# 隠れたU(1)ゲージ場を媒介として光と パリティを破った相互作用をする フェルミオンの４光子有効ラグランジアンと 真空複屈折実験での探索について

山下 公子（お茶の水女子大学）

共同研究者:
樊 星（ハーバード大学）、上岡 修星、浅井 祥仁（東京大学）実験
菅本 晶夫（お茶の水女子大学、放送大学）理論

arXiv:1707.03308（PTEPへ掲載予定）、1707.03609

2017年11月20日
京都産業大学 益川塾

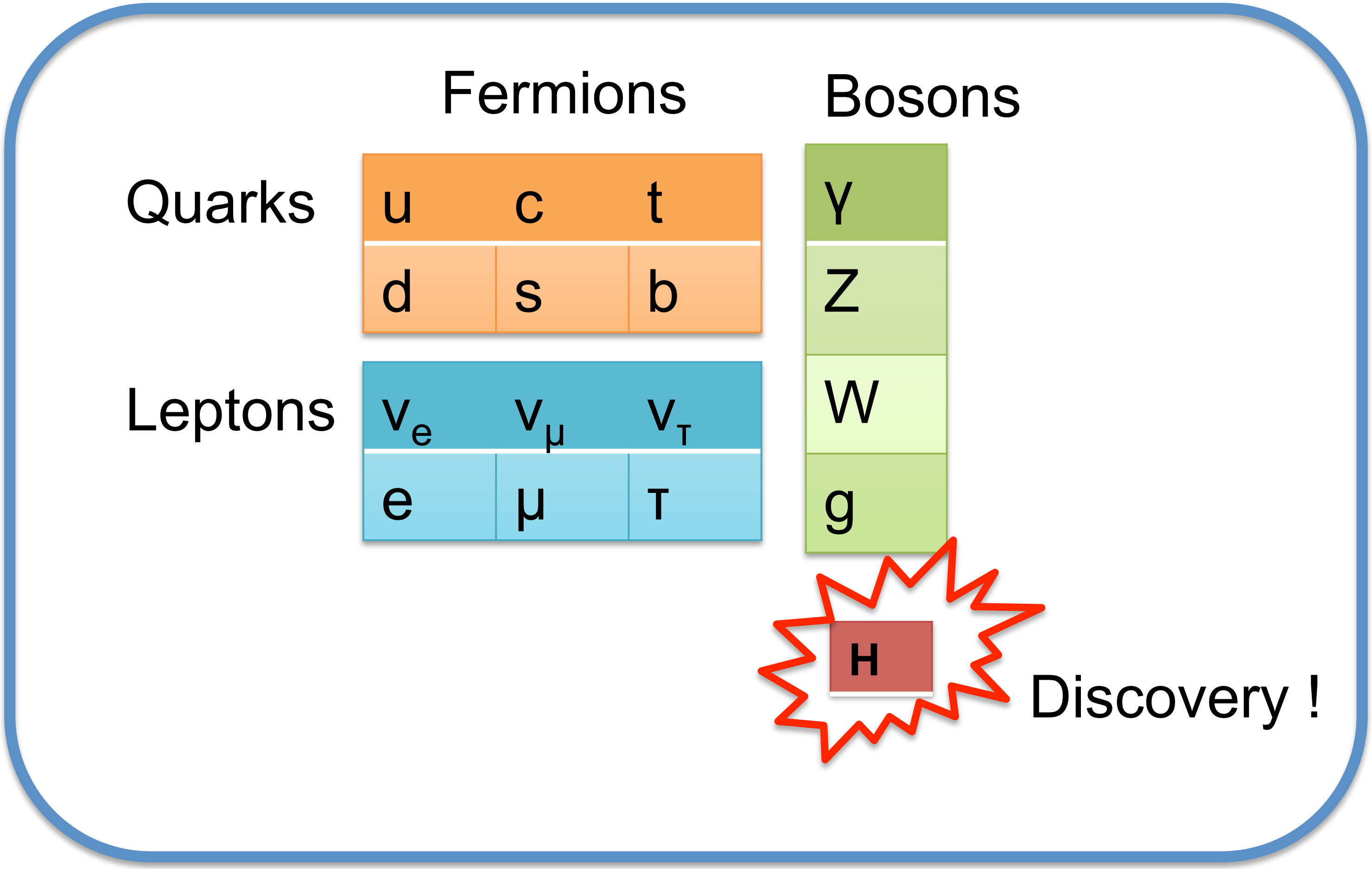
# my research experiences

1. Kaluza-Klein Graviton Search at the LHC
   - via forward detector
     G. C. Cho, T. Kono, K. Mawatari, K. Y, Phys. Rev. D. **91**, no. 11, 115015 (2015)
   - as a mediator of dark matter
     S. Kraml, U. Laa, K. Mawatari, K. Y, Eur. Phys. J. C **77**, no. 5, 326 (2017)
2. Baryogenesis
   - a model by using a dynamics of a rotating forced pendulum.
     K. Bamba, N. D. Barrie, A. Sugamoto, T. Takeuchi, K. Y, arXiv: 1610.03268 [hep-ph]
3. Model of Monopolium and its Search at the LHC
   N. D. Barrie, A. Sugamoto, K. Y, PTEP **2016**, no. 11, 113B02 (2016).
4. Dark Sector Search at a low energy experiment
   - my recent work: today's talk
     X. Fan, S. Kamioka, K. Y, S. Asai, A. Sugamoto, arXiv: 1707.03609 [hep-ph]
     K. Y, X. Fan, S. Kamioka, S. Asai, A. Sugamoto, arXiv: 1707.03308 [hep-ph]

# Table of contents

# Table of contents

# 1. Introduction: Standard Model




Standard model is successful theory

# 1. Introduction: Problems of the Standard Model

- Gravitational interaction
- Gauge hierarchy problem
- Flavor hierarchy problem
- Electric charge quantization
- Dark matter
- Dark energy
- Baryon asymmetry

⋮

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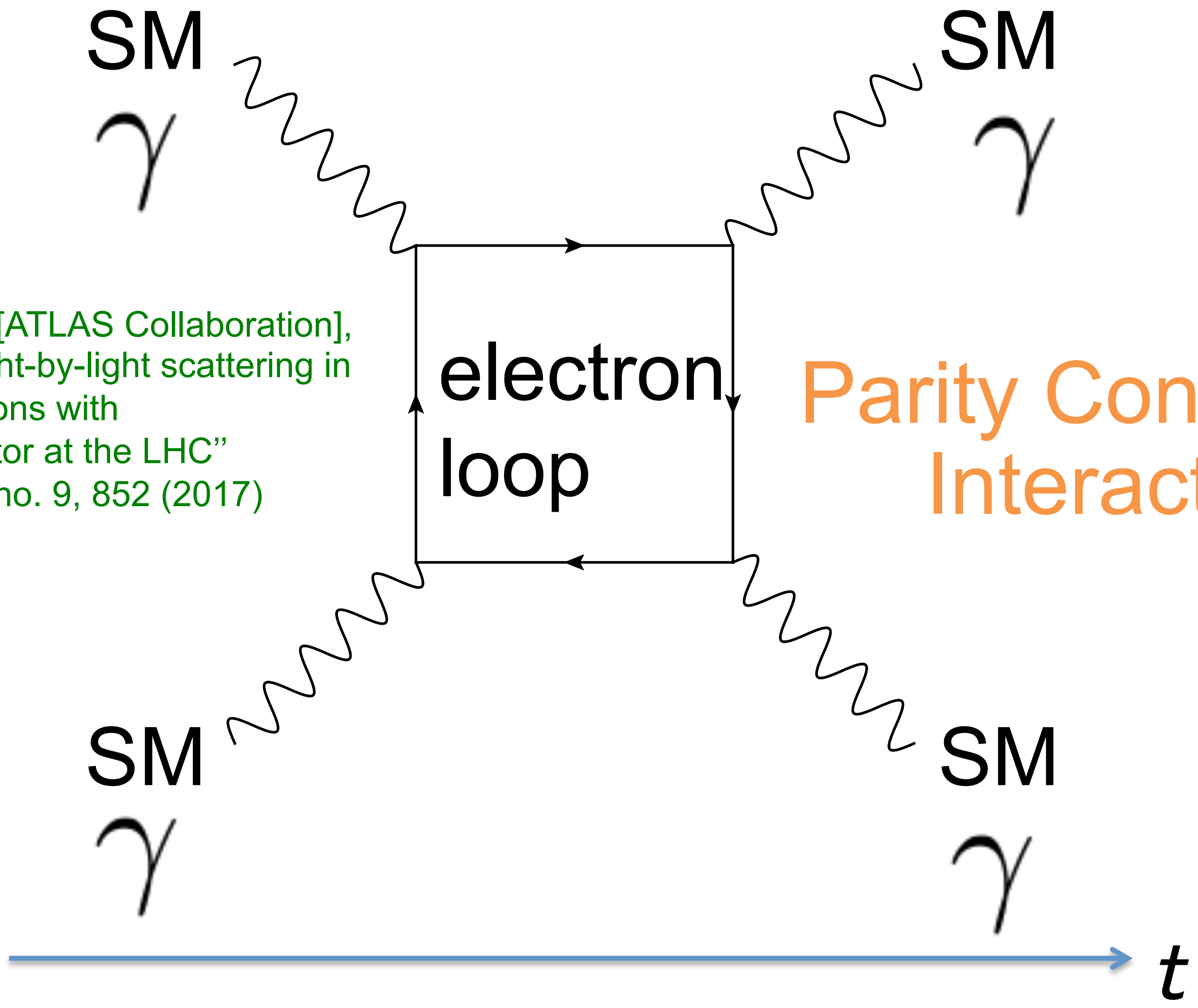
# 1. Introduction: QED interaction

