F-理論におけるフラックスコンパクト化

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with

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Outline

- Introduction
- Flux compactification in type IIB string
- Flux compactification in F-theory

i) F-theory
ii) Setup
iii) Flux compactification

- Conclusion

Introduction

The standard model of particle physics

Gauge group : SU(3) × SU(2) × U(1)

Matter content :

	spin1/2	$SU(3)_c$, $SU(2)_L$, $U(1)_Y$
quarks (×3 families)	$Q^i=(u_L,\ d_L)^i$	(3, 2, 1/6)
	u_R^i	($\bar{3}$, 2, -2/3)
	d_R^i	($\bar{3}$, 1, 1/3)
leptons (×3 families)	$L^i=(\nu,\ e_L)^i$	(1, 2, -1/2)
	e_R^i	(1, 1, 1)
	spin0	
Higgs	$H=(H^+,\ H^0)$	(1, 2, -1/2)

	spin1	$SU(3)_c$,$SU(2)_L$,$U(1)_Y$
gluon	g	(8, 1, 0)
W bosons	$W^\pm$ W^0	(1, 3, 0)
B boson	B^0	(1, 1, 0)

Introduction

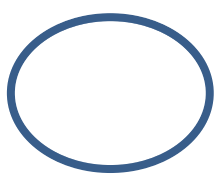
Problem:

No gravitational interaction in the standard model

String theory

A good candidate for the unified theory of the gauge and gravitational interactions

Closed string



→Graviton

Dp-brane

Ramond-Ramond field: C_{p+1}

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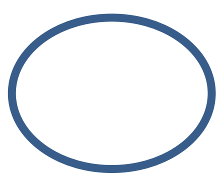

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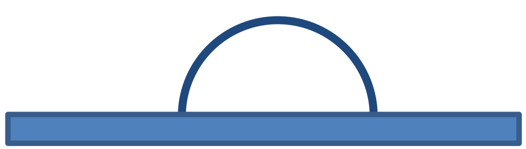


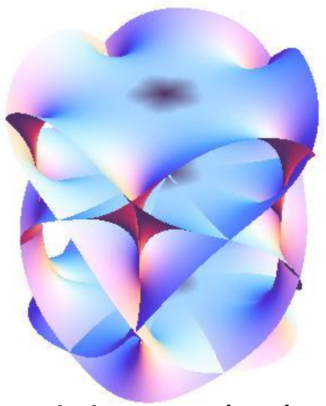
Dp-brane

Ramond-Ramond field: C_{p+1}

→Gauge fields

(Perturbative) superstring theory requires the extra 6 dimension.

10＝4＋6



6D Calabi-Yau (CY)Manifold

$R_{ij} = 0$

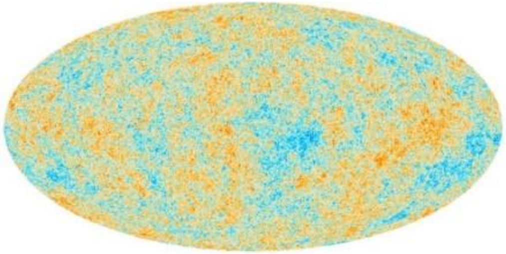
〇The geometric parameters of extra 6D dimensions
→4D scalar fields (called moduli)

〇Unless they are stabilized, it will lead to unobserved fifth forces.

〇Stabilization of the extra dimensional space
→ <u>Moduli stabilization (creating a moduli potential)</u>

Moduli are ubiquitous in string compactifications

○ Good candidate of inflaton



○ Supersymmetry breaking

○ Moduli cosmology

Moduli interact with matter fields gravitationally.

→Such long-lived particles affects the cosmology of the early Universe

(e.g., dark matter abundance, baryon asymmetry,…)

○ Yukawa couplings

In string theory as well as the higher-dimensional theory,
<u>Yukawa couplings $\simeq$ Overlap integral of matter wave functions</u>

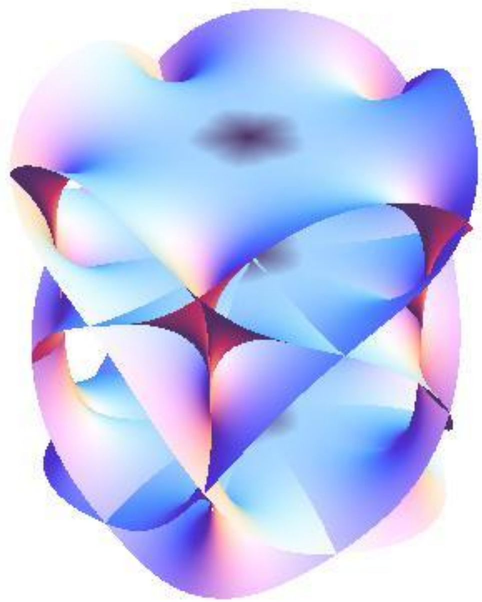
$$\lambda_{\mathrm{Yukawa}} = \int_{CY} \psi \phi \psi$$

Yukawa couplings depend on the moduli fields.

From such phenomenological points of view,
it is quite important to discuss the moduli dynamics.

Two types of moduli fields (4D massless scalar fields) :

①Closed string moduli



i) Dilaton (τ)

