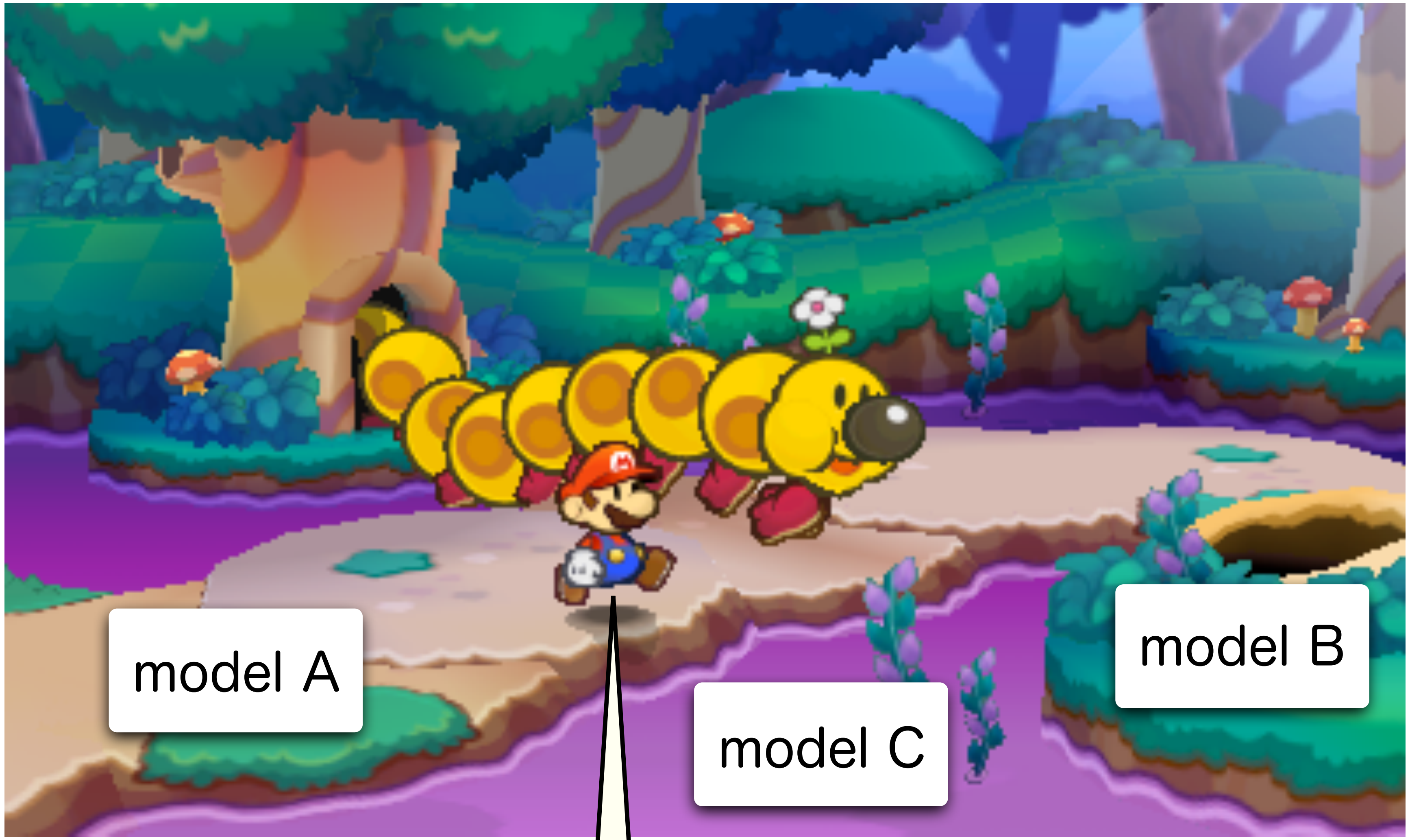
a complementary view of landscape

[Vafa ’05]

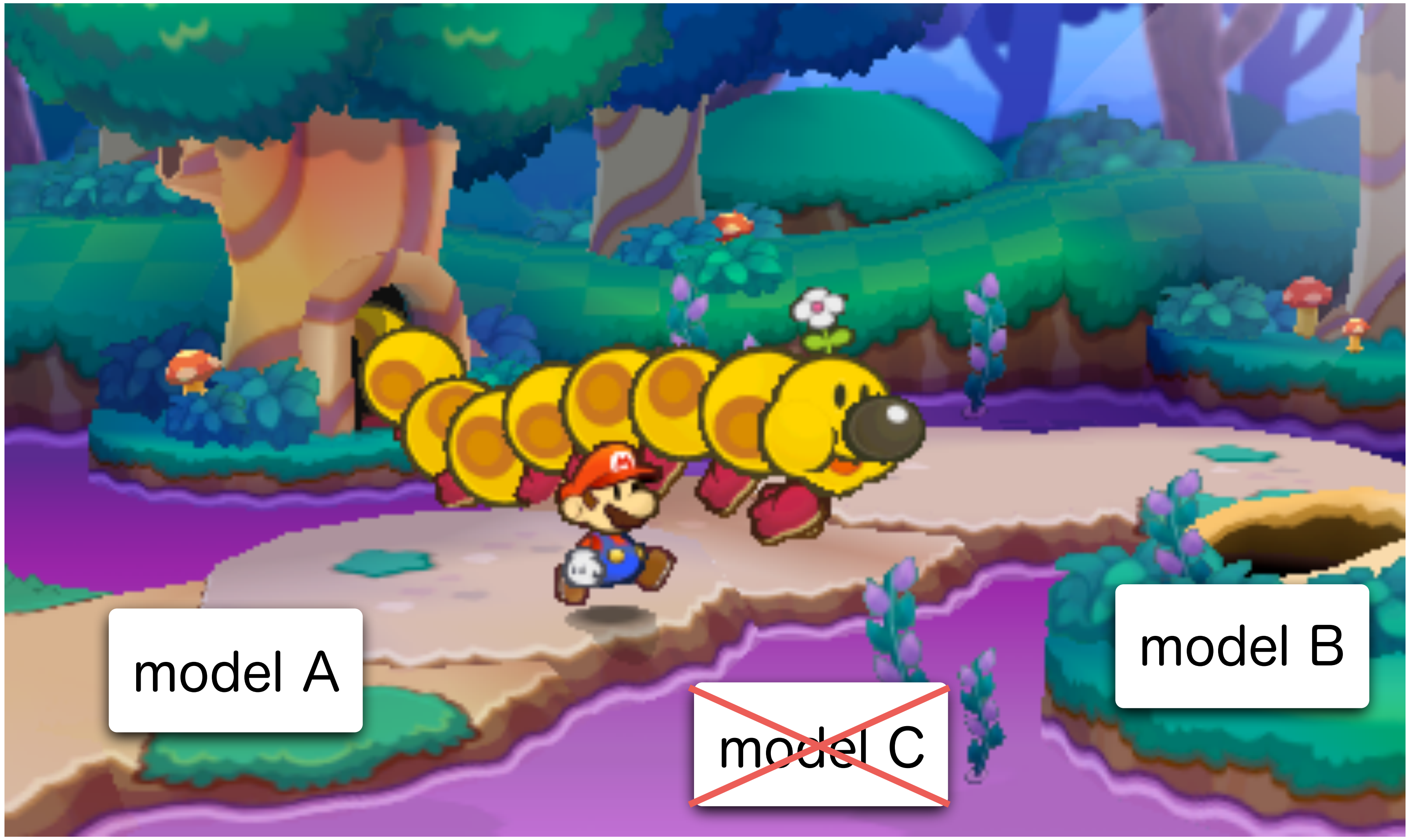
two key words: Landscape and Swampland（沼地）