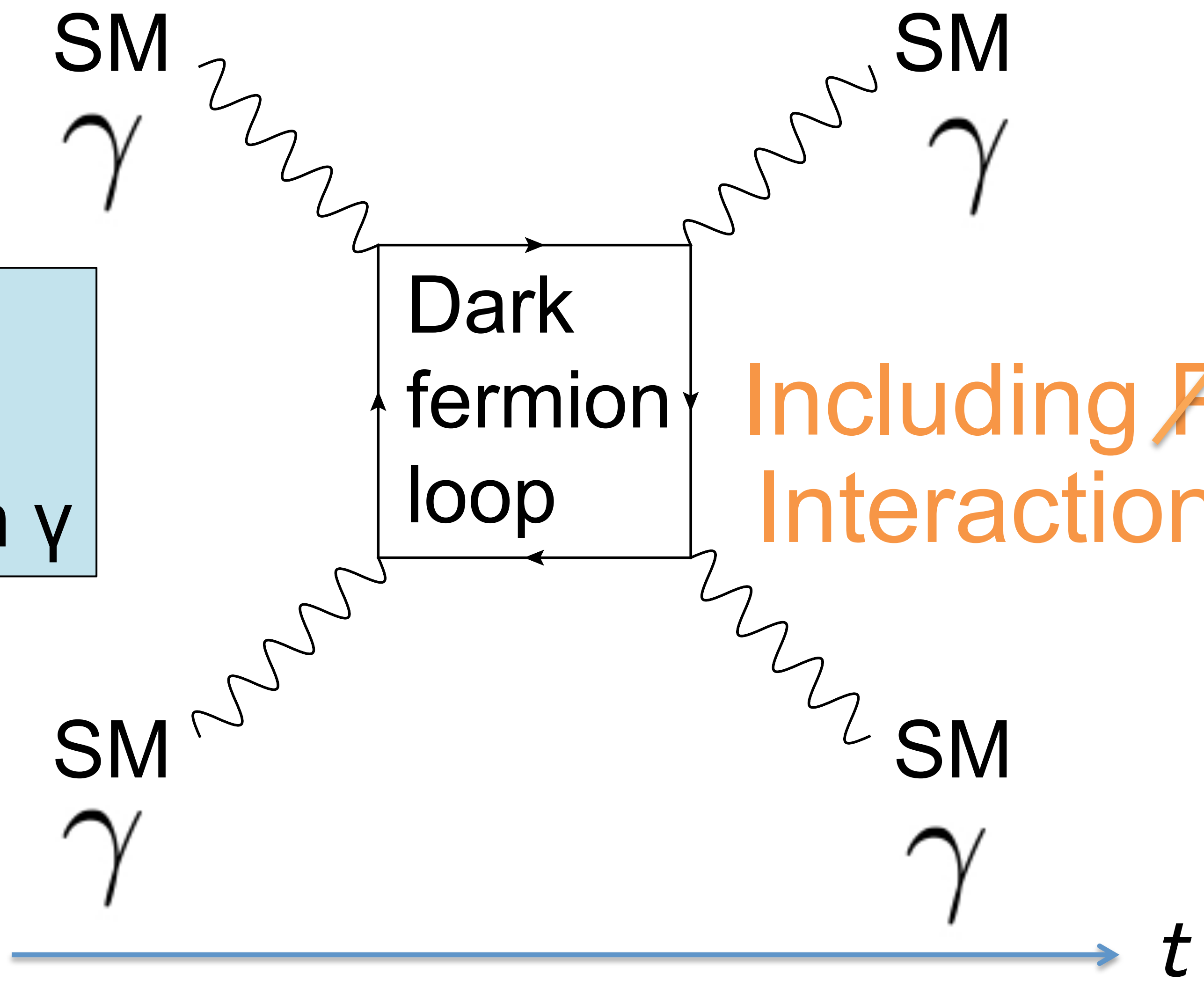



M. Aaboud *et al*. [ATLAS Collaboration], ``Evidence for light-by-light scattering in heavy-ion collisions with the ATLAS detector at the LHC'' Nature Phys.**13**, no. 9, 852 (2017)

Parity Conserving Interaction

# 1. Introduction: Dark Matter Search

DM:
- $\psi \times 1$
- $\not{P}$ with $\gamma$




Including $\not{P}$ Interaction

Need to Calculate Effective Lagrangian

→ Vacuum Birefringence Experiment

# 1. Introduction: QED Case

already known

W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936)

Heisenberg-Euler Lagrangian:

$$\mathfrak{L} = -\mathfrak{F} - \frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3} \exp(-m^2 s) \times \left[ (es)^2 \mathfrak{G} \frac{\mathrm{Re}\cosh esX}{\mathrm{Im}\cosh esX} - 1 - \tfrac{2}{3}(es)^2 \mathfrak{F} \right]$$

$$= \tfrac{1}{2}(\mathbf{E}^2 - \mathbf{H}^2) + \frac{2\alpha^2}{45}\frac{(\hbar/mc)^3}{mc^2} \times [(\mathbf{E}^2 - \mathbf{H}^2)^2 + 7(\mathbf{E}\cdot\mathbf{H})^2] + \cdots$$

$$X = \sqrt{2(\mathcal{F} + i\mathcal{G})},$$

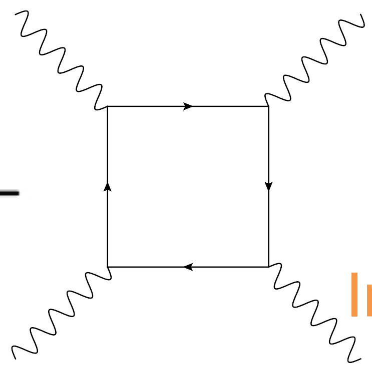
$$\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}\left(\vec{H}^2 - \vec{E}^2\right)$$

$$\mathcal{G} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{H}$$

$$\tilde{F}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}$$

from J. Schwinger, Phys. Rev. **82**, 664 (1951)

$$\mathcal{L}_{\mathrm{eff}} = -\tfrac{1}{4} F_{\mu\nu}^2 +$$

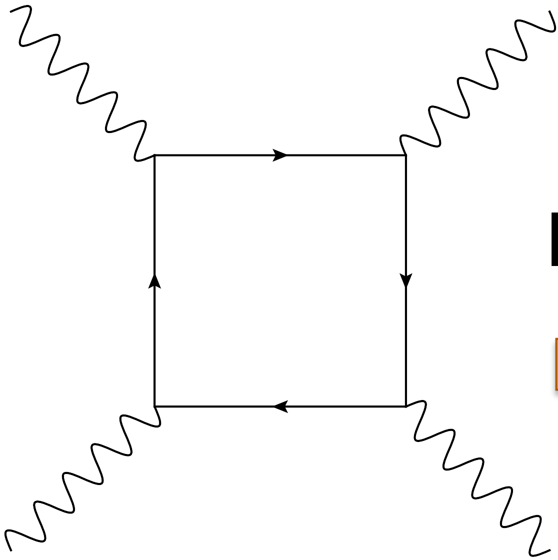


$+ \cdots$

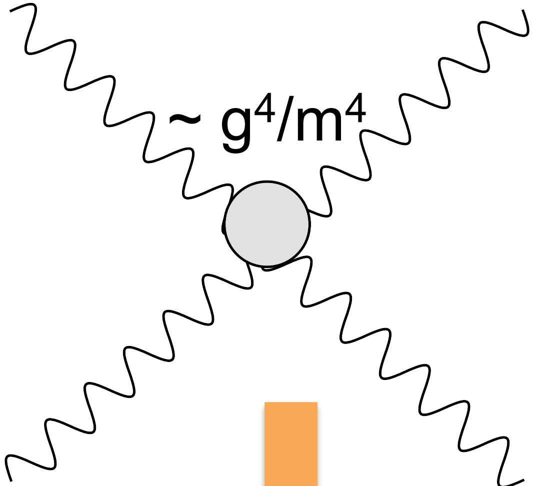
Including this

Our work
arXiv:1707.03308

# 1. Introduction: Dark Sector Case



Photon Energy
^
^
Dark Fermion Mass




We calculated here including coefficients a,b,c

$E_\gamma \ll m_{DM}$

$$\mathcal{L}_{\rm eff} = -\mathcal{F} + \boxed{a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}}$$

Vacuum Birefringence Experiment

# Table of contents

# 2-1. Effective Action in Proper-time Representation

DM:
- ψ×1
- P with γ

~~P~~ Action:

Include axial current coupling

$$S_\psi(m) = \int d^4x \; \bar{\psi} \left[ \gamma^\mu \left( i\partial_\mu - (g_V + g_A \gamma_5) A_\mu \right) - m \right] \psi$$