$\langle \mathrm{Im}\tau \rangle = g_s^{-1}$

g_s: string coupling

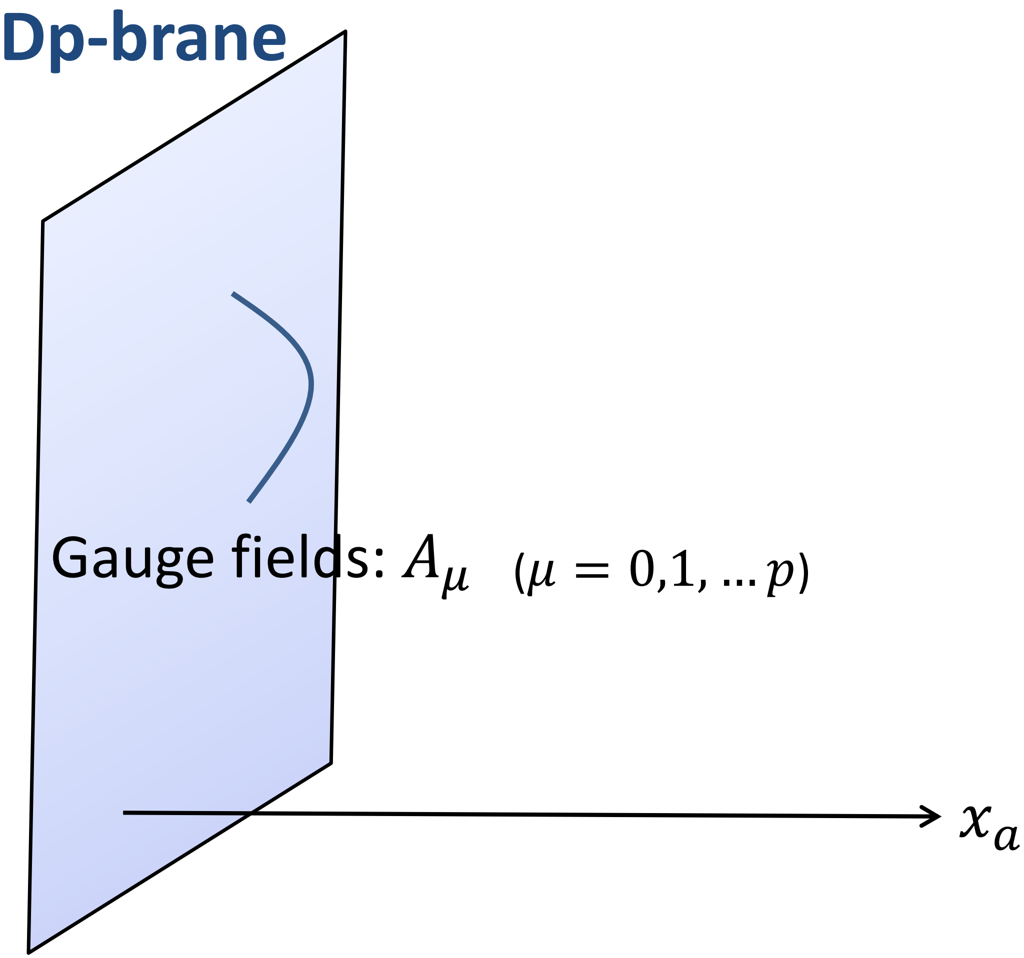
ii) Kähler moduli ($\delta g_{i\bar{j}}$)

Size of the internal cycles

iii) Complex structure moduli ($\delta g_{\bar{i}\bar{j}}$)

Shape

Two types of moduli fields (4D massless scalar fields) :



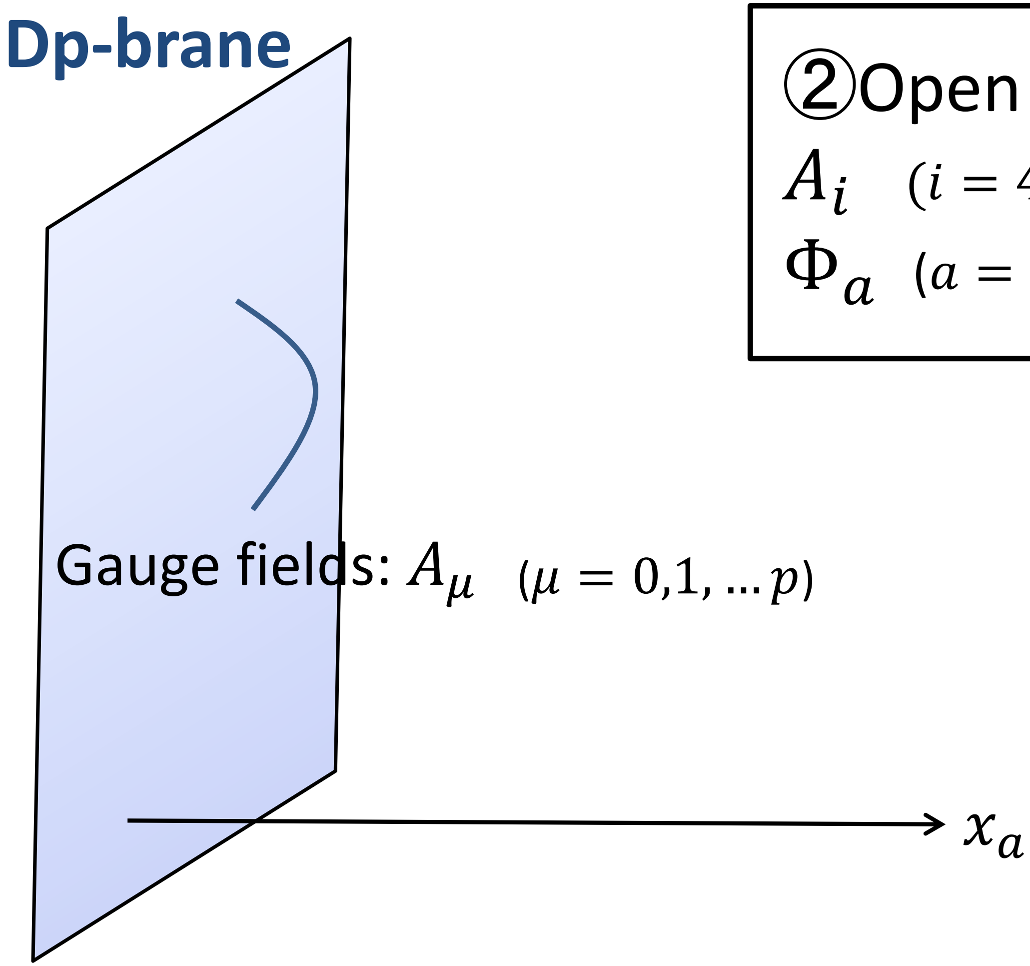
Scalar field : Φ_a $(a = p+1,\ldots,9)$

Two types of moduli fields (4D massless scalar fields) :

②Open string moduli

$A_i \quad (i = 4, 5, .., p)$

$\Phi_a \quad (a = p+1, \ldots, 9)$



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Two types of moduli fields (4D massless scalar fields)：

①Closed string moduli
②Open string moduli

Moduli dynamics will give significant effects to our Universe.

In this talk, we consider the stabilization of both the open and closed string moduli based on F-theory (“non-perturbative” description of IIB string).

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- **Flux compactification in F-theory**

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Flux compactification

Flux compactification is useful to stabilize the moduli fields.

Let us consider higher-dimensional Maxwell's theory on $R^{1,3} \times M$,

$$\int_{R^{1,3}\times M} F_p \wedge * F_p$$

When then exists a magnetic flux F_p in a cycle Σ_p of M

$$\int_{\Sigma_p} F_p = n \in Z$$

It generates a potential depending on the <u>metric</u> of extra dimension.