Q. Is my QFT model consistent with quantum gravity?

landscape :

models with healthy UV completion



swampland :

apparently consistent, but problematic

boundaries!

clarifying boundaries of landscape and swampland

is important for both the theory and phenomenology

- "consistency requirements" on phenomenological models
- if the nature favors what we think in swamplands,

  we need to change our criteria to construct UV theories

Weak Gravity Conjecture is a typical example

for criteria to distinguish swampland from landscape

※ relevant to axion inflation, dark matter scenarios, ⋯

in the rest of my talk

1. Weak Gravity Conjecture

 - a criterion to distinguish landscape from swampland

2. WGC vs positivity bounds

 - possible connections to other QFT principles

 - our proposal: Tower Weak Gravity Conjecture

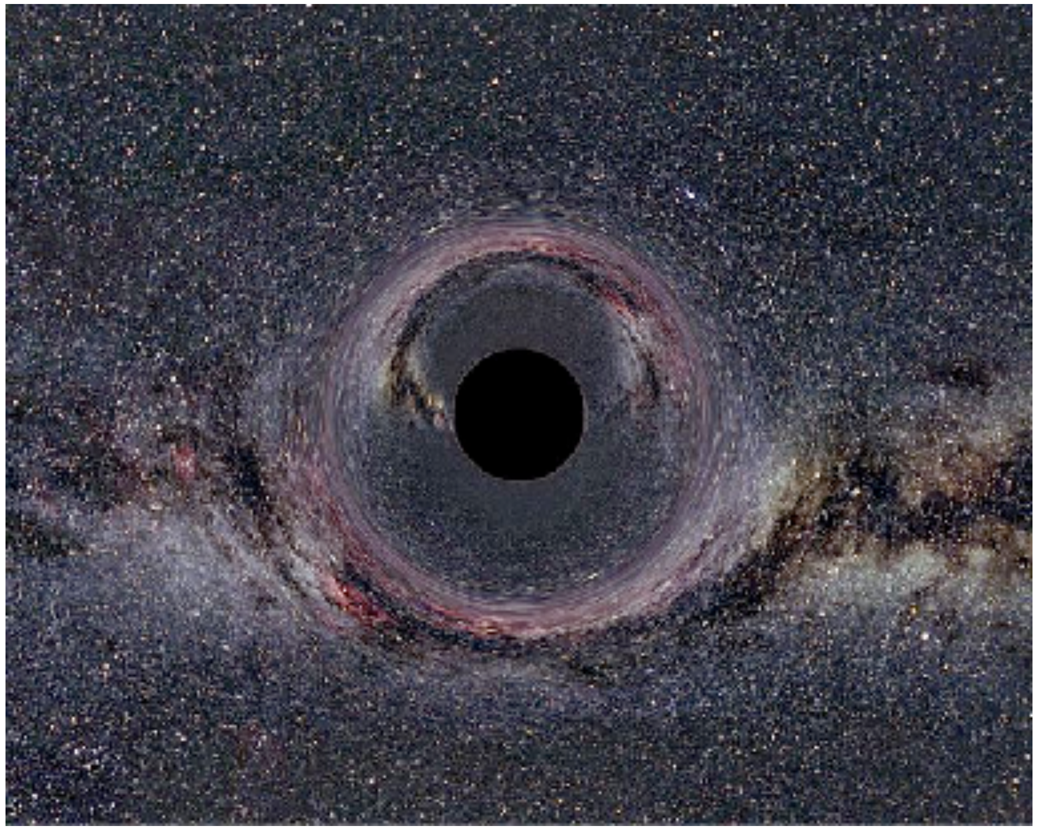
1. Weak Gravity Conjecture

to motivate Weak Gravity conjecture,

let me start with a widely accepted statement:

no continuous global symmetry in quantum gravity

black hole entropy



BH enjoys thermodynamic properties

[Bekenstein, Hawking,...]

in particular, its entropy S is

$$S = \frac{A}{4} \quad (A : \text{horizon area})$$

in quantum gravity (= microscopic description of gravity)

we expect that BH entropy is statistical entropy $S = -\mathrm{tr}(\rho \ln \rho)$

indeed, string theory explicitly showed that it is the case

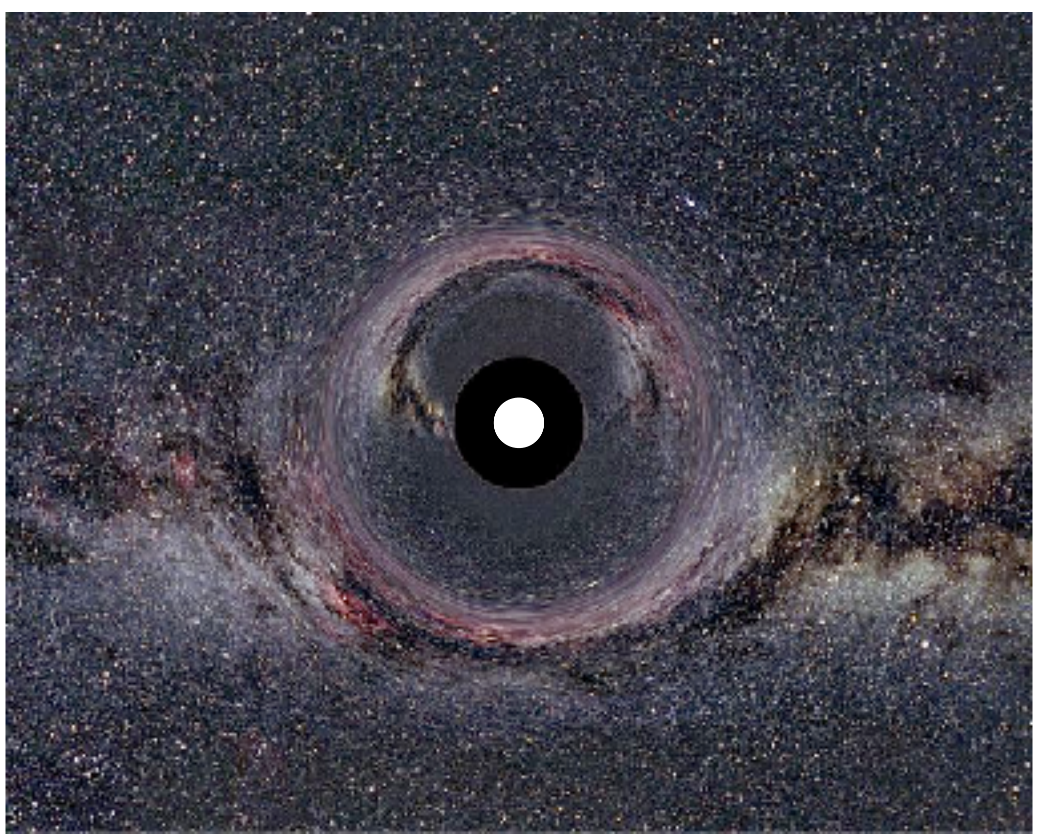
at least for certain black holes [Strominger-Vafa '96]

