A New Mechanism for generating particle number asymmetry through interaction

両角 卓也 (広島大学, Core-U.)

益川塾セミナー (2017 年 11 月 27 日)

長尾　桂子

(National Institute of Tech., Niihama College)

Apriadi Salim Adam (Hiroshima Univ.)

高田　浩行 (Tomsk Pedagogical Univ.)

arXiv:1709:08781/ arXiv:1609:0299v1

Contents of this work

1. **新しい粒子数生成機構**
   時間に依存するスカラー場の凝縮と複素スカラー場の間の相互作用によって粒子数生成を行う。(e.g. 重い粒子の崩壊とは異なる機構)

2. **粒子数の時間発展は、密度行列を使った非平衡の場の量子論 (実時間形式) を用いる.**
   (密度行列 を用いることで確率的, 統計的な意味での場の始状態を指定できる.)

3. この形式を使って, 粒子数の**時間発展**を調べることができる.
(宇宙膨張の効果も取り入れることができる.)

4. 本研究では相互作用の leading order（結合定数の一次), 膨張の効果の leading order $H(t_0)(x^0 - t_0) \leq 1$ を考慮した計算結果を示す。

粒子数の破れを含むスカラーモデル

$$S=\int d^4x\sqrt{-g}\,(\mathcal{L}_{\rm free}+\mathcal{L}_{\rm int})\,,$$

$$\mathcal{L}_{\rm free}=g^{\mu\nu}\nabla_\mu\phi^\dagger\nabla_\nu\phi-m_\phi^2|\phi|^2$$

$$+\frac{g^{\mu\nu}}{2}\nabla_\mu N\nabla_\mu N-\frac{M_N^2}{2}N^2+$$

$$+\frac{B^2}{2}(\phi^2+\phi^{\dagger 2})+\left(\frac{\alpha_2}{2}\phi^2+h.c.\right)R+\alpha_3|\phi|^2R,$$

N 中性スカラー　ϕ複素スカラー

$$g_{\mu\nu}=(1,-a^2(x^0),-a^2(x^0),-a^2(x^0)).$$

相互作用 $\mathcal{L}_{\rm int.} = A\phi^2 N + A^*\phi^{*2} N + A_0|\phi|^2 N$

実場形式 ($\phi = \dfrac{\phi_1 + i\phi_2}{\sqrt{2}}$ $\phi_3 = N$ **) の Lagrangian:**

$$\mathcal{L} = \frac{g^{\mu\nu}}{2}(\nabla_\mu\phi_i\nabla_\nu\phi_i) - \frac{m_i^2}{2}\phi_i^2 + \sum_{ijk=1}^{3}\frac{A_{ijk}}{3}\phi_i\phi_j\phi_k$$

$m_{1,2}^2 = m_\phi^2 \mp B^2$	**U(1) violation**
$A_{113} = \dfrac{A_0}{2} + {\rm Re.}(A)$	$A_{223} = \dfrac{A_0}{2} - {\rm Re.}(A)$
$A_{113} - A_{223} = 2{\rm Re.}(A)$	**U(1) violation**
$A_{123} = -{\rm Im.}(A)$	**U(1), CP violation**

初期条件 $(x^0 = t_0)$ 密度行列を用いた混合状態;

$$\rho(t_0) = \frac{e^{-\beta H_0}}{\mathrm{Tr} e^{-\beta \mathrm{H}_0}}, \quad \beta = \frac{1}{T} \quad (T = \text{温度})$$

$$H_0 = \frac{a(t_0)^3}{2} \sum_{i=1}^{3} \int d^3\mathrm{x} \ \left[\pi_{\phi_i} \pi_{\phi_i} + \frac{\nabla \phi_i \cdot \nabla \phi_i}{a(t_0)^2} \right.$$

$$\left. + \sum_{i=1}^{3} \tilde{m}_i^2 (\phi_i - v_i \delta_{i3})^2 \right].$$

場の初期期待値

$$\mathrm{Tr}(\phi_i(0,\mathrm{x})\rho(0)) = v_i\delta_{i3}$$

Green 関数の初期条件

$$\mathrm{Tr}((\phi_j(0,\mathrm{y}) - v_j)(\phi_i(0,\mathrm{x}) - v_i)\rho(0))$$
$$= \delta_{ij}\int \frac{d^3k}{(2\pi)^3 2\omega_i(\mathrm{k})a(t_0)^3}\frac{\sinh\beta\omega_i(\mathrm{k})}{\cosh\beta\omega_i(\mathrm{k}) - 1}e^{-i\mathrm{k}\cdot(\mathrm{x}-\mathrm{y})}$$

