Three-generation model and flavor symmetry in string theory

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1406.4660 [hep-th], 1502.00789 [hep-ph]

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Plan of Talk

1. Introduction
2. Heterotic string compactification
3. Three-generation models
4. Flavor symmetry at symmetry enhanced point
5. U(1) flavor model
6. Conclusion

Introduction

- String → Standard Model
  - String theory
    -- A candidate which describe quantum gravity and unify four forces
    -- Is it possible to realize phenomenological properties of Standard Model ?
  - (Supersymmetric) Standard Model
    -- We have to realize all properties of Standard model

      Four-dimensions,
      N=1 supersymmetry,
      Standard model group(SU(3)*SU(2)*U(1)),
      Three generations,
      Quarks, Leptons and Higgs,
      No exotics,
      Yukawa hierarchy,
      Proton longevity,
      R-parity,
      Doublet-triplet splitting,
      Moduli stabilization,
      ...

If we believe string theory as the fundamental theory of our nature, we have to realize standard model as the effective theory of string theory !

Introduction

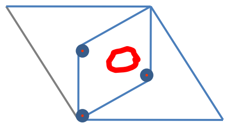
● String → Standard Model ----- String compactification : 10-dim → 4-dim

Orbifold compactification, Calabi-Yau, Intersecting D-brane, Magnetized D-brane, F-theory, M-theory, …

● (Symmetric) orbifold compactification

Dixon, Harvey, Vafa, Witten '85,'86
Ibanez, Kim, Nilles, Quevedo '87

- SM or several GUT gauge symmetries
- N=1 supersymmetry
- Chiral matter spectrum



● MSSM searches in symmetric orbifold vacua :
Embedding higher dimensional GUT into string

Three generations,
Quarks, Leptons and Higgs,
No exotics,
Top Yukawa,
Proton longevity,
R-parity,
Doublet-triplet splitting,
...

Kobayashi, Raby, Zhang '04
Buchmuller, Hamaguchi, Lebedev, Ratz '06
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter '07
Kim, Kyae '07
......

Introduction

- Asymmetric orbifold compactification of heterotic string theory Narain, Sarmadi, Vafa '87

  Generalization of orbifold action (Non-geometric compactification)

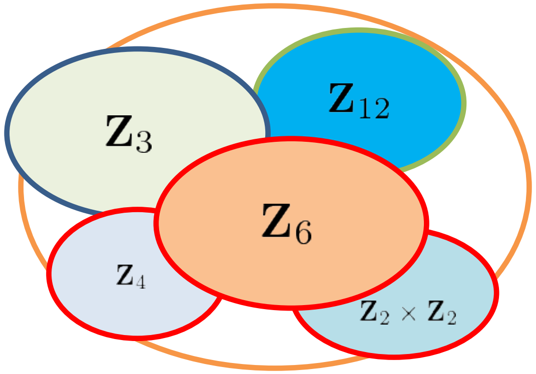
  - SM or several GUT gauge symmetries
  - N=1 supersymmetry
  - Chiral matter spectrum
  - Increase the number of possible models (symmetric → asymmetric)

    ⇨ All Yukawa hierarchies ?

  - A few/no moduli fields (non-geometric)

    ⇨ Moduli stabilization ?

  However, in asymmetric orbifold construction, a systematic search for SUSY SM or other GUT extended models has not been investigated so far.



Goal : Search for SUSY SM in heterotic asymmetric orbifold vacua

Introduction

- SUSY SM in asymmetric orbifold vacua
  - First step for model building: Gauge symmetry + Three generations

Four-dimensions,
N=1 supersymmetry,
Standard model group(SU(3)xSU(2)xU(1)),
Three generations,
Quarks, Leptons and Higgs,
No exotics,
Yukawa hierarchy,
Proton stability,
R-parity,
Doublet-triplet splitting,
Moduli stabilization,
...

What types of gauge symmetries can be derived in these vacua ?

- SM group ?
- GUT group ?
- Flavor symmetry ?
- Hidden sector ?