Effective Action:

$$S_{\text{eff}}[A_\mu] = \int d^4x \; \mathcal{L}_{\text{eff}}[A_\mu] = -i \ln \left[ \int \mathcal{D}\psi(x) \mathcal{D}\bar{\psi}(x) e^{iS_\psi(m)} \right]$$

$$= (-i)\frac{1}{2} \text{Tr} \ln(\hat{H} + m^2)$$

Integrated out

$$A_\mu(x) = \tfrac{1}{2} x^\lambda F_{\lambda\mu}$$

$E_\gamma << m_{DM}$ $F_{\mu\nu}$: Constant

→ easier to get $L_{eff}$

$$\hat{H} = -\left( i\partial_\mu - g_V \frac{1}{2} x^\nu F_{\nu\mu}(x) \right)^2 - \frac{1}{4} x^\mu \left( g_A^2 F_{\mu\lambda} F^{\lambda\nu} \right) x_\nu$$
$$+ \frac{1}{2}(g_V + g_A\gamma_5)\sigma^{\mu\nu} F_{\mu\nu} + i\frac{1}{2}\sigma^{\mu\nu} g_A \gamma_5 (x^\lambda F_{\lambda\mu} \, i\partial_\nu - x^\lambda F_{\lambda\nu} \, i\partial_\mu)$$

# 2-1. Effective Action in Proper-time Representation

x(0) = x(s)

$$S_{\text{eff}}(A) = (-i)\frac{1}{2}\text{Tr}\ln(\hat{H}+m^2)$$

traces of $x^\mu$ and spin

Proper time description:

$$= \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s}\,\text{Tr}(e^{-i\hat{H}s})$$

transition amplitude

$$\int d^4x \ \text{tr}\ \langle x(s), a(s) | x(0), b(0)\rangle$$

V. Fock,
Physik. Z. Sowjetunion, **12**, 404 (1937),
Y. Nambu,
Prog. Theor. Phys. **5**, 82 (1950)

Quantum mechanics of a point particle with position $x^\mu(s)$ at a proper time s

# 2-2. Path Integral Representation

$$S_{\rm eff}(A) = \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s} \int d^4x\, \underline{{\rm tr}\, \langle x(s), a(s) | x(0), b(0)\rangle}$$

$$\parallel$$

$$\underline{\langle x(s)|x(0)\rangle' \times \left\langle 2\left(\cos \bar{\boldsymbol{X}}'_+(s) + \cos \bar{\boldsymbol{X}}'_-(s)\right)\right\rangle'}$$

$$\langle x(s)|x(0)\rangle' = \int_{x^\mu(0)=x^\mu}^{x^\mu(s)=x^\mu} \mathcal{D}x^\mu(s')\; e^{i\int_0^s ds' A(s')}$$

$$\left\langle 2\left(\cos \bar{\boldsymbol{X}}'_+(s) + \cos \bar{\boldsymbol{X}}'_-(s)\right)\right\rangle' = 2\left(\cos \bar{\boldsymbol{X}}'_+(s) + \cos \bar{\boldsymbol{X}}'_-(s)\right)[x^\lambda(s') \to (-i)\delta/\delta j_\lambda(s')]$$

$$\times\; e^{-i\int_0^s ds' \int_0^s ds'' \sum_{\alpha\beta} j^\alpha(s')\Delta(s'-s'')_{\alpha\beta} j^\beta(s'')}\Big\}\Big|_{j_\lambda=0}$$

$$\boxed{\mathcal{L}_{\rm eff}(x) = \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s}\, \langle x(s)|x(0)\rangle' \times \left\langle 2\left(\cos \bar{\boldsymbol{X}}'_+(s) + \cos \bar{\boldsymbol{X}}'_-(s)\right)\right\rangle'}$$

$$\bar{A}(s) = \int_0^s ds'\left[-\frac{1}{4}(\dot{x}^\mu)^2 + \frac{1}{2}g_V x^\mu(F_{\mu\nu})\dot{x}^\nu - \frac{1}{2}g_A^2 x^\mu(F_{\mu\lambda}F^{\lambda\nu})x_\nu\right] \quad \bar{B}_{\mu\nu}(s) = \int_0^s ds'\left[g_A\frac{1}{2}\epsilon_{\mu\nu\beta\gamma}x_\alpha F^{\alpha\beta}\dot{x}^\gamma - (g_V F_{\mu\nu} - i g_A \tilde{F}_{\mu\nu})\right]$$

$$\bar{\boldsymbol{X}}'_\pm(s) = \sqrt{2\left(\bar{\boldsymbol{\mathcal{F}}}'(s) \pm i\bar{\boldsymbol{\mathcal{G}}}'(s)\right)} \quad \bar{\boldsymbol{\mathcal{F}}}'(s) = \frac{1}{4}\bar{B}_{\mu\nu}(s)\bar{B}^{\mu\nu}(s),\quad \bar{\boldsymbol{\mathcal{G}}}'(s) = \frac{1}{4}\bar{B}_{\mu\nu}(s)\tilde{\bar{B}}^{\mu\nu}(s).$$

# 2-2. Path Integral Representation

$$\mathcal{L}_{\rm eff}(x) = \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s}\,\langle x(s)|x(0)\rangle' \times \left\langle 2\left(\cos \bar{\boldsymbol{X}}'_+(s) + \cos \bar{\boldsymbol{X}}'_-(s)\right)\right\rangle'$$

Calculated free part

Generalized Heisenberg-Euler formula:

$$\mathcal{L}_{\rm eff} = -\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s}\frac{(g_+ s)^2\mathcal{G}}{Im\cosh(g_+ X s)} \times \frac{(g_- s)^2\mathcal{G}}{Im\cosh(g_- X s)} \times \frac{1}{2}\left\langle\left(\cos \bar{\boldsymbol{X}}'_+(s\to -is) + \cos \bar{\boldsymbol{X}}'_-(s\to -is)\right)\right\rangle'$$

$$X = \sqrt{2(\mathcal{F}+i\mathcal{G})},$$

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right)$$

$$\mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

# 2-2. Path Integral Representation

$g_V$=-e, $g_A$=0 ($g_+$=0, $g_-$=-e)
->reproduced QED case

= 1 in QED

$g_-$ = -e

$$\mathcal{L}_{\text{eff}} = -\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s} \frac{(g_+ s)^2 \mathcal{G}}{Im\cosh(g_+ X s)} \times \frac{(g_- s)^2 \mathcal{G}}{Im\cosh(g_- X s)}$$

$$\times \frac{1}{2}\left\langle\left(\cos \bar{X}'_+(s \to -is) + \cos \bar{X}'_-(s \to -is)\right)\right\rangle'$$

= Re cosh(esX) in QED

$$g_\pm = \frac{1}{2}(g_V \pm \sqrt{g_V^2 + 2g_A^2})$$

$$X = \sqrt{2(\mathcal{F} + i\mathcal{G})},$$

$$\mathfrak{L} = -\mathfrak{F} - \frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3} \exp(-m^2 s) \times \left[(es)^2 \mathfrak{G} \frac{\text{Re cosh}\, esX}{\text{Im cosh}\, esX} - 1 - \tfrac{2}{3}(es)^2 \mathfrak{F}\right]$$

from J. Schwinger, Phys. Rev. **82**, 664 (1951)

# 3. Effective Lagrangian of Fourth Order

$$\mathcal{L}_{\rm eff} = -\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s} \frac{(g_+ s)^2 \mathcal{G}}{Im \cosh(g_+ X s)} \times \frac{(g_- s)^2 \mathcal{G}}{Im \cosh(g_- X s)}$$

$$\times \frac{1}{2}\left\langle\left(\cos \bar{\boldsymbol{X}}'_+(s \to -is) + \cos \bar{\boldsymbol{X}}'_-(s \to -is)\right)\right\rangle'$$

$$\|$$

$$1 - \langle \bar{\mathcal{F}}'(s) \rangle' + \frac{1}{6}\langle (\bar{\mathcal{F}}'(s))^2 - (\bar{\mathcal{G}}'(s))^2 \rangle' + \cdots$$

# 3. Effective Lagrangian of Fourth Order

We derived as follows by using diagrams:

$$\langle \bar{\mathcal{F}}'(s)\rangle' = \frac{1}{3}\, g_A^2 \left\{2s^2\mathcal{F} + \frac{1}{15}s^4\left((4g_V^2+2g_A^2)\mathcal{F}^2 - 3g_V^2\mathcal{G}^2\right)\right\} + s^2\left((g_V^2+g_A^2)\mathcal{F} - 2ig_V g_A\mathcal{G}\right),$$

$$\langle(\bar{\mathcal{F}}'(s))^2\rangle' = \frac{1}{16}s^4\left\{\frac{32}{9}g_A^4(3\mathcal{F}^2-\mathcal{G}^2) + \frac{8}{45}g_A^4(5\mathcal{F}^2+\mathcal{G}^2) + \frac{64}{3}g_A^2\left((g_V^2+g_A^2)\mathcal{F}^2 - 2ig_V g_A\mathcal{F}\mathcal{G}\right) + 16\left((g_V^2+g_A^2)\mathcal{F} - 2ig_V g_A\mathcal{G}\right)^2\right\},$$

$\mathcal{F} \to \mathcal{G}$

$\mathcal{G} \to -\mathcal{F}$

$$\langle(\bar{\mathcal{G}}'(s))^2\rangle' = \frac{1}{16}s^4\left\{\frac{32}{9}g_A^4(-\mathcal{F}^2+3\mathcal{G}^2) + \frac{8}{45}g_A^4(\mathcal{F}^2+5\mathcal{G}^2) + \frac{64}{3}g_A^2\left((g_V^2+g_A^2)\mathcal{G}^2 + 2ig_V g_A\mathcal{F}\mathcal{G}\right) + 16\left((g_V^2+g_A^2)\mathcal{G} + 2ig_V g_A\mathcal{F}\right)^2\right\}$$

