# Inflation through hidden Yukawa couplings in the extra dimension

Tetsutaro Higaki (Keio University)



Based on work (in progress ) with Y. Tatsuta (Waseda).

#### What we want to focus on

# An inflation model $\leftrightarrow$ Symmetries

# Planck unit in this talk

$$M_{\rm Pl} \simeq 2.4 \times 10^{18} {\rm GeV} \equiv 1.$$

#### Contents

- 1. Introduction: Review of inflation
- 2. Flat inflaton potential & symmetry
- 3. A UV completion in the string theory
- 4. Conclusion & discussion

# 1. Introduction: Review of inflation

# Cosmic Microwave Background (CMB)



where T  $\sim$  2.7 K.

[Planck collaborations]



# Inflation: origin of CMB

#### Accelerating expansion of the universe



#### Planck satellite



# Why inflation?

- Generating <u>density fluctuations</u> : ΔT/T ~ 10<sup>-5</sup>
   = seeds of galaxies (= those of us)
- Solutions for fine-tuning problems by the expansion
  - Flatness problem:  $\Omega_{curvature} << 1$
  - Horizon problem : T  $\sim$  2.7K in CMB all over the sky



#### (Flat) Freedman-Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

a(t): Scale factor $H = \frac{\dot{a}}{a}$ : Hubble parameter

# Inflation driven by an inflaton $\boldsymbol{\varphi}$

• EOM

 $\ddot{\phi} + 3H\dot{\phi} + V' = 0$ 

• Friedman Eq.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right)$$

• Slow-roll conditions



Inflation!!

$$V(\phi)$$
Slow roll
$$V_{inf}$$

$$\phi = \phi_{inf}$$

$$\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad \eta = \frac{V''}{V} \ll 1$$

 $a \propto e^{Ht}$ 

# Inflation driven by an inflaton $\varphi$

• EOM  $\dot{\phi} \simeq -\frac{V'}{\mathbf{3}H}$ <sub>^</sub> V(φ) Slow roll • Friedman Eq.  $H^2 = \left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{1}{3}V$  $V_{\mathsf{inf}}$  $\phi = \phi_{inf}$  Slow-roll conditions  $\frac{\dot{H}}{H^2} \ll 1, \quad \ddot{\phi} \ll H \dot{\phi}$  $\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad \eta = \frac{V''}{V} \ll 1$  $a \propto e^{Ht}$ Inflation!!

Φ

#### Metric perturbation

• Perturbations to FRW metric:

$$ds^{2} = -dt^{2} + a(t)^{2} e^{2\zeta(t,\vec{x})} [\delta_{ij} + h_{ij}(t,\vec{x})] dx^{i} dx^{j}$$

#### $\zeta$ : Scalar perturbation from inflaton

#### h: Tensor perturbation

(gravitational wave itself from inflation energy  $V_{inf}$ )  $\rightarrow$  B-mode polarization in CMB photon

# $\zeta$ = Inflaton's quantum fluctuation $\delta \varphi$

$$\left( \zeta \sim \frac{\delta \rho_{\phi}}{\rho_{\phi}} \sim H \delta t \sim \frac{H}{\dot{\phi}} \delta \phi \sim \frac{V^{3/2}}{V'} \right)$$



# CMB fluctuation generated by inflaton



#### Theory and observations

$$P_{\zeta} \simeq \frac{2V}{3\pi^{2}r} \left(\frac{k}{k_{0}}\right)^{n_{s}-1}$$

$$n_{s} \simeq 1 - 3\left(\frac{V'}{V}\right)^{2} + 2\left(\frac{V''}{V}\right)$$

$$r \simeq 8\left(\frac{V'}{V}\right)^{2}$$

$$P_{\zeta}^{\text{obs}}(k = k_{0}) \simeq 2.5 \times 10^{-9}$$

$$k_{0} = 0.002 \text{Mpc}^{-1}$$

**P**<sub>ζ</sub>: Power spectrum = (ΔT/T)<sup>2</sup> from scalar,  $P_{\zeta} \simeq (\Delta T/T)^2_{\text{scalar}}$ 

 $n_s$ : Spectral index = scale dependence of  $P_{\zeta}$ 

r: Tensor to scalar ratio = ( $\Delta T/T$ )<sup>2</sup> ratio of gravitational wave to scalar,  $r \simeq \frac{(\Delta T/T)_{\text{tensor}}^2}{(\Delta T/T)_{\text{scalar}}^2}$ 