粒子数密度の期待値 粒子数 U(1) カレント

$$j_\mu = \frac{1}{2}\left(\phi_2 \overleftrightarrow{\partial}_\mu \phi_1 - \phi_1 \overleftrightarrow{\partial}_\mu \phi_2\right)$$

$$\langle j_0(x)\rangle = \mathrm{Re.}\left[\left(\frac{\partial}{\partial x^0} - \frac{\partial}{\partial y^0}\right) G^{12}_{12}(x,y)\Big|_{y\to x} + \bar{\phi}^{2,*}_2(x)\overleftrightarrow{\partial}_0 \bar{\phi}^1_1(x)\right],.$$

$$\phi(x) = \varphi(x) + \bar{\phi}(x)$$

$$G^{12}_{12}(x,y) = \mathrm{Tr}(\varphi_2(x)\varphi_1(y)\rho(0)).$$

Closed Time Path 形式と and 2 PI 有効作用の導入

$G^{ab}_{ij}(x,y)$ and $\bar{\phi}_i$ は 2 PI 有効作用から得られる.

与えられた初期密度行列に対して,Green 関数や場の期待値の時間発展を求める。

Green 関数の場合;

$$G^{ab}_{ij}(x,y) = \begin{pmatrix} G^{11}_{ij}(x,y) & G^{12}_{ij}(x,y) \\ G^{21}_{ij}(x,y) & G^{22}_{ij}(x,y) \end{pmatrix}$$

Contour ordered Green functions, time ordered, anti-time ordered,non-ordered (Keldysh)

$$G^{11}_{ij}(x,y) = Tr(T(\varphi_j(y)\varphi_i(x))\rho(t_0)))$$

$$G^{22}_{ij}(x,y) = Tr(\rho(t_0)\tilde{T}(\varphi_j(y)\varphi_i(x)))$$

$$G^{12}_{ij}(x,y) = Tr(\varphi_j(y)\varphi_i(x)\rho(t_0))$$

$$G^{21}_{ij}(x,y) = G^{12}_{ji}(y,x).$$

CTP (Closed time path) and Green functions

$$G^{12}(x,y) \equiv Tr(\phi_H(y_0,\mathrm{y})\phi_H(x_0,\mathrm{x})\rho(0))$$

$$= \frac{\delta^2}{-i\delta J^2(y) i\delta J^1(x)} \int d\phi^1(0) d\phi^2(0) < \phi^1(0)|\rho(0)|\phi^2(0) >$$

$$< \phi^2(0)|U^\dagger_{J^2}(\infty,0) U_{J^1}(\infty,0)|\phi^1(0) >$$

$$\int d\phi^a(C) < \phi^1(0)|\rho(0)|\phi^2(0) > \exp[iS_J[\phi^a]]$$

$$S_J[\phi^a] = \int d^4x \frac{c^{ab}}{2} \partial_\mu \phi^a \partial^\mu \phi^b + c^{ab} \int J^a \phi^b, \quad c^{11} = -c^{22} = 1$$

$G^{ab}_{ij}(x,y)$ and $\bar{\phi}_i$ は 2 PI 有効作用から得られる.

$$\Gamma[G,\bar{\phi},g] = S[\bar{\phi},g] + \frac{i}{2}\mathrm{TrLnG}^{-1}$$

$$+\frac{1}{2}\int d^4x d^4y \frac{\delta^2 S[\bar{\phi}]}{\delta\bar{\phi}^a_i(x)\delta\bar{\phi}^b_j(y)} G^{ab}_{ij}(x,y) + \Gamma^{2PI}_Q[G].$$

Schwinger Dyson Eqs. for $\bar{\phi}(x^0)$ and Green functions.