# 3. Effective Lagrangian of Fourth Order

$$\mathcal{L}_{\text{eff}} = -\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s} \frac{(g_+ s)^2 \mathcal{G}}{Im \cosh(g_+ X s)} \times \frac{(g_- s)^2 \mathcal{G}}{Im \cosh(g_- X s)}$$

$$\times \frac{1}{2}\left\langle\left(\cos \bar{X}'_+(s \to -is) + \cos \bar{X}'_-(s \to -is)\right)\right\rangle'$$

$$\parallel$$

$$1 - \langle \bar{\mathcal{F}}'(s)\rangle' + \frac{1}{6}\langle (\bar{\mathcal{F}}'(s))^2 - (\bar{\mathcal{G}}'(s))^2\rangle' + \cdots$$

dimension 4

extract $s^4$ terms

$O(s^4)$ corresponds to $O(F^4)$

($sF_{\mu\nu}$ makes no dimension)

$$\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right)$$

$$\mathcal{G} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \vec{E} \cdot \vec{B}$$

$$a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}$$

# 3. Effective Lagrangian of Fourth Order

$$S_\psi(m) = \int d^4x\ \bar{\psi}_{\rm DM} \left[\gamma^\mu \left(i\partial_\mu - (g_V + g_A\gamma_5)A'_\mu\right) - m\right] \psi_{\rm DM}$$

$$\mathcal{L}_{\rm eff} = -\mathcal{F} + a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}$$

$\not{P}$

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \quad \mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

$$a = \frac{1}{(4\pi)^2 m^4}\left(\frac{8}{45}\ g_V^4 - \frac{4}{5}\ g_V^2 g_A^2 - \frac{1}{45}\ g_A^4\right)$$

$$b = \frac{1}{(4\pi)^2 m^4}\left(\frac{14}{45}\ g_V^4 + \frac{34}{15}\ g_V^2 g_A^2 + \frac{97}{90}\ g_A^4\right)$$

$$c = \frac{1}{(4\pi)^2 m^4}\left(\frac{4}{3}\ g_V^3 g_A + \frac{28}{9}\ g_V g_A^3\right)$$

c=0 when $g_A$ or $g_V$ is 0

We followed a method developed by Schwinger

J. Schwinger, Phys. Rev.**82**, 664 (1951)

# Table of contents

# 4. Dark Matter Model

SM + U'(1)$_{Y'}$ + 1 Complex Scalar

$$\mathcal{L}_S = \left|\left(i\partial_\mu - g_1 Y_s B_\mu - g_1' Y_s' B_\mu'\right) S(x)\right|^2$$

spontaneously broken $\langle S \rangle = v_s/\sqrt{2}$,

$$\mathcal{L}_{\rm mixing} = \frac{1}{2} m_{B'}^2 \left(\varepsilon^2 B_\mu B^\mu + 2\varepsilon B_\mu B'^\mu + B_\mu' B'^\mu\right)$$

$$m_{B'} = g_1' Y_s' v_s \qquad \varepsilon \equiv \frac{g_1 Y_s}{g_1' Y_s'}$$

$$m_{B'} = g'_1 Y'_s v_s$$

# 4. Dark Matter Model

$$\varepsilon \equiv \frac{g_1 Y_s}{g'_1 Y'_s}$$

$$\mathcal{L}_{\text{mixing}} = \frac{1}{2} m_{B'}^2 \left( \varepsilon^2 B_\mu B^\mu + 2\varepsilon B_\mu B'^\mu + B'_\mu B'^\mu \right)$$

mass diagonalization

$$(m_{\tilde{A}})^2 = 0, \;\; (m_{\tilde{Z}})^2 = \frac{1}{4} v^2 (g_1^2 + g_2^2) + \varepsilon^2 \frac{g_1^2}{g_1^2 + g_2^2 - \alpha'} (m_{B'})^2, \;\; \text{and}$$

$$(m_{\tilde{B}'})^2 = (m_{B'})^2 \left( 1 + \varepsilon^2 \frac{g_2^2 - \alpha'}{g_1^2 + g_2^2 - \alpha'} \right).$$

$$\tilde{A}_\mu = \frac{g_1 A_\mu^3 + g_2 B_\mu}{\sqrt{g_1^2 + g_2^2}} - \varepsilon \frac{g_2}{\sqrt{g_1^2 + g_2^2}} B'_\mu$$

We assume $\epsilon \ll 1$

# 4. Dark Matter Model

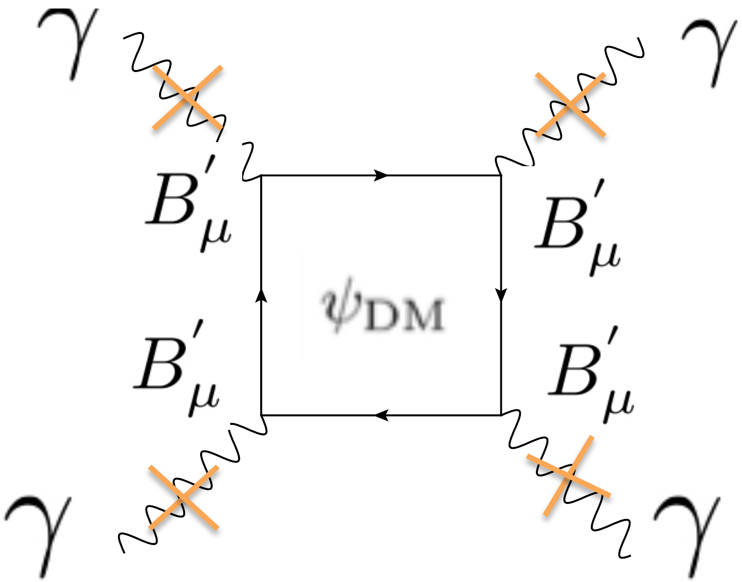
$$\varepsilon \equiv \frac{g_1 Y_s}{g_1' Y_s'}$$

$$\tilde{A}_\mu = \frac{g_1 A_\mu^3 + g_2 B_\mu}{\sqrt{g_1^2 + g_2^2}} - \varepsilon \frac{g_2}{\sqrt{g_1^2 + g_2^2}} B'_\mu$$

$\chi$

$$\mathcal{L}'_{\text{eff}} = \chi^4 \left\{ a\,\mathcal{F}^2 + b\,\mathcal{G}^2 + ic\,\mathcal{F}\mathcal{G} \right\}$$

$$S_\psi(m) = \int d^4x\ \bar{\psi}_{\text{DM}} \left[ \gamma^\mu \left( i\partial_\mu - (g_V + g_A \gamma_5)\, B'_\mu \right) - m \right] \psi_{\text{DM}}$$