Flux compactification in type IIB string on CY

Type IIB string on $R^{1,3} \times \mathrm{CY}$,

$$\int_{R^{1,3}\times \mathrm{CY}} G_3 \wedge * \, G_3$$

$G_3 = F_3 - \tau H_3$: three-form

The fluxes on the three-cycle of CY (Σ_3) generate the moduli potential,

$$\int_{\Sigma_3} F_3 \qquad \int_{\Sigma_3'} \mathrm{H}_3$$

In the 4D low-energy effective action,
Flux-induced superpotential:

[Gukov-Vafa-Witten '99]

$$W(\tau, z) = \int_{\mathrm{CY}} G_3 \wedge \Omega(z)$$

$\Omega(z)$: hol. (3,0) form of CY
z: Complex structure moduli

Low-energy effective action described by 4D N=1 SUGRA

$$K = -\ln\left(i\int \Omega\wedge\overline{\Omega}\right) - \ln\left(-i(\tau-\bar{\tau})\right) - 2\ln(V(T))$$

$$W(\tau,z) = \int_{\mathrm{CY}} G_3(\tau)\wedge\Omega(z)$$

$\Omega(z)$: hol. (3,0) form of CY

$V(T)$: CY Volume

$$V = e^K\left(\sum_{I,J=\tau,z} K^{I\bar{J}} D_I W D_{\bar{J}} W + \underbrace{(K^{T\bar{T}}K_T K_{\bar{T}} - 3)}_{=\,\mathbf{0}}|W|^2\right)$$

No-scale structure

$$D_I = \partial_I + K_I$$
$$K_I = \partial_I K$$

in the reduced Planck unit $M_{\mathrm{pl}} = 1$

Dilaton and complex structure moduli are stabilized at

$$D_\tau W = D_z W = 0$$

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G_3-fluxes are constrained as the imaginary self-dual fluxes:

$$G_3 = i *_6 G_3$$

$G_3 = F_3 - \tau H_3$: three-form

Tadpole condition for C_4:

$$\int_{CY} H_3 \wedge F_3 + Q_{D3} = 0$$

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$G_3 = F_3 - \tau H_3$: three-form

Tadpole condition for C_4:

$$\int_{10\mathrm{D}} C_4 \wedge G_3 \wedge \overline{G_3} \qquad \int C_4$$

$$\int_{CY} H_3 \wedge F_3 + Q_{D3} = 0$$

How do we compute the flux-induced superpotential?

$$W = \int_{\mathrm{CY}} G_3 \wedge \Omega$$

$G_3 = F_3 - \tau H_3$: three-form

Let us expand G_3 and Ω on the integral symplectic basis (α_a, β^a) of $H^3(CY, Z)$,

$\int \alpha_a \wedge \beta^b = \delta_a^b$

$a, b = 0,1, \dots, h^{2,1}$

$$G_3 = \underbrace{(M^a \alpha_a - N_a \beta^a)}_{F_3} - \tau \underbrace{(\widetilde{M}^a \alpha_a - \widetilde{N}_a \beta^a)}_{H_3}$$

Quantized fluxes

$$\Omega = X^a \alpha_a - F_a \beta^a$$

$F_a = \frac{\partial F}{\partial X^a}$

Period vector :

$$\Pi^t = \left(\int_{\mathrm{A}_a} \Omega, \int_{B^a} \Omega \right) = (X^a, F_a)$$

(A_a, B^a: basis of 3-cycles in CY)

How do we compute the flux-induced superpotential?

$$W = \int_{\mathrm{CY}} G_3 \wedge \Omega = \left(n^F - \tau n^H\right) \cdot \Pi$$

$G_3 = F_3 - \tau H_3$: three-form

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Period integral

$$\Pi^t = \left(\int_{A_a} \Omega, \int_{B^a} \Omega\right) = (X^a, F_a)$$

can be exactly calculated by solving the Picard-Fuchs (PF) equation.

For mirror quintic CY ($h^{2,1} = 1$)

$$P(\psi) = \sum_{i=1}^{5} x_i^5 - \psi x_1 x_2 x_3 x_4 x_5 = 0$$

in an orbifold of CP^4

$(x_1, x_2, x_3, x_4, x_5) \sim (\lambda x_1, \lambda x_2, \lambda x_3, \lambda x_4, \lambda x_5)$

Complex structure moduli

PF equation:

$$z \equiv (5\psi)^{-5}$$

$$\left[\left(z\frac{d}{dz}\right)^4 - 5z\left(5z\frac{d}{dz}+1\right)\left(5z\frac{d}{dz}+2\right)\left(5z\frac{d}{dz}+3\right)\left(5z\frac{d}{dz}+4\right)\right]\Pi = 0$$

$$\Omega = \int_{P=0} \frac{\Delta}{P(\psi)}$$

Δ: 4-form in CP^4

[Candelas et al, '91]

Period integral

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E.g., in the large complex structure limit $\psi \gg 1$ $(z \ll 1)$,

$$\Pi_0 \simeq 1 + \cdots \qquad \Pi_2 \simeq \frac{5}{2}\frac{\ln^2(z)}{(2\pi i)^2} + \cdots$$

$$\Pi_1 \simeq \frac{\ln(z)}{2\pi i} + \cdots \qquad \Pi_3 \simeq \frac{5}{6}\frac{\ln^3(z)}{(2\pi i)^3} + \cdots$$

[Candelas et al, '91]

4D effective potential:

$$W = \int_{\mathrm{CY}} G_3 \wedge \Omega = \left(n^F - \tau n^H\right) \cdot \Pi$$

$$K = -\ln\left(\Pi_i \Sigma^{ij} \Pi_j\right) - \ln\left(-i(\tau - \bar{\tau})\right) - 2\ln(V(T))$$

Σ^{ij}: Symplectic matrix

→ Stabilization of complex structure moduli and dilaton

○ Previous mirror quintic CY can be engineered by 2D N=(2,2) [Witten '93]
U(1) Gauged Linear Sigma Model with 6 chiral superfields $\Phi_{0,1,\ldots,5}$

U(1) charges (Toric charges)

$$l = (l_0, l_1, l_2, l_3, l_4, l_5) = (-5,1,1,1,1,1)$$

○ Picard-Fuchs operator:

$$D = a_0^{-1}\left(\Pi_{l_i>0}\left(\frac{\partial}{\partial a_i}\right)^{l_i} - \Pi_{l_i<0}\left(\frac{\partial}{\partial a_i}\right)^{-l_i}\right) \qquad z \equiv (-1)^{l_0}\Pi_{i=0}^{n} a_i^{l_i}$$

[Hosono-Klemm-Theisen-Yau, '93]

Comment on the Kähler Moduli stabilization

The remaining Kähler moduli (T) can be stabilized by the non-perturbative effects.

$$W =< W_{\rm flux} > + Ae^{-aT}$$

A, a : Constants

○ <u>KKLT scenario</u> ($< W_{\rm flux} > \ll 1$) [Kachru-Kallosh-Linde-Trivedi '03]

○ <u>LARGE volume scenario</u> ($< W_{\rm flux} > \sim O(1)$) [Balasubramanian-Berglund-Conlon-Quevedo '05]

De Sitter vacua can be realized by introducing the anti D3-branes.