no global symmetry in quantum gravity

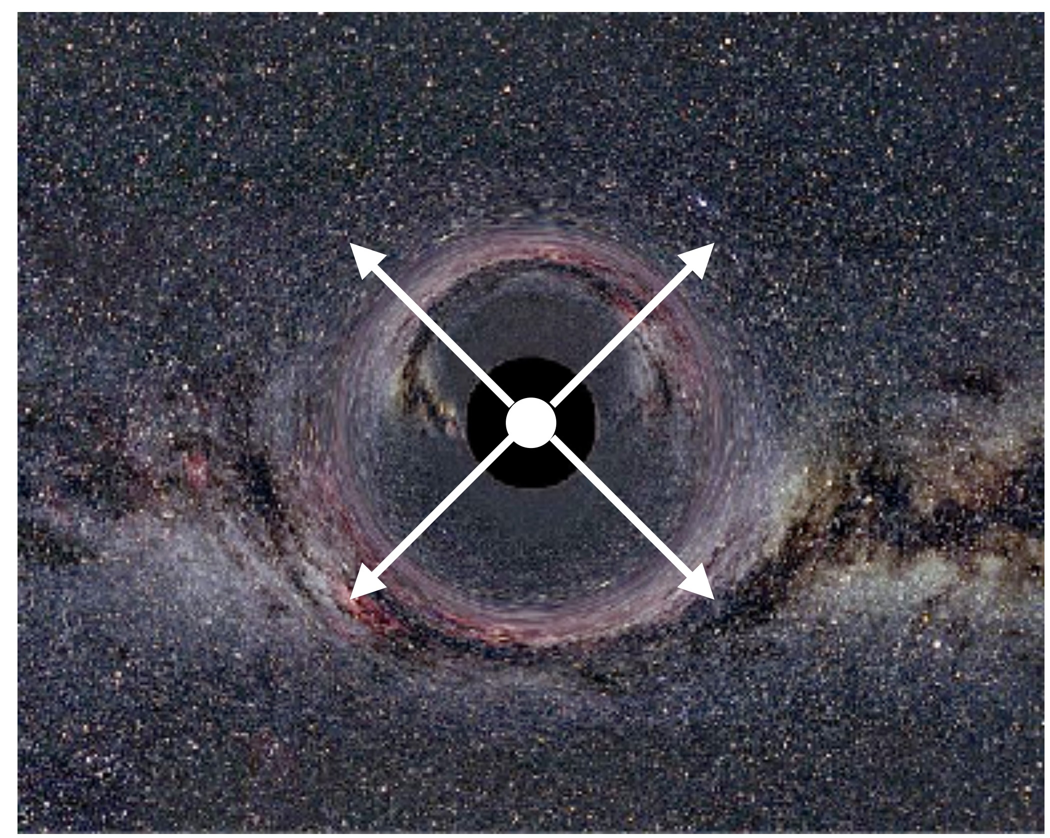
no-hair theorem:

event horizon → global symmetry charge cannot be observed

cf. elemag charge is observable via background gauge field



global symmetry



gauge symmetry

no global symmetry in quantum gravity

no-hair theorem:

event horizon → global symmetry charge cannot be observed

cf. elemag charge is observable via background gauge field

statistical BH entropy in theories w/continuous global symmetry

require ensemble of states wth ∀ global charge

→ generically large degeneracy & divergent entropy

→ no continuous global symmetry in quantum gravity!?

※ consistent with string theory, AdS/CFT etc

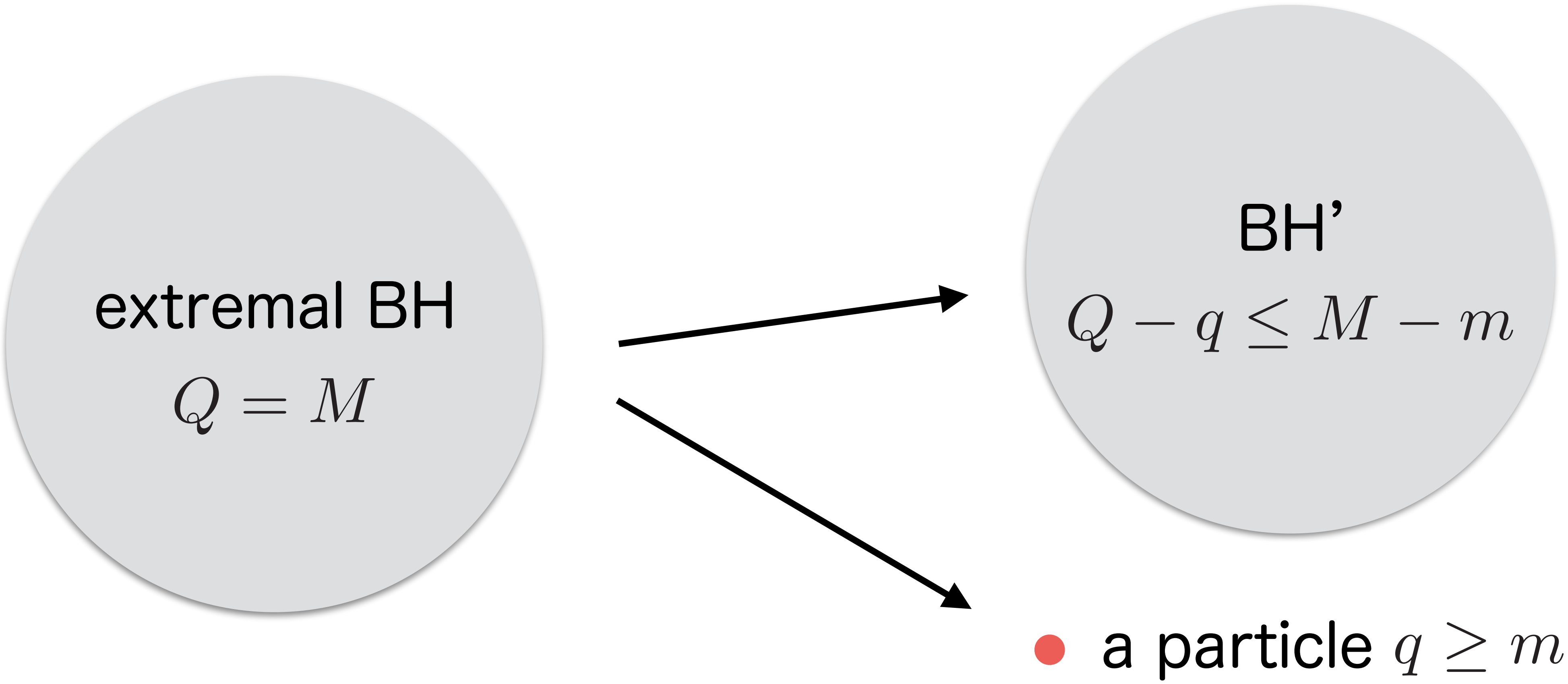
[ex. Susskind 95', Banks-Seiberg 10']

global symmetry = gauge symmetry at $g = 0$

$\rightarrow$ natural to expect a lower bound on the gauge coupling

Weak Gravity Conjecture

[ArkaniHamed-Motl-Nicolis-Vafa 06']