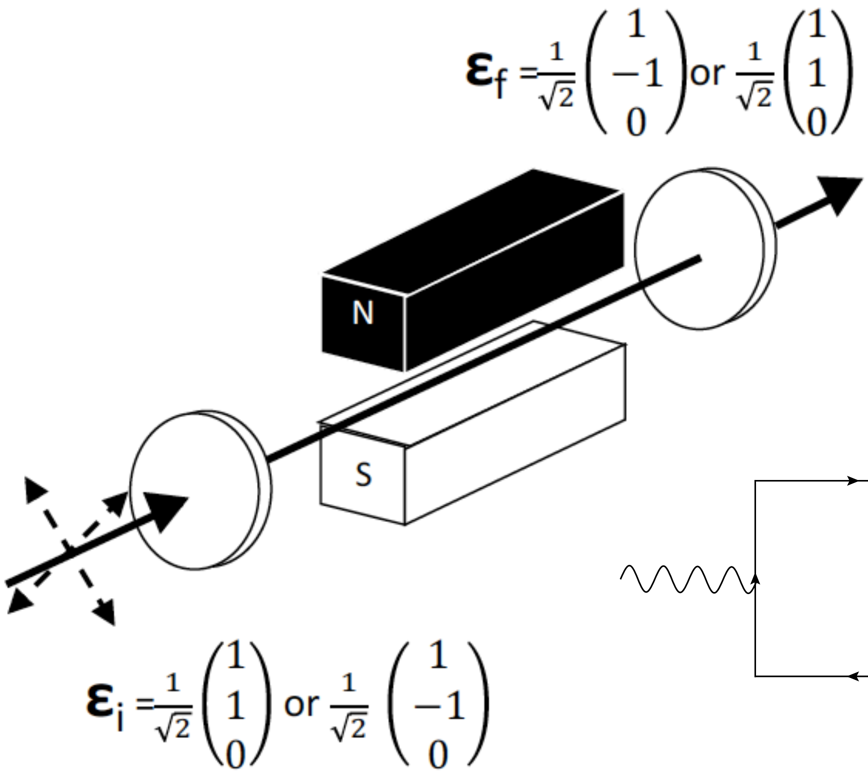


# 5. Relation to the low energy experiment: Vacuum Magnetic Birefringence Experiment

arXiv:1705.00495

- OVAL (Observing Vacuum with Laser) experiment

Conventional

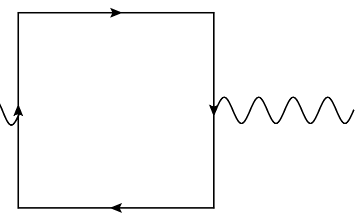


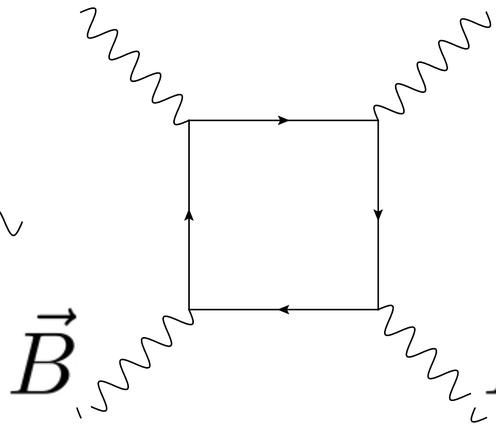

Eur. Phys. J. D (2014) **68**: 16

- BMV experiment

Eur. Phys. J. C (2016) **76**: 24

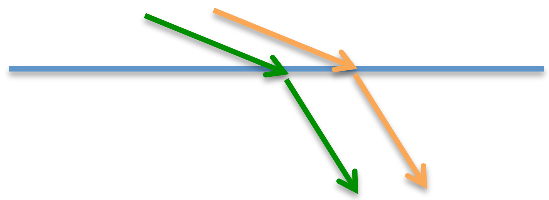
- PVLAS experiment



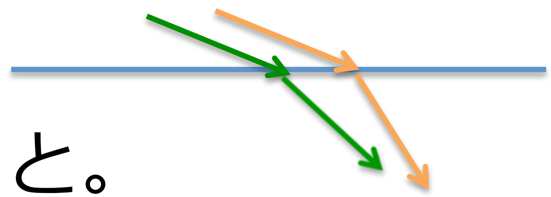


# 5. Relation to the low energy experiment: Vacuum Magnetic Birefringence Experiment

屈折：
光の位相速度が変化



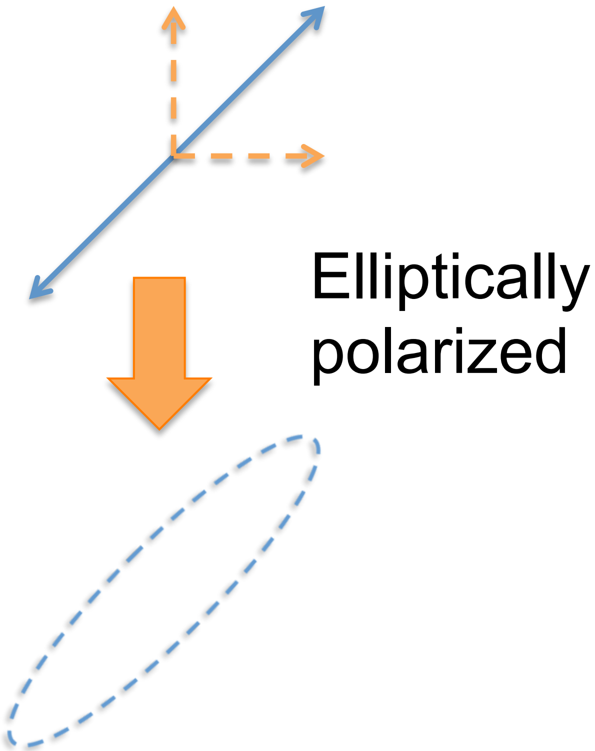
複屈折とは:
光の2つの偏極ごとに、
位相速度が異なった変化をすること。



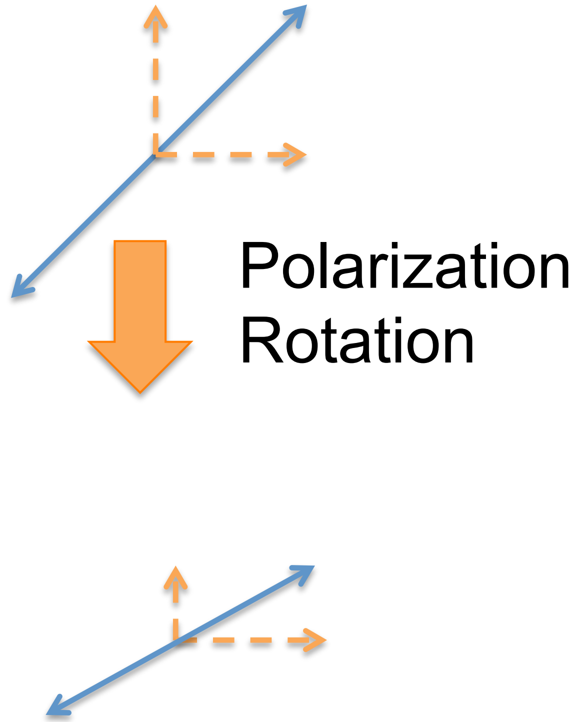
# 5. Vacuum Magnetic Birefringence Experiment

Polarization State: 2 parameters

**Ellipticity**



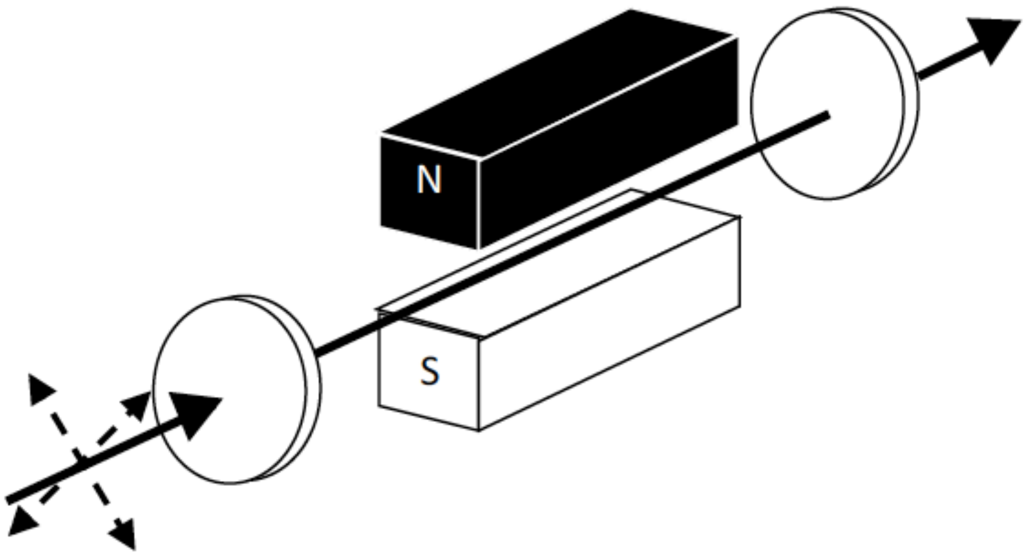

**Direction of the long axis of an ellipse**



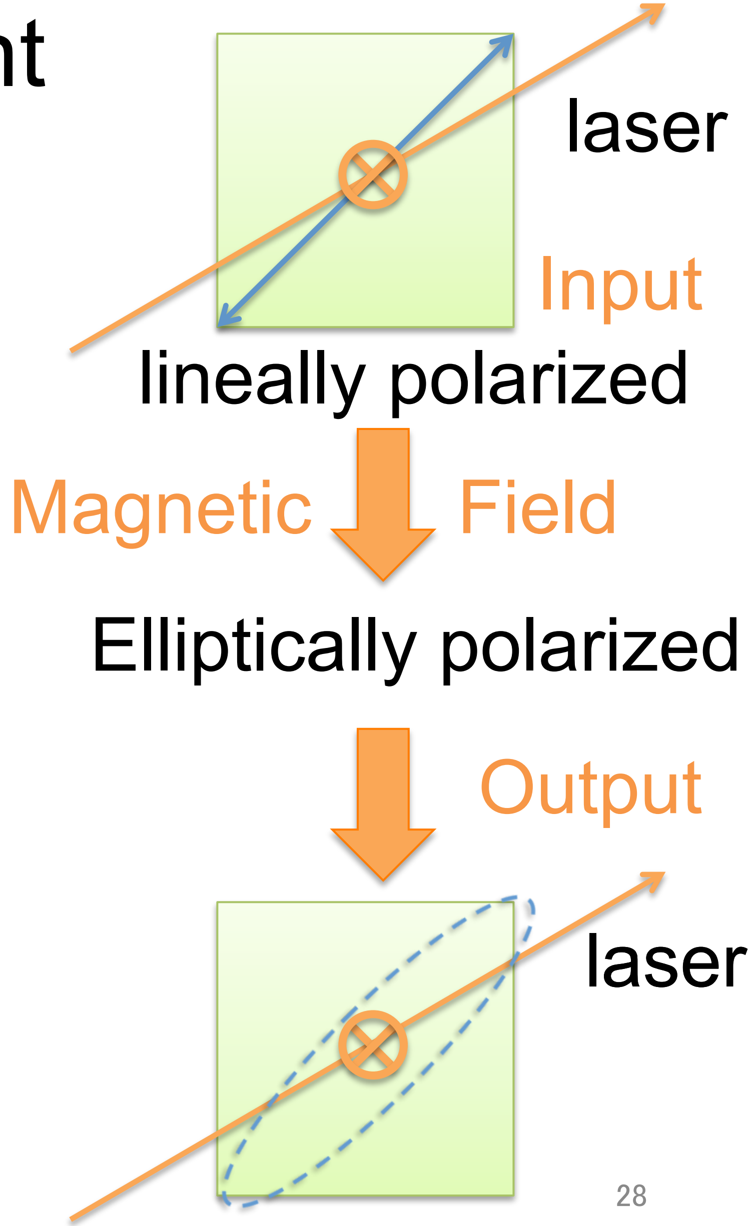
# 5. Vacuum Magnetic Birefringence Experiment