○ <u>Radiative moduli stabilization scenario</u>

[Kobayashi-Omoto-Otsuka-Tatsuishi '18]

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Type IIB action in Einstein frame(with other fields set to 0):

$$L_{IIB} = \sqrt{g}\left(R - \frac{|\partial\tau|^2}{2(\mathrm{Im}\,\tau)^2} \right)$$

where $\tau = C_0 + ie^{-\phi}$, ($\langle \mathrm{Im}\tau \rangle = g_s^{-1}$, g_s: string coupling)

This action is invariant under $SL(2,Z)$:

$$\tau \to \tau + 1\ , \quad \tau \to -1/\tau$$

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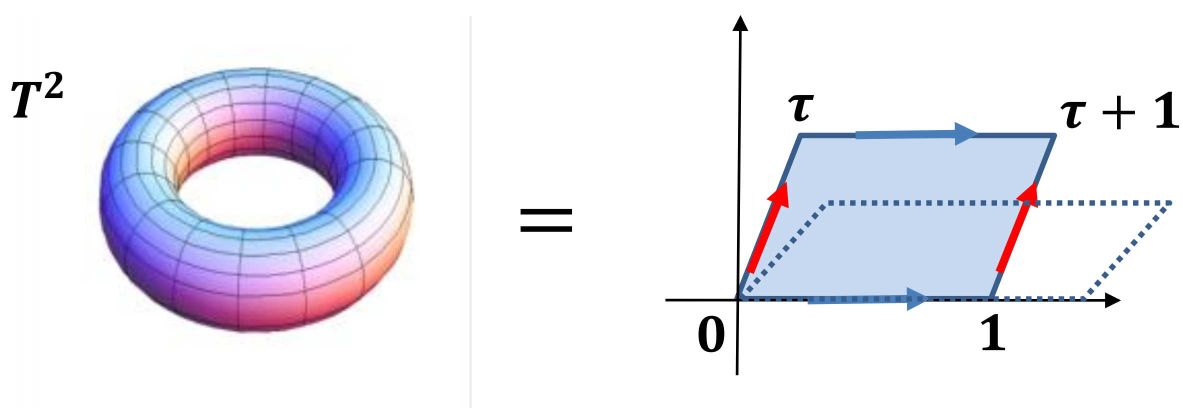
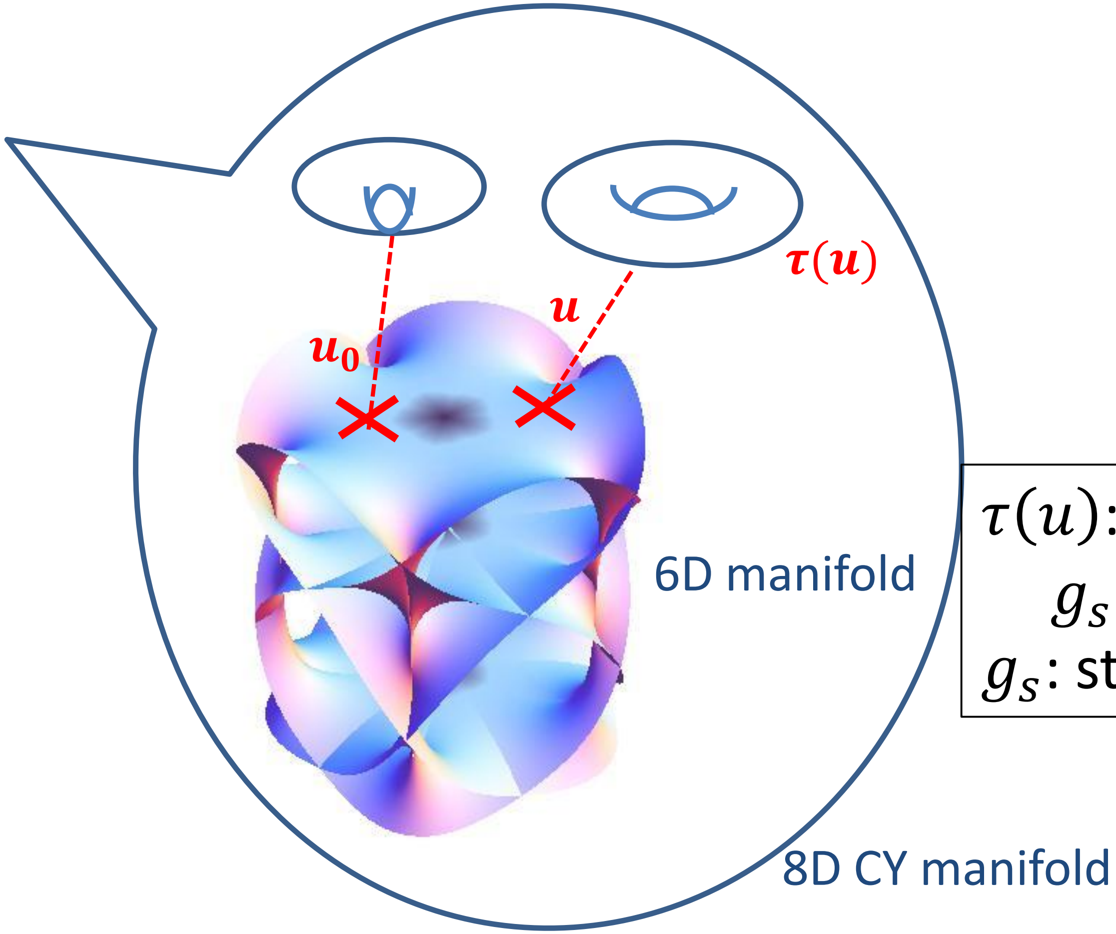
Interpret τ as complex structure of auxiliary torus T^2 (Vafa '96)



F-theory is defined in “12”D spacetime

12=4+8



$\tau(u)$: dilaton
$$g_s = \langle \mathrm{Im}\,\tau \rangle^{-1}$$
g_s: string coupling

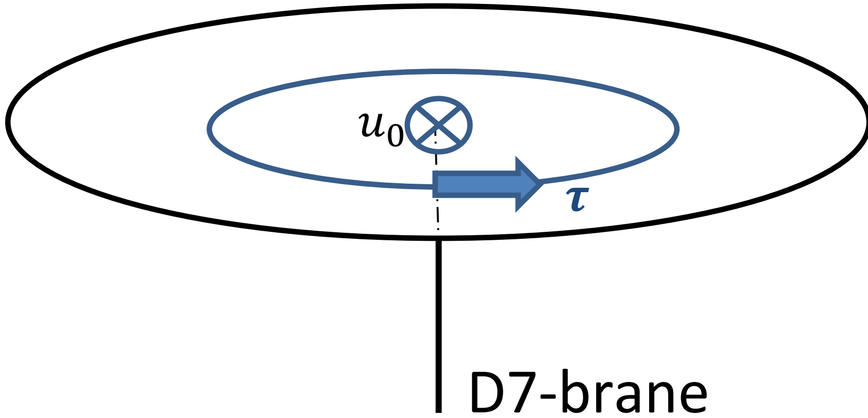
String coupling can be taken as $g_s = \langle \mathrm{Im}\,\tau \rangle^{-1} > 1$.
F-theory = “non-perturbative” description of type IIB