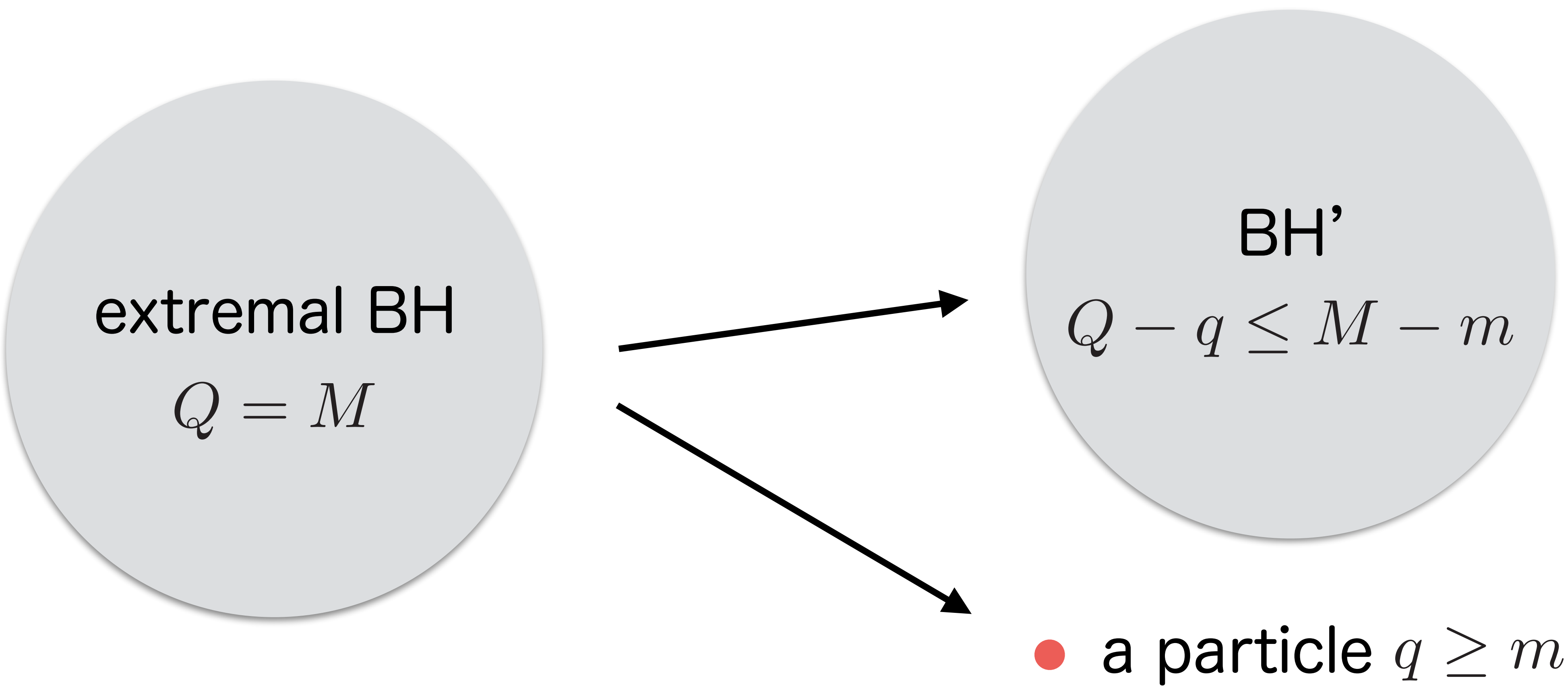
weak gravity conjecture provides a quantitative bound by postulating finiteness of the # of stable states

※ to make extremal BH (no hawking radiation) unstable,
require existence of a particle satisfying $q \geq m$

work in the unit $Q_{\rm ext} = M_{\rm ext}$

Weak Gravity Conjecture

[ArkaniHamed-Motl-Nicolis-Vafa 06']



weak gravity conjecture provides a quantitative bound by postulating finiteness of the # of stable states

※ to make extremal BH (no hawking radiation) unstable,

require existence of a particle satisfying $gq \geq \text{``}1\text{''} \cdot \dfrac{m}{M_{\rm Pl}}$

work in the unit $Q_{\rm ext} = M_{\rm ext}$

- no rigorous proof, so it is still a conjecture
- but consistent with all known examples in string theory
- if true, various phenomenological implications

  ex. mili-charged dark matter, axion inflation, axion DM, ⋯

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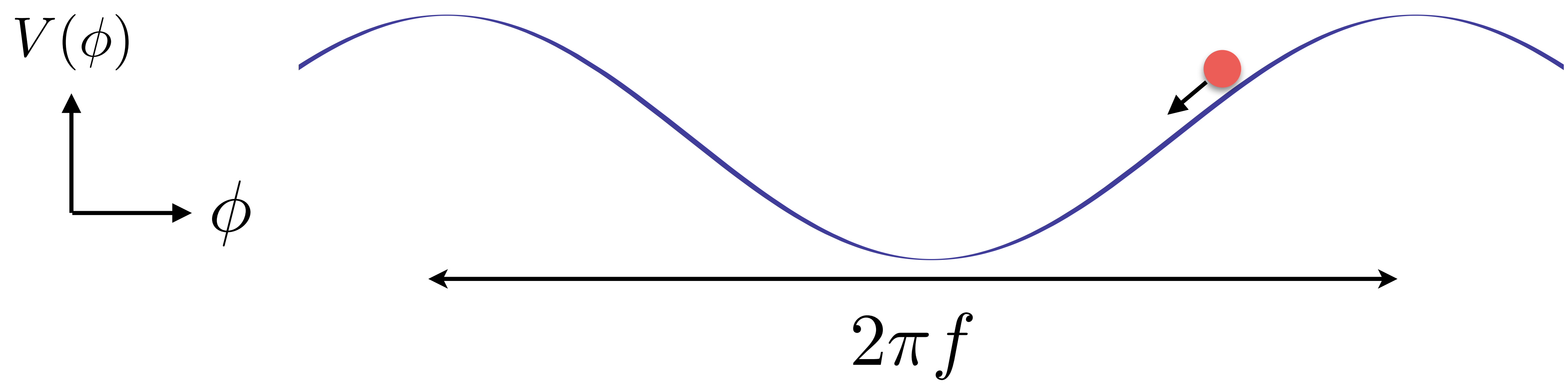
ex. mili-charged dark matter, axion inflation, axion DM, $\cdots$

"charge > mass" $\leftrightarrows \; \frac{1}{f} > \frac{S_{\rm inst}}{M_{\rm Pl}} \; \leftrightarrows \; \frac{f}{M_{\rm Pl}} \cdot S_{\rm inst} < 1$

implications to axion inflation



inflaton potential has to be flat enough (slow-roll condition)

$$V(\phi) \propto e^{-S_{\rm inst}}\left(1-\cos\frac{\phi}{f}\right) + \cancel{\sum_{n\geq 2} e^{-nS_{\rm inst}}\left(1-\cos\frac{n\phi}{f}\right)}$$

