Ghost condensation as the simplest Higgs phase of gravity

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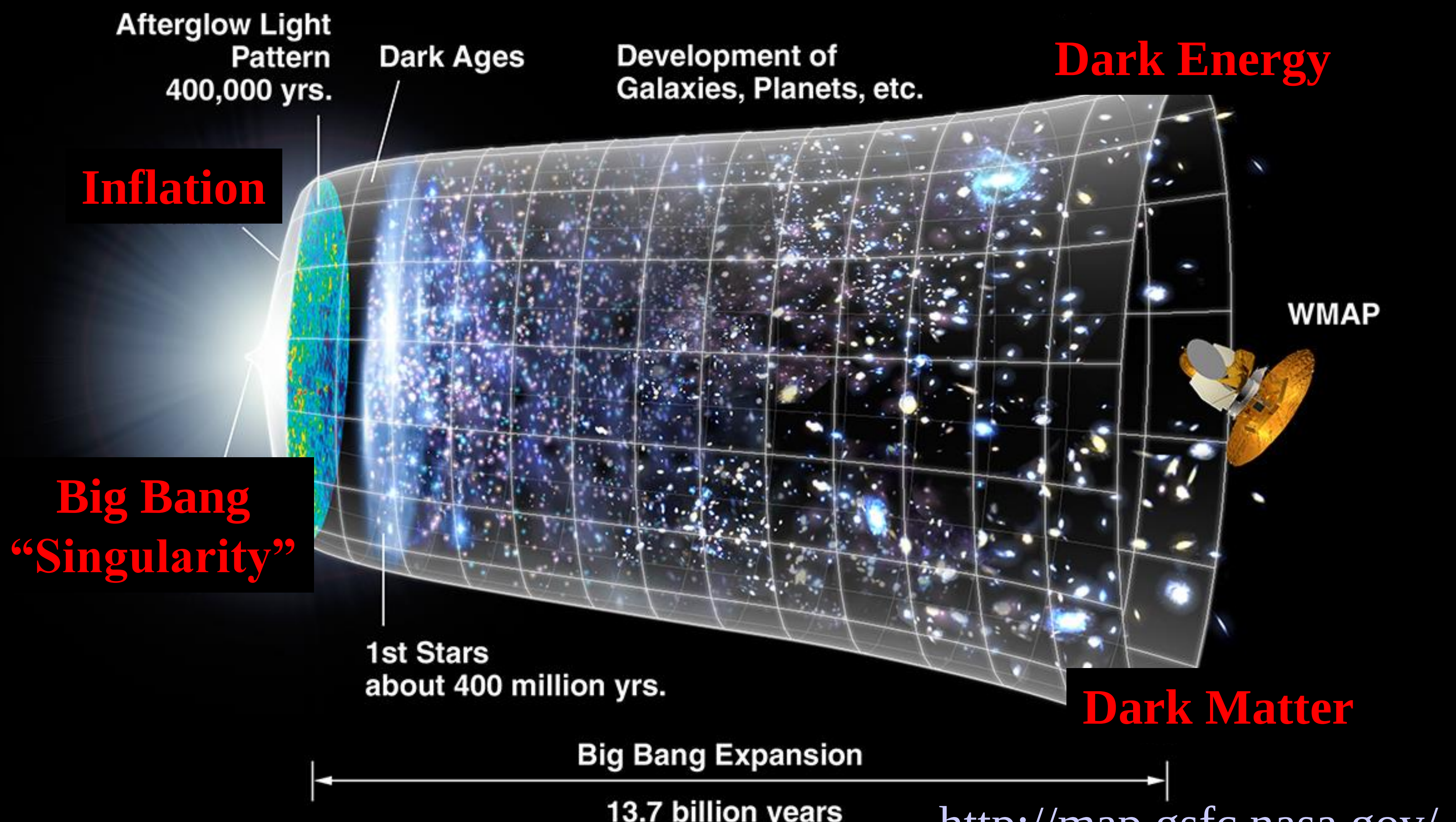
Based on collaborations with
N. Arkani-Hamed, H.-C. Cheng, P. Creminelli, T. Furukawa, K. Ichiki, K. Izumi, S. Jazayeri, M. Luty, R. Saitou, N. Sugiyama, J. Thaler, Y. Watanabe, T. Wiseman, M. Zaldarriaga

Why alternative gravity theories?



http://map.gsfc.nasa.gov/

Three conditions for good alternative theories of gravity (my personal viewpoint)

1. Theoretically consistent
   e.g. no ghost instability
2. Experimentally viable
   solar system / table top experiments
3. Predictable
   e.g. protected by symmetry

Some examples

I. Ghost condensation
IR modification of gravity
motivation: dark energy/matter

II. Nonlinear massive gravity
IR modification of gravity
motivation: "Can graviton have mass?"

III. Horava-Lifshitz gravity
UV modification of gravity
motivation: quantum gravity

IV. Superstring theory
UV modification of gravity
motivation: quantum gravity, unified theory

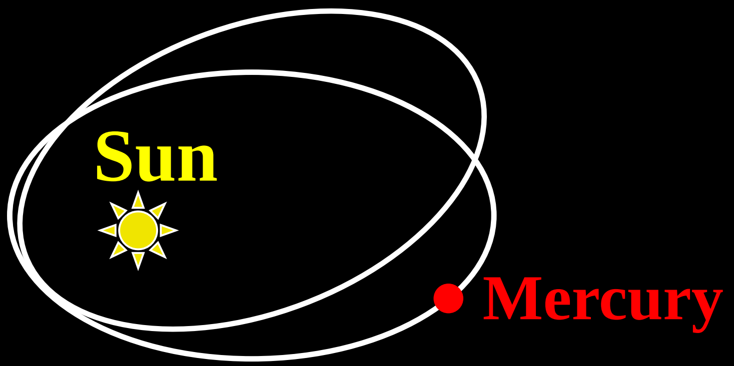
A motivation for IR modification

- Gravity at long distances
  Flattening galaxy rotation curves
  extra gravity
  Dimming supernovae
  accelerating universe

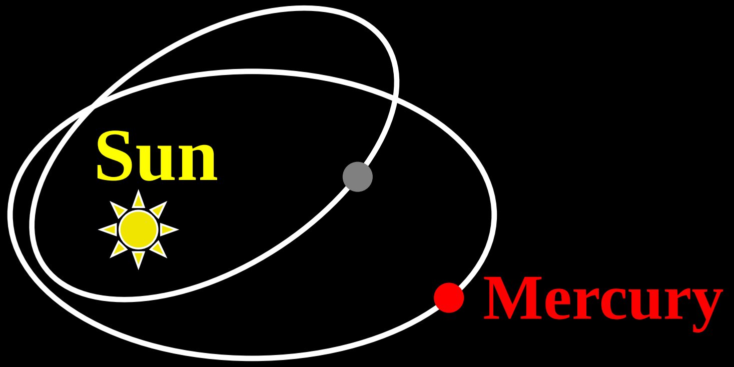
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion observed in 1800's…



which people tried to explain with a "dark planet", Vulcan,



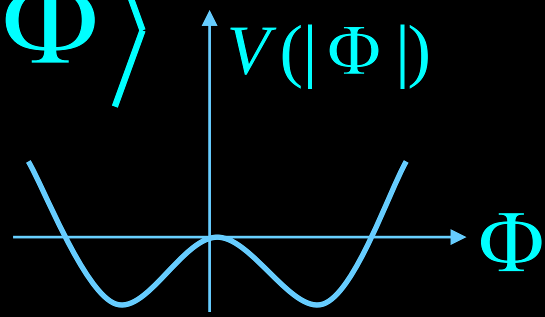
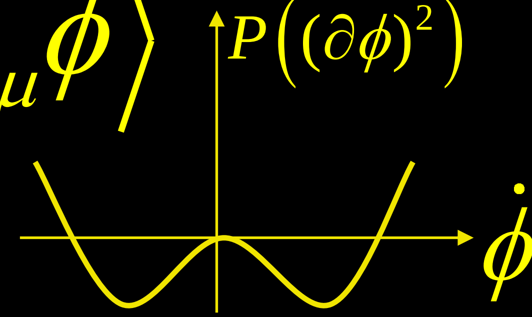
But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

Can we change gravity in IR?