D7-brane looks like “cosmic string” in ambient space

(Greene, Shapere, Vafa, Yau, ‘89)

Metric:

$$ds_{10}^2 = \underbrace{-dt^2 + \sum_{i=1}^{7} dx_i^2}_{\text{D7}} + H(u,\bar{u})dud\bar{u}$$



D7-brane has magnetic charge under C_0

$$1 = \oint_{u=u_0} dC_0 = C_0\left(ue^{2\pi i}\right) - C_0(u)$$

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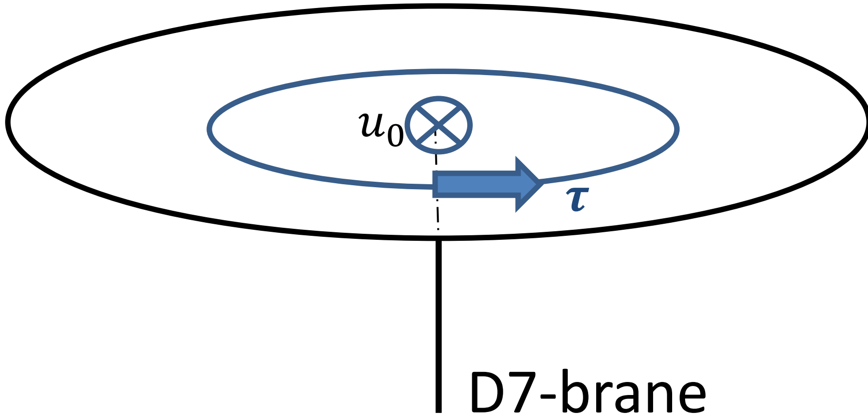

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$C_0 = \mathrm{Re}\tau$

Near D7-brane : $\tau \simeq \frac{1}{2\pi i}\ln(u - u_0)$

D7-brane location : $\tau(u_0) \to i\infty$

(T^2 degenerate at $u = u_0$.)

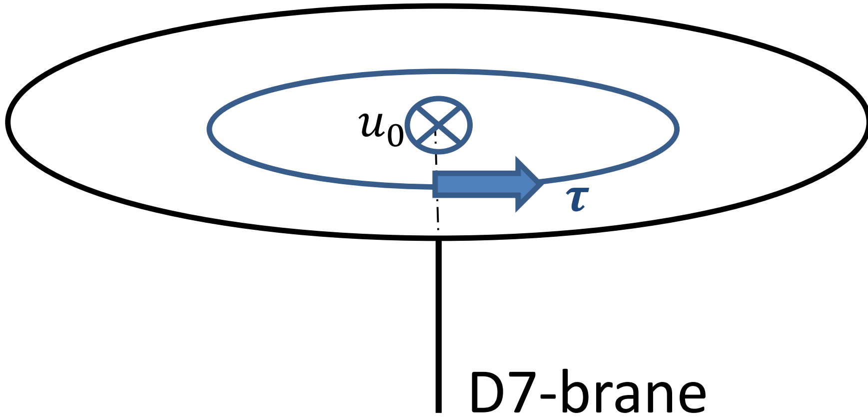
D7-brane looks like "cosmic string" in ambient space

(Greene, Shapere, Vafa, Yau, '89)

Metric:

$$ds_{10}^2 = -dt^2 + \sum_{i=1}^{7} dx_i^2 + H(u,\bar{u})dud\bar{u}$$

D7



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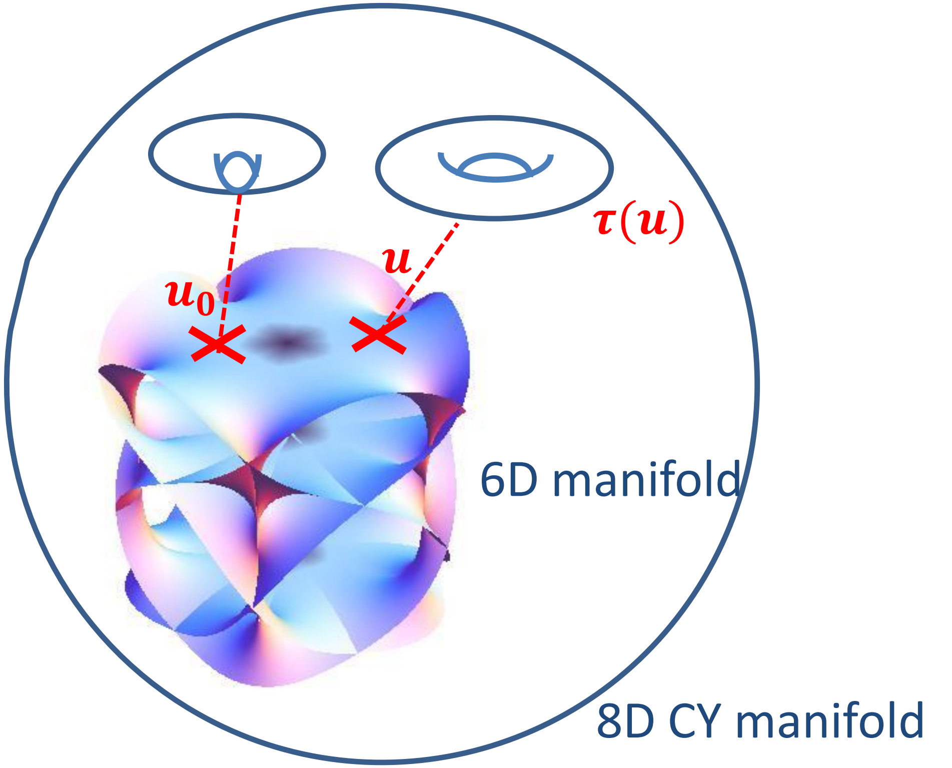