# (n<sub>s</sub>, r)-contour: focus on small r

[Planck 2015 collaborations]

0.25Planck 2013 Planck TT+lowP Planck TT, TE, EE+lowP 0.20Conver Natural inflation Hilltop quartic model Tensor-to-scalar ratio ( $r_{0.002}$ ) 0.10 0.15  $\alpha$  attractors Concave  $n_s = 0.968 \pm 0.006$ Power-law inflation Low scale SB SUSY  $R^2$  inflation  $V \propto \phi^3$ r < 0.11 (95% CL) $V \propto \phi^2$  $V \propto \phi^{4/3}$  $V \propto \phi$ 0.05 $V \propto \phi^{2/3}$ 1  $N_{*} = 50$ N<sub>\*</sub>=60 0.000.94 0.960.981.00Primordial tilt  $(n_s)$ 

## (n<sub>s</sub>, r)-contour: focus on small r

[Planck 2015 collaborations]



# 2. Flat inflaton potential & symmetry

# Inflaton potential: very small slope and curvature



# Control of potential flatness by shift symmetry

• Shift symmetry  $\phi \rightarrow \phi + \text{const.}$  for a flat inflaton potential:

$$\left[ V(\phi) = 0 \right]$$

if the symmetry is exact.

# Control of potential flatness by shift symmetry

• Shift symmetry  $\phi \rightarrow \phi + const.$  for a flat inflaton potential:

 $V(\phi) = 0$  if the symmetry is exact.

 $\succ$  Chaotic (monodromy) inflation: softly-broken only by a single scale  $\mu$ 

[Linde]; [Silverstein, Westphal]; [McAllister, Silverstein, Westphal]

$$V(\phi) = \mu^{4-n} \phi^n$$

## Control of potential flatness by shift symmetry

• Shift symmetry  $\phi \rightarrow \phi + const.$  for a flat inflaton potential:

 $V(\phi) = 0$  if the symmetry is exact.

 $\succ$  Chaotic (monodromy) inflation: softly-broken only by a single scale  $\mu$ 

[Linde]; [Silverstein, Westphal]; [McAllister, Silverstein, Westphal]

$$V(\phi) = \mu^{4-n} \phi^n.$$

 $\blacktriangleright$  Natural inflation: broken but a discrete shift symmetry below  $\Lambda$  [Freese, Frieman, Olinto]

$$\phi \to \phi + 2\pi f; \quad V(\phi) = \Lambda^4 \cos\left(\frac{\phi}{f}\right).$$

#### Natural inflation & discrete shift symmetry

Natural inflation: well-controlled by a discrete shift symmetry

$$\phi \to \phi + 2\pi f; \quad V = \Lambda^4 \cos\left(\frac{\phi}{f}\right).$$

#### Natural inflation & discrete shift symmetry

Natural inflation: well-controlled by a discrete shift symmetry

$$\phi \to \phi + 2\pi f; \quad V = \Lambda^4 \cos\left(\frac{\phi}{f}\right).$$



#### Natural inflation and f-dependence

• f $\rightarrow$ small:  $\phi_{inf}$  is near hilltop for a long slow-roll:

$$\phi_{\inf} \sim \pi f \to \epsilon \sim 0 \ll |\eta|.$$

•  $f \rightarrow$  large: Chaotic inflation

$$V = \frac{m^2}{2}\phi^2$$
,  $r = 0.16\left(\frac{50}{N}\right)$ ,  $\epsilon = \eta = \frac{1}{2N}$ 

$$N = \log(a_f/a_{\inf}) = \int_{t_{\inf}}^{t_f} H dt \simeq \int_{\phi_f}^{\phi_{\inf}} \frac{V}{V'} d\phi \quad :\text{e-folding}$$