- **Change Theory?**
  Massive gravity **Fierz-Pauli 1939**
  DGP model **Dvali-Gabadadze-Porrati 2000**

- **Change State?**
  Higgs phase of gravity
  The simplest: Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004.

	Higgs mechanism	***Ghost condensate*** Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V'=0, V''>0	P'=0, P''>0
Broken symmetry	Gauge symmetry	Time translational symmetry
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

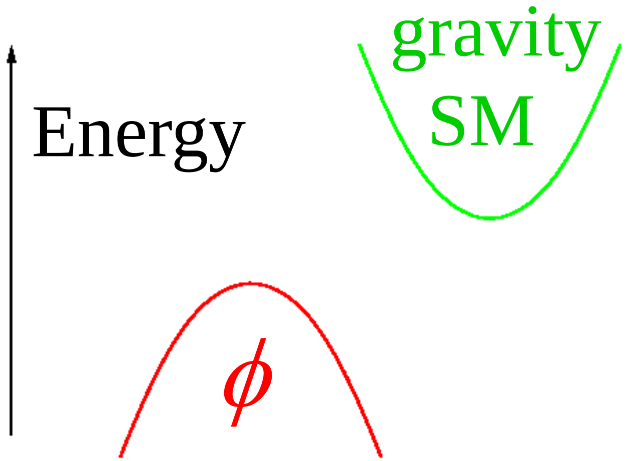
Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

Suppose

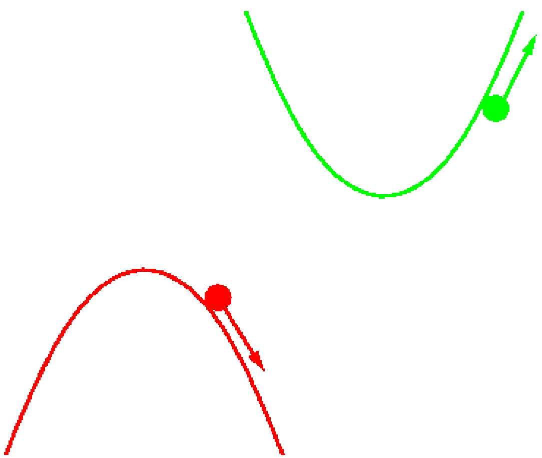
$$L = L_{grav} + L_{SM} + L_{\phi}$$

$$L_{\phi} = -(\partial\phi)^2 + \cdots$$



Any coupling $\Rightarrow$ instability

Vacuum $\Rightarrow$



In analogy with Higgs potential, can this be stabilized?

$$L_\phi = M^4\left[-(\partial\phi)^2 + (\partial\phi)^4 \cdots\right] \quad ?$$

$(+---)$

Clearly $(\partial\phi)^4 \cdots$ are higher dim ops. Really we should consider

$$L_\phi = P\left((\partial\phi)^2\right) + \tilde{P}\left((\partial\phi)^2\right)Q(\Box\phi) + \cdots$$

(Shift symmetry is assumed.)

Naively no sensible EFT.

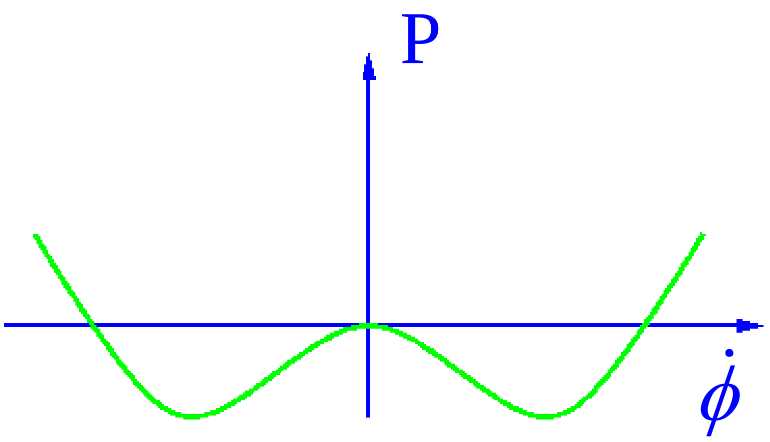
<u>NOT THE CASE.</u>

There is a sensible EFT + derivative expansion.

For simplicity

$$L_\phi = P\left((\partial\phi)^2\right)$$

in flat background.



Eq of motion

$$\partial_\mu \left[P' \cdot \partial^\mu \phi\right] = 0$$

Clearly any $\partial_\mu \phi =$ constant is a solution!
Suppose $\partial_\mu \phi =$constant + timelike.
Go to frame where

$$\dot{\phi} = c, \partial_i \phi = 0 \Rightarrow \phi = ct$$

Solution for any c!

In this language, we can have a good EFT because the background sits at some value c.

$\Longrightarrow$ doesn't sample entire function $P\left((\partial\phi)^2\right)$.
Taylor expansion of $P\left((\partial\phi)^2\right)$ around $(\partial\phi)^2 = c^2$ makes perfect sense.

Small fluctuations controlled by small # of parameters at low energies.

Look at small fluctuations

$$\phi = ct + \pi$$

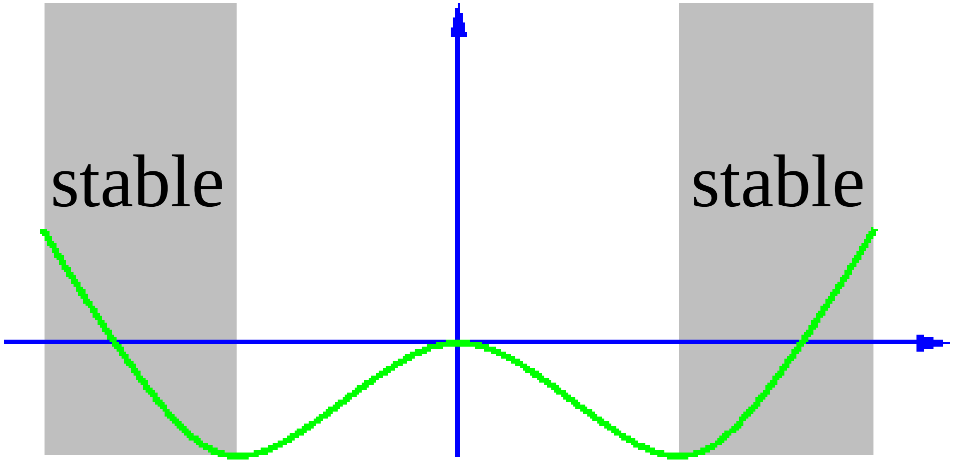
$$S = \int d^4x \left[\left(P'(c^2) + 2c^2 P''(c^2) \right) \dot{\pi}^2 - P'(c^2) \cdot (\nabla \pi)^2 \right]$$

So for $P'(c^2) + 2c^2 P''(c^2) > 0$

$$P'(c^2) > 0$$

NORMAL sign kinetic terms.