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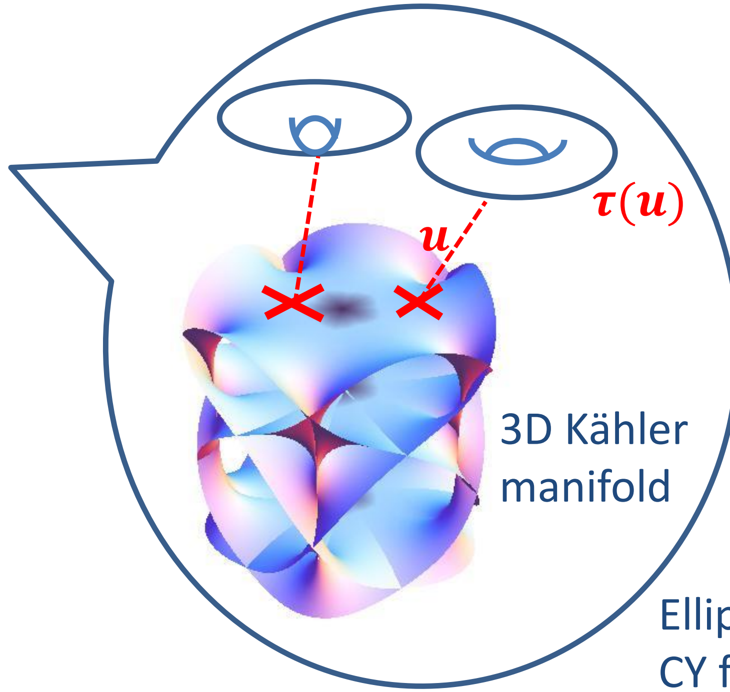

$\tau(u)$: dilaton
$g_s = \langle \mathrm{Im}\, \tau \rangle^{-1}$
g_s: string coupling

① 7-branes exist at the singular limit of torus

② String coupling > 1
(“Non-perturbative” description of type IIB superstring)

③ Both open and closed string moduli are involved.

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Brane Superpotential:

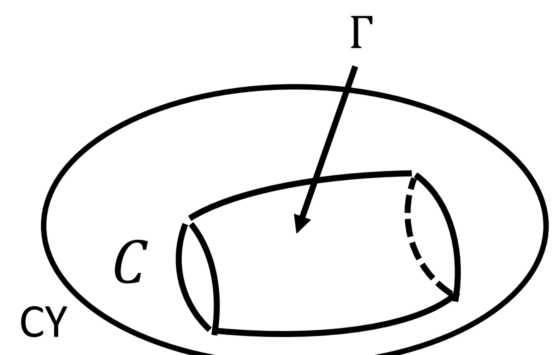
$$W_{\mathrm{brane}} = \int_{\Gamma,\partial\Gamma=C} \Omega$$

For branes wrapping on the whole CY, open string partition function is given by holomorphic Chern-Simons theory [Witten '92]:

$$W = \int_{CY} \Omega \wedge \mathrm{Tr}[\mathrm{A} \wedge \bar{\partial} A + \frac{2}{3} A \wedge A \wedge A]$$

Lower dimensional branes wrapping on holomorphic submanifold C can be obtained by dimensional reduction $A \to \phi$ [Aganagic-Vafa '00]

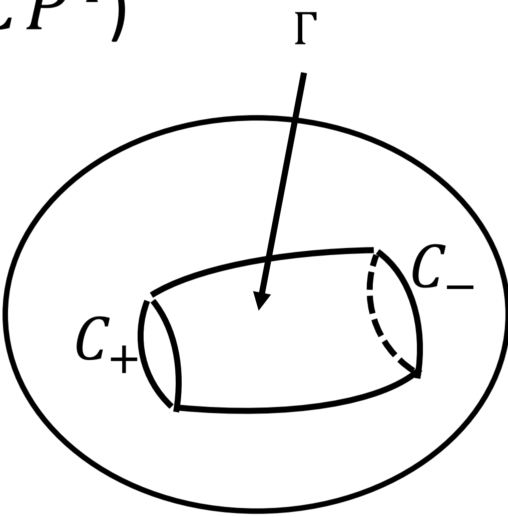
$$W_{\mathrm{brane}}(\psi,\phi) = \int_{\Gamma,\partial\Gamma=C} \Omega$$



Brane superpotential (Open mirror symmetry)

Mirror Quintic CY3 (degree 5 hypersurface in CP^4)

$$P(\psi) = \sum_{i=1}^{5} x_i^5 - 5\psi x_1 x_2 x_3 x_4 x_5 = 0$$



Mirror Quintic CY3

Let us consider holomorphic 2-cycles where the brane wraps [Morrison-Walcher '07]

$C_{\pm}: x_1 + x_2 = 0, \mathrm{x}_3 + \mathrm{x}_4 = 0,$

$$x_5^2 \pm \sqrt{5\psi} x_1 x_3 = 0$$

$$W = \int_{\Gamma} \Omega$$

No moduli dependence at fixed $C_{\pm}$!

<u>Brane deformation : $\partial\Gamma$ into (generically non-holomorphic) curve surrounded by a holomorphic divisor</u>

Brane superpotential (Open mirror symmetry)

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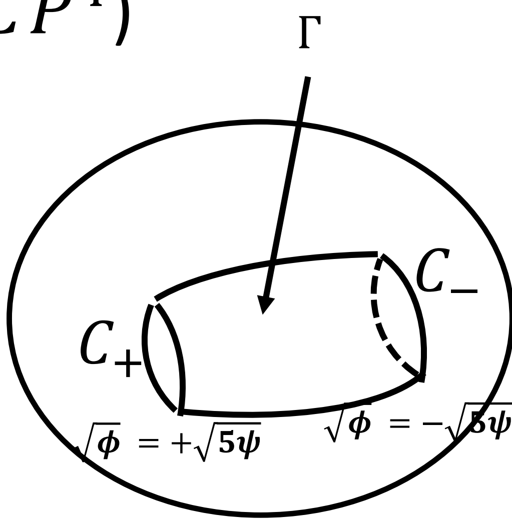


Mirror Quintic CY3

$$P(\psi) = \sum_{i=1}^{5} x_i^5 - 5\psi x_1 x_2 x_3 x_4 x_5 = 0$$

Continuous deformation of $C_\pm$:

(Hol. divisor defined by a degree 4 polynomial)

$$Q(\phi) = x_5^4 - 5\phi x_1 x_2 x_3 x_4 = 0$$

Brane deformation

Brane superpotential:

$$W_{\text{brane}}(\psi, \phi) = \int_\Gamma \Omega(\psi, \phi) = \int_{\hat{\Gamma}, \partial\hat{\Gamma} = Q(\phi)} F \wedge \Omega$$

which is related to D7-brane with magnetic flux F

[Grimm-Ha-Klemm-Klevers '09]

Toric charges of the previous system,

$$l = (-5,1,1,1,1,1;0,0) \qquad \tilde{l} = (-1,0,0,0,0,1;1,-1)$$

l: Quintic CY3
$\tilde{l}$: brane deformation

The period integral

$$\Pi_i = \int_{\Gamma_i} \Omega(\psi, \phi)$$

[Jockers-Soroush ’08]

can be computed by solving the corresponding Picard-Fuchs equation.

○Brane and geometry cannot be distinguished.

○The above system is a noncompact CY4.
(CY3 fibered over $\mathbb{C}$)

○Compactification $\mathbb{C} \longrightarrow CP^1$ leads to a compact CY4.