Introduction

- Flavor structure of quarks and leptons in standard model
  - Hierarchical masses and mixings
  - The key is flavor symmetry
  - Flavor model based on non-Abelian discrete flavor symmetry

$$S_3, S_4, D_4, A_4, \Delta(27), \Delta(54), \cdots$$

  - Some discrete flavor symmetries have string origin
  - We consider orbifold string models at symmetry enhanced point in moduli space

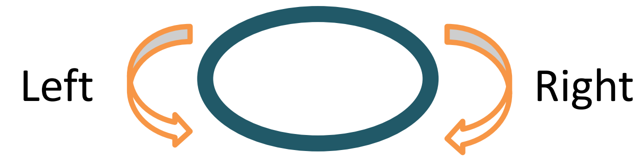
$\longrightarrow$ Gauge origin of non-Abelian discrete symmetry

  - Applications to phenomenological models

Heterotic string compactification

Heterotic String Theory

- Heterotic string theory
  - Heterotic string for our starting point
  - Degrees of freedom
    - --- Left mover 26 dim. Bosons X_{L}
    - --- Right mover 10 dim. Bosons and fermions $X_{\mathrm{R}}\ \Psi_{\mathrm{R}}$
  - Extra 16 dim. have to be compactified
  - Consistency (Modular invariance) → If 10D N=1, E8 X E8 or SO(32)



Ex.) E8 Root Lattice Γ_{E8}

$$\Gamma_{E8} \equiv \sum_{i=1}^{8} n_i \alpha_i \quad (n_i \in \mathbf{Z}^8)$$

$\alpha_i \quad (i = 1 \sim 8)$: Simple roots of E8

Left-moving momentum $p_{\mathrm{L}} \in \Gamma_{E8}$

	10dim	8dim	8dim
Left		E_8	E_8
Right			

Heterotic Orbifold Compactification

- $\mathbf{Z}_3$ heterotic orbifold compactification
  - Heterotic string theory
  - External 6 dim. → Assuming as Orbifold
  - Strings on orbifold

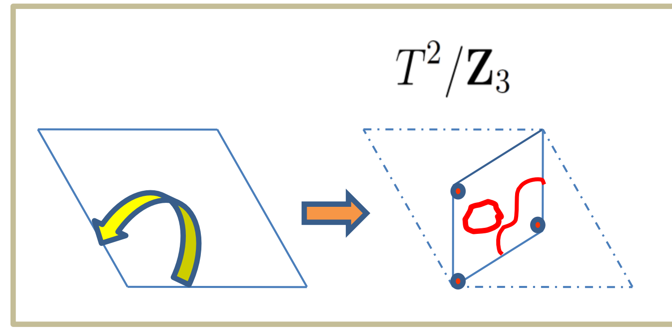
    --- Untwisted sector

    --- Twisted sector (Fixed points)
  - Consistency condition (Modular invariance)