STABLE



So far, possible backgrounds are labeled by continuous parameter c.

Situation changes in presence of gravity, in expanding universe.

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2$$

$$S = \int d^4x\sqrt{-g}\,P(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi)$$

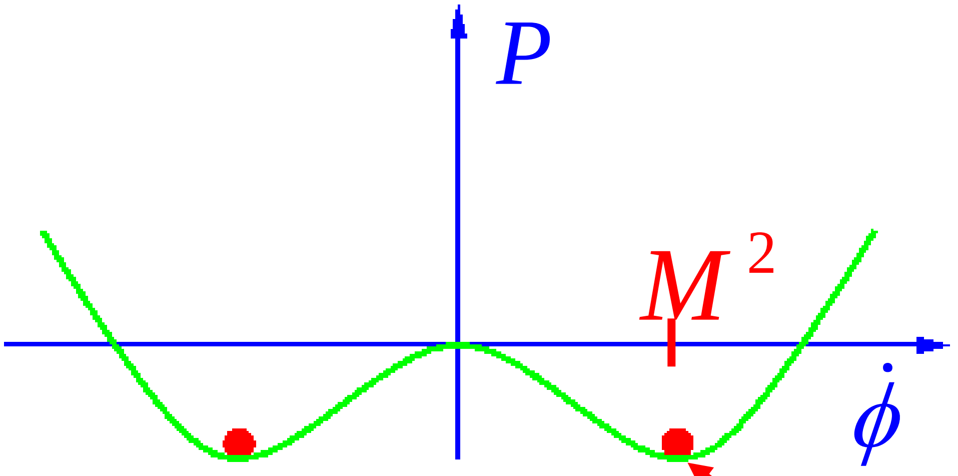
$$T_{\mu\nu} \sim \; Pg_{\mu\nu} + P'\partial_\mu\phi\partial_\nu\phi$$

E.O.M.

$$\partial_t[a^3P'\cdot\dot{\phi}] = 0 \;\Longrightarrow\; P'\dot{\phi} \to 0 \text{ as } a \to \infty$$

$$\Longrightarrow \quad \dot{\phi} = 0 \qquad \text{or} \qquad \boxed{P'(\dot{\phi}^2) = 0}$$

(unstable ghost background)



In this language, we can have a good EFT because the background sits at the special value.

$\Rightarrow$ doesn't sample entire function $P\left((\partial\phi)^2\right)$.
Taylor expansion of $P\left((\partial\phi)^2\right)$ around $(\partial\phi)^2 = M^4$ makes perfect sense.

Small fluctuations controlled by small # of parameters at low energies.

Look at small perturbations $\phi = M^2 t + \pi$

$$\int d^4x \left[\left(P'(M^4) + M^4 P''(M^4) \right) \dot{\pi}^2 - P'(M^4)(\nabla \pi)^2 \right]$$

⇨ No spatial kinetic term for π!

Other terms $\tilde{P}\left((\partial\phi)^2\right) Q(\Box\phi) + \cdots$

do contain spatial kinetic terms but at least

$$(\nabla^2 \pi)^2 + \cdots$$

⇨ Low energy

$$S = \int d^4x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha}{M^2}(\nabla^2\pi)^2 + \cdots \right]$$

$$\omega^2 \approx \frac{k^4}{M^2}$$

Positive definite Hamiltonian $\int d^3x \left[\frac{1}{2}\dot{\pi}^2 + \frac{\alpha}{M^2}(\nabla^2\pi)^2 + \cdots \right]$

⇨ STABLE!

Systematic construction of Low-energy effective theory

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

Backgrounds characterized by

✧ $\langle \partial_\mu \phi \rangle \neq 0$ and timelike

✧Background metric is maximally symmetric, either Minkowski or dS.

⇨

$$L_{eff} = L_{EH} + M^4 \left\{ \left(h_{00} - 2\dot{\pi}\right)^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2 \pi\right)^2 - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi\right)\left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi\right) + \cdots \right\}$$

Gauge choice: $\phi(t,\vec{x}) = t.$ $\pi \equiv \delta\phi = 0$

(Unitary gauge)

Residual symmetry: $\vec{x} \to \vec{x}'(t,\vec{x})$

⇨ Write down most general action invariant under this residual symmetry.

(⇨ Action for π: undo unitary gauge!)

Start with flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Under residual ξ^i

$$\delta h_{00} = 0, \delta h_{0i} = \partial_0 \xi_i, \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ^i

$$\begin{cases} (h_{00})^2 & \textbf{OK} \\ \cancel{(h_{0i})^2} & \\ K^2, K^{ij}K_{ij} & \textbf{OK} \end{cases}$$

Beginning at quadratic order, since we are assuming flat space is good background.

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4 \left\{ (h_{00})^2 - \frac{\alpha_1}{M^2} K^2 - \frac{\alpha_2}{M^2} K^{ij} K_{ij} + \cdots \right\}$$

Action for π

$$\xi^0 = \pi \quad \begin{cases} h_{00} \to h_{00} - 2\partial_0 \pi \\ K_{ij} \to K_{ij} + \partial_i \partial_j \pi \end{cases}$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4 \left\{ (h_{00} - 2\dot{\pi})^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2 \pi\right)^2 - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi\right)\left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi\right) + \cdots \right\}$$

$$\left.\begin{aligned} E &\to rE \\ dt &\to r^{-1} dt \\ dx &\to r^{-1/2} dx \\ \pi &\to r^{1/4} \pi \end{aligned}\right\}$$

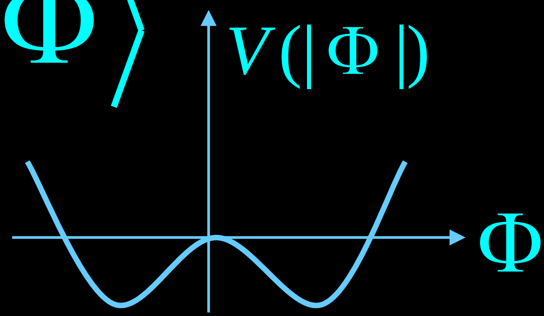
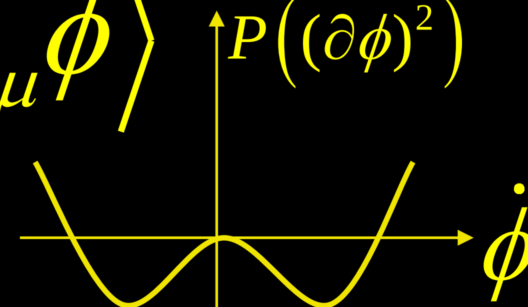
Make invariant ➡ $\int dt\, d^3x \left[\frac{1}{2} \dot{\pi}^2 - \frac{\alpha (\vec{\nabla}^2 \pi)^2}{M^2} + \cdots \right]$