$$(\delta_{ij}\Box + \tilde{m}^2_{ij})\bar{\phi}^d_j(x) = J^d_i(x) + \int d^4z K^{de}_{ij}(x,z)c^{ef}\sqrt{-g(z)}\bar{\phi}^f_j(z)$$
$$+c^{da}D_{abc}A_{ijk}\left\{\bar{\phi}^b_j(x)\bar{\phi}^c_k(x) + G^{bc}_{jk}(x,x)\right\},$$

$$(\overrightarrow{\Box}_x + \tilde{m}_i^2) G^{ab}_{ij}(x,y) = -i\delta_{ij} \frac{c^{ab}}{\sqrt{-g_x}} \delta(x-y)$$
$$+ 2(c \cdot ((D \otimes A) \cdot \bar{\phi}))^{ac}_{ik,x} G^{cb}_{kj,xy} + O(A^2)$$
$$+ \int d^4z K^{ae}_{ik}(x,z) \sqrt{-g_z} c^{ef} G^{fb}_{kj}(z,y)$$

$$(\overrightarrow{\Box}_y + \tilde{m}_j^2) G^{ab}_{ij}(x,y) = -i\delta_{ij} \delta(x-y) \frac{c^{ab}}{\sqrt{-g_y}}$$
$$+ 2G^{ac}_{ik,xy}((D \otimes A) \cdot \bar{\phi}) \cdot c)^{cb}_{kj,y} + O(A^2)$$
$$+ + \int d^4z G^{ae}_{ik}(x,z) c^{ef} \sqrt{-g(z)} K^{fb}_{kj}(z,y)$$

K and J are related to the initial density matrix which acts as source terms for the field and Green functions.

初期密度行列の汎関数表示 及び　ソース項

$$\rho(0) = Ce^{-\beta H_0} \quad \frac{1}{C} = Tr(e^{-\beta H_0})$$

Functional representation

$$\langle \phi^1 | \rho(t_0) | \phi^2 \rangle = C$$

$$\exp\left[i \frac{1}{2} \int \int d^4x \; d^4y \sqrt{-g} \phi_i^a(x) c^{ab} K_{ij}^{bd}(x-y) c^{de} \phi_j^e(y) \sqrt{-g} \right]$$

$$\exp\left[i \int d^4x \sqrt{-g(x^0)} \phi_i^a(x) c^{ab} J_i^b(x) \right]$$

Summary of the source terms at initial time

$$K^{ab}_{ij}(x,y) = -i\delta(x^0 - t_0)\delta(y^0 - t_0)\kappa^{ab}_{ij}(\mathbf{x}-\mathbf{y})$$

$$J^a_i(x) = -i\delta(x^0 - t_0)j^a_i$$

$$\kappa^{ab}_{ij}(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\kappa^{ab}_{ij}(k)e^{-i\mathbf{k}\cdot\mathbf{x}}$$

$$\kappa^{aa}_{ij}(\mathbf{k}) = -\frac{1}{a(t_0)^3}\frac{\omega_i(\mathbf{k})\cosh\beta\omega_i(\mathbf{k})}{\sinh\beta\omega_i(\mathbf{k})}\delta_{ij}$$

$$\kappa^{12}_{ij}(\mathbf{k}) = \kappa^{21}_{ij}(\mathbf{k}) = -\frac{1}{a(t_0)^3}\frac{\omega_i(\mathbf{k})}{\sinh\beta\omega_i(\mathbf{k})}\delta_{ij}$$

$$\omega_i(\mathbf{k}) = \sqrt{\frac{\mathbf{k}^2}{a(t_0)^2} + \tilde{m_i}^2}$$

$$j^b_i = -a(t_0)^3\kappa^{bd}_{ij}(\mathbf{k}=0)c^{de}v^e_j$$

粒子数密度 (Particle Number Asymmetry= PNA

$$
\begin{aligned}
&\langle j_0(x^0)\rangle \\
&= \frac{2}{a(x^0)^3}\hat{\varphi}_{3,t_0}\int\frac{d^3\mathrm{k}}{(2\pi)^3}\int_{t_0}^{x^0}\hat{A}_{123,t}(-\bar{K}'_{3,tt_0,0}) \\
&\Bigg[\Bigg\{\frac{1}{2\omega_{2,\mathrm{k}}(t_0)}\coth\frac{\beta\omega_{2,\mathrm{k}}(t_0)}{2} \\
&\times\Big[\dot{\bar{K}}_{1,x^0t,\mathrm{k}}\bar{K}'_{2,x^0t_0,\mathrm{k}}\bar{K}'_{2,tt_0,\mathrm{k}}-\bar{K}_{1,x^0t,\mathrm{k}}\dot{\bar{K}}'_{2,x^0t_0,\mathrm{k}}\bar{K}'_{2,tt_0,\mathrm{k}} \\
&+\omega^2_{2,\mathrm{k}}(t_0)(\dot{\bar{K}}_{1,x^0t,\mathrm{k}}\bar{K}_{2,x^0t_0,\mathrm{k}}-\bar{K}_{1,x^0t,\mathrm{k}}\dot{\bar{K}}_{2,x^0t_0,\mathrm{k}})\bar{K}_{2,tt_0,\mathrm{k}}]\Bigg\} \\
&-\{1\leftrightarrow 2 \text{ for lower indices}\}\Bigg].
\end{aligned}
$$