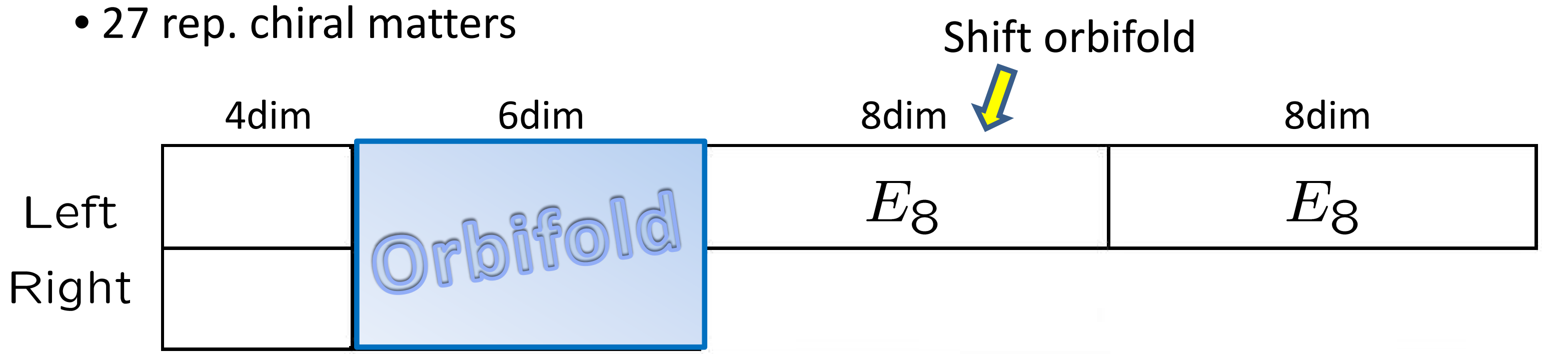
    ⇒ Shift orbifold for E8

    ⇒ Gauge symmetry is broken !

$$E_8 \times E_8 \to E_6 \times SU(3) \times E_8$$



- Project out suitable right-moving fermionic states $\mathcal{N}=4 \to \mathcal{N}=1$
- 27 rep. chiral matters



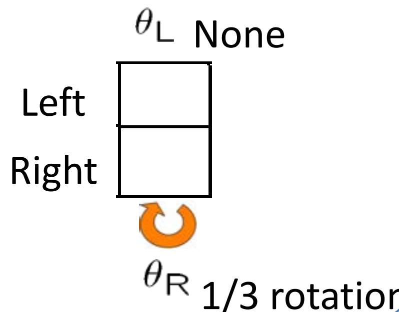
Asymmetric Orbifold Compactification

- Asymmetric orbifold compactification
  - Generalization of orbifold action
  - Orbifold action $\theta = (\theta_{\mathrm{L}}, \theta_{\mathrm{R}})$ (Twist, Shift)

Left mover : $X_{\mathrm{L}} \to \theta_{\mathrm{L}} X_{\mathrm{L}}$

Right mover : $X_{\mathrm{R}} \to \theta_{\mathrm{R}} X_{\mathrm{R}}$, $\Psi_{\mathrm{R}} \to \theta_{\mathrm{R}} \Psi_{\mathrm{R}}$

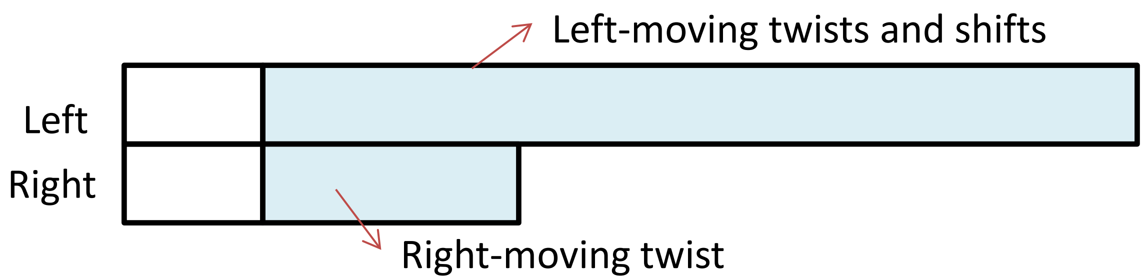
Ex.) $\mathbf{Z}_3$ action



Orbifold actions for left and right movers can be chosen independently

$$\theta = (\theta_{\mathrm{L}}, \theta_{\mathrm{R}}) \quad \theta_{\mathrm{L}} \neq \theta_{\mathrm{R}}$$

- Starting points should be suitable (22,6)-dimensional Narain lattices $\Gamma_{22,6}$