## Conventional

$$\boldsymbol{\varepsilon}_f = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix} \text{or } \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$$



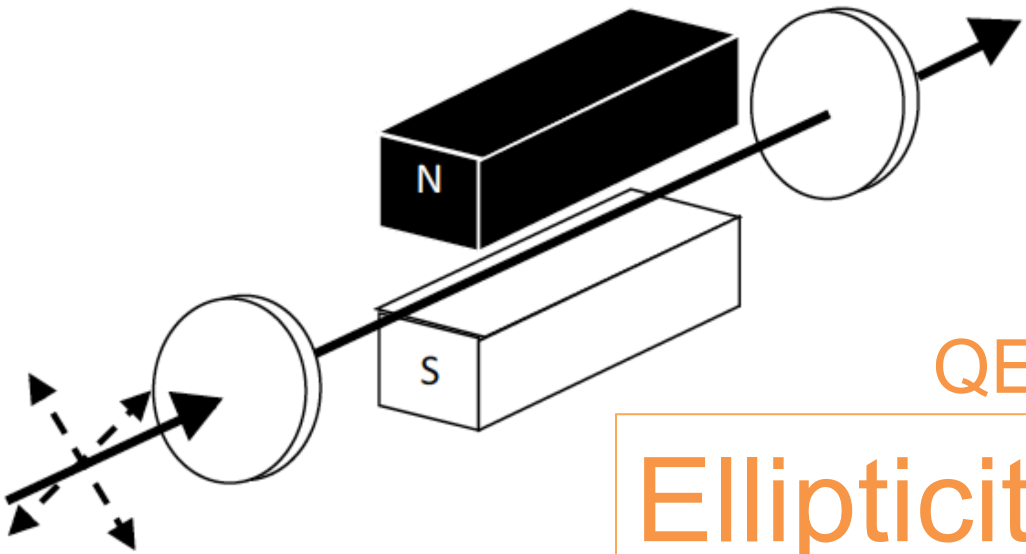

$$\boldsymbol{\varepsilon}_i = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix} \text{or } \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$$



# 5. Vacuum Magnetic Birefringence Experiment

## Conventional

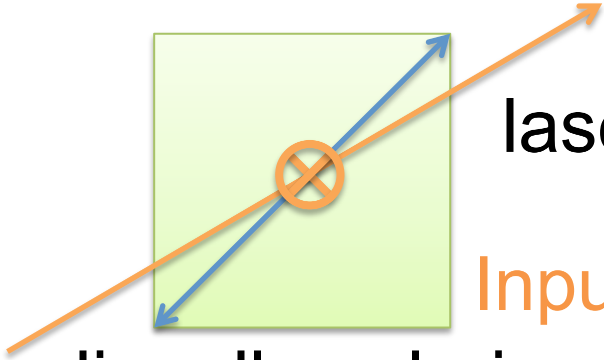
$$\boldsymbol{\varepsilon}_f = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix} \text{or } \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$$




$$\boldsymbol{\varepsilon}_i = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix} \text{or } \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$$

QED

**Ellipticity**

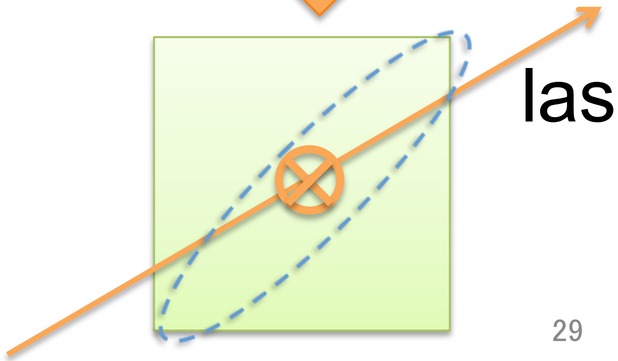



lineally polarized

Magnetic Field

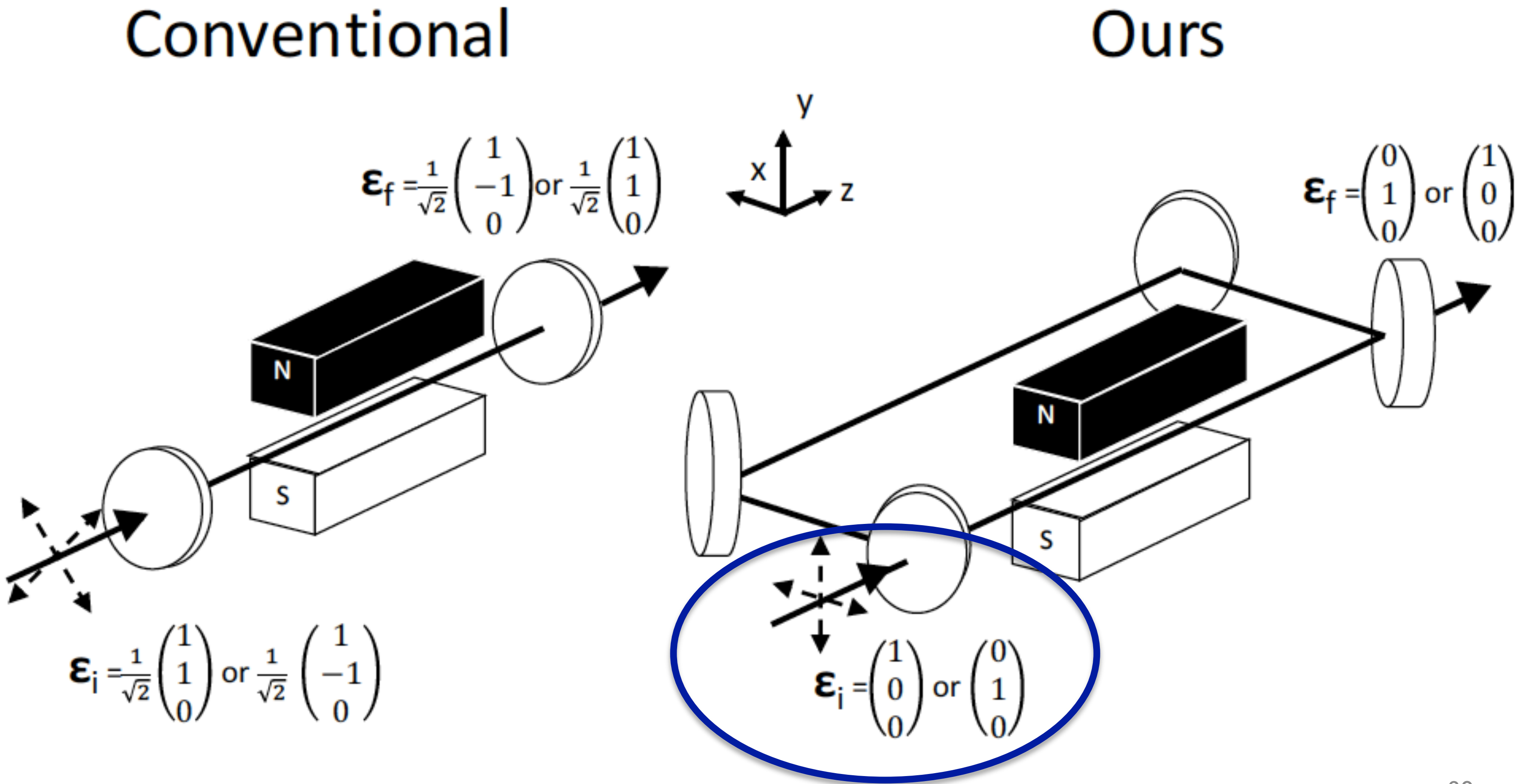
Elliptically polarized

Output



# 5. Vacuum Magnetic Birefringence Experiment

Detecting ~~P~~ Interaction



# 5. Vacuum Magnetic Birefringence Experiment

$$\mathcal{L}_{\text{eff}} = -\mathcal{F} + a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}$$

Equation of Motion

reflection constant: 1/(phase velocity of the laser):

$$n_{\pm} = 1 + \frac{1}{2}\boldsymbol{B}^2 \left\{ (a+b) \pm \sqrt{(a-b)^2 - c^2} \right\}.$$

=D

# 5. Vacuum Magnetic Birefringence Experiment

After a distance L though the magnetic field

Conventional: ε(45°) for QED

$$\epsilon(45^\circ) \rightarrow \begin{cases} (\cos(\Psi - 2\phi)\epsilon(45^\circ) - i\sin\Psi\epsilon(-45^\circ))\,/\,\cos 2\phi & (D > 0) \\ ((\cosh\theta\sinh\Psi - \cosh\Psi)\epsilon(45^\circ) - i\sinh\theta\sinh\Psi\epsilon(-45^\circ))\,/\,\cosh\theta & (D < 0) \end{cases}$$