Leading nonlinear operator in infrared $\int dt\, d^3x \frac{\dot{\pi} (\nabla \pi)^2}{\tilde{M}^2}$

has scaling dimension 1/4. **(Barely) irrelevant**

⇨ Good low-E effective theory

Robust prediction

	Higgs mechanism	***Ghost condensate*** Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
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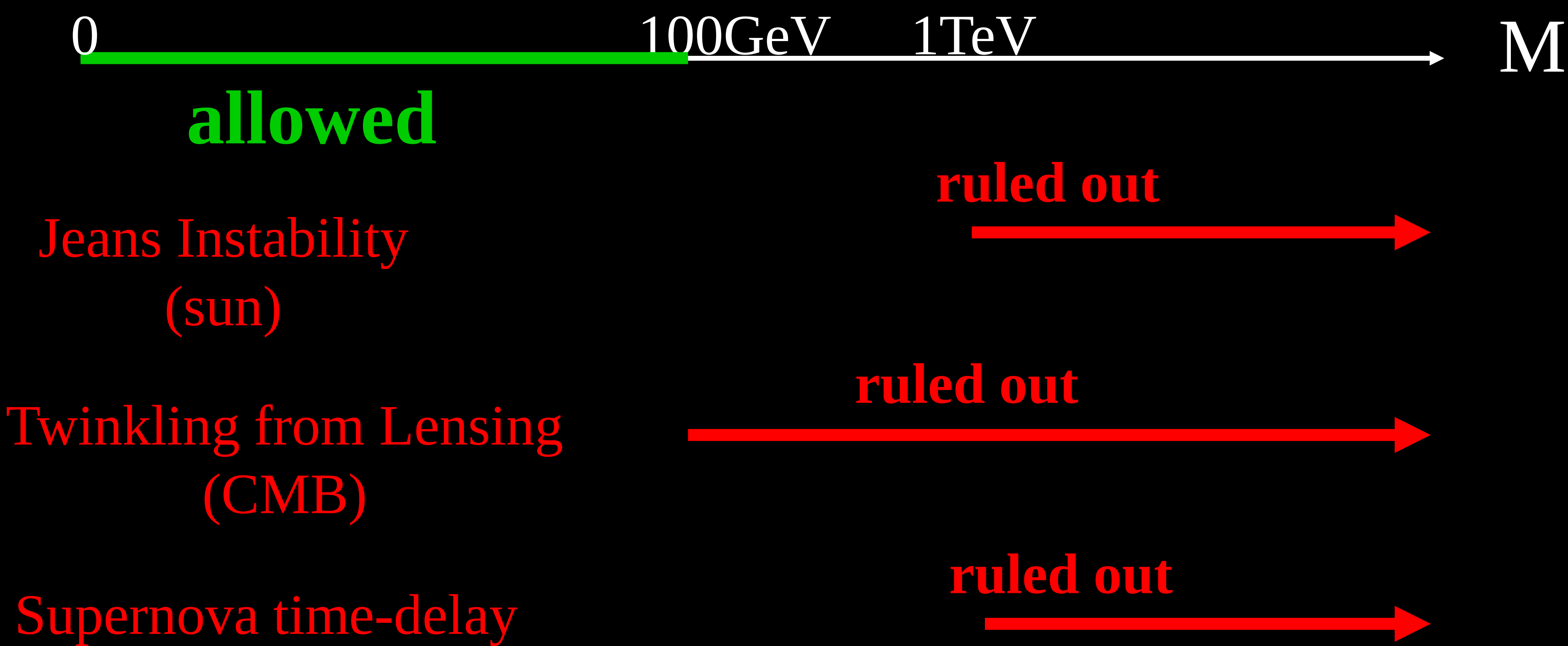
Bounds on symmetry breaking scale M

Arkani-Hamed, Cheng, Luty and Mukohyama and Wiseman, JHEP 0701:036,2007



So far, there is no conflict with experiments and observations if M < 100GeV.

Ghost condensation

- Ghost condensation is the simplest Higgs phase of gravity.
- The low-E EFT is determined by the symmetry breaking pattern. No ghost in the EFT.
- Gravity is modified in IR.
- Consistent with experiments and observations if $M < 100\text{GeV}$.

Can be an alternative to DE/DM?

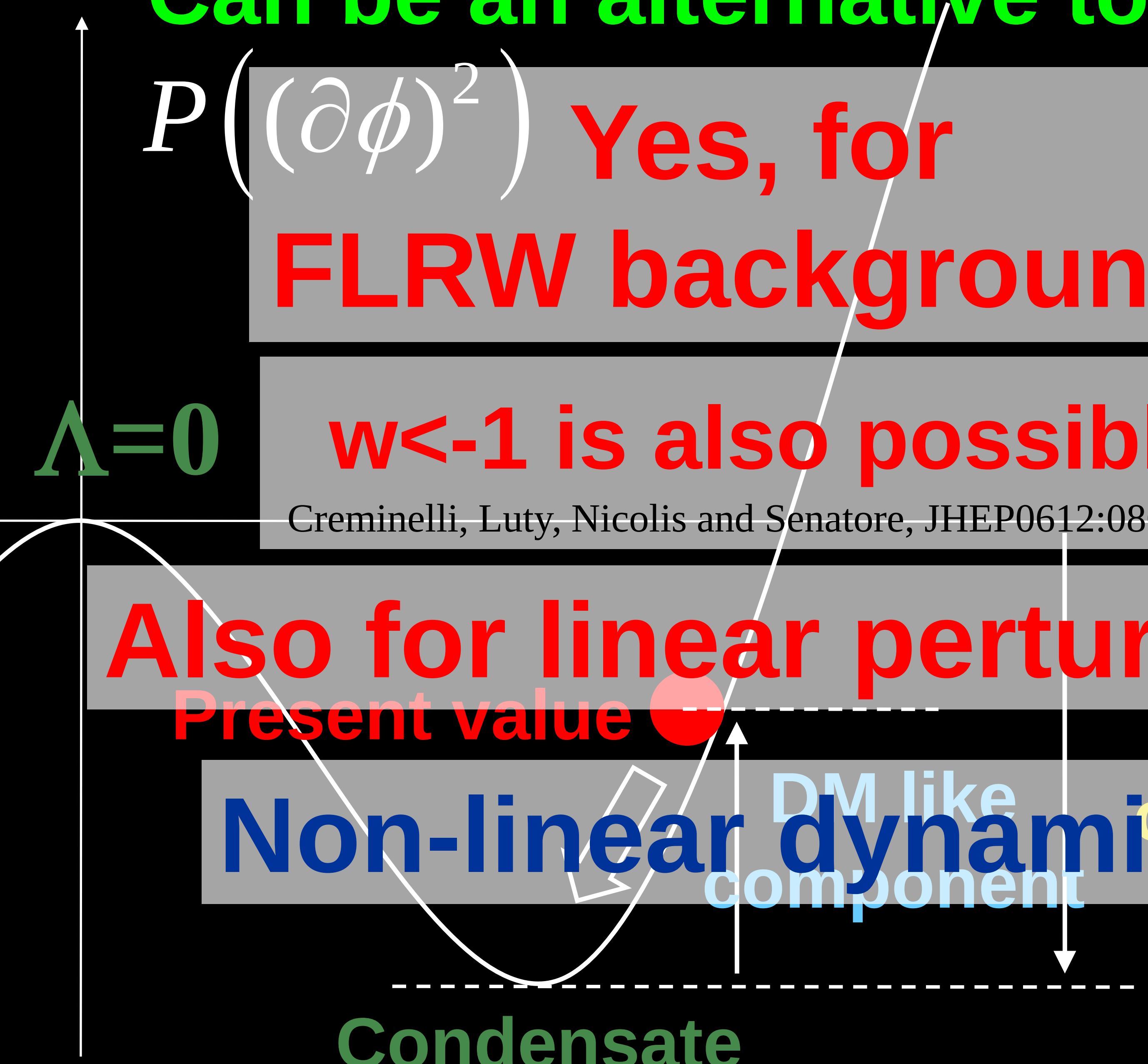
Yes, for FLRW background!