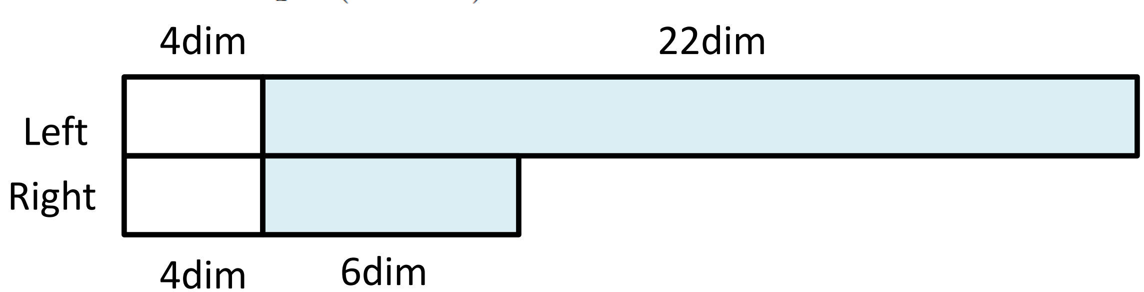


Asymmetric Orbifold Compactification

- Narain lattices
  - (22,6)-dimensional Narain lattices $\Gamma_{22,6}$
  - General flat compactification of heterotic string
    --- Left : 22 dim
    --- Right : 6 dim
  - 4D N=4 SUSY
  - Left-right combined momentum $(p_{\rm L}, p_{\rm R})$ are quantized on some momentum lattices which are described in terms of group theory (An, Dn, E6, E7, E8)

Mode expansion

$$X_{\rm L} = x_{\rm L} + p_{\rm L}(\tau + \sigma) + {\rm Oscillator}$$
$$X_{\rm R} = x_{\rm R} + p_{\rm R}(\tau - \sigma) + {\rm Oscillator}$$



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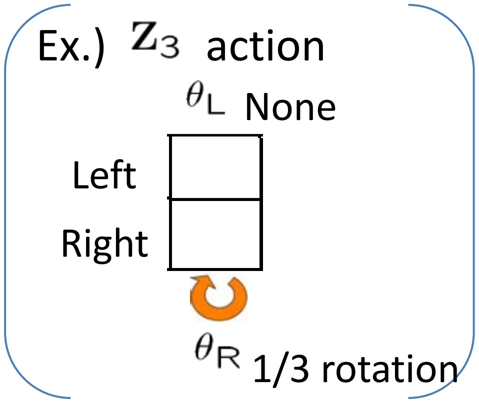
    Left mover : $X_{\mathrm{L}} \to \theta_{\mathrm{L}} X_{\mathrm{L}}$

    Right mover : $X_{\mathrm{R}} \to \theta_{\mathrm{R}} X_{\mathrm{R}}$, $\Psi_{\mathrm{R}} \to \theta_{\mathrm{R}} \Psi_{\mathrm{R}}$



Orbifold actions for left and right movers can be chosen independently

$$\theta = (\theta_{\mathrm{L}}, \theta_{\mathrm{R}}) \quad \theta_{\mathrm{L}} \neq \theta_{\mathrm{R}}$$

- Starting points should be suitable (22,6)-dimensional Narain lattices $\Gamma_{22,6}$
- A few/no moduli fields because of asymmetric action
- Rich source of hidden gauge symmetries

→ Moduli stabilization in heterotic string theory ?

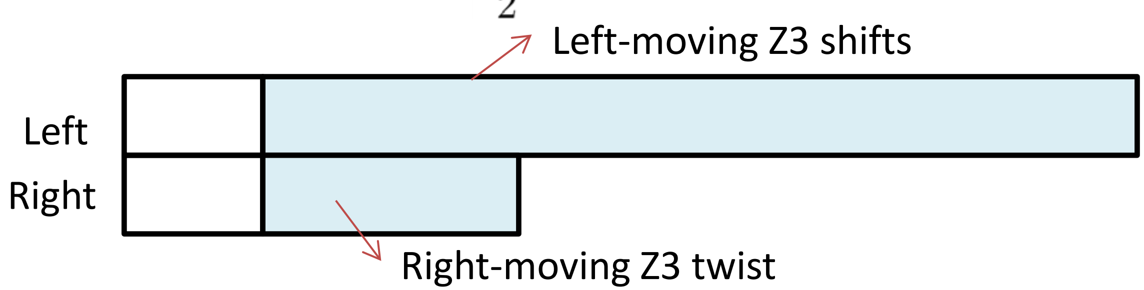
Z3 Asymmetric Orbifold Compactification

Asymmetric orbifold compactification = 4D Heterotic string theory on Narain lattice $\Gamma_{22,6}$
+ Asymmetric orbifold action

