Color confinement due to violation of non-Abelian Bianchi identity

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T.Suzuki, arXiv:1402.1294 (2014)

T. Suzuki et al, P.R. D80, 054504 (2009)

T.Suzuki, K.Ishiguro, V.Bornyakov, arXiv:1712.05941, P.R. D97 (2018) 034501

T. Suzuki, arXiv:1712.06812(2017), P.R. D97 (2018) 034509

1. Introduction

Color confinement problem:

Almost half a century history !!!

1. 1963: Quark model (Gell-Mann and Zweig): fractionally charged quarks confined?
2. 1971: Quark confinement as a superconductor (Nambu)
3. 1972: Quantum Chromogynamics (QCD) as non-Abelian gauge theory $SU_c(3)$ (Fritzsch and Gell-Mann): quark confinement as color confinement?
4. 1974-75: Idea of dual superconductor (electric $\leftrightarrow$ magnetic) as the color-confinement mechanism ('tHooft-Mandelstam): Something color magnetic must be condensed.
5. So far two ideas proposed. Color magnetic monopole and color magnetic vortex

Not yet solved !!! One of the biggest problems unsolved in QCD!!!

To clarify color confinement mechanism = To find a theory in QCD such as BCS in usual superconductor

Difficulties of gluodynamics in comparison with the BCS theory: (Let me neglect dynamical quarks in the following.)

- Non-Abelian $SU(3)$ gauge symmetry in gluodynamics vs Abelian $U(1)$ of the BCS theory
- Scale invariance with no dimensional constants (in BCS, electrons with mass)

Important targets in gluodynamics:

1. To find a theoretical framework like the Higgs mechanism in superconductor without breaking non-Abelian $SU(3)$ symmetry
2. To find a key magnetic quantity which condenses like the Cooper pair from massless gluons alone
3. To show analytically the existence of mass gaps, namely, string tension, glueball mass, .. from the scale-invariant theory $\longleftarrow$ One of 'The millennium problems : the seven greatest unsolved mathematical puzzles of our time'
4. To analytically explain the value of T_c
5. To analytically explain the properties of finite-temperature transition, i.e., the 2nd-order phase transition in SU2 gluodynamics and the 1st-order phase transition in SU3 gluodynamics

2. 'tHooft idea of monopole in QCD: Abelian projection

G. 'tHooft, N.P. B190 (1981) 455.

SU2 gluodynamics coupled to adjoint Higgs fields has a classical monopole solution. But there is no Higgs field in QCD. 'tHooft's proposed an idea of Abelian projection:

1. Consider a composite operator $X(A_\mu)$ transforming as an adjoint operator.

2. X can be diagonalized by a gauge transformation: $X' = V(x)XV^\dagger(x) = \lambda(x)\sigma_3$

3. There is a $U(1)$ ambiguity w.r.t $V(x)$ corresponding to the maximal torus group of $SU(2)$. Hence fix the form of $V(x)$, say, as $V(x) = \exp(ia_1\sigma_1 + ia_2\sigma_2)$ and consider the transformation property of V under an arbitrary $SU2)$ matrix $W(x)$. Then $V(x)$ transforms as

$$
\begin{aligned}
V^W(x) &= d(x)V(x)W^\dagger(x) \\
V^W(x)X^W V^{W\dagger}(x) &= \lambda(x)\sigma_3
\end{aligned}
$$

where $d(x) \in U(1)$ keeps the form of V^W as fixed.

4. Then the gauge transformed gauge field

$$\begin{aligned} A'_\mu &= VA_\mu V^\dagger - \frac{i}{g}\partial_\mu V V^\dagger \\ &\xrightarrow{W} dVW^\dagger(WA_\mu W^\dagger - \frac{i}{g}\partial_\mu W W^\dagger)WV^\dagger d^\dagger \\ &\quad -\frac{i}{g}\partial_\mu(dVW^\dagger)WV^\dagger d^\dagger \\ &= dA'_\mu d^\dagger - \frac{i}{g}\partial_\mu d d^\dagger \end{aligned}$$

Only the diagonal component of $A'_\mu(x)$ transforms as a photon with respect to the remaining $U(1)$ and other components transform as a charged matter.

5. Define $\hat{Y} \equiv V^\dagger \sigma_3 V$ and $a_\mu = \mathrm{Tr}\hat{Y}A_\mu$:

$$\begin{aligned} & A'_\mu = VA_\mu V^\dagger - \frac{i}{g}\partial_\mu V V^\dagger \\ \rightarrow \qquad & A'^3_\mu = a_\mu - \frac{1}{g(1+\hat{Y}^3)}\epsilon_{3ab}\hat{Y}^a\partial_\mu\hat{Y}^b \end{aligned}$$

There is a line singularity on $\hat{Y}^3(x) = -1$. Note $\hat{Y}^2 = 1$.

6. Monopole appears at the point where the gauge fixing matrix $V(x)$ is not defined, i.e., $X(x) = 0$.

$$
\begin{aligned}
f^3_{\mu\nu} &\equiv \partial_\mu A'^3_\nu - \partial_\nu A'^3_\mu \\
&= \partial_\mu a_\nu - \partial_\nu a_\mu - \frac{1}{g}\epsilon_{abc}\hat{Y}^a \partial_\mu \hat{Y}^b \partial_\nu \hat{Y}^c \\
{}^*f^3_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} f^3_{\rho\sigma} \\
\partial_\mu {}^*f^3_{\mu\nu} &= k^3_\nu = \frac{1}{2g}\epsilon_{\mu\nu\rho\sigma}\epsilon_{abc}\partial_\nu \hat{Y}^a (\partial_\rho \hat{Y}^b)(\partial_\sigma \hat{Y}^c) \\
g_m &\equiv \int_{V_r} d^3x k^3_0 = \frac{1}{2g}\int_S \epsilon_{ijk}\epsilon_{abc}\hat{Y}^a \partial_j \hat{Y}^b \partial_k \hat{Y}^c (d^2\sigma)_i \\
&= \frac{N}{g}\int_S A d^2\xi,
\end{aligned}
$$

where A is the Jacobian of $\hat{Y}^2 = 1 \Leftrightarrow S^2$ and $d^2\xi$ is the surface element describing the space-time surface $S(\xi^1, \xi^2)$.

$$
g g_m = 4\pi N \longleftarrow \Pi_2(SU(2)/U(1) = S^2) = Z
$$

'tHooft conjectured that the monopoles appeared here condense in the vacuum, so that color confinement occurs.

Many interesting numerical results were obtained from 1987 until now, suggesting the relevance of 'tHooft's monopoles especially by adopting a special smooth gauge called maximally Abelian gauge (MAG).