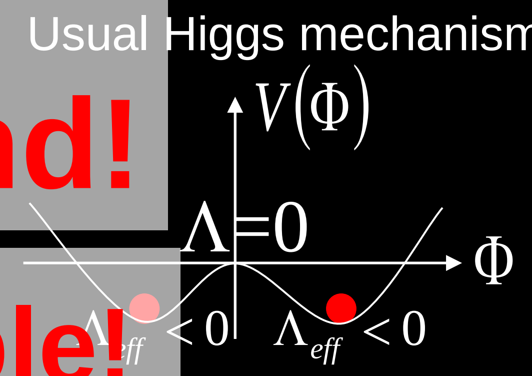
w<-1 is also possible!

Creminelli, Luty, Nicolis and Senatore, JHEP0612:080, 2006

Also for linear perturbation!

Non-linear dynamics?

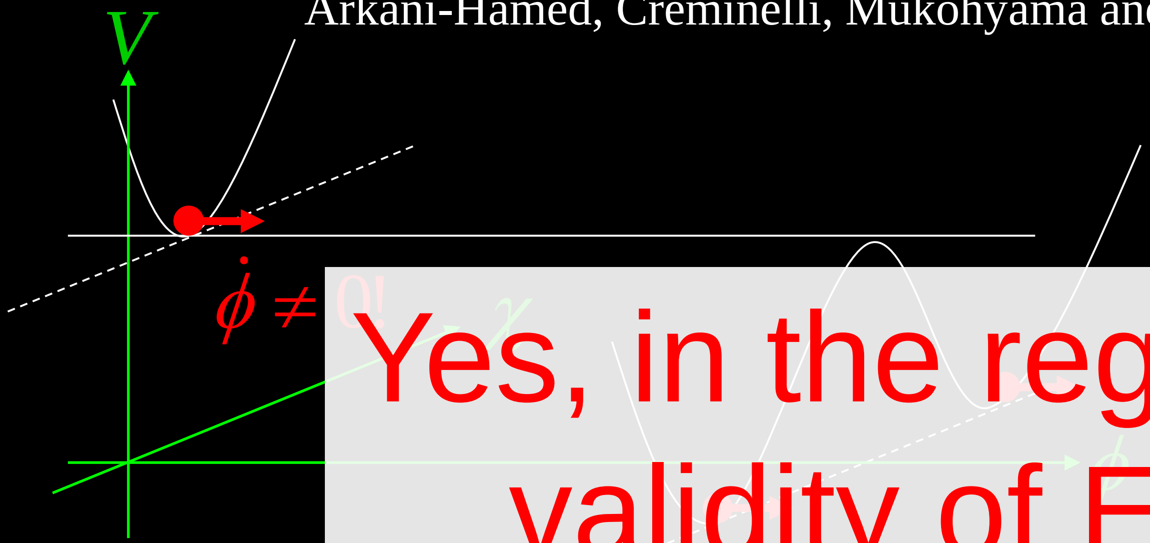




Can drive inflation?

Ghost inflation:

Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga, JHEP 0404:001,2004



Similar to hybrid inflation but <u>NOT SLOW ROLL</u>

Yes, in the regime of validity of EFT!

Scale-invariant perturbations

cf. tilted ghost inflation, Senatore (2004)

scaling dim of π

$$\frac{\delta\rho}{\rho} \sim \frac{H\delta\pi}{\dot{\phi}} \sim \left(\frac{H}{M}\right)^{5/4}$$

$$\delta\pi \sim M \cdot (H/M)^{1/4} \qquad \dot{\phi} \sim M^2$$

[compare $\dfrac{H}{M_{Pl}\sqrt{\varepsilon}}$]

Prediction of Large non-Gauss.

Leading non-linear interaction $\beta \dfrac{\dot{\pi}(\nabla\pi)^2}{M^2}$

non-G of $\sim \beta \left(\dfrac{H}{M}\right)^{1/4}$ ← scaling dim of op.

$\sim \beta \left(\dfrac{\delta\rho}{\rho}\right)^{1/5}$

$$\int dt d^3x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2\pi)^2}{M^2} + \cdots\right]$$

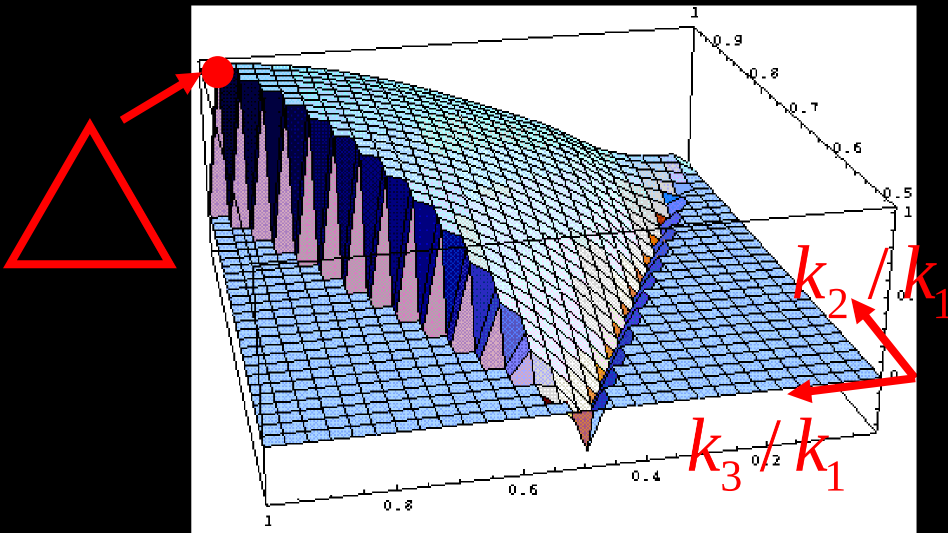
[Really "0.1" $\times(\delta\rho/\rho)^{1/5} \sim 10^{-2}$. VISIBLE.
In usual inflation, non-G ~ $(\delta\rho/\rho) \sim 10^{-5}$ too small.]