We consider Z3 Abelian orbifold action

A Z3 asymmetric orbifold model is specified by

- a (22,6)-dimensional Narain lattice $\Gamma_{22,6}$ which contains a right-moving $\overline{E}_6$ or $\overline{A}_2^3$ lattice (compatible with Z3 automorphism)
- a Z3 shift action $V = (V_{\mathrm{L}}, 0)$
- a Z3 twist action (N=4 SUSY → N=1 SUSY)
- Consistency condition: $\frac{3V_{\mathrm{L}}^2}{2} \in \mathbf{Z}$



Lattice and gauge symmetry

- Our starting point → Narain lattice

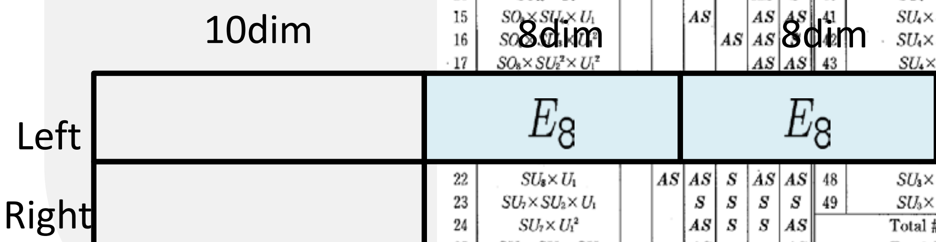
Symmetric orbifolds

Lattice: E8 x E8, SO(32)

Gauge symmetry breaking pattern: Classified

No.	Gauge Group	Z_3	Z_4	Z_6	Z_7	Z_8	Z_{12}	No.	Gauge Group
0	E_8	*	*	*	*	*	*	26	$SU_6\times SU_3\times U_1$
1	$E_7\times SU_2$		AS	AS		AS	AS	27	$SU_6\times SU_2^2\times U_1$
2	$E_7\times U_1$	AS	AS	AS	S	AS	AS	28	$SU_6\times SU_2\times U_1^2$
3	$E_6\times SU_3$	AS		AS			AS	29	$SU_6\times U_1^3$
4	$E_6\times SU_2\times U_1$		AS	S	S	AS	AS	30	$SU_5\times SU_4\times U_1$
5	$E_6\times U_1^2$			AS	S	S	AS	31	$SU_5\times SU_3\times SU_2\times U_1$
6	SO_{16}		AS	AS		AS	AS	32	$SU_5\times SU_3\times U_1^2$
7	$SO_{14}\times U_1$	AS	AS	AS	S	AS	AS	33	$SU_5\times SU_2^2\times U_1^2$
8						S	AS	34	$SU_5\times SU_2\times U_1^3$
9						S	AS	35	[illegible]
10						S	AS	36	$SU_4^2\times SU_2\times U_1$
11							S	37	$SU_4^2\times U_1^2$
12	$SO_{10}\times SU_2^2\times U_1$			AS		S	AS	38	$SU_4\times SU_3\times SU_2^2\times U_1$
13	$SO_{10}\times SU_2\times U_1^2$			AS	S	S	AS	39	$SU_4\times SU_3\times SU_2\times U_1^2$
14	$SO_{10}\times U_1^3$					AS	S	40	$SU_4\times SU_3\times U_1^3$
15	[illegible]			AS		AS	AS	41	$SU_4\times SU_2^3\times U_1^2$
16	[illegible]				AS	AS	[illegible]	42	$SU_4\times SU_2^2\times U_1^3$
17	$SO_8\times SU_2^2\times U_1^2$					AS	AS	43	$SU_4\times SU_2\times U_1^4$
22	$SU_8\times U_1$		AS	AS	S	AS	AS	48	[illegible]
23	$SU_7\times SU_2\times U_1$			S	S	S	S	49	[illegible]
24	$SU_7\times U_1^2$			AS	S	S	AS		Total # of A
25	$SU_6\times SU_3\times SU_2$			AS			AS		Total # of S