(coefficient of ε(-45°)) / (coefficient of ε(45°))

ellipticity *

$$\sin\Psi / \cos(\Psi - 2\phi) \quad \text{for} \quad \epsilon_i = \epsilon(45^\circ)\ (D > 0)$$

$$\sinh\theta\sinh\Psi / (\cosh\Psi - \cosh\theta\sinh\Psi) \quad \text{for} \quad \epsilon_i = \epsilon(45^\circ)\ (D < 0)$$

( * ) D > 0 in QED

# 5. Vacuum Magnetic Birefringence Experiment

After a distance L though the magnetic field

Detecting P̸ interaction: $\epsilon_{||}$

0!

$$\epsilon_{||} \rightarrow \begin{cases} \left( (-i\sin\Psi + \cos 2\phi \cos\Psi)\epsilon_{||} + \sin\Psi \sin 2\phi \epsilon_{\perp} \right) / \cos 2\phi & (D>0) \\ (\cosh\Psi + i\sinh\theta\sinh\Psi)\epsilon_{||} - \cosh\theta\sinh\Psi\epsilon_{\perp} & (D<0) \end{cases}$$

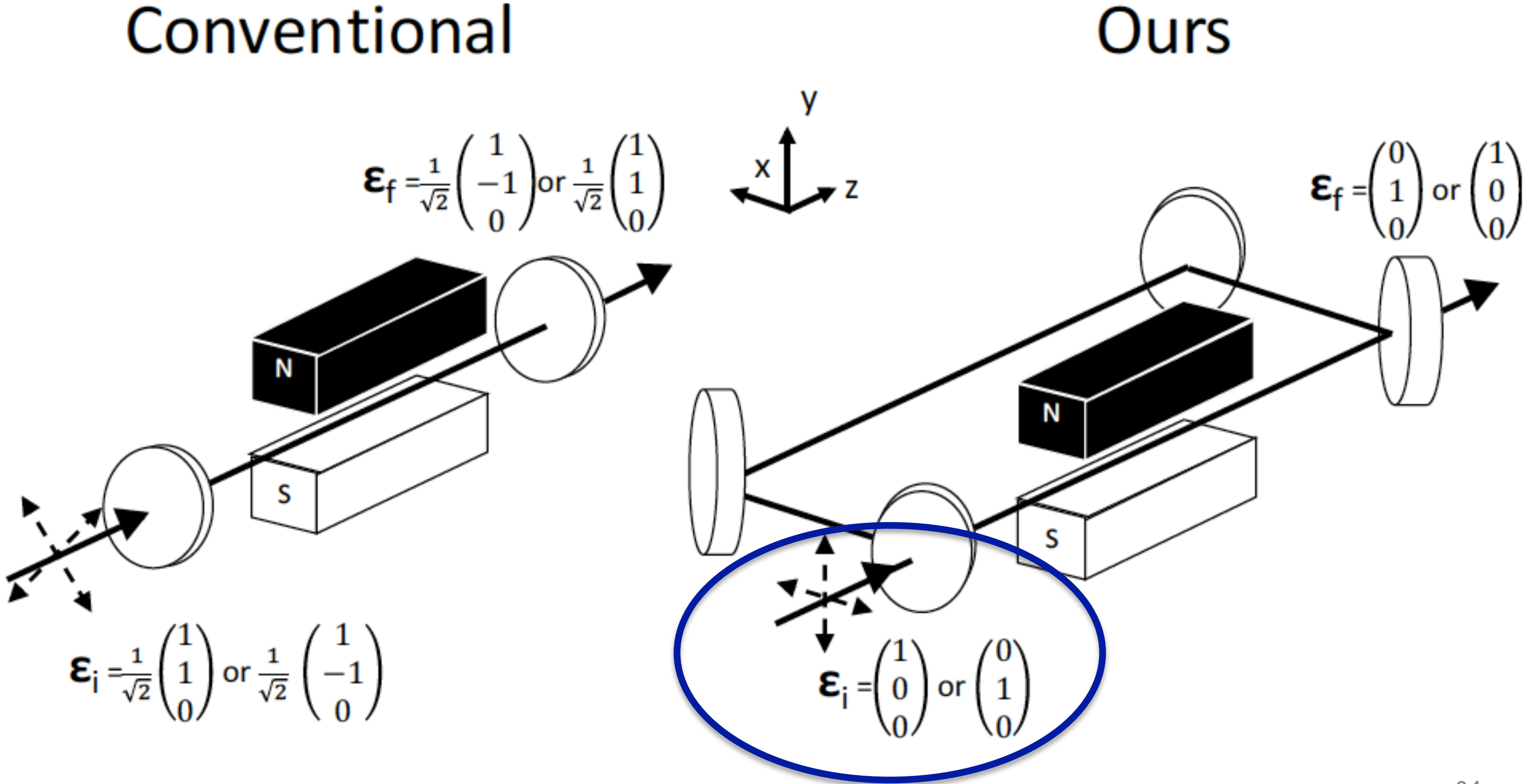
$$\epsilon_{\perp} \rightarrow \begin{cases} \left( \sin 2\phi \sin\Psi \epsilon_{||} + (i\sin\Psi + \cos 2\phi\cos\Psi)\epsilon_{\perp} \right) / \cos 2\phi & (D>0) \\ -\cosh\theta\sinh\Psi\epsilon_{||} + (\cosh\Psi - i\sinh\theta\sinh\Psi)\epsilon_{\perp} & (D<0) \end{cases}$$

0 !