CY3+brane →CY4 without brane

○In the toric language, the previous system corresponds to

A-model : Quintic CY3 over CP^1

[Berglund-Mayr '98,
Grimm-Ha-Klemm-Klevers '09,
Jockers-Mayr-Walcher '09]

$$l_1 = (-4,0,1,1,1,1,-1,-1,0)$$
$$l_2 = (-1,1,0,0,0,0,1,-1,0)$$
$$l_3 = (0,-2,0,0,0,0,0,1,1)$$

$l_1 + l_2$: Quintic CY3
l_2: brane deformation
l_3: base CP^1

B-model : Elliptically fibered CY4

[Berglund-Mayr '98]

F-theory compactification on elliptically fibered CY4

$l_1 = (-4,0,1,1,1,1,-1,-1,0)$	$l_1 + l_2$: Quintic CY3
$l_2 = (-1,1,0,0,0,0,1,-1,0)$	l_2: brane deformation
$l_3 = (0,-2,0,0,0,0,0,1,1)$	l_3: base CP^1

○Period vector of CY4 in the large complex structure limit

$\Pi_i = \int_{\gamma^i} \Omega$: Fourfold periods

γ^i : Homology basis of $H_4^H(CY4, \boldsymbol{Z})$

z: Quintic modulus

S: Dilaton

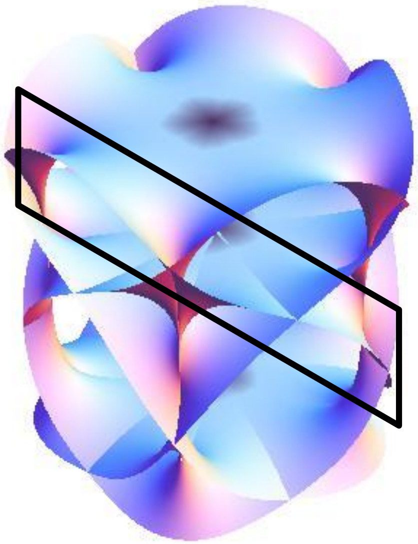
z_1: Open string modulus

$$\Pi_1 = 1,\ \Pi_2 = z,\ \Pi_3 = -z_1,\ \Pi_4 = S,$$

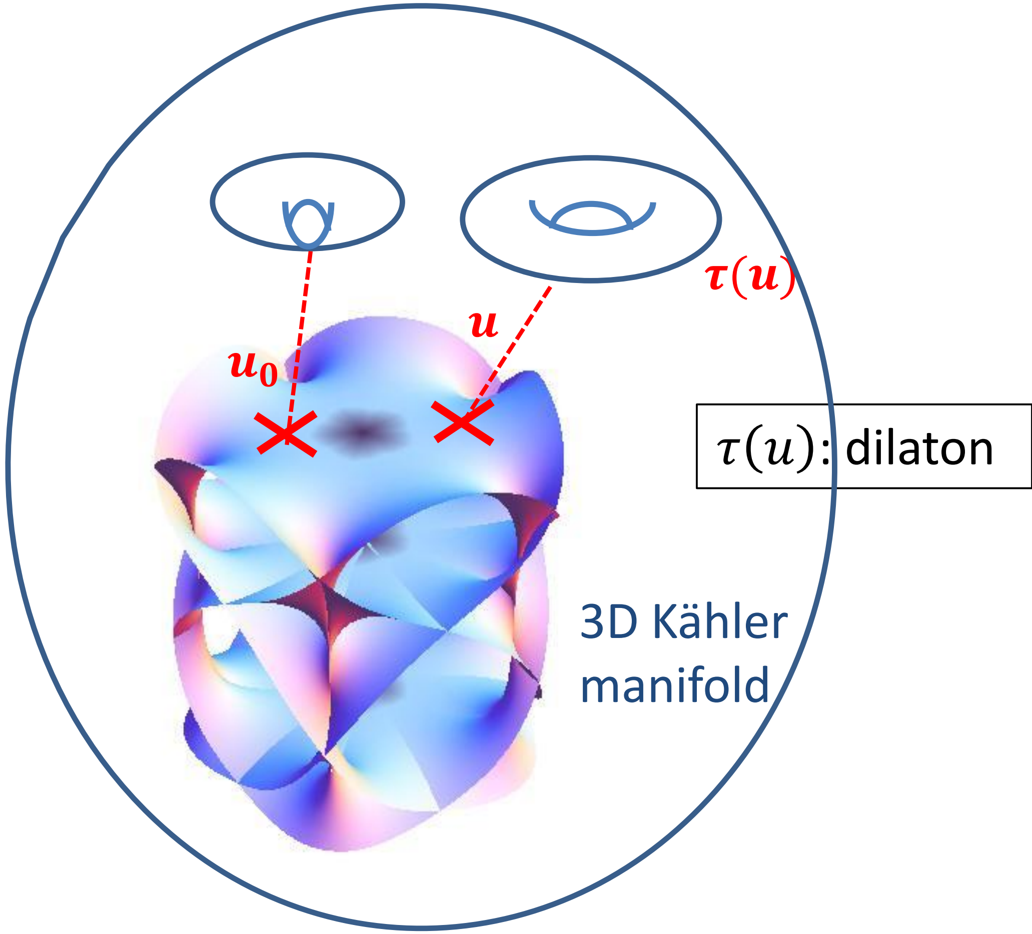
$$\Pi_5 = 5Sz,\ \Pi_6 = \frac{5}{2}z^2,\ \Pi_7 = 2z_1^2,\ \Pi_8 = -\frac{5}{2}Sz^2 - \frac{5}{3}z^3,$$

$$\Pi_9 = -\frac{2}{3}z_1^3,\ \Pi_{10} = -\frac{5}{6}z^3,\ \Pi_{11} = \frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4,$$

F-theory compactification on CY4



CY3+branes



Elliptically fibered CY4

Complex structure moduli of CY3
Dilaton
Open string (position) moduli

Complex structure moduli of CY4

Outline

- Introduction
- Flux compactification in type IIB string
- Flux compactification in F-theory

i) F-theory
ii) Setup
iii) Flux compactification

- **Conclusion**

F-theory on elliptically fibered CY4 → 4D N=1 supergravity

In 4D N=1 SUGRA

Kähler potential:

$$K = -\ln \int_{\mathrm{CY4}} \Omega \wedge \bar{\Omega} - 2\ln V$$
$$= -\ln(\Pi_i \eta^{ij} \bar{\Pi}_j) - 2\ln V$$

Superpotential:

$$W = \int_{\mathrm{CY4}} G_4 \wedge \Omega = n_i \eta^{ij} \Pi_j$$

[Gukov-Vafa-Witten, '99]