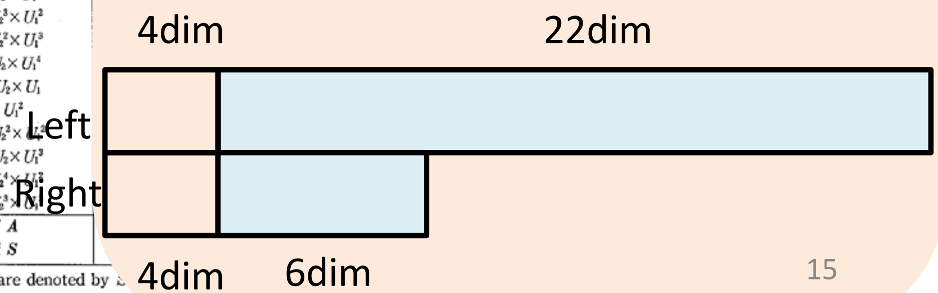
Gauge groups realized by the shift (automorphism) of E_8 lattice are denoted by



Asymmetric orbifolds

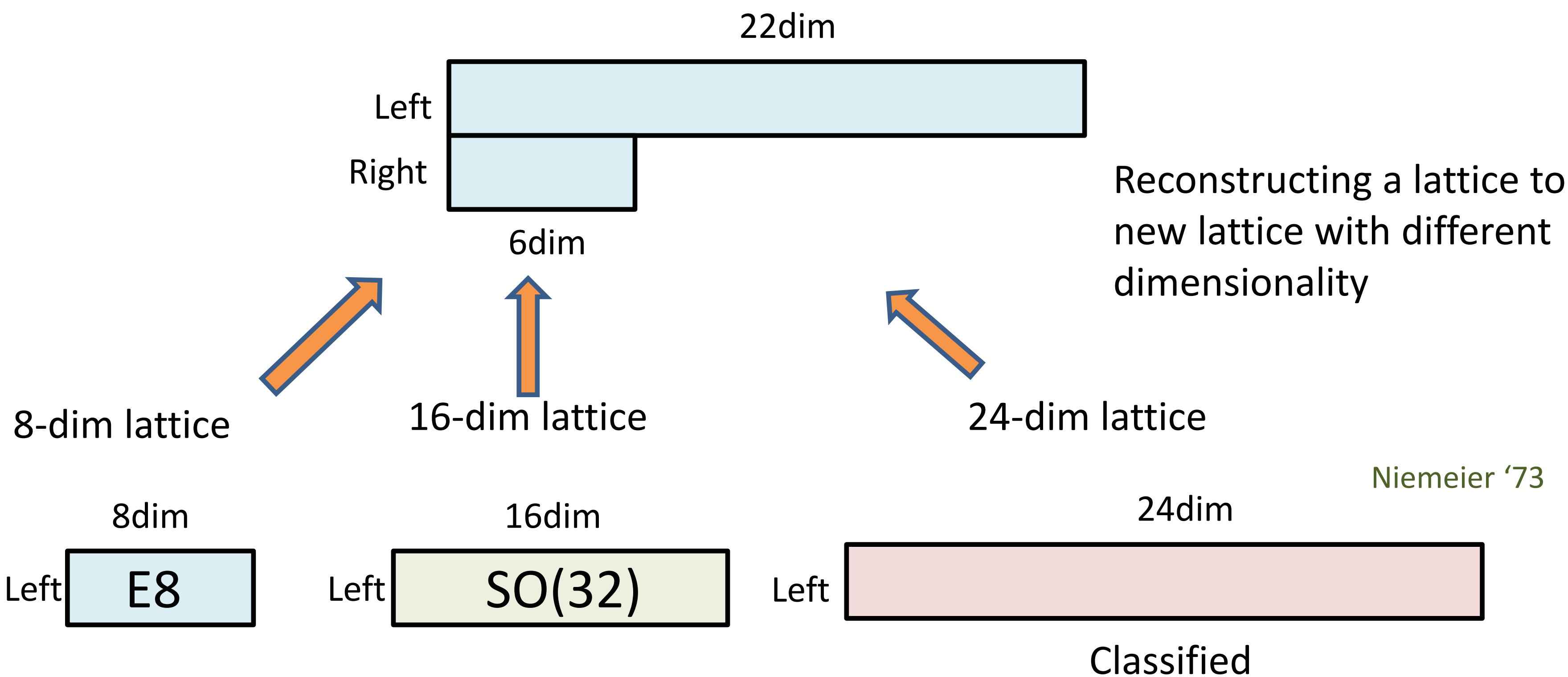
What types of (22,6)-dimensional Narain lattices can be used for starting points ?