#### Natural inflation & discrete shift symmetry

Natural inflation: well-controlled by a discrete shift symmetry

$$\phi \to \phi + 2\pi f; \quad V = \Lambda^4 \cos\left(\frac{\phi}{f}\right).$$



# Natural inflation for a small r?



# Multi-natural inflation: a bottom-up approach

[Czerny, Takahashi]; [Czerny, TH, Takahashi]; [TH, Takahashi]; [Kobayashi, Takahashi]; [Czerny, Kobayashi, Takahashi]

• Modification for it: Adding cosine function(s) to natural inflation

$$V(\phi) = V_0 - \Lambda^4 \left[ \cos\left(\frac{\phi}{f_1}\right) + B\cos\left(\frac{\phi}{f_2} + \theta\right) \right].$$

# Multi-natural inflation: a bottom-up approach

[Czerny, Takahashi]; [Czerny, TH, Takahashi]; [TH, Takahashi]; [Kobayashi, Takahashi]; [Czerny, Kobayashi, Takahashi]

Modification for it: Adding cosine function(s) to natural inflation

$$V(\phi) = V_0 - \Lambda^4 \left[ \cos\left(\frac{\phi}{f_1}\right) + B\cos\left(\frac{\phi}{f_2} + \theta\right) \right].$$



- r < 0.11
- $f_1, f_2 < M_{Pl}$  via a tuning:  $B \sim (f_2/f_1)^2, \ \theta \sim -\pi (f_1/f_2)$ 
  - $\rightarrow$  Good against weak gravity conjecture.

Cf. Possible to have a modulation for  $f_1 >> f_2 \& B << 1$  (Running  $n_s$ ).

# A UV completion and compactification

• Q. How is this model controlled? :

> What are discrete symmetries for control?

> What are their origins?

• A. Discrete symmetry from compactification of extra dimension



# 3. A UV completion in the string theory

# String theory as the origin of forces & matter



# String theory compactification on torus

• Let 10D = 4D spacetime + 2d torus (T<sup>2</sup>) + X<sub>4</sub> (something)



# String theory compactification on torus

• Let 10D = 4D spacetime + 2d torus (T<sup>2</sup>) + X<sub>4</sub> (something)



Consider intersecting D6-branes in IIA model; just one direction of D6 on T<sup>2</sup>

(Similarly, possible to consider magnetized D-branes in IIB model)

# String theory compactification on torus

• Let 10D = 4D spacetime + 2d torus (T<sup>2</sup>) + X<sub>4</sub> (something)



- Consider intersecting D6-branes in IIA model; just one direction of D6 on T<sup>2</sup> (Similarly, possible to consider magnetized D-branes in IIB model)
- Inflation energy: SUSY-breaking by Izawa-Yanagida-Intriligator-Thomas (IYIT)

$$W = y_{ijk} X^{i} Y^{j} \Phi^{k} \quad \rightarrow \quad y_{ijk} \mathcal{M}^{ij} \Phi^{k} + Z \Big[ \mathsf{Pf}(\mathcal{M}) - \Lambda^{4}_{SU(2)} \Big]$$

# An example of D6-brane configuration

Spacetime	0	1	2	3	4	5	6	7	8	9			
D6 <sub>a</sub> D6 <sub>b</sub>	0 0	0 0	0 0	0 0	O ×	× O	O ×	× O	0 0	× ×		T <sup>2</sup>	





4

#### An example of D6-branes on $T^2$





[Cremades-Ibanez-Marhesano]

- Branes (gauge theories) = lines
- Matter = Intersection points
- Yukawa coupling = Sum of triangles (Winding modes)

#(b•c) = 5 (k =0,1,2,3,4)

#### Example: Parts of D6-branes on T<sup>2</sup>





[Cremades-Ibanez-Marhesano]

- Branes (gauge theories) = lines
- Matter = Intersection points
- Yukawa coupling = Sum of triangles (Winding modes)

#(b•c) = 5 (k =0,1,2,3,4)