$$\hat{\varphi}_{3,t_0}=v_3.$$

ゼロでない PNA $\leftrightarrow$ $v_3A_{123}\neq 0$ $\tilde{m}_1\neq\tilde{m}_2$ (ϕ_1, ϕ_2 の質量の非縮退)

膨張の３効果

The effect	The origin
Dilution	The increase of volume of the universe due to expansion, $\frac{1}{a(x^0)^3} - \frac{1}{a_{t_0}^3}$
Freezing interaction	The decrease of the strength of the cubic interaction $\hat{A}$ as $\hat{A}_{123} - A_{123}$.
Redshift	The effective energy of particle $\frac{\mathrm{k}^2}{a(x^0)^2} + \bar{m}_i^2(x^0)$.

以下，宇宙膨張の効果について以下の近似を行う

$((x^0 - t_0) \leq \frac{1}{H(t_0)})$

$$\frac{a(x^0)}{a(t_0)} \sim 1 + (x^0 - t_0)H(t_0), \quad H(t_0) = \frac{\dot{a}}{a}\Big|_{x^0 = t_0}$$

$$\begin{aligned} \hat{A}_{123}(t) &= A_{123}\left(\frac{a(t_0)}{a(t)}\right)^{\frac{3}{2}} \\ &= A_{123}(1 - \frac{3}{2}(t - t_0)H(t_0)). \end{aligned}$$

$$\bar{K}_i[x^0, y^0] = \bar{K}_i^{(0)}[x^0, y^0] + \bar{K}_i^{(1)}[x^0, y^0]$$

$$\bar{K}_i^{(0)}[x^0, y^0] = \frac{\sin[\omega_{i,\mathbf{k}}(x^0 - y^0)]}{\omega_{i,\mathbf{k}}},$$

$$\bar{K}_i^{(1)}[x^0, y^0] = \frac{H(t_0)}{2} \frac{\mathbf{k}^2}{\omega_{i,\mathbf{k}}^2 a(t_0)^2} (x^0 + y^0 - 2t_0)$$

$$\times \left(\frac{\sin[\omega_{i,\mathbf{k}}(x^0 - y^0)]}{\omega_{i,\mathbf{k}}} - (x^0 - y^0) \cos[\omega_{i,\mathbf{k}}(x^0 - y^0)] \right).$$

$\dot{\bar{K}}_i$, $\dot{\bar{K}}'_i$ etc.

$$\dot{\bar{K}}_i[x^0, y^0] := \frac{\partial \bar{K}[x^0, y^0]}{\partial x^0}, \bar{K}'_i[x^0, y^0] := \frac{\partial \bar{K}[x^0, y^0]}{\partial y^0},$$

$$\dot{\bar{K}}'_i[x^0, y^0] := \frac{\partial^2 \bar{K}[x^0, y^0]}{\partial x^0 \partial y^0}.$$