1. Maximally Abelian gauge (MA):

A.S. Kronfeld *et al.*, P.L. **198B** (1987) 516; N.P. **B293** (1987) 461

The 'tHooft scenario depends on the choice of partial gauge fixing. The maximally Abelian gauge to maximize $R = \sum_{s,\hat{\mu}} \mathrm{Tr}\Big(\sigma_3 U(s,\mu)\sigma_3 U^\dagger(s,\mu)\Big)$.

2. Abelian dominance:

T. Suzuki and I. Yotsuyanagi, P.R. **D42** (1990) 4257

3. Monopole dominance:

H. Shiba and T. Suzuki, P.L.333B (1994) 461, J. Stack et al., P.R. D50 (1994) 3399

4. Abelian dual Meissner effect:

Y. Koma et al., P.R. D68 (2003) 114504

Figure 1: Color electric field squeezing

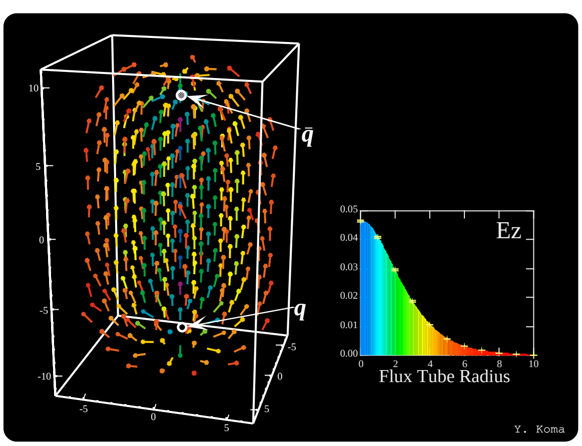
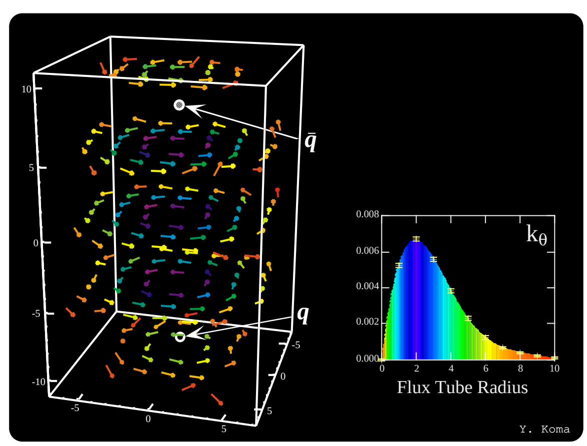


Figure 2: Monopole currents



5. Blockspin transformation and infrared effective monopole action:

H. Shiba and T. Suzuki

6. Monopole action in the continuum space-time:

M. Chernodub et al., P.R. D62 (2000) 094506

7. Monopoles and dynamical mass generation:

S. Kitahara et al., N.P. B533 (1998) 576.

8. Finte-temperature transition and monopole:

S. Ejiri et al., Phys. Lett. B400 (1997) 163

9. Monopoles and the anomaly $\geq T_c$:

M. Chernodub et al.,PoS (LATTICE 2007) 174

10. Monopole importance at high temperature:

K. Ishiguro et al., JHEP 0201 (2002) 038 and references therein.

11. Monopoles and chiral symmetry breaking:

A. Ramamuruti and E. Schuryak, arXiv:1801.06922.

12. Monopoles in other gauges:

T. Sekido et al., Phys. Rev. D76 (2007) 031501. M. Chernodub, P.R.**D69** (2004) 094504

Although numerically interesting in MA gauge, the 'tHooft idea of Abelian projection is theoretically very very unsatisfactory.

1. In non-perturabative QCD, any gauge-fixing is not necessary at all. 'tHooft scheme needs to introduce an additional partial gauge-fixing. There are infinite ways of such a partial gauge-fixing. the 't Hooft scheme looks gauge dependent !!!

2. After an Abelian projection, only one (in $SU(2)$) or two (in $SU(3)$) gluons are photon-like with respect to the residual $U(1)$ or $U(1)^2$ symmetry and the other gluons are massive charged matter fields. Such an asymmetry among gluons is unnatural.

3. Charge confinement but whether non-Abelian color is confined or not is unknown.

3. Breakthrough in monopole in QCD

Note the Jacobi identities:

$$\epsilon_{\mu\nu\rho\sigma}[D_\nu, [D_\rho, D_\sigma]] = 0,$$

where $D_\mu \equiv \partial_\mu - igA_\mu$. Calculate explicitly:

$$\begin{aligned}
[D_\rho, D_\sigma] &= [\partial_\rho - igA_\rho, \partial_\sigma - igA_\sigma] \\
&= -ig(\partial_\rho A_\sigma - \partial_\sigma A_\rho - ig[A_\rho, A_\sigma]) + [\partial_\rho, \partial_\sigma] \\
&= -igG_{\rho\sigma} + [\partial_\rho, \partial_\sigma]
\end{aligned}$$

$[\partial_\rho, \partial_\sigma]$ can not be neglected in general!!

$D_\nu G^*_{\mu\nu} = 0 \rightarrow$ Non-Abelian Bianchi identity (NABI):

$\partial_\nu f^*_{\mu\nu} = 0 \rightarrow$ Abelian-like Bianchi identity:

$f_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\sigma^a/2$

Jacobi identity + $[D_\nu, G_{\rho\sigma}] = D_\nu G_{\rho\sigma}$

$$
\begin{aligned}
\Longrightarrow D_\nu G^*_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma} \\
&= -\frac{i}{2g}\epsilon_{\mu\nu\rho\sigma}[D_\nu, [\partial_\rho, \partial_\sigma]] \\
&= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\partial_\rho, \partial_\sigma] A_\nu = \partial_\nu f^*_{\mu\nu} \\
J_\mu = \frac{1}{2} J^a_\mu \sigma^a = D_\nu G^*_{\mu\nu} &= \partial_\nu f^*_{\mu\nu} = \frac{1}{2} k^a_\mu \sigma^a = k_\mu
\end{aligned}
$$

$k^a_\mu \neq 0 \rightarrow$ color magnetic Abelian-like monopole
$J^a_\mu \neq 0 \rightarrow$ Violation of NABI

Color magnetic monopoles= Violation of non-Abelian Bianchi identity (VNABI)

$$
[\partial_\rho, \partial_\sigma] A_\nu \neq 0
$$
$$
\Downarrow
$$

Line singularities existing in gauge fields $A_\mu(x)$ themselves!!! are the origin of the QCD monopoles and $N^2 - 1$ monopoles exist in $SU(N)$.