• Discrete symmetries from "Torus property × D-brane configuration":



$$W = y_{ijk} X^i Y^j \Phi^k$$

 $SL(2,\mathbb{Z}) \times (\text{periodicity}) \times (\mathbb{Z}_2)^2$ 

relevant to control  $y_{ijk}$  . (preliminary result)

• Torus property × D-brane configuration ~ SL(2,Z) × periodicity ×  $(Z_2)^2$ 



$$W = y_{ijk} X^i Y^j \Phi^k$$

$$y_{ijk} \sim \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) = \sum_{l=-\infty}^{\infty} e^{\pi i (a+l)^2 \tau} e^{2\pi i (a+l)(\nu+b)}$$
  
 $\tau = (B+iA)/\alpha'$ 

**B:** NS B-field axion, A: torus area v + b: brane position moduli

a: brane intersection #-dependent number

• Torus property × D-brane configuration ~ SL(2,Z) × periodicity ×  $(Z_2)^2$ 



$$W = y_{ijk} X^i Y^j \Phi^k$$

$$y_{ijk} \sim \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) = \sum_{l=-\infty}^{\infty} e^{\pi i (a+l)^2 \tau} e^{2\pi i (a+l)(\nu+b)}$$
  
 $\tau = (B+iA)/\alpha'$ 

Periodicity : 
$$u 
ightarrow 
u + n + m au, \quad m,n \in \mathbb{Z}.$$

$$\cdot SL(2,\mathbb{Z}): \ au o rac{a au+b}{c au+d}, \quad ad-bc=1, \quad a,b,c,d\in\mathbb{Z}.$$

(T-dual of complex structure on T<sup>2</sup>)

• Torus property × D-brane configuration ~ SL(2,Z) × periodicity ×  $(Z_2)^2$ 



• Torus property  $\times$  D-brane configuration  $\sim$  SL(2,Z)  $\times$  periodicity  $\times$  (Z<sub>2</sub>)<sup>2</sup>



$$W = y_{ijk} X^i Y^j \Phi^k$$

 $(\mathbb{Z}_2 imes \mathbb{Z}_2)$  invariant Yukawa coupling  $y_{ijk}$  :

1. 
$$(X^i, Y^j, \Phi^k) \rightarrow (-X^i, -Y^j, \Phi^k)$$
  
2.  $\rightarrow (X^i, -Y^j, -\Phi^k)$ 

#### Low energy W & explicit form of Yukawas

• Low energy W in IYIT model:

$$W = y_{ijk} \mathcal{M}^{ij} \Phi^k$$

with taking  $Pf(\mathcal{M}) = -\mathcal{M}_{13}\mathcal{M}_{24} = \Lambda^4$ .

 $(|\mathcal{M}_{13}| = |\mathcal{M}_{24}|)$ 





 $A/\alpha' \sim 1.81$ ;  $f \sim 0.39 M_{\mathsf{Pl}}$  for Planck results

# 4. Review of other attempts for natural inflation

#### Natural inflation & discrete shift symmetry

Natural inflation: well-controlled by a discrete shift symmetry

$$\phi \to \phi + 2\pi f; \quad V = \Lambda^4 \cos\left(\frac{\phi}{f}\right).$$



## Aligned natural inflation for $f > M_{Pl}$

• Two axions:  $\phi_1 \rightarrow \phi_1 + 2\pi f_1$ ,  $\phi_2 \rightarrow \phi_2 + 2\pi f_2$ 

[Kim, Nilles, Peloso]

$$V = \Lambda_1^4 \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right) + \Lambda_2^4 \cos\left(m_1 \frac{\phi_1}{f_1} + m_2 \frac{\phi_2}{f_2}\right) \qquad n_i, \ m_i \in \mathbb{Z}$$

For 
$$\Lambda_1 >> \Lambda_2$$
,  $\phi \equiv \frac{f_1 f_2}{\sqrt{(n_1 f_1)^2 + (n_2 f_2)^2}} \left( -n_2 \frac{\phi_1}{f_2} + n_1 \frac{\phi_2}{f_1} \right)$  becomes inflaton:

$$V_{\text{eff}} = \Lambda_2^4 \cos\left(\frac{\phi}{f_{\text{eff}}}\right) \qquad \qquad f_{\text{eff}} = \frac{\sqrt{(n_1 f_1)^2 + (n_2 f_2)^2}}{|n_1 m_2 - n_2 m_1|}$$