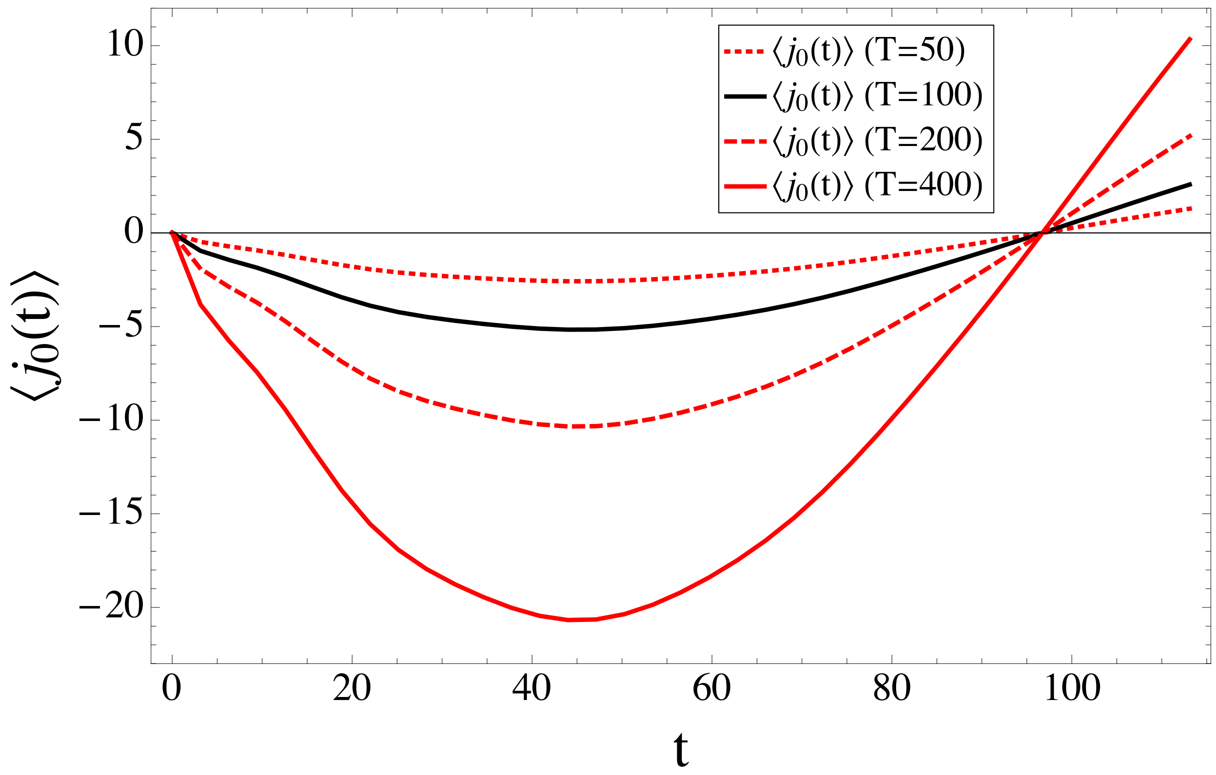

Figure 1: T **dependence.** $t = 0.35(x^0 - t_0)$ $(\tilde{m}_1, \tilde{m}_2, B, H_{t_0}, \omega_{3,0})=(0.04, 0.05, 0.02, 10^{-3}, 0.0035)$.