This is completely different from the 'tHooft definition of monopoles, where original gauge fields are assumed regular and the additional singular gauge transformation leads to a line singularity and monopole.

Note:

$$
\begin{aligned}
\partial_\mu J^a_\mu &= \partial_\mu k^a_\mu = \partial_\mu \partial_\nu f^{*a}_{\mu\nu} = 0 \\
D_\mu J_\mu &= D_\mu D_\nu G^*_{\nu\mu} = \frac{1}{2}[D_\mu, D_\nu] G^*_{\nu\mu}. \\
&= \frac{ig}{4}\epsilon_{\nu\mu\rho\sigma}[G_{\nu\mu}, G_{\rho\sigma}] = 0
\end{aligned}
$$

$$
\begin{aligned}
D_\mu J_\mu - \partial_\mu J_\mu &= -ig[A_\mu, J_\mu] \\
&= \epsilon_{\mu\nu\rho\sigma}[A_\mu, \partial_\nu \partial_\rho A_\sigma] \\
&= -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu \partial_\mu [A_\rho, A_\sigma] \\
&= \frac{i}{g}(\partial_\mu \partial_\nu G^*_{\mu\nu} - \partial_\mu \partial_\nu f^*_{\mu\nu}) = 0.
\end{aligned}
$$

VNABI satify also Abelian conservation rule!
$N^2 - 1$ conserved charges exist in SU(N)

Each $N^2 - 1$ current satisfies the Dirac quantization condition:

Consider a space-time point O where VNABI$\neq 0$ and 3d sphere V with a radius r from O.

Note $k_4 = J_4$ is given by the total derivative. When $r \to \infty$, the non-Abelian field strength should vanish since otherwise the action diverges. Then the magnetic charge could be evaluated by a gauge field described by a pure gauge $A_\mu = \Omega\partial_\mu\Omega^\dagger/ig$.

$SU(2)$

$$
\begin{aligned}
g_m^d &= \int_V d^3x k_4^d = \int d^3x \frac{1}{2}\epsilon_{4\nu\rho\sigma}\partial_\nu(\partial_\rho A_\sigma^d - \partial_\sigma A_\rho^d) \\
&= \int_V d^3x \frac{1}{2ig}\epsilon_{ijk}\partial_i \mathrm{Tr}\sigma^d(\partial_j\Omega\partial_k\Omega^\dagger - \partial_k\Omega\partial_j\Omega^\dagger \\
&\quad +\Omega[\partial_j, \partial_k]\Omega^\dagger) \\
&= \int_V d^3x \frac{1}{2g}\epsilon_{ijk}\epsilon^{abc}\partial_i(\hat{\phi}^a\partial_j\hat{\phi}^b\partial_k\hat{\phi}^c) \\
&= \int_{\partial V} d^2S \frac{1}{2g}\epsilon_{ijk}\epsilon^{abc}\hat{\phi}^a\partial_j\hat{\phi}^b\partial_k\hat{\phi}^c,
\end{aligned}
$$

$\hat{\phi}$ is a Higgs-like field defined as $\hat{\phi} = \hat{\phi}^i\sigma^i = \Omega\sigma^d\Omega^\dagger$.

$\hat{\phi}^2 = 1$ is shown easily.
Since the field $\hat{\phi}$ is a single-valued function, the integral is given by the wrapping number n characterizing the homotopy class of the mapping between the spheres described by $\hat{\phi}^2 = (\hat{\phi}^1)^2 + (\hat{\phi}^2)^2 + (\hat{\phi}^3)^2 = 1$ and $\partial V = S^2$: $\pi_2(S^2) = Z$. Namely

$$g_m^d g = 4\pi n. \quad d = 1, 2, 3$$

This is just the Dirac quantization condition. Note that the minimal color electric charge in any color direction is $g/2$.

$SU(3)$

Ω is a 3×3 gauge transformation matrix of $SU(3)$.:
Consider a magnetic charge in the λ^1 direction: Then

$$
\begin{aligned}
g_m^1 &= \int_V d^3 x k_4^1 \\
&= \int_{\partial V} d^2 S \frac{1}{2g} \epsilon_{ijk} \epsilon^{abc} \hat{\phi}^a \partial_j \hat{\phi}^b \partial_k \hat{\phi}^c,
\end{aligned}
$$

where $\hat{\phi} = \hat{\phi}^i \lambda^i = \Omega \lambda^1 \Omega^\dagger$.

$$
\hat{\phi}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
$$

This gives us

$$
\begin{aligned}
(\hat{\phi}^1)^2 + (\hat{\phi}^2)^2 + (\hat{\phi}^3)^2 &= 1, \\
\hat{\phi}^4 = \hat{\phi}^5 = \hat{\phi}^6 = \hat{\phi}^7 = \hat{\phi}^8 &= 0.
\end{aligned}
$$

The subspace composed of $(\hat{\phi}^1, \hat{\phi}^2, \hat{\phi}^3)$ is a sphere S^2 and the mapping is just like that in the case of color $SU(2)$. Hence

$$
g_m^1 g = 4\pi n.
$$

Proposal of a new color confinement scheme

Now new monopoles are found to exist in QCD as VNABI coming from the line singularities of original gauge fields.

However the monopoles transform as an adjoint operator having a color charge. Hence if all color components of the monopole make condensation in the vacuum independently, the electric color symmetry is broken spontaneously as well as the color magnetic symmetry. This is contradictory to the usual picture of color confinement where electric color symmetry is not broken.

Note $J_\mu(x)$ can be diagonalized by a unitary matrix $V_d(x)$:

$$V_d(x)J_\mu(x)V_d^\dagger(x) = \lambda_\mu(x)\frac{\sigma_3}{2},$$

where $\lambda_\mu(x)$ is the eigenvalue. Note in the continuum space-time, the unitary matrix $V_d(x)$ does not depend on the direction μ of the current as shown by Coleman-Mandula theorem.