⇒ $f_{NL} \sim 82\ \beta\alpha^{-4/5}$, equilateral type

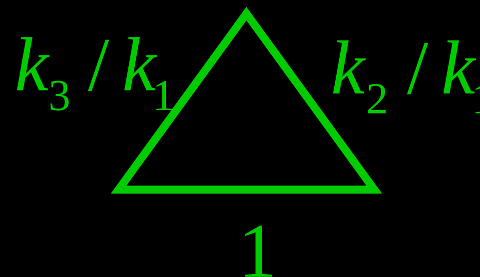
Planck 2015 constraint (equilateral type)

$f_{NL} = -4 \pm 43$ (68% CL statistical) → $-0.6 \leq \beta\alpha^{-4/5} \leq 0.5$

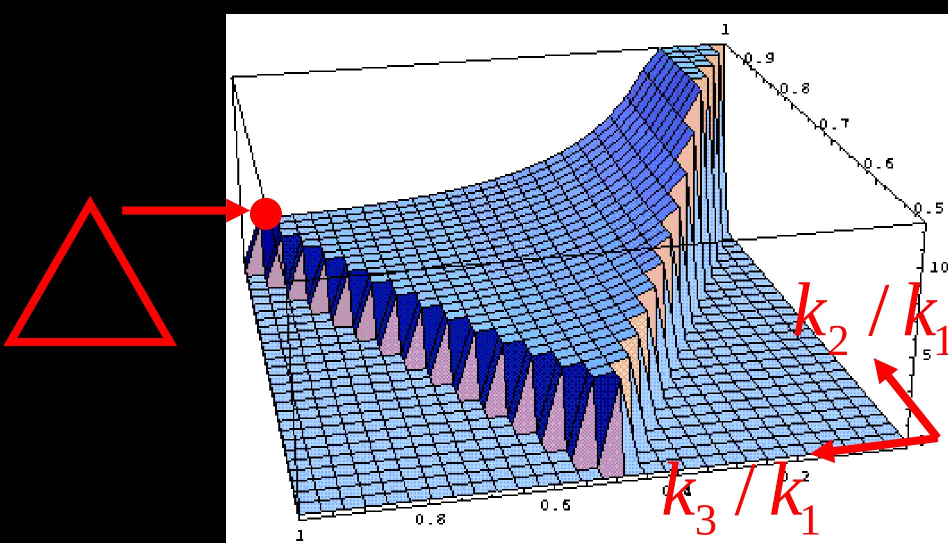
3-point function for ghost inflation



$$F(k_1, k_2, k_3) = \frac{1}{k_1^6} F\left(\frac{k_2}{k_1}, \frac{k_3}{k_1}\right)$$



3-point function for “local” non-G



$$\varsigma = \varsigma_G - \frac{3}{5} f_{NL} \cdot \left(\varsigma_G^2 - \langle \varsigma_G^2 \rangle\right)$$

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[Really "0.1" $\times (\delta\rho / \rho)^{1/5}$ ~ 10^{-2}. VISIBLE.
In usual inflation, non-G ~ $(\delta\rho / \rho)$ ~ 10^{-5} too small.]

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Summary so far

- Ghost condensation is the simplest Higgs phase of gravity.
- The low-E EFT is determined by the symmetry breaking pattern. No ghost in the EFT.
- Gravity is modified in IR.
- Consistent with experiments and observations if $M < 100\text{GeV}$.
- Behaves like DE/DM for FLRW background and large-scale linear perturbation.
- Ghost inflation predicts large non-Gaussianity and is still consistent with Planck data.

Ghost inflation and de Sitter entropy bound

S.Jazayeri, S.Mukohyama, R.Saitou, Y.Watanabe 2016

- **Black holes & cosmology** in gravity theories are **as important as Hydrogen atoms & blackbody radiation** in quantum mechanics
- Provide **non-trivial tests** for theories of gravity e.g. black-hole entropy in string theory
- **Does the theory of ghost condensation pass those tests?**
- **Ghost condensation can be consistent with BH thermodynamics (Mukohyama 2009, 2010)**
- **How about de Sitter thermodynamics?**

de Sitter thermodynamics

- de Sitter (dS) spacetime is one of the three spacetimes with maximal symmetry
- dS horizon has temperature $T_H = H/(2\pi)$
- In quantum gravity, a dS space is probably unstable (e.g. KKLT, Susskind, …). So, let's consider a dS space as a part of inflation
- Friedmann equation → 1st law with entropy $S = A/(4G_N) = \pi/(G_N H^2)$ (This is in contrast with analogue gravity systems.)

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Slow roll inflation (non-eternal)

$$\dot{H} = -4\pi G_{\rm N}\dot{\phi}^2$$

$$S = \pi/(G_{\rm N}H^2) \qquad dN = H dt$$

$$\frac{\delta\rho}{\rho} \sim \frac{\delta a}{a} \sim H\delta t \sim H\frac{\delta\phi}{\dot{\phi}} \sim \frac{H^2}{\dot{\phi}}$$

$$\frac{dS}{dN} = \frac{8\pi^2\dot{\phi}^2}{H^4} \sim \left(\frac{\delta\rho}{\rho}\right)^{-2}$$

$|\delta\rho/\rho| \lesssim 1$ for non-eternal inflation

$$N_{\rm tot} \lesssim S_{\rm end} - S_{\rm beginning} < S_{\rm end}$$

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Eternal inflation

$$\delta\rho/\rho \gtrsim 1 \Rightarrow \Delta N \gtrsim \Delta S.$$

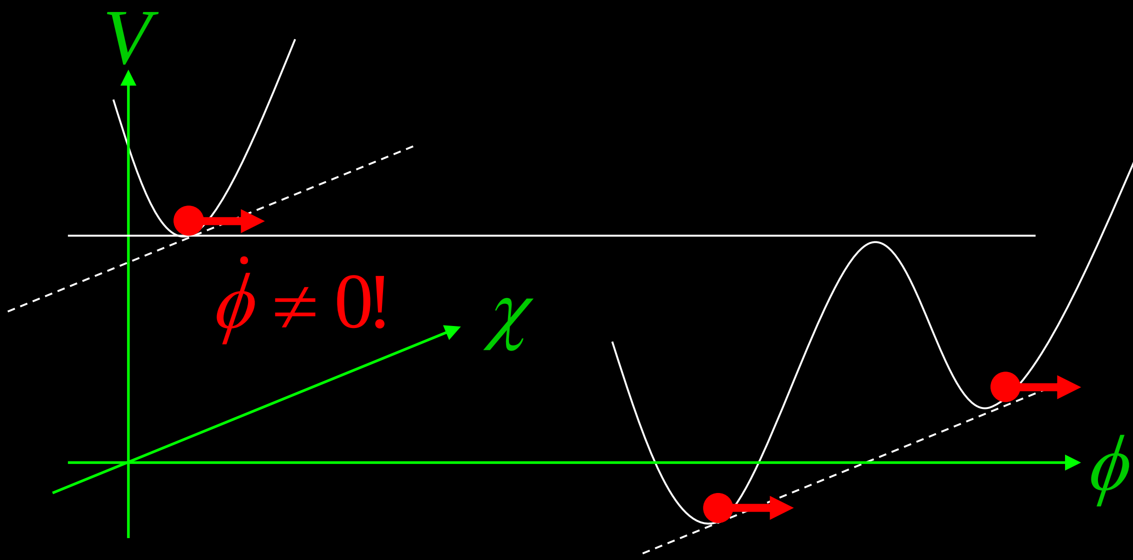
- Fluctuation generated during eternal epoch would collapse to form BH → unobservable!

$$\Rightarrow \boxed{N_{\rm obs} \lesssim S_{\rm end}}$$

- This bound holds for a large class of models of inflation
- Does ghost inflation satisfy the bound?