- negligible higher harmonics ($n \geq 2$) $\rightarrow$ $S_{\rm inst} > 1$
- long enough periodicity $\rightarrow$ $f > M_{\rm Pl}$

※ inconsistent with WGC $\frac{f}{M_{\rm Pl}} \cdot S_{\rm inst} < 1$

recent directions:

1. how to evade WGC and realize axion inflation models

[De la Fuente et al ’14, Bachlechner et al ’15, Choi-Kim ‘15, Conlon-Krippendorf ’16, …]

2. constraints on particle physics models (ex. neutrino masses)

[Ooguri-Vafa ’16, Ibanez, MartinLozano-Valenzuela ’17, Hamada-Shiu ’17]

3. better understanding & towards a proof of WGC

- lessons from string theory examples

[Brown et al ’15, Heidenreich et al ’15, Hebecker-Soler ’17, Montero et al ’17]

- use of AdS/CFT (holography)

[Nakayama-Nomura ’15, Harlow ’15, Benjamin et al ’16, Montero et al ’16]

- relation to positivity bounds

[Cheung-Remmen ’14, Andriolo-Junghans-TN-Shiu ’18] recent directions:

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2. WGC vs positivity bounds

[Cheung-Remmen '14, Andriolo-Junghans-TN-Shiu '18] consistency such as unitarity, analyticity and causality

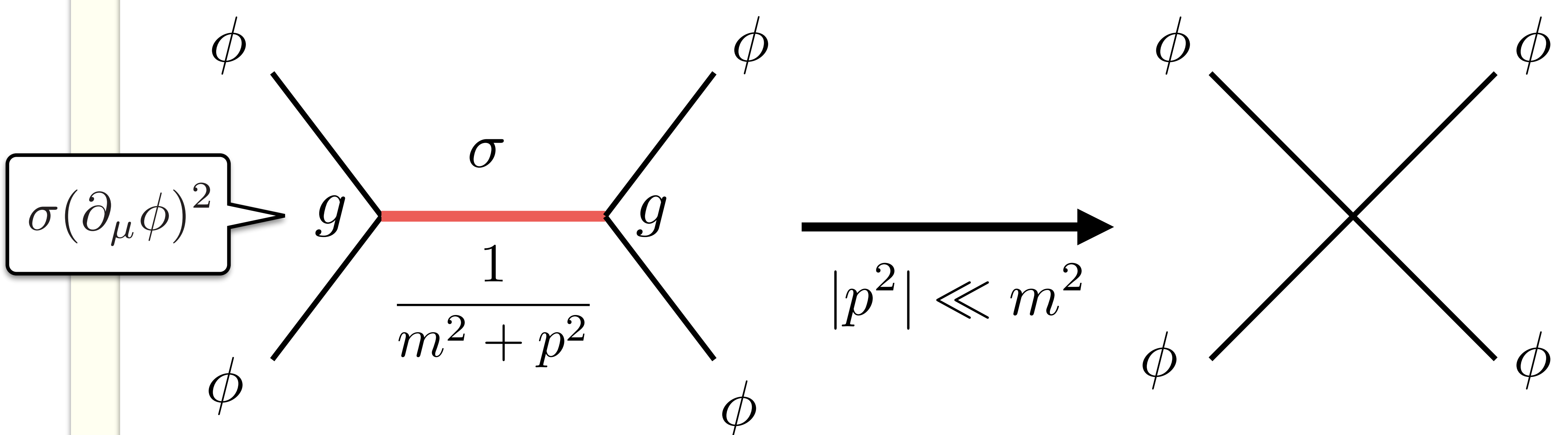
$\rightarrow$ generically constrain signs of effective interactions

an illustrative example for positivity

a scalar EFT with a shift symmetry $\phi \to \phi + \mathrm{const}$

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 + \frac{\alpha}{\Lambda^4}(\partial_\mu \phi)^4 + \dots$$

※ α shows up, e.g., after integrating out a heavy field σ



the effective coupling is $\alpha = \dfrac{g^2}{2m^2} \geq 0$

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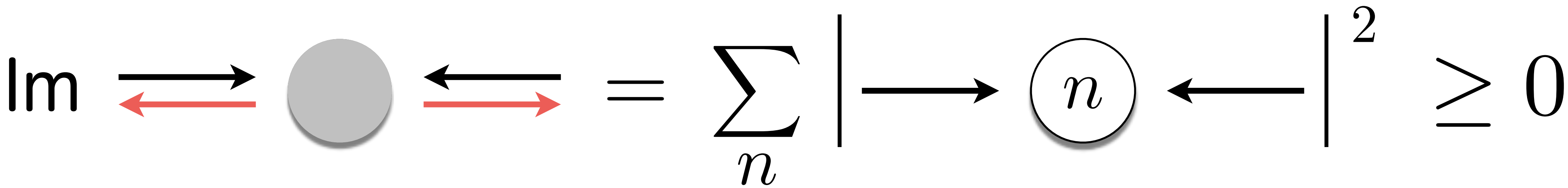
more generally, positivity of α follows only from

- unitarity of UV completion



- analyticity & locality of scattering amplitudes

[Adams-Arkani Hamed-Dubovsky-Nicolis-Rattazzi '06]

Such a positivity is better understood than WGC

$\rightarrow$ Is there any relation between the two?

photon + graviton + massive charged particles

$\Downarrow$ integrate out matters

IR effective theory of photon & graviton

Q. What the positivity of this EFT implies?

1-loop effective action for photon & graviton

$$\mathcal{L}_{\mathrm{eff}} = \frac{M_{\mathrm{Pl}}^2}{2} R - \frac{1}{4} F_{\mu\nu}^2$$
$$+ \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 + \alpha_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \alpha_3 F_{\mu\nu} F_{\rho\sigma} W^{\mu\nu\rho\sigma} + \dots$$

- positivity implies $\alpha_1 + \alpha_2 \geq 0$

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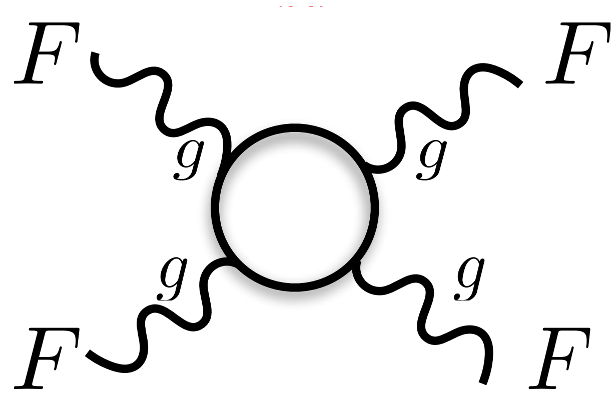
- α_i depends on mass and charge of particles integrated out

$\alpha_i =$



$$+ \underbrace{\mathcal{O}(g^2) + \mathcal{O}(g^0)}_{\text{gravitational effects}}$$