S. Coleman and J. Mandula, P.R. 159 (1967) 1251.

Then one gets

$$\begin{aligned}
\Phi(x) &\equiv V_d^\dagger(x)\sigma_3 V_d(x) \\
J_\mu(x) &= \frac{1}{2}\lambda_\mu(x)\Phi(x), \\
\sum_a (J_\mu^a(x))^2 &= \sum_a (k_\mu^a(x))^2 = (\lambda_\mu(x))^2.
\end{aligned}$$

The color electrically charged part $\Phi(x)$ and the magnetically charged part $\lambda_\mu(x)$ are separated out. Also

$$\begin{aligned}
\partial_\mu J_\mu(x) &= \frac{1}{2}(\partial_\mu\lambda_\mu(x)\Phi(x) + \lambda_\mu(x)\partial_\mu\Phi(x)) \\
&= 0.
\end{aligned}$$

Since $\Phi(x)^2 = 1$,

$$\begin{aligned}
\partial_\mu\lambda_\mu(x) &= -\lambda_\mu(x)\Phi(x)\partial_\mu\Phi(x) \\
&= 0.
\end{aligned}$$

The gauge-invariant eigenvalue $\lambda_\mu(x)$ also satisfies the Abelian conservation rule.

It is possible to prove that

$$\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu f'_{\mu\nu}(x) \quad = \quad \lambda_\mu(x)\frac{\sigma_3}{2},$$

where

$$\begin{aligned} f'_{\mu\nu}(x) &= \partial_\mu A'_\nu(x) - \partial_\nu A'_\mu(x) \\ A'_\mu &= V_d A_\mu V_d^\dagger - \frac{i}{g}\partial_\mu V_d V_d^\dagger, \\ &\equiv \frac{A'^a_\mu \sigma^a}{2}. \end{aligned}$$

Namely,

$$\begin{aligned} \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu f'^{1,2}_{\rho\sigma}(x)(x) &= 0 \\ \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu f'^{3}_{\rho\sigma}(x)(x) &= \lambda_\mu(x). \end{aligned}$$

The singularity appears only in the diagonal component of the gauge field A'_μ.

If one considers for large r

$$
\begin{aligned}
A'_\mu &\to \Omega \partial_\mu \Omega^\dagger / ig, \\
\hat{\phi} &= \hat{\phi}^i \sigma^i = \Omega \sigma^3 \Omega^\dagger,
\end{aligned}
$$

one can easily see that the magnetic charge from the eigenvalue λ_μ also satisfies the Dirac quantization condition $gg_m = 4\pi n$.

The condensation of the gauge-invariant magnetic currents λ_μ does not give rise to a spontaneous breaking of the color electric symmetry.

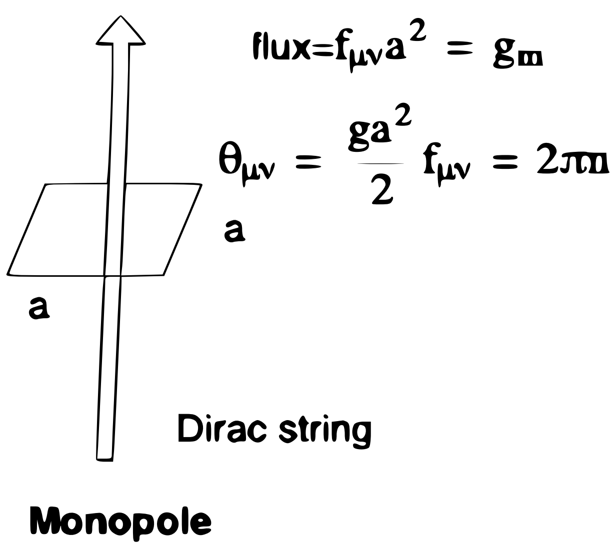
Condensation of the color invariant magnetic currents λ_μ may be a key mechanism of the physical confining vacuum.

4. Monopole dominance and the dual Meissner effect

T. Suzuki et al., P.R. D77 (2008) 034502, T. Suzuki et al., P.R. D80 (2009) 054504.

Define each color-component of lattice VNABI as a lattice monopole following DeGrand and Toussaint without any additional gauge-fixing

$$
\begin{aligned}
\theta^a_\mu(s) &= \arctan(U^a_\mu(s)/U^0_\mu(s)) \quad (|\theta^a_\mu(s)| < \pi, \quad a = 1,2,3) \\
\theta^a_{\mu\nu}(s) &\equiv \partial_\mu \theta^a_\nu(s) - \partial_\nu \theta^a_\mu(s) = \bar{\theta}^a_{\mu\nu}(s) + 2\pi n^a_{\mu\nu}(s) \quad (|\bar{\theta}^a_{\mu\nu}| < \pi) \\
k^a_\mu(s) &= -(1/2)\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha \bar{\theta}^a_{\beta\gamma}(s+\hat{\mu}) = (1/2)\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha n^a_{\beta\gamma}(s+\hat{\mu})
\end{aligned}
$$



integer $n_{\mu\nu}$: Dirac quantization condition

Abelian conservation rule: $\sum_\mu \left(k^a_\mu(s) - k^a_\mu(s-\mu)\right) = 0$

Gauge invariant eigenvalue: $(\lambda_\mu(s))^2 = \sum_{a=1}^3 (k^a_\mu(s))^2$

Lattice monopole is not gauge-covariant and various techniques smoothing the vacuum are necessary to extract physical moopole in general. But evaluating correlations between a gauge-invariant quantities and monopoles can be done without any gauge-fixing. Note Elitzer's theorem saying that only the gauge-invariant part is extracted by Monte-Carlo average of gauge-variant quantities.

S. Elitzur, P.R. D12 (1975) 3978.

Monopole dominance of the string tension

$$V_{\rm mon}(R) = -\frac{1}{aN_t}\ln\langle P_{\rm mon}(0)P^*_{\rm mon}(R)\rangle \ .$$

$$P_{\rm A} = \exp[i\sum_{k=0}^{N_t-1}\theta_4(s+k\hat{4})] = P_{\rm ph}\cdot P_{\rm mon} \ ,$$

$$P_{\rm ph} = \exp\{-i\sum_{k=0}^{N_t-1}\sum_{s'}D(s+k\hat{4}-s')\partial'_\nu\bar{\Theta}_{\nu 4}(s')\} \ ,$$

$$P_{\rm mon} = \exp\{-2\pi i\sum_{k=0}^{N_t-1}\sum_{s'}D(s+k\hat{4}-s')\partial'_\nu n_{\nu 4}(s')\}$$

Take average over $4000 \sim 7000$ thermalized vacua and their random gauge-transformed vacua $1000 \sim 4000$ for each one (totally around 10 million vacua).

Table 1: Best fitted values of the string tension σa^2, the Coulombic coefficient c, and the constant μa for the potentials $V_{\rm NA}$, $V_{\rm A}$, $V_{\rm mon}$ and $V_{\rm ph}$.