Ghost inflation

Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga, JHEP 0404:001,2004



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  $$\Rightarrow \quad \boxed{N_{\rm obs} \lesssim S_{\rm end}}$$

- This bound holds for a large class of models of inflation

- Does ghost inflation satisfy the bound?
  The answer appears to be “no” since N_{tot} can be arbitrarily large. Swampland?

Lower bound on Λ?

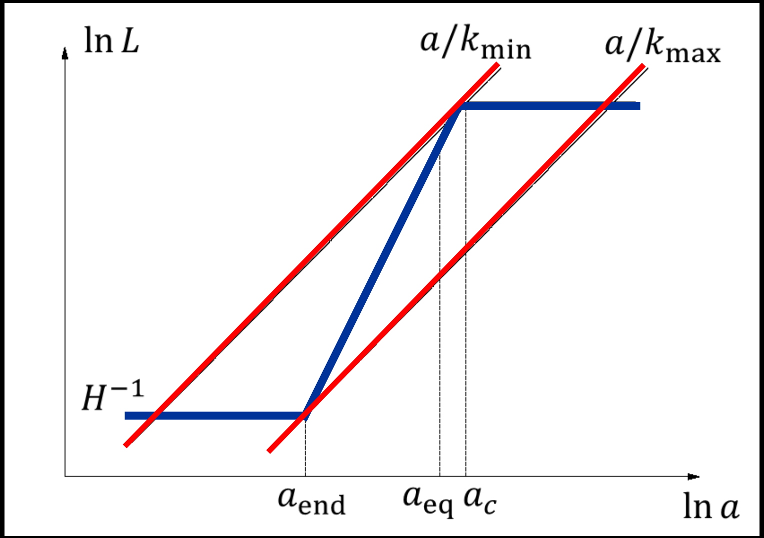
S.Jazayeri, S.Mukohyama, R.Saitou, Y.Watanabe 2016

- Tiny Λ prevents earlier inflationary modes from being observed.

$$\frac{a_{\rm end}}{a_{\rm reh}} \sim \left(\frac{\rho_{\rm reh}}{\rho_{\rm inf}}\right)^{1/3} \qquad \frac{a_{\rm reh}}{a_{\rm eq}} \sim \left(\frac{s_{\rm eq}}{s_{\rm reh}}\right)^{1/3}$$

$\ddot{a}(t=t_c)=0$ with

$$6M_{\rm Pl}^2\frac{\ddot{a}}{a} = -\rho_{\rm m}^{\rm eq}\left(\frac{a_{\rm eq}}{a}\right)^3 + 2\rho_\Lambda$$



- $N_{\rm obs} \sim \ln(k_{\rm max}/k_{\rm min}) \lesssim S = \pi/(G_N H^2)$

$$\Rightarrow \quad \Omega_\Lambda \gtrsim \exp\left[-10^{42}\left(\frac{M}{100\,{\rm GeV}}\right)^{-2}\right] \qquad M \lesssim 100\,{\rm GeV}$$

- In our universe, Ω_Λ=O(1) and thus the bound is well satisfied.

Cosmological Page time

S.Jazayeri, S.Mukohyama, R.Saitou, Y.Watanabe 2016

- Hawking rad from BH → $S_{rad} = S_{ent}$ increases but S_{BH} ($\geqq S_{ent}$) decreases → semi-classical description should break down @ Page time, when S_{BH} ~ half of $S_{BH,init}$
- After inflation, we expect to see O(1) deviation from semi-classical description @ Page time, when $N_{obs} \sim S_{end}$
- For example, if Λ decays at $a=a_{decay}$ then

$$\frac{a_{\rm Page}}{a_{\rm decay}} \sim \left(\frac{M}{100\,{\rm GeV}}\right)^{-1} \left(\frac{a_{\rm decay}}{a_{\rm eq}}\right)^{2} \exp\left[10^{42}\left(\frac{M}{100\,{\rm GeV}}\right)^{-2}\right]$$

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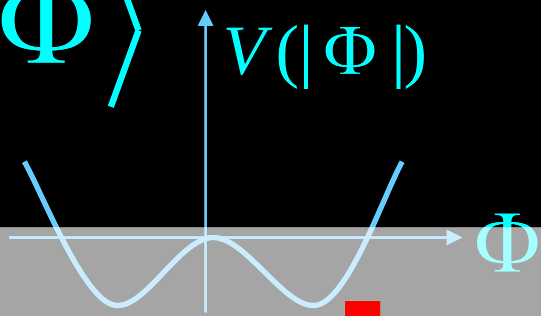
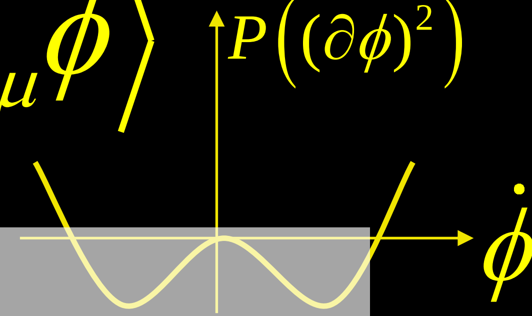
- Ghost condensation is the simplest Higgs phase of gravity.
- The low-E EFT is determined by the symmetry breaking pattern. No ghost in the EFT.
- Gravity is modified in IR.
- Consistent with experiments and observations if M < 100GeV.
- Behaves like DE/DM for FLRW background and large-scale linear perturbation.
- Ghost inflation predicts large non-Gaussianity and is still consistent with Planck data.
- It appears easy but is actually difficult to violate the generalized 2nd law by ghost condensate. (Mukohyama 2009, 2010)
- de Sitter entropy bound appears to be violated but is actually satisfied by ghost inflation in our universe with tiny DE/cc.
- If the DE/cc decays to zero in the future then O(1) deviation from EFT is expected only after the universe becomes at least $\exp(10^{42})$ times bigger than now.

Thank you very much!