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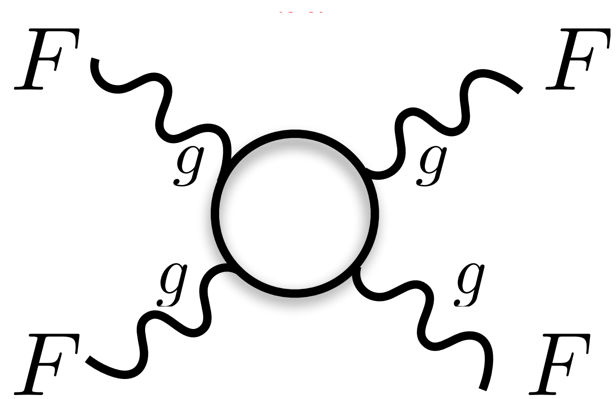


$$+\underbrace{\mathcal{O}(g^2) + \mathcal{O}(g^0)}_{\text{gravitational effects}}$$

- Cheung-Remmen found positivity implies $z^4 - z^2 + \gamma \geq 0$

※ $z = \dfrac{qg}{m/M_{\rm Pl}}$, γ is a UV sensitive $\mathcal{O}(z^0)$ coefficient

(free parameter in the EFT framework)

positivity of photon-graviton EFT implies $z^4 - z^2 + \gamma \geq 0$

$\rightarrow$ at lest one of the following two should be satisfied

1) WGC type lower bound on charge-to-mass ratio

   in particular when $\gamma = 0$, WGC $z^2 \geq 1$ is reproduced!

2) not so small value of UV sensitive parameter γ

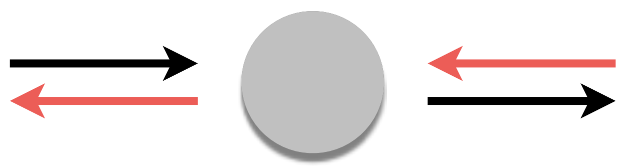
in [Andriolo-Junghans-TN-Shiu '18], we discussed

- multiple U(1) extension
- implications from KK reduction

multiple U(1) extension

for example, let us consider $U(1)_1 \times U(1)_2$

a new ingredient is positivity of $\gamma_1 + \gamma_2 \to \gamma_1 + \gamma_2$

Im  ≥ 0 implies $z_1^2 z_2^2 - z_1^2 - z_2^2 \geq 0$

- $z_i = q_i/m$ is the charge-to-mass ratio for each U(1)

- we set $\mathcal{O}(z^0) = 0$ for illustration (same as γ = 0 before)

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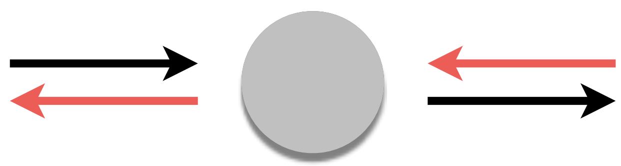
Im  ≥ 0 implies $z_1^2 z_2^2 - z_1^2 - z_2^2 \geq 0$

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the punchline here:

positivity bound cannot be satisfied unless $z_1^2 z_2^2 \neq 0$

$\to$ requires existence of a bifundamental particle!

implications from KK reduction

\# S^1 compactify d+1 dim Einstein-Maxwell with single U(1)

into d dim Einstein-Maxwell with $U(1) \times U(1)_{\mathrm{KK}}$

d+1 dim charged particle (q,m)

$\rightarrow$ KK tower with the charged-to-mass ratios

$$(z, z_{\mathrm{KK}}) = \left(\frac{q}{\sqrt{m^2 + n^2 m_{\mathrm{KK}}^2}}, \ \frac{n}{\sqrt{(m/m_{\mathrm{KK}})^2 + n^2}} \right)$$

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in the small radius limit $m_{\rm KK} \rightarrow \infty$,

the lowest mode (n = 0): $(z, z_{\rm KK}) = (q/m, 0)$

KK modes (n ≠ 0): $(z, z_{\rm KK}) \simeq (0, 1)$

※ no bifundamentals → ~~positivity bound~~ generically a solution to make the theory healthy is