$24^3 \times 4$	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.181(8)	0.25(15)	0.54(7)	3.9 - 8.5	1.00
$V_{\rm A}$	0.183(8)	0.20(15)	0.98(7)	3.9 - 8.2	1.00
$V_{\rm mon}$	0.183(6)	0.25(11)	1.31(5)	3.9 - 6.7	0.98
$V_{\rm ph}$	$-2(1) \times 10^{-4}$	0.010(1)	0.48(1)	4.9 - 9.4	1.02
$24^3 \times 6$	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.072(3)	0.49(6)	0.53(3)	4.0 - 9.0	0.99
$V_{\rm A}$	0.073(4)	0.41(7)	1.09(3)	3.7 - 10.9	1.00
$V_{\rm mon}$	0.073(4)	0.44(10)	1.41(4)	3.9 - 9.3	1.00
$V_{\rm ph}$	$-1.7(3) \times 10^{-4}$	0.0131(1)	0.4717(3)	5.1 - 9.4	0.99
$36^3 \times 6$	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.072(3)	0.48(9)	0.53(3)	4.6 - 12.1	1.03
$V_{\rm A}$	0.073(2)	0.47(6)	1.10(2)	4.3 - 11.2	1.03
$V_{\rm mon}$	0.073(3)	0.46(7)	1.43(3)	4.0 - 11.8	1.01
$V_{\rm ph}$	$-1.0(1) \times 10^{-4}$	0.0132(1)	0.4770(2)	6.4 - 11.5	1.03
$24^3 \times 8$	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.0415(9)	0.47(2)	0.46(8)	4.1 - 7.8	0.99
$V_{\rm A}$	0.041(2)	0.47(6)	1.10(3)	4.5 - 8.5	1.00
$V_{\rm mon}$	0.043(3)	0.37(4)	1.39(2)	2.1 - 7.5	0.99
$V_{\rm ph}$	$-6.0(3) \times 10^{-5}$	0.0059(3)	0.46649(6)	7.7 - 11.5	1.02

Abelian dual Meisnner effect

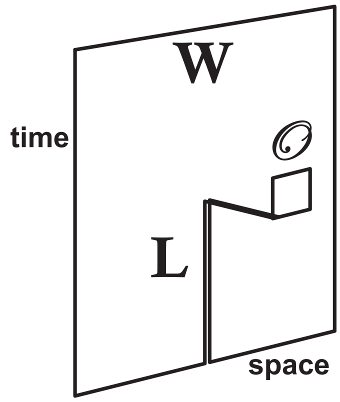


Figure 3: A schematic figure for the connected correlation function.

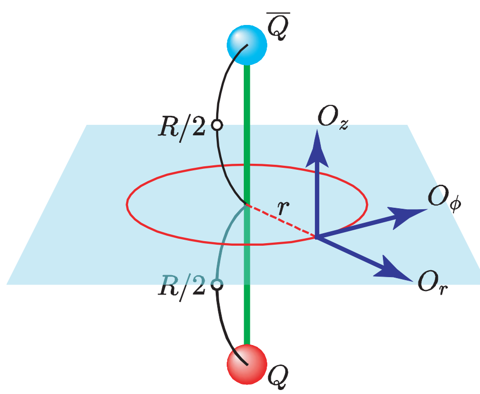


Figure 4: Definition of the cylindrical coordinate (r, ϕ, z) along the q-$\bar{q}$ axis.

$$\langle \mathcal{O}(r) \rangle_W = \frac{\langle \mathrm{Tr}\left[L W(R,T) L^\dagger \sigma^1 \mathcal{O}(r) \right] \rangle}{\langle \mathrm{Tr}\left[W(R,T) \right] \rangle} .$$

Iwasaki gauge action at $\beta = 1.10$ and 1.28 on the 32^4 lattice, and $\beta = 1.40$ on the 40^4 lattice.

1) Penetration length:

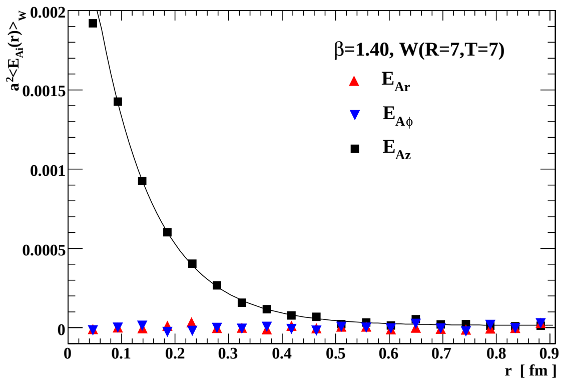


Figure 5: The profile of the color-electric field $\vec{E}_{\rm A}$ at $\beta = 1.40$

2) The dual Ampère law

$$\vec{\nabla} \times \vec{E}_{\rm A} = \partial_4 \vec{B}_{\rm A} + 2\pi \vec{k} \ ,$$

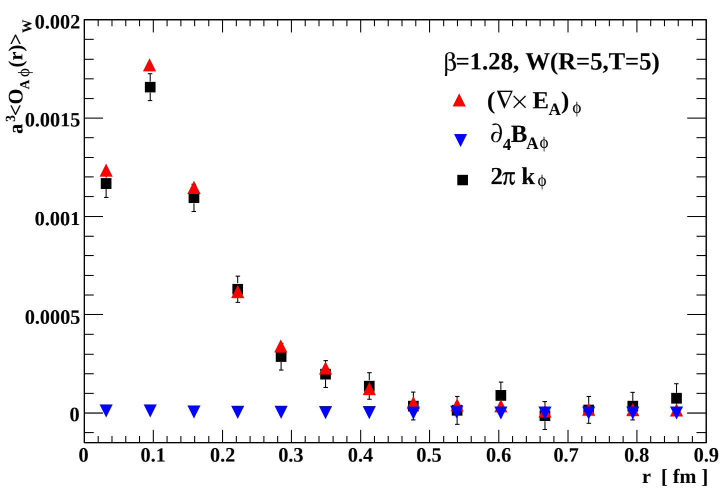


Figure 6: Tests of the dual Ampère law at $\beta = 1.28$ for $W(R = 5, T = 5)$

3) Coherence length and the Ginzburg-Landau parameter
Coherence length can be fixed from the correlation function between the squared monopole density $\mathcal{O}(s) = k_\mu^2(s)$ and the Wilson loop ('05 Chernodub).