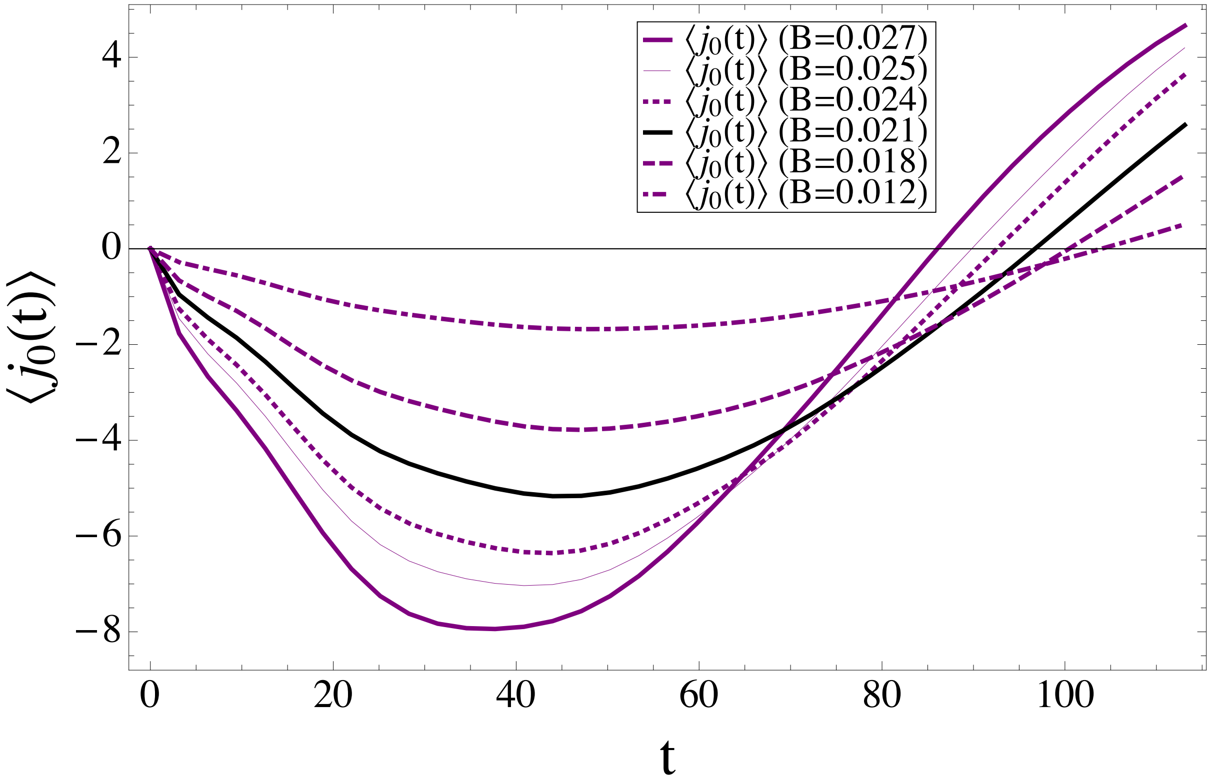

Figure 2: B **dependence.** $t = 0.35(x^0 - t_0)$. $(\tilde{m}_2, T, H_{t_0}, \omega_{3,0})=(0.05, 100, 10^{-3}, 0.0035)$.

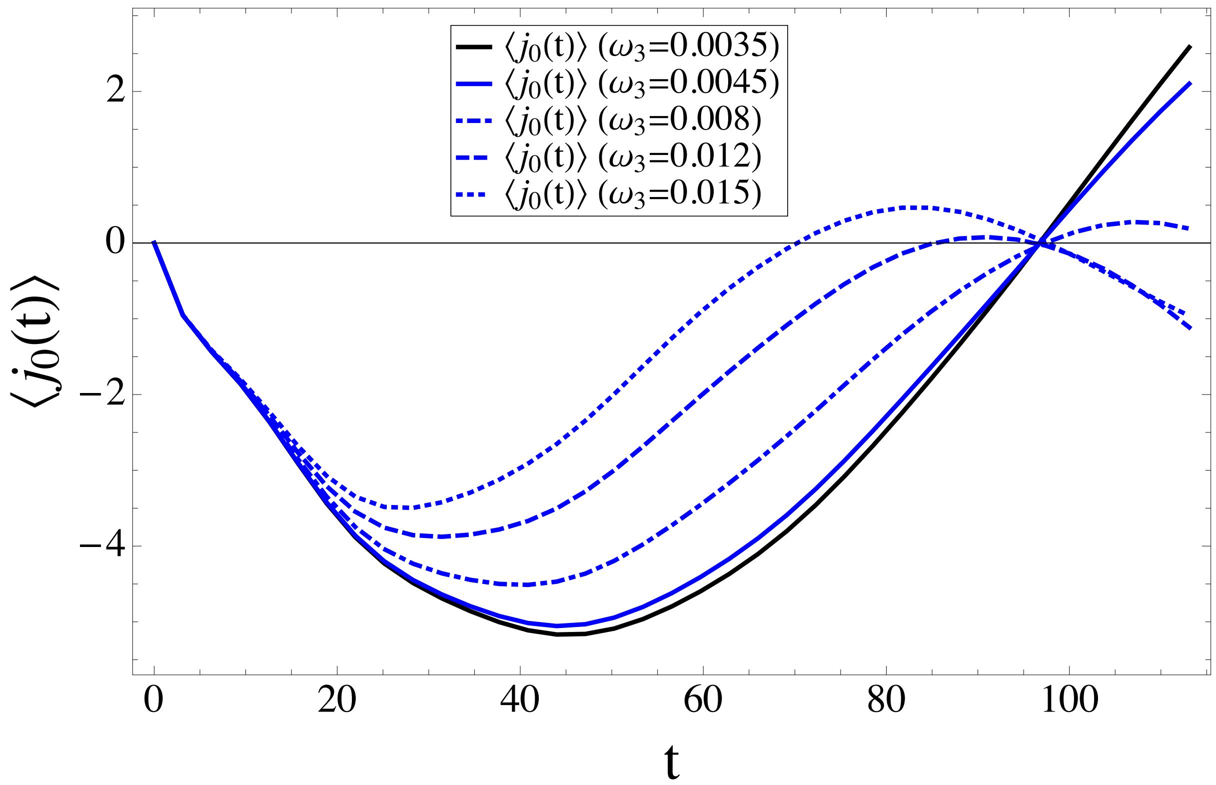

Figure 3: The $\omega_{3,0}$ dependence. $t = 0.35(x^0 - t_0)$.$(\tilde{m}_1, \tilde{m}_2, B, T, H_{t_0})$=$(0.04, 0.05, 0.021, 100, 10^{-3})$.