	Higgs mechanism	Ghost condensate Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V'=0, V''>0	P'=0, P''>0
Broken symmetry	Gauge symmetry	Time translational symmetry
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

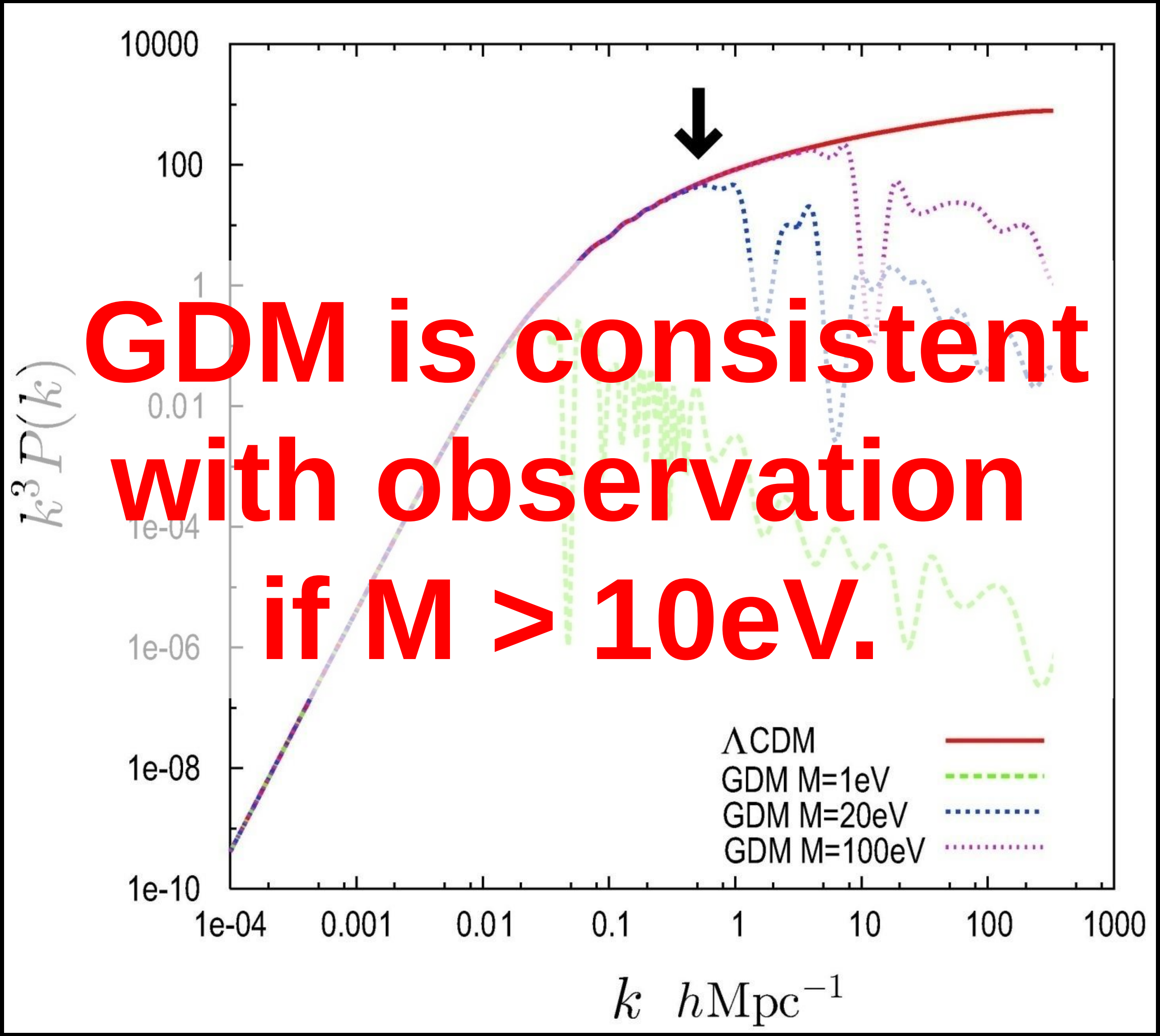
BACKUP SLIDES

GDM as all DM?

Furukawa, Yokoyama, Ichiki, Sugiyama and Mukohyama 2010

- Standard CDM has P=0 and c_s^2=0.
- $P_{GDM} \sim \rho_{GDM}^2/M^4$ and $c_s^2 \sim 2\rho_{GDM}/M^4$ are small but non-zero.
- Let's suppose that ghost dark matter is responsible for all DM in the universe.
- $P_{GDM}/\rho_{GDM} \sim \rho_{GDM}/M^4 \ll 1$ all the way to matter-radiation equlity → M > 1eV
- $L_J \sim c_s/(aH) < 1$Mpc all the way to matter-radiation equality → M > 10eV

Matter power spectrum



GDM is consistent with observation if M > 10eV.

Extension to FLRW background

Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

- Action invariant under $x^i \to x^i(t,x)$
- Ingredients

  $g_{\mu\nu}$ $g^{\mu\nu}$ $R_{\mu\nu\rho\sigma}$ ∇_μ

  t & its derivatives

- 1^{st} derivative of t

$$\partial_\mu t = \delta^0_\mu \qquad n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu}\partial_\mu t \partial_\nu t}} = \frac{\delta^0_\mu}{\sqrt{-g^{00}}}$$

$$g^{00} \qquad h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

- 2^{nd} derivative of t

$$K_{\mu\nu} \equiv h^\rho_\mu \nabla_\rho n_\nu$$

Unitary gauge action

$$I = \int d^4x\sqrt{-g}L(t, \delta^0_\mu, K_{\mu\nu}, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, R_{\mu\nu\rho\sigma})$$

derivative & perturbative expansions

$$\begin{aligned} I &= M_{Pl}^2 \int dx^4\sqrt{-g}\left[\frac{1}{2}R + c_1(t) + c_2(t)g^{00}\right. \\ &\left. + L^{(2)}(\tilde{\delta}g^{00}, \tilde{\delta}K_{\mu\nu}, \tilde{\delta}R_{\mu\nu\rho\sigma}; t, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu)\right] \end{aligned}$$

$$\begin{aligned} L^{(2)} &= \lambda_1(t)(\tilde{\delta}g^{00})^2 + \lambda_2(t)(\tilde{\delta}g^{00})^3 + \lambda_3(t)\tilde{\delta}g^{00}\tilde{\delta}K^\mu_\mu \\ &+ \lambda_4(t)(\tilde{\delta}K^\mu_\mu)^2 + \lambda_5(t)\tilde{\delta}K^\mu_\nu\tilde{\delta}K^\nu_\mu + \cdots \end{aligned}$$

NG boson

- Undo unitary gauge $t \to \tilde{t} = t - \pi(\tilde{t}, \vec{x})$

$$
\begin{aligned}
H(t) &\to H(t+\pi), \quad \dot{H}(t) \to \dot{H}(t+\pi), \\
\lambda_i(t) &\to \lambda_i(t+\pi), \quad a(t) \to a(t+\pi), \\
\delta^0_\mu &\to (1+\dot{\pi})\delta^0_\mu + \delta^i_\mu \partial_i \pi,
\end{aligned}
$$

- NG boson in decoupling (subhorizon) limit

$$
\begin{aligned}
I_\pi = M_{Pl}^2 \int dt d^3\vec{x}\, a^3 \Bigg\{ & -\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) \\
& -\dot{H}\left(\frac{1}{c_s^2} - 1\right)\left(\frac{c_3}{c_s^2}\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i \pi)^2}{a^2}\right) + O(\pi^4, \tilde{\epsilon}^2) + L^{(2)}_{\tilde{\delta}K, \tilde{\delta}R} \Bigg\}
\end{aligned}
$$

$$
\frac{1}{c_s^2} = 1 - \frac{4\lambda_1}{\dot{H}}, \quad c_3 = c_s^2 - \frac{8 c_s^2 \lambda_2}{-\dot{H}}\left(\frac{1}{c_s^2} - 1\right)^{-1}
$$