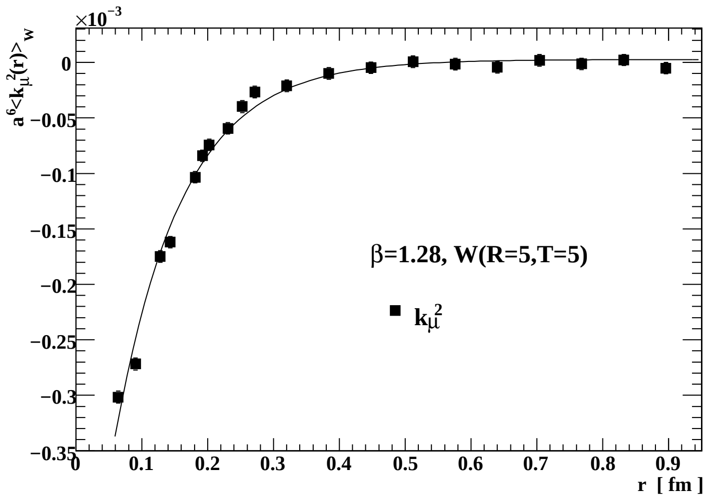


Figure 7: The profile of the squared monopole currents at 1.28

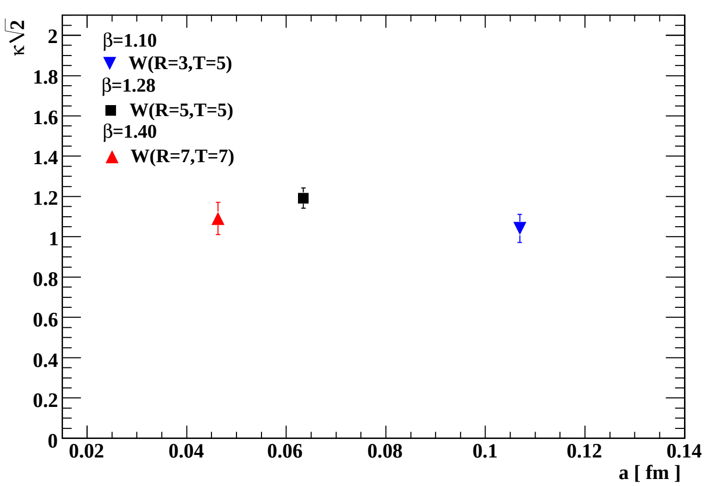


Figure 8: The GL parameters $\sqrt{2}\kappa = \lambda/\xi$ as a function of the lattice spacing $a(\beta)$.

5. Existence of the continuum limit

Does the continuum limit of $\lambda_\mu(s)$ exist?

The lattice vacuum is contaminated with large amount of lattice artifact monopoles. To reduce lattice artifacts, various techniques smoothing the vacuum are introduced.
1. Tadpole improved action:

$$S = \beta \sum_{pl} S_{1\times 1} - \frac{\beta}{20u_0^2} \sum_{rt} S_{1\times 2}, \quad u_0 = (\langle \frac{1}{2} \mathrm{tr} U_{pl} \rangle)^{1/4}$$

48^4 at $\beta = 3.0 \sim 3.9$ and 24^4 at $\beta = 3.0 \sim 3.7$

2. Introduction of smooth gauge-fixings

1) Maximal center gauge: Maximization of
$R = \sum_{s,\mu} (\mathrm{Tr} U(s,\mu))^2$ SU(2) $\to$ Z(2)

2) Direct Laplacian center gauge (DLCG)

3) Maximal Abelian Wilson loop gauge (AWL):
Maximization of $R = \sum_{s,\mu\neq\nu} \sum_a (cos(\theta^a_{\mu\nu}(s))$

4) Maximal Abelian and $U(1)$ Landau gauge (MAU1):
Maximization of $R = \sum_{s,\hat{\mu}} \mathrm{Tr}\Big(\sigma_3 U(s,\mu) \sigma_3 U^\dagger(s,\mu)\Big) \to$
Maximization of $\sum_{s,\mu} cos\theta^3_\mu(s)$ global color violating

Table 2: A typical example of monopole loop distributions (Loop length (L) vs Loop number (No.)) on 24^4 lattice at $\beta = 3.6$. $a(\beta = 3.6) = 0.13(\sigma_{phys}^{-1/2})$, $24a(\beta = 3.6) = 1.4$fm

NGF		MCG		DLCG	
L	No	L	No	L	No
4	154	4	166	4	164
6	20	6	64	6	66
8	7	8	30	8	28
10	2	10	13	10	15
14	1	12	11	12	10
16	1	14	4	14	3
407824	1	16	5	16	6
		18	1	18	2
		22	2	20	1
		24	2	22	1
		28	1	24	2
		30	1	26	3
		32	1	30	1
		34	2	36	1
		36	1	44	1
		44	1	48	1
		46	1	54	1
		48	1	58	1
		58	1	124	1
		124	1	1106	1
		2254	1	1448	1
AWL		MAU1 I=1		MAU1 I=3	
L	No	L	No	L	No
4	142	4	73	4	190
6	66	6	32	6	80
8	36	8	13	8	22
10	8	10	11	10	15
12	7	12	6	12	2
14	3	14	3	14	3
16	3	16	2	16	1
18	1	18	3	18	3
20	1	20	2	20	3
22	3	22	1	24	1
26	3	30	2	36	1
28	1	34	2	42	1
30	2	58	1	60	1
32	1	148	1	66	1
34	1	5188	1	146	1
40	1			318	1
46	1			722	1
58	1				
120	1				
308	1				
1866	1				

Study of the continuum limit based on the blockspin transformation.