to introduce a tower of d+1 dim U(1) charged particles

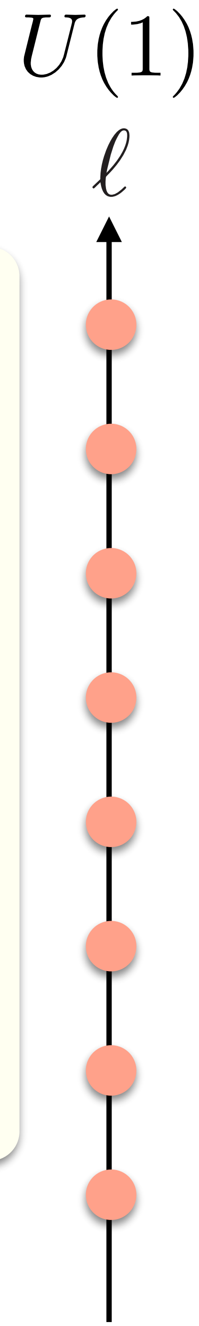
<u>d+1 dim</u>

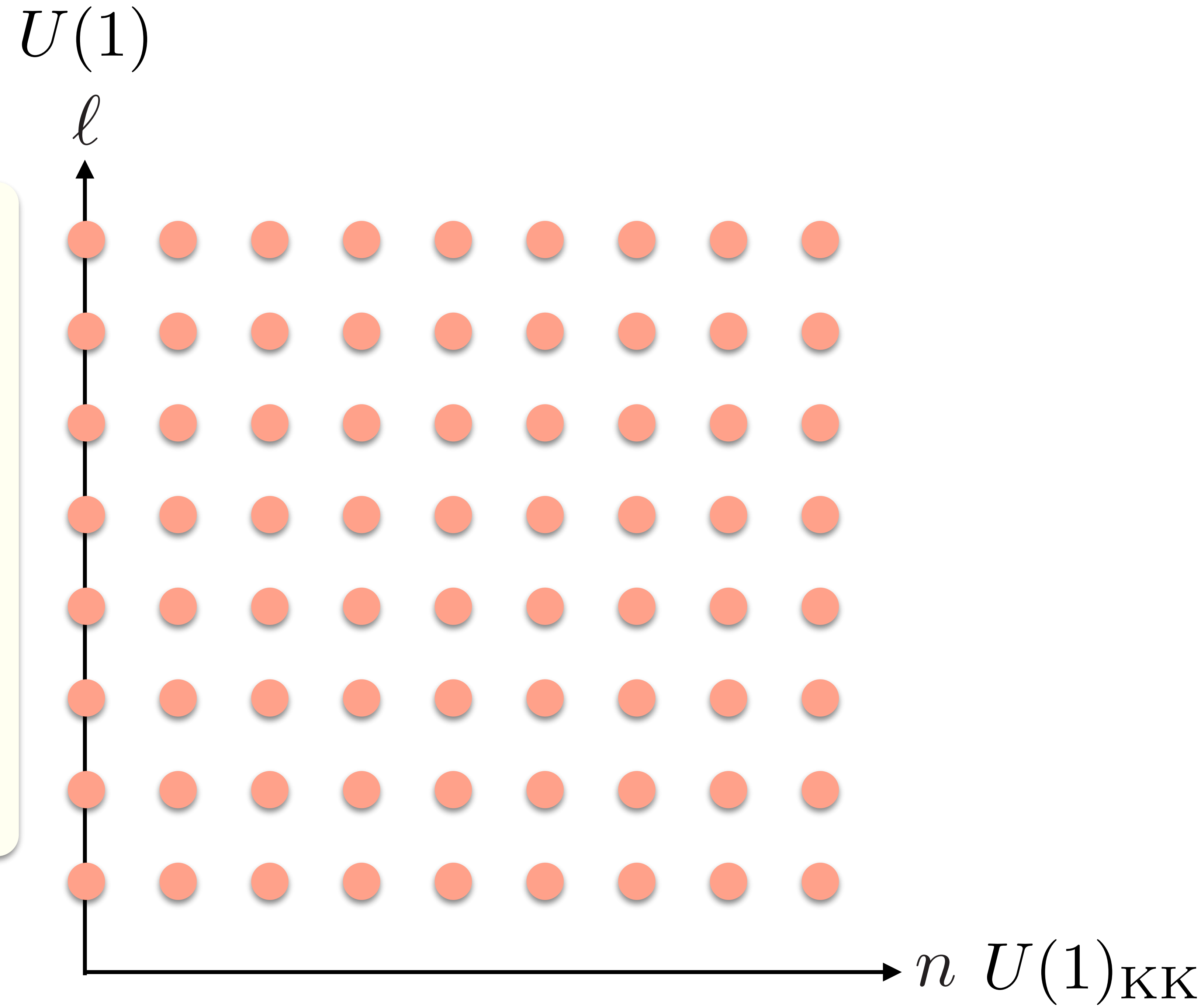
charged particles

labeled by $\ell = 1, 2, \ldots$

$$(q, m) = (\ell\, q_*, \ell\, m_*)$$

s.t. $z_* = \dfrac{q_*}{m_*} = \mathcal{O}(1)$





<u>d+1 dim</u>

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d dim charged particles

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<u>d+1 dim</u>

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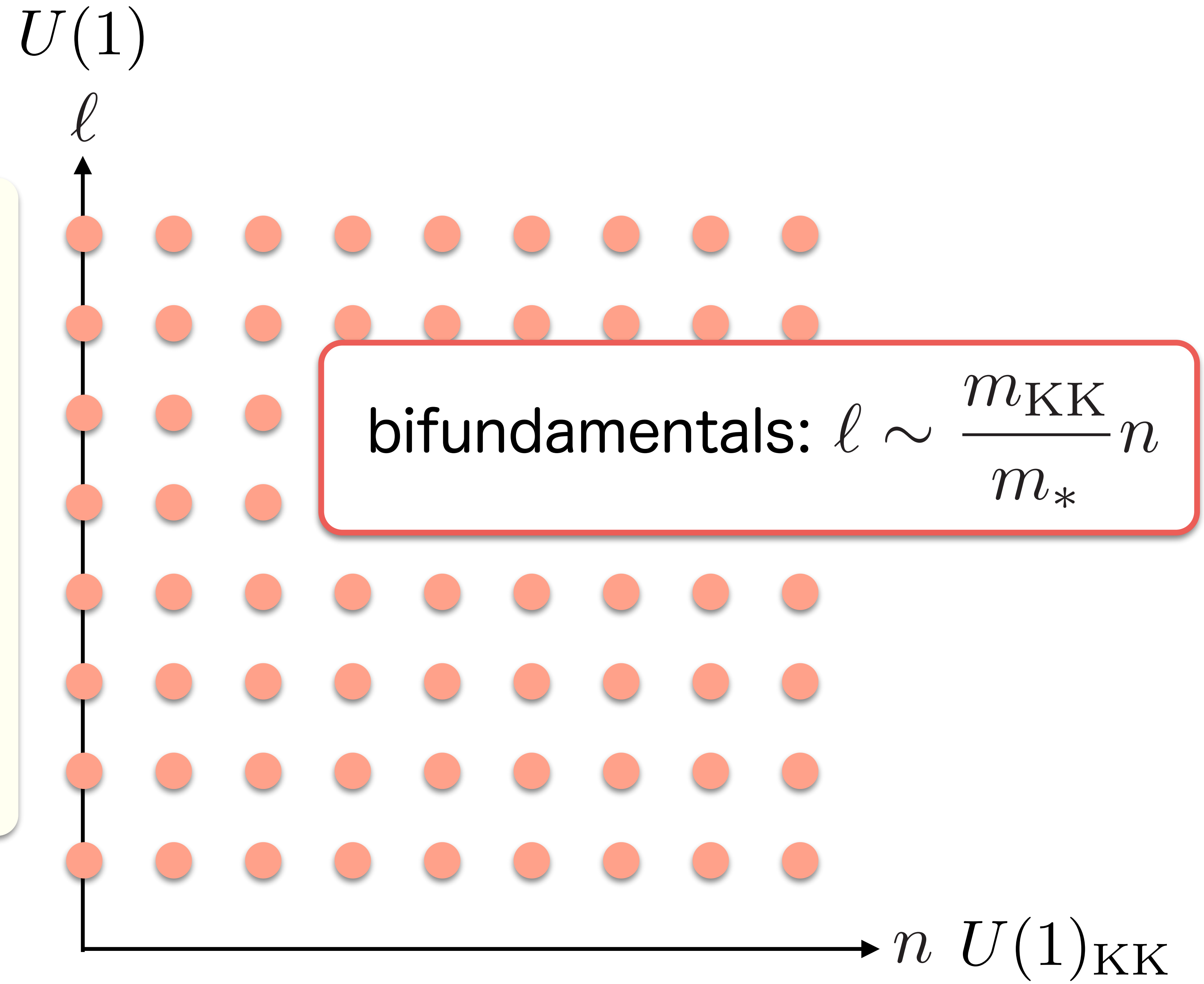