$$\phi = 0(\mathrm{QED})$$

# 5. Vacuum Magnetic Birefringence Experiment

Detecting P Interaction

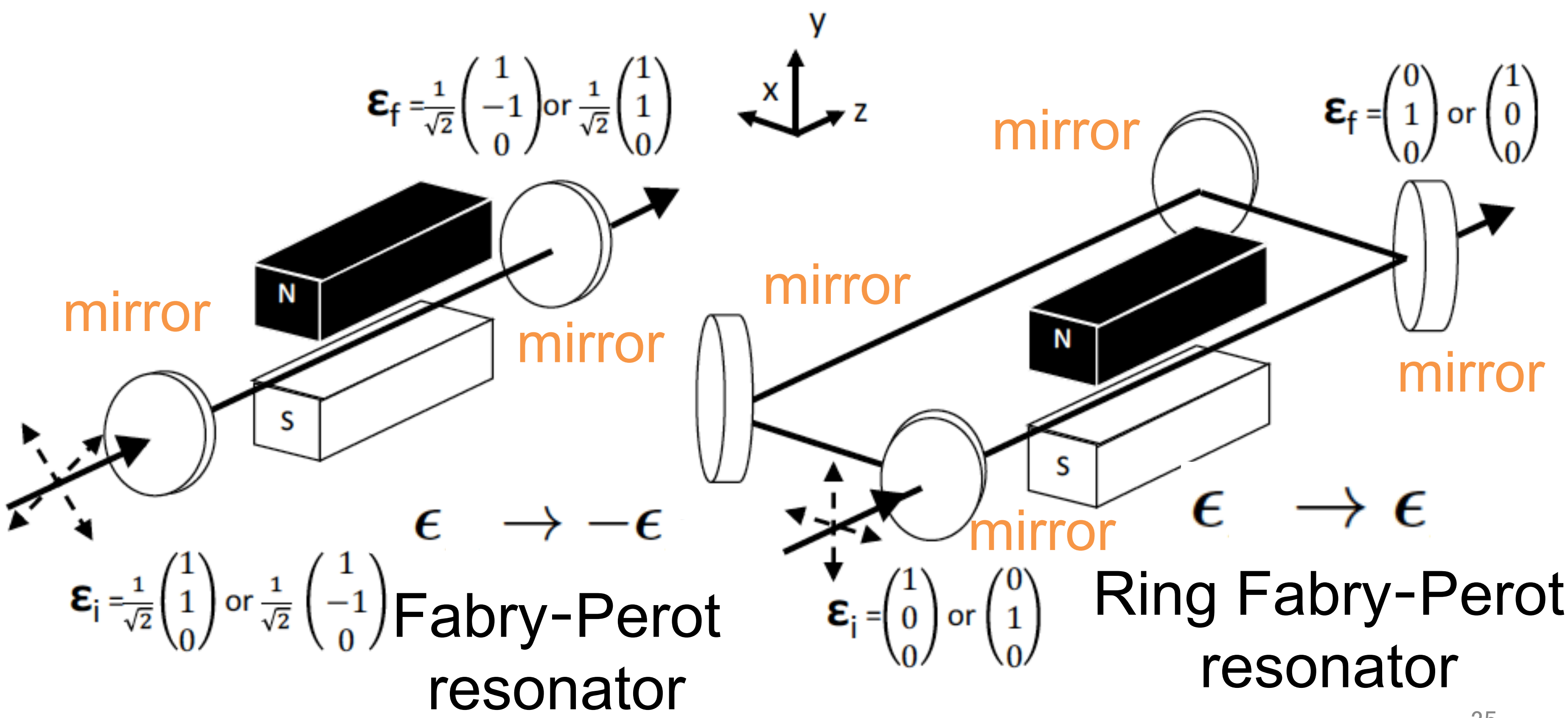


# 5. Vacuum Magnetic Birefringence Experiment

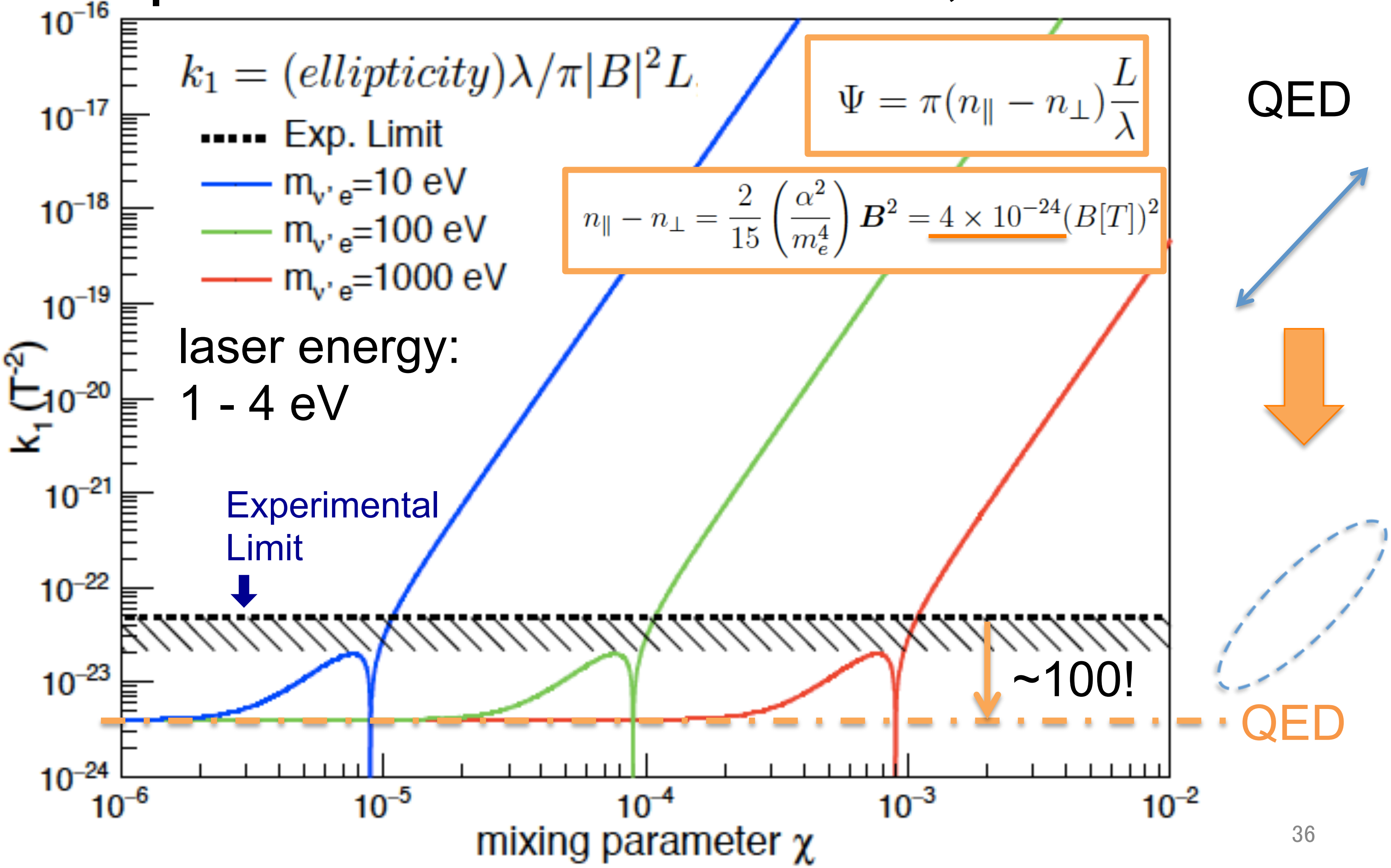
Detecting $\not{P}$ Interaction

Conventional

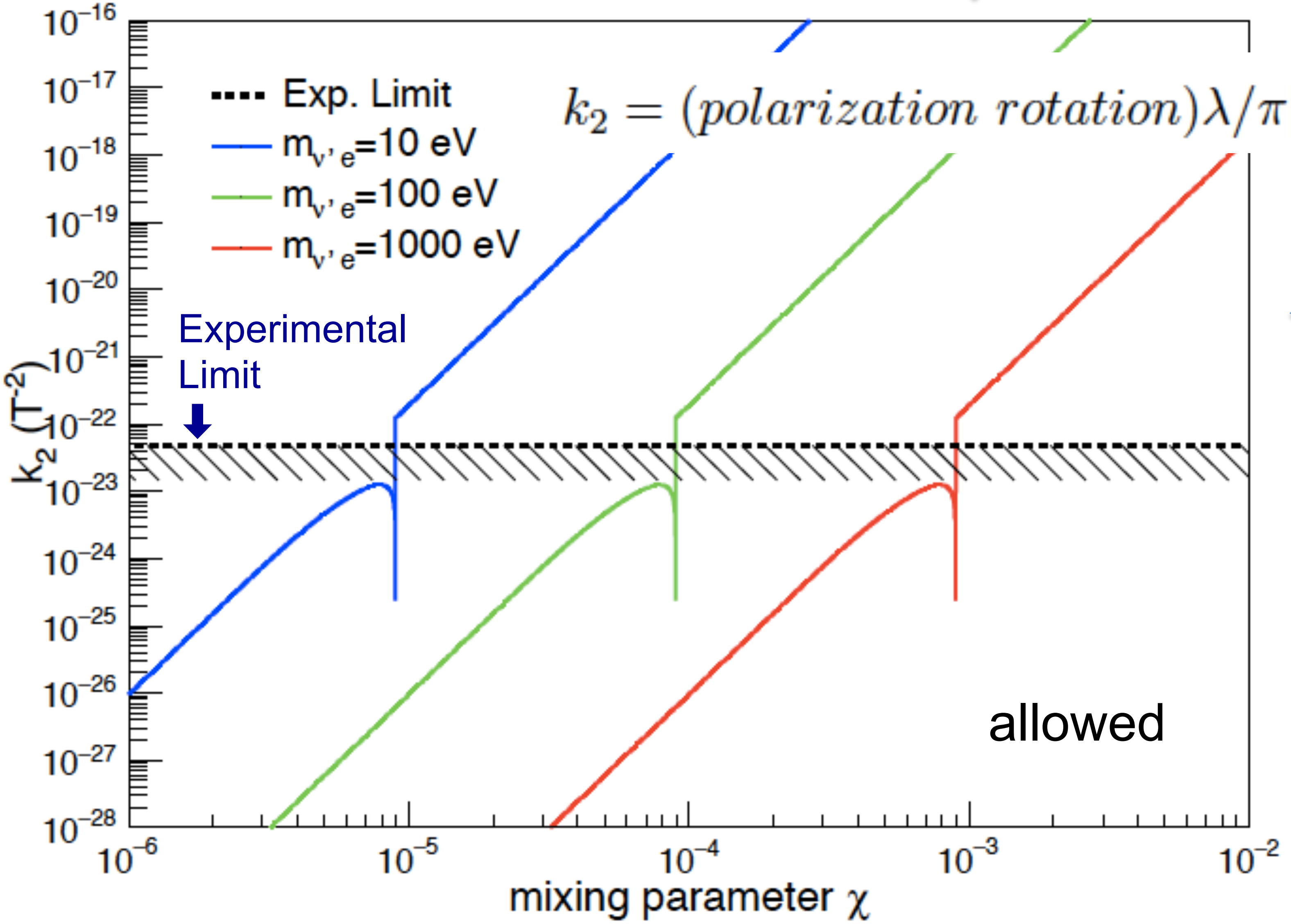
Ours



# 5. Vacuum Magnetic Birefringence Experiment – Conventional, QED/DM

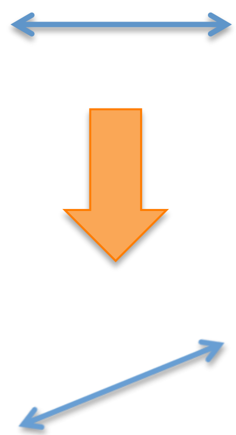


# 5. Vacuum Magnetic Birefringence Experiment – New Set Up, P̸ DM only




$$k_2 = (polarization\ rotation)\lambda/\pi|B|^2L$$

P̸



# 6. Summary

1. We considered Parity violated dark sector model, and derived generalized Heisenberg-Euler formula
2. Our focus lay on light-by-light scattering effective Lagrangian of fourth order and gave a result:

$$\mathcal{L}_{\text{eff}} = -\mathcal{F} + a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}$$

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \quad \mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

$$a = \frac{1}{(4\pi)^2 m^4}\left(\frac{8}{45}\ g_V^4 - \frac{4}{5}\ g_V^2 g_A^2 - \frac{1}{45}\ g_A^4\right)$$

$$b = \frac{1}{(4\pi)^2 m^4}\left(\frac{14}{45}\ g_V^4 + \frac{34}{15}\ g_V^2 g_A^2 + \frac{97}{90}\ g_A^4\right)$$

$$c = \frac{1}{(4\pi)^2 m^4}\left(\frac{4}{3}\ g_V^3 g_A + \frac{28}{9}\ g_V g_A^3\right)$$

3. We focus on Vacuum Magnetic Birefringence Experiment to probe the dark sector and propose new polarization state and the ring resonator in stead of the usual Fabry-Perot resonator to measure the Parity violated term