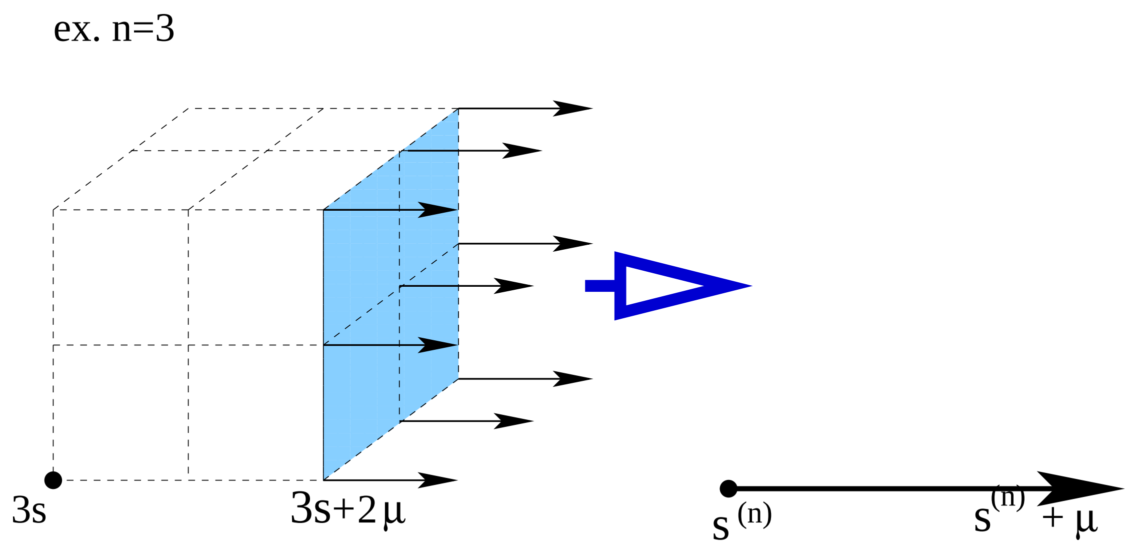


Figure 9: Blockspin definition of monopoles

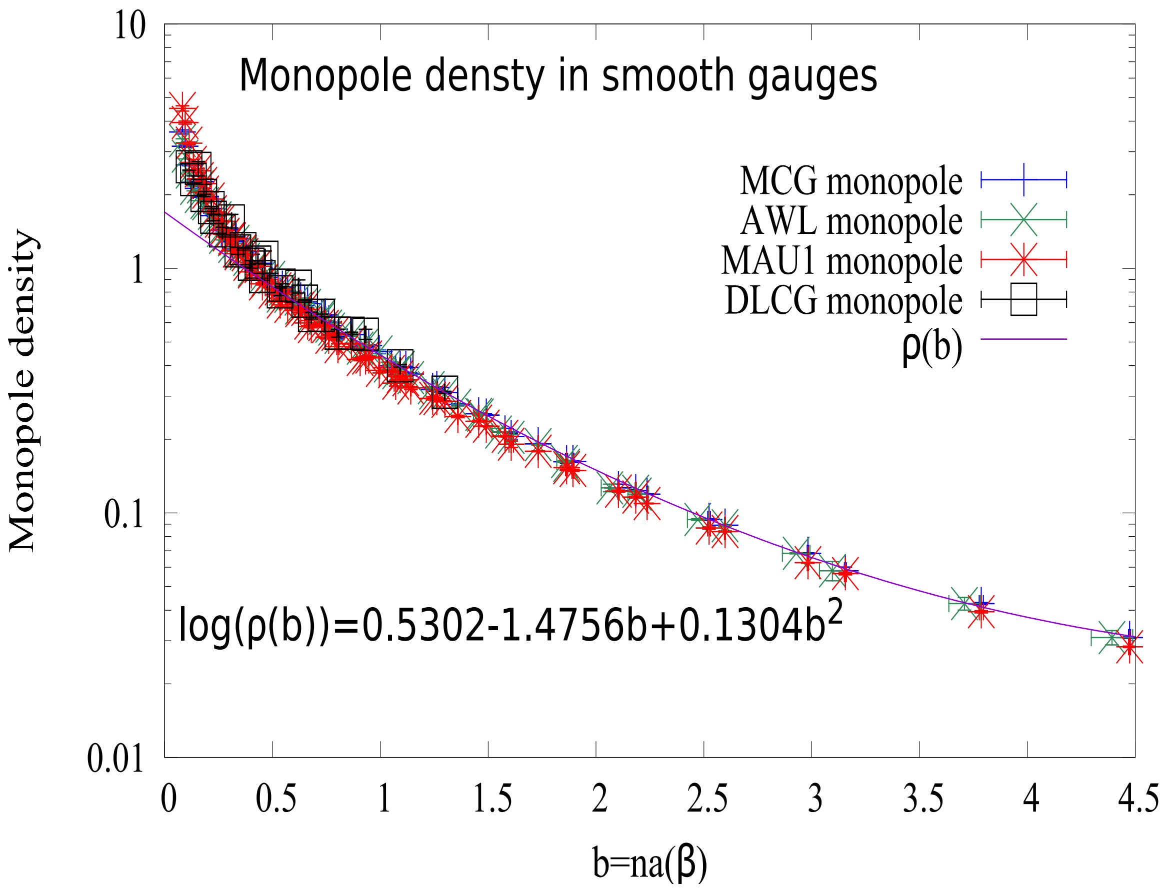
Monopole is defined on a a^3 cube and the n-blocked monopole is defined on a cube with a lattice spacing $b = na$

$$
\begin{aligned}
k_\mu^{(n)}(s_n) &= \sum_{i,j,l=0}^{n-1} k_\mu(ns_n + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma}) \\
s(n,i,j,l) &\equiv ns_n + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma}
\end{aligned}
$$

Consider a gauge-invariant density of the n-blocked monopole:

$$\rho = \frac{\sum_{\mu,s_n}\sqrt{\sum_a (k_\mu^{(n)a}(s_n))^2}}{4\sqrt{3}V_n b^3}$$

Figure 10: Comparison of the VNABI (Abelian-like monopoles) densities versus $b = na(\beta)$ in MCG, AWL, DLCG and MAU1 cases.



Summary

1. Clear scaling behaviors are observed up to the 12-step blockspin transformations for $\beta = 3.0 \sim 3.9$. The density $\rho(a(\beta), n)$ is a function of $b = na(\beta)$ alone, i.e. $\rho(b)$. $n \to \infty$ means $a(\beta) \to 0$ for fixed $b = na$. Existence of the continuum limit!
2. If the vacuum becomes smooth enough shown here in MCG, DLCG, AWL, MAU1, the same $\rho(b)$ is obtained. Gauge independence!
   This is as expected in the continuum limit.

6. Derivation of infrared effective monopole action:

$$
\begin{aligned}
e^{-\mathcal{S}[k]} &= \int DU(s,\mu)e^{-S(U)} \\
&\times \prod_a \delta(k^a_\mu(s) - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu n^a_{\rho\sigma}(s+\hat{\mu})) \\
\mathcal{S}[k] &= \sum_i F(i)\mathcal{S}_i[k]
\end{aligned}
$$

Inverse Monte-Carlo method:

('84 Swendsen, '94 Shiba-Suzuki)

Start from the vacuum ensemble $\left\{k^{(n)}_\mu(s_n)\right\} \rightarrow$ Determine $\mathcal{S}[k^n]$