What types of gauge symmetries can be realized ?



(22,6)-dim lattices from 8, 16, 24-dim lattices

- We construct (22,6)-dim Narain lattices from 8, 16, 24-dim lattices by lattice engineering technique.



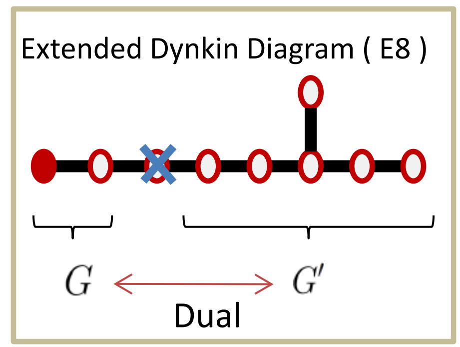
24 types of lattices

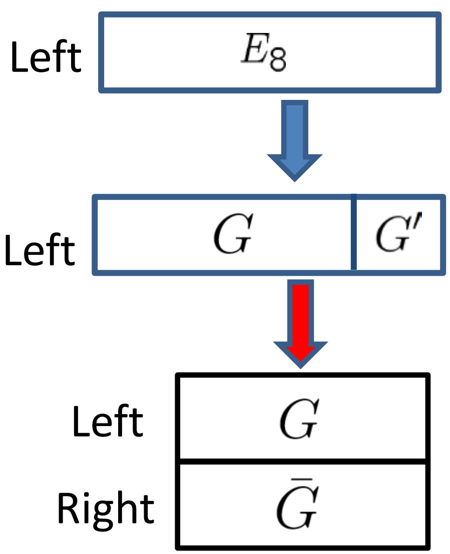
$$D_{24}, D_{16}\times E_8, E_8^3, A_{24}, D_{12}^2,$$
$$A_{17}\times E_7, D_{10}\times E_7^2, A_{15}\times D_9, D_8^3,$$
$$A_{12}^2, A_{11}\times D_7\times E_6, E_6^4, A_9^2\times D_6, D_6^4, A_8^3,$$
$$A_7^2\times D_5^2, A_6^4, A_5^4\times D_4, D_4^6, A_4^6, A_3^8, A_2^{12}, A_1^{24}.$$

Lattice Engineering Technique

- Lattice engineering technique — Lerche, Schellekens, Warner '88
  - We can construct new Narain lattice from known one.
  - We can replace one of the left-moving group factor with a suitable right-moving group factor.