#### The weak gravity conjecture

[Arkani-hamed, Motl, Nicolis, Vafa]

The conjecture: "The gravity is the weakest force."

$$\left( \begin{array}{c} q \gtrsim \frac{m}{M_{\text{Pl}}} \\ M_{\text{Pl}} \end{array} \right) \quad (F_{U(1)} = \frac{q^2}{r^2} \gtrsim F_g = G_N \frac{m^2}{r^2} \right)$$

- But, one might have an axion interaction of  $M_{Pl} < f: \mathcal{L} = \frac{\phi}{f} \mathcal{O} < \frac{\phi}{M_{Pl}} \mathcal{O}$ .
- What if we have a weaker force than gravity?

 $M_P \equiv 1$ 

#### The Weak Gravity Conjecture

[Slide from G. Shiu]

Take a U(1) and a single family with q < m (WGC)</li>



All these (BH) states are stable. Trouble w/ remnants Susskind '95;

(  $G_N \rightarrow 0$  via Bekenstein-Hawking formula)

Need a light state into which they can decay

$$\frac{q}{m} \ge "1" \equiv \frac{Q_{Ext}}{M_{Ext}}$$

#### The weak gravity conjecture for axion

[Arkani-hamed, Motl, Nicolis, Vafa]

The conjecture for axion potential via one instanton effect:

$$\left(\frac{1}{f} \gtrsim \frac{S}{M_{\rm Pl}}\right) \qquad V_{\rm 1-inst} = e^{-S} M_{\rm Pl}^4 \cos\left(\frac{\phi}{f}\right)$$

• Axion interaction for 
$$M_{PI} < f: \mathcal{L} = \frac{\phi}{f} \mathcal{O} < \frac{\phi}{M_{PI}} \mathcal{O}.$$

# A possible loophole

[Slide from G. Shiu]

The WGC requires f·m<1 for ONE instanton, but not ALL</li>

$$V = e^{-m} \left[ 1 - \cos\left(\frac{\Phi}{F}\right) \right] + e^{-M} \left[ 1 - \cos\left(\frac{\Phi}{f}\right) \right]$$

With  $1 < m \ll M$ ,  $F \gg M_P > f$ ,  $M \times f \ll 1$ 

#### **Multi-natural inflation!**

 The second instanton fulfills the WGC, but is negligible, an "spectator". Inflation is governed by the first term.

# Potential with modulations



## Recent studies of potentials with modulation

• Potential via WGC = Multi-natural inflation!:





# 4. Conclusion & discussion

#### Conclusion

- Planck result can suggest a small r (< 0.11).
- Discrete symmetries control an inflation model; **3** multi-instantons.
- UV completion: compactification periodicity and brane configuration:  $SL(2,\mathbb{Z}) \times (\text{periodicity}) \times (\mathbb{Z}_2)^2 + \text{constraint on torus area} (A/\alpha' \sim 2).$ (preliminary result)
- General message:

Compactification may help slow roll inflation due to discrete symmetries.

#### Discussion

• Moduli stabilization during inflation required:

 $H_{inf} \sim gravitino mass < Heavy moduli.$ 

Flux + racetrack (KL) model would help this issue.

Otherwise, slow roll inflation may be broken by a large inflaton mass via:

- Heavy moduli-inflaton mixing
- > Quantum corrections from SUSY-breaking.

# Appendix

#### The Weak Gravity Conjecture

[Slide from G. Shiu]

Arkani-Hamed et al. '06

For bound states to decay, there must a particle w/

$$\frac{q}{m} \ge "1" \equiv \frac{Q_{Ext}}{M_{Ext}}$$

Strong-WGC: satisfied by *lightest* charged particle

Weak-WGC: satisfied by <u>any</u> charged particle