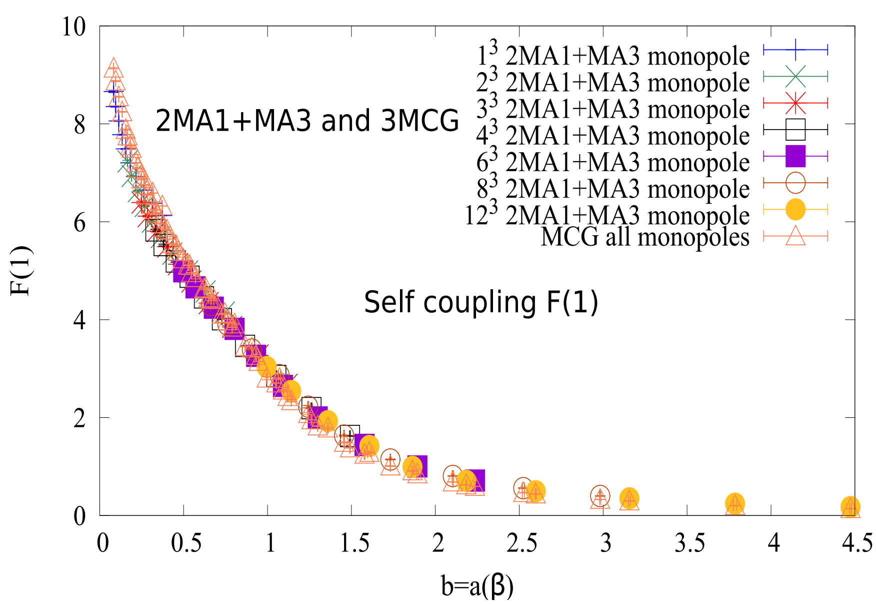
Assume $S[k] = \sum_{i=1}^{10} F(i)S^{(2)}_i(k)$:

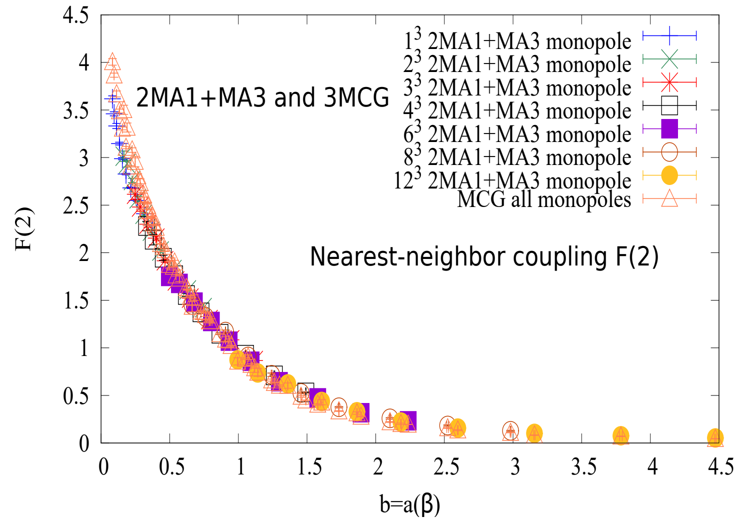
$S^{(2)}_1 = \sum_{\mu,s} k_\mu(s)k_\mu(s)$,

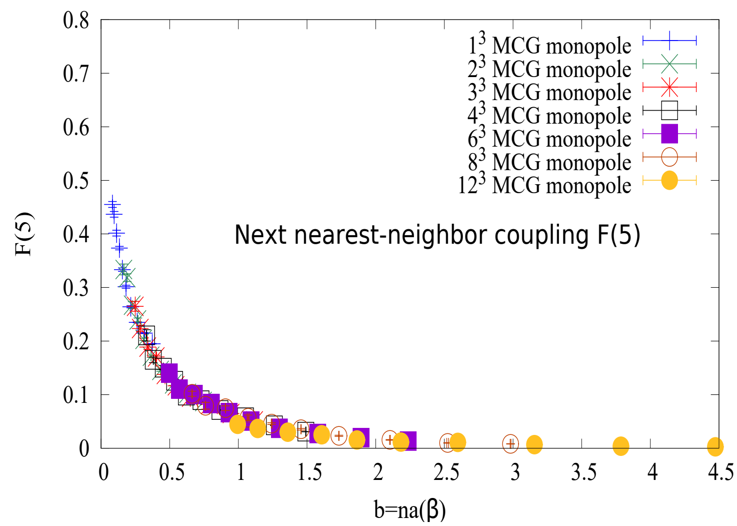
$S^{(2)}_2 = \sum_{\mu,s} k_\mu(s)k_\mu(s+\mu)$,

$S^{(3)}_2 = \sum_{\mu,\nu,s} k_\mu(s)k_\mu(s+\nu)$,

$\cdots$







Summary:

1. $F(i)$ $(i = 1 \sim 10)$ are fixed very beautifully for $3.0 \leq \beta \leq 3.9$ and $1 \leq n \leq 12$.
2. Scaling $F(n, a(\beta)) = F(b = na)$ Existence of the continuum limit
3. Gauge independence of $\sum_{\mu,s}(\lambda_\mu(s)^2)$ in MCG, DLCG, AWL, MAU1.

Comments:

1. $S(k)$:the continuum action, but still a lattice action with a finite lattice spacing b. Hence the Lorentz invariance is not apparently seen.

To derive the action in the continuum space-time reproducing the same physics at the scale b is desirable.

2. Using the 'Blocking from the continuum method' ('96 Bietenholz-Wiese, '99 Fujimoto et al.) it is possible when the monopole action is composed of two-point interactions alone.

3. $S(k)$ on b can be transformed into the action of the string model.

4. Using the strong-coupling expansion of the string model, we can evaluate non-perturbative quantities like the string tension, the lowest scalar glueball mass and the monopole density within about 30% error coming from the simple 10 two-point monopole interactions assumption.

7. Outlook

What is 'BCS theory' in QCD?

My answer:

It must be the dual Abelian Higgs model (the dual Ginburg-Landau model)!! in a general case with ϕ^4 interactions corresponding to the Abelian eigenvalue magnetic currents $\lambda_\mu(s)$ in SU2 and $\lambda_\mu^{3,8}(s)$ in SU3. There are one (SU2) and three complex scalar fields with a constraint. (SU3)

$$
\begin{aligned}
L(SU2) &= -\frac{1}{4}H_{\mu\nu}^2 + |(\partial_\mu + ig_m C_\mu)\chi|^2 - \lambda(|\chi|^2 - v^2), \\
H_{\mu\nu} = &\quad \partial_\mu C_\nu - \partial_\nu C_\mu + \epsilon_{\mu\nu\lambda\sigma}\int d^4y n_\lambda (n\cdot\partial)^{-1}(x-y) j_\sigma^{ext}
\end{aligned}
$$

Mass gap problem of YM thoery

To solve this problem, one has to take into account the existence of line singularities $[\partial_\mu, \partial_\nu]A_\alpha(x) \neq 0$ and monopoles.