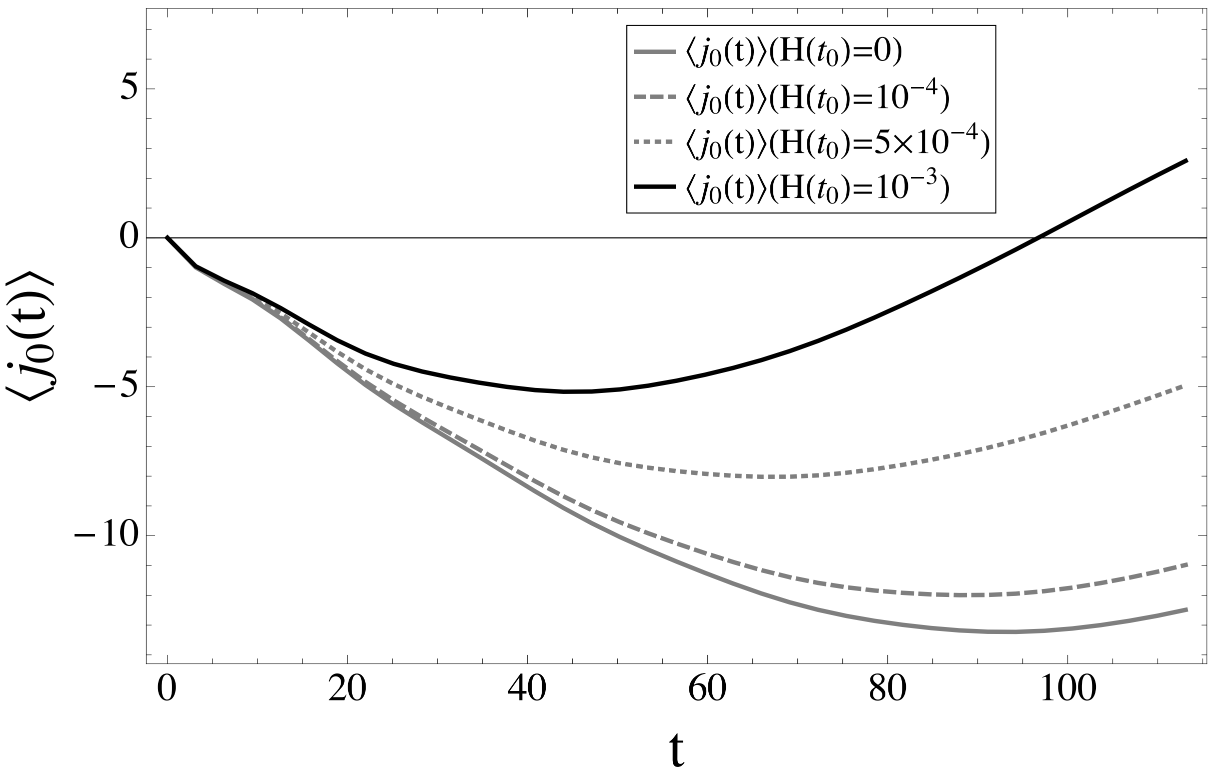

Figure 4: H_{t_0} dependence. $t = 0.35(x^0 - t_0)$ $(\tilde{m}_1, \tilde{m}_2, B, T, \omega_{3,0})$=$(0.04, 0.05, 0.021, 100, 0.0035)$.