Scalar potential:

$$V = e^K \left(\sum_{I,J} K^{I\bar{J}} D_I W D_{\bar{J}} W \right)$$

$\Pi_i = \int_{\gamma^i} \Omega$: Fourfold periods

$n_i = \int_{\gamma^i} G_4$: Quantized four-form fluxes

γ^i : Homology basis of $H_4^H(CY4, \boldsymbol{Z})$

η^{ij}: Topological intersection matrix

V : Volume of 3D Kähler base

Flux compactification in F-theory on CY4

○ G_3-flux superpotential + brane superpotential in type IIB
=G_4-flux superpotential in F-theory

[Grimm-Ha-Klemm-Klevers '09,…]

$$W = \int_{\mathrm{CY4}} G_4 \wedge \Omega$$

○ Imaginary self-dual three-form fluxes in type IIB
=correspond to self-dual G_4-fluxes

[Gukov-Vafa-Witten '99]

$$G_4 = * G_4$$

○ Tadpole conditions

[Becker-Becker '96]

[Sethi-Vafa-Witten '96]

$$\frac{\chi}{24} = n_{D3} + \frac{1}{2}\int_{CY4} G_4 \wedge G_4$$

χ: Euler number of CY4
n_{D3}: # of D3

F-theory compactification on elliptically fibered CY4

z: Quintic modulus

S: Dilaton

z_1: Open string modulus

n_i: Quantized fluxes

$$\Pi_1 = 1,\ \Pi_2 = z,\ \Pi_3 = -z_1,\ \Pi_4 = S,$$

$$\Pi_5 = 5Sz,\ \Pi_6 = \frac{5}{2}z^2,\ \Pi_7 = 2z_1^2,\ \Pi_8 = -\frac{5}{2}Sz^2 - \frac{5}{3}z^3,$$

$$\Pi_9 = -\frac{2}{3}z_1^3,\ \Pi_{10} = -\frac{5}{6}z^3,\ \Pi_{11} = \frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4,$$

Kähler potential:

$$K = -\ln\left[-i(S-\overline{S})\right] - \ln\left[\frac{5i}{6}(z-\overline{z})^3 + \boxed{\frac{i}{S-\overline{S}}\left(-\frac{1}{6}(z_1-\overline{z_1})^4 + \frac{5}{12}(z-\overline{z})^4\right)}\right] - 2\ln\mathcal{V}$$

NLO in g_s correction

Superpotential:

$$W = n_{11} + n_{10}S + n_8 z + n_6 Sz + \frac{5}{2}\left(\frac{n_5}{5} + \frac{2n_6}{5}\right)z^2 - \frac{5n_4}{6}z^3 - n_2\left(\frac{5}{2}Sz^2 + \frac{5}{3}z^3\right) - n_9 z_1 - \frac{n_7}{2}z_1^2 - \frac{2n_3}{3}z_1^3 + n_1\left(\frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4\right)$$

F-theory compactification on elliptically fibered CY4

z: Quintic modulus

S: Dilaton

z_1: Open string modulus

n_i: Quantized fluxes

$$\Pi_1 = 1,\ \Pi_2 = z,\ \Pi_3 = -z_1,\ \Pi_4 = S,$$
$$\Pi_5 = 5Sz,\ \Pi_6 = \frac{5}{2}z^2,\ \Pi_7 = 2z_1^2,\ \Pi_8 = -\frac{5}{2}Sz^2 - \frac{5}{3}z^3,$$
$$\Pi_9 = -\frac{2}{3}z_1^3,\ \Pi_{10} = -\frac{5}{6}z^3,\ \Pi_{11} = \frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4,$$

Kähler potential:

$$K = -\ln\left[-i(S-\overline{S})\right] - \ln\left[\frac{5i}{6}(z-\overline{z})^3 + \boxed{\frac{i}{S-\overline{S}}\left(-\frac{1}{6}(z_1-\overline{z_1})^4 + \frac{5}{12}(z-\overline{z})^4\right)}\right] - 2\ln\mathcal{V}$$

NLO in g_s correction

Superpotential:

$$W = n_{11} + n_6 Sz + \frac{5}{2}\left(\frac{n_5}{5} + \frac{2n_6}{5}\right)z^2 - \frac{n_7}{2}z_1^2 + n_1\left(\frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4\right)$$

The self-dual G_4 fluxes

Vacuum structure of F-theory

As a consequence of the self-dual condition to G_4 fluxes, all the moduli fields are stabilized at

z: Quintic modulus
S: Dilaton
z_1: Open string modulus
n_i: Quantized fluxes

$$D_S W = D_z W = D_{z_1} W = 0$$

VEVs:

$$\mathrm{Re}z = \mathrm{Re}z_1 = \mathrm{Re}S = 0$$

$$\mathrm{Im}z = \left(\frac{6n_{11}}{5n_1}\right)^{1/4} \frac{2\sqrt{n_6}}{(8n_6(n_5+n_6)-5n_7^2)^{1/4}},$$

$$\mathrm{Im}z_1 = \left(\frac{30n_{11}}{n_1}\right)^{1/4} \frac{\sqrt{n_7}}{(8n_6(n_5+n_6)-5n_7^2)^{1/4}},$$

$$\mathrm{Im}S = \left(\frac{6n_{11}}{5n_1}\right)^{1/4} \frac{n_5}{\sqrt{n_6}(8n_6(n_5+n_6)-5n_7^2)^{1/4}}$$

Vacuum structure of F-theory

Although the fluxes are constrained by the tadpole condition,

$$\frac{\chi}{24} = n_{D3} + \frac{1}{2}\int_{CY4} G_4 \wedge G_4$$

χ=1860: Euler number of CY4
n_{D3}: # of D3

we find the consistent F-theory vacuum, e.g.,

$$n_1 = 1, n_5 = 15, n_6 = 10, n_7 = 2, n_{11} = 28$$

$$n_{\rm D3} = 0$$

All the moduli fields can be stabilized around the LCS point of CY fourfold

$$\begin{aligned} &{\rm Re}z = {\rm Re}z_1 = {\rm Re}S = 0, \\ &{\rm Im}z \simeq 2.28, \quad {\rm Im}z_1 \simeq 1.14, \quad {\rm Im}S \simeq 1.71 \end{aligned}$$

The masses of all the moduli fields are positive definite.

Comment on other models in F-theory on CY4

○The orientifold limit of F-theory

[Dasgupta-Rajesh-Sethi ‘99, Denef-Douglas-Florea-Grassi-Kachru ‘05]

○K3 × K3 background [Berglund-Mayr ‘13]

○Elliptically fibered CY4 in the large complex structure limit

[Honma-Otsuka ‘17]

Conclusion

○Mirror symmetry techniques can be applied to the F-theory compactifications.

○We explicitly demonstrate the moduli stabilization around the large complex structure point of the F-theory fourfold.

○All the complex structure moduli can be stabilized at the Minkowski minimum.

Discussion

○Quantum corrections to the moduli potential
○Other CY4
○Particle spectra in global F-theory models
○Heterotic/F-theory duality