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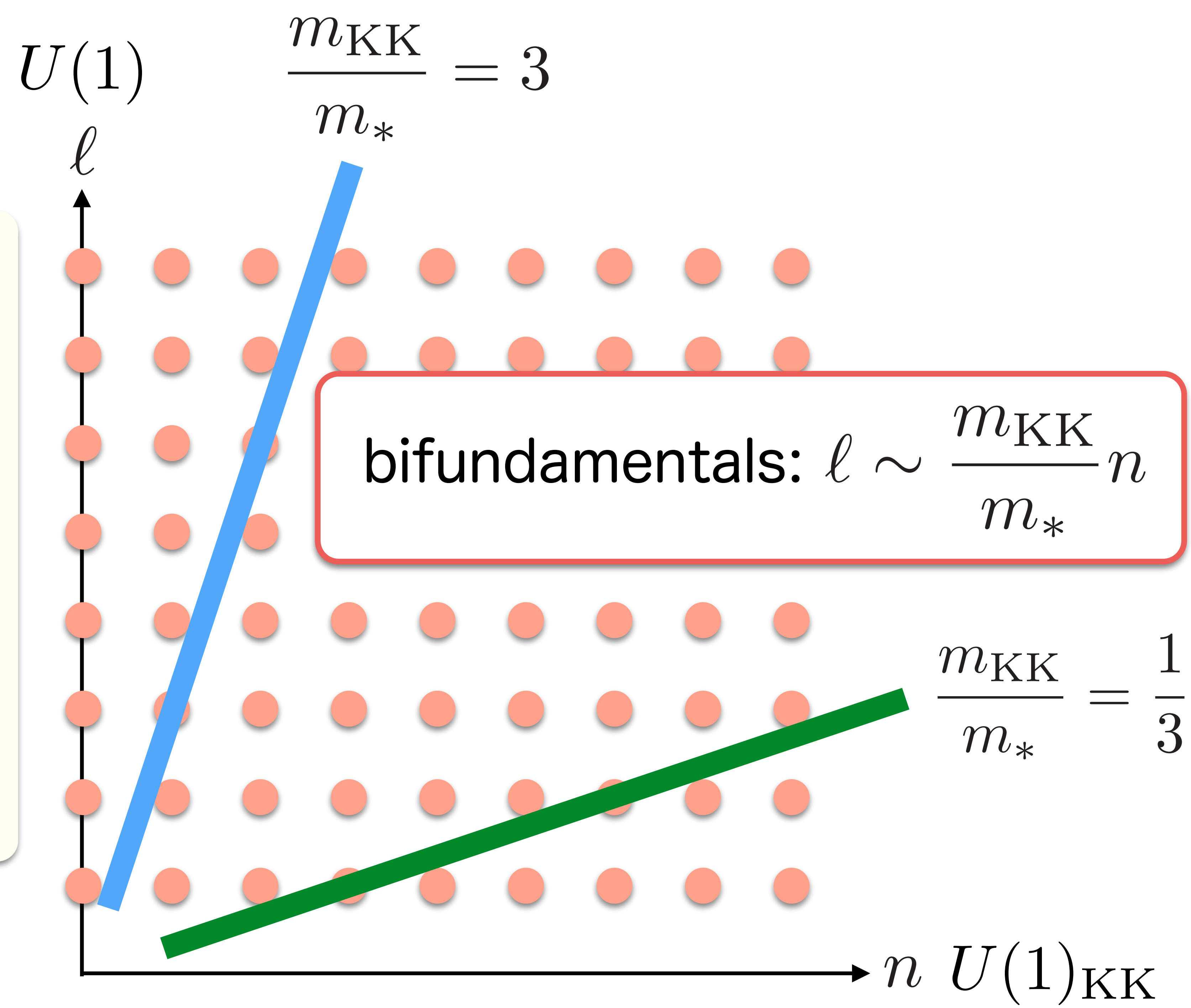

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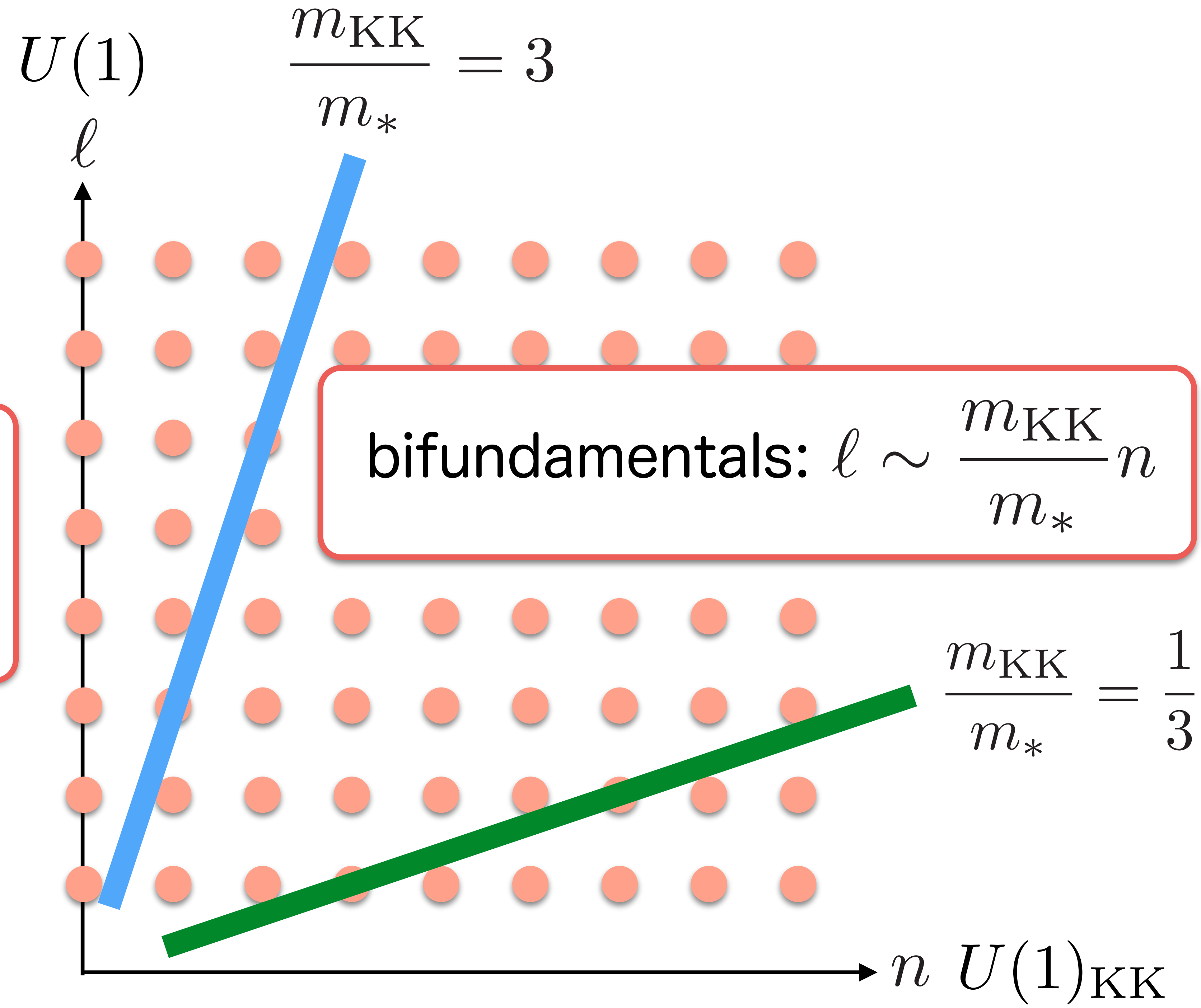
bifundamentals

for $\forall$ KK scale $m_{\rm KK}$!



d dim charged particles

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in this way, consistency with KK reduction

seems to imply a tower of d+1 dim U(1) charged particles

→ Tower Weak Gravity Conjecture!

※ a similar conjecture “lattice WGC” was proposed

based on BH argument [Heidenreich-Reece-Rudelius ’15]

summary and prospects

summary

Weak Gravity Conjecture

- requires existence of a superextremal particle
- upper bound on axion decay constant

  → relevant to axion inflation, axion DM, ⋯

positivity bound

- signs of effective interactions are generically constrained by unitarity, analyticity and causality

argued possible connection between the two

- bifundamental particles when we have multiple U(1)'s
- KK reduction implies a tower of charged particles

prospects

phenomenological implications of Tower WGC

ex. axion potential generated by infinite instanton species

$$V(\phi) = \sum_i e^{-S_i} \cos\left(\frac{\phi}{f_i} + \alpha_i\right)$$

i: label of instanton species

relation to other consistency requirements

ex. entropy bounds on higher derivatives corrections

Thank you!