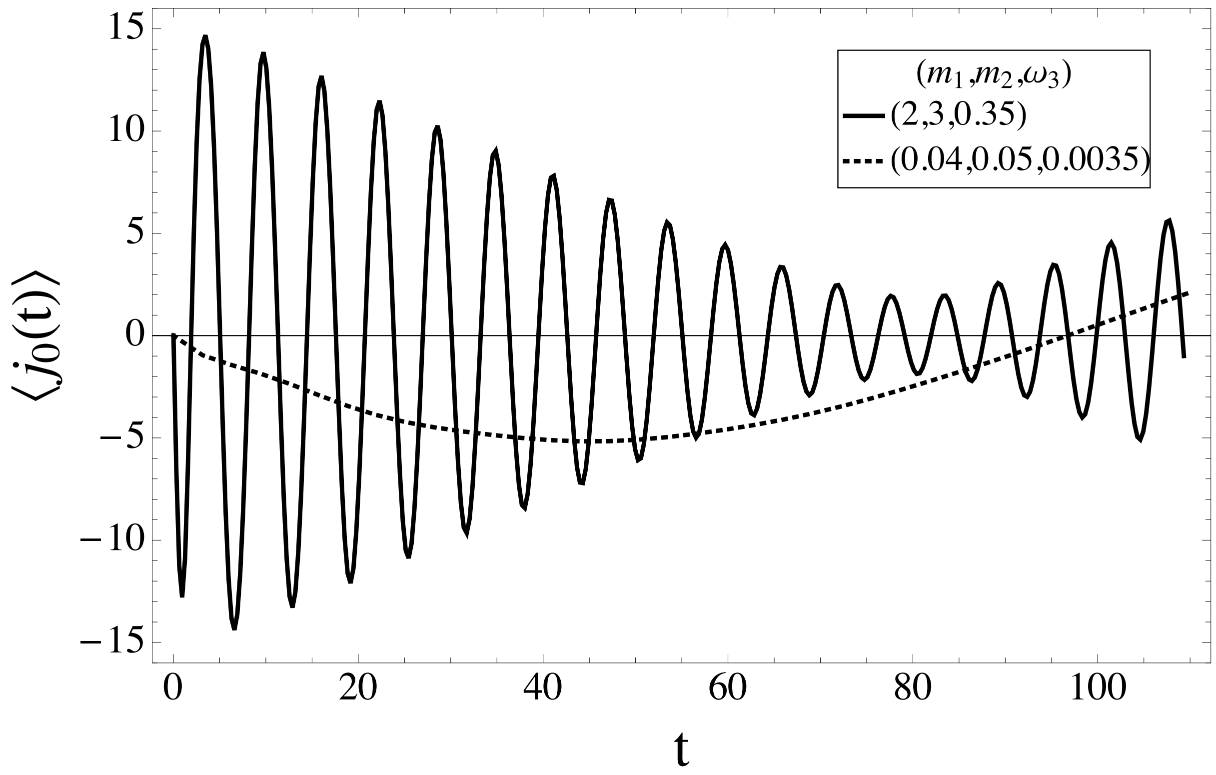

Figure 5: $t = 0.35(x^0 - t_0)$. $(T, H_{t_0}) = (100, 10^{-3})$. **The black (dotted)lines correspond to** $B = 1.58(B = 0.021)$.**(our approximation will break down after** $t = 80$**).**

数値計算の結果

Fig.1 温度依存性 T 温度依存性は初期密度行列のボーズ分布関数に含まれているもので PNA の振幅は温度の増加とともに増大する

Fig.2 B 依存性 $2B^2 = \tilde{m_2}^2 - \tilde{m_1}^2$ が増加すると振幅は増大し振動の周期は短くなる傾向

Fig.3 $\omega_{3,0}$(場の凝縮の振動数) 依存性; $\varphi_3(x^0) = v_3 \cos(\omega_{3,0} x^0)$
$\omega_{3,0} > \tilde{m_2} - \tilde{m_1}$ の場合, 新しい節の形成

Fig.4 $H(t_0)$ 依存性. $H(t_0)$ が増加すると体積膨張による密度の減少が顕著になる.

Fig.5 Fig.1-4 に比べ短周期の場合

全体のまとめ

- 中性スカラー場 (N) の真空期待値の振動が複素スカラ-場に結合することで粒子数（PNA) が相互作用により生成するメカニズムを提案
- PNA の実時間の発展を研究
- PNA の振幅は温度, 相互作用の結合定数 A_{123}（CPV,U(1) の破れ）および $\tilde{m}_2 - \tilde{m}_1$(もともと 1 個の複素スカラー場として導入した場の質量固有状態の質量の非縮退に比例
- 膨張率 $H(t_0)$ や温度 (T) や模型のパラメーターを変えることにより、時間依存性:振幅、振動周期の変化などを数値計算 $\rightarrow$（今後）バリオン数や Dark Matter Anti-Dark Matter の生成機構の理解へ