	Replace	
Left-mover	$\longleftrightarrow$	Right-mover
G	$\longleftrightarrow$	$\mathrm{Dual}(G) = G'$





(Decomposition $E_8 \to G \times G'$)

(Replace left G'
→ Right $\bar{G}(= G'_{\mathrm{dual}})$)

The resulting lattice is also modular invariant (modular transformation properties of G' part and $\bar{G}$ part are similar)

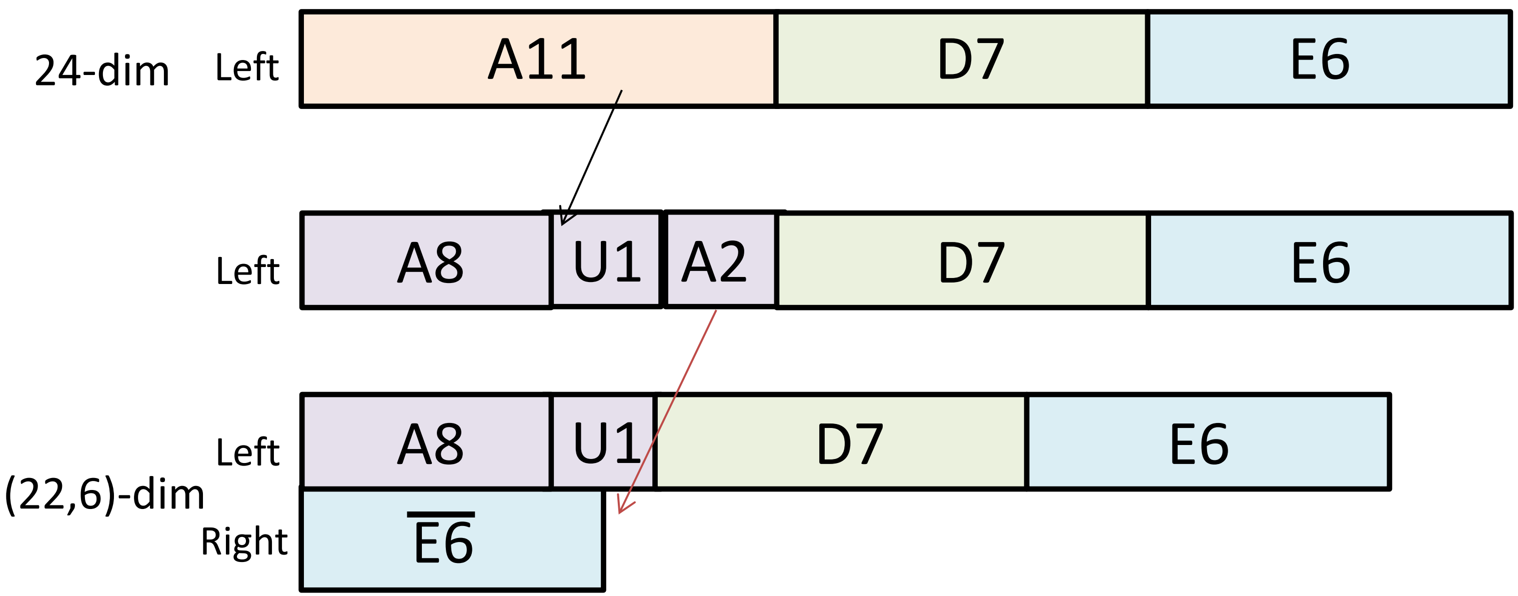
G_{L}	c_{L}	$\overline{G}_{\mathrm{R}}$	c_{R}
E_6	(1)	$\overline{A}_2$	(1)
D_4	(v) (s)	$\overline{D}_4$	(v) (s)
A_2	(1)	$\overline{E}_6$	(1)
A_2^2	$(1,0)$ $(1,2)$	$\overline{A}_2^2$	$(1,2)$ $(2,0)$
$U(1)^2$	$(1/3,1/2)$ $(1/4,1/4)$	$\overline{D}_4 \times \overline{A}_2$	$(s,1)$ $(c,0)$

(22,6)-dim lattices from 8, 16, 24-dim lattices

Example :

$A_{11} \times D_7 \times E_6$ 24-dim lattice

Gauge symmetry : SU(12) x SO(14) x E6



$D_7 \times E_6 \times A_8 \times U(1) \times \overline{E}_6$ (22,6)-dim lattice

Gauge symmetry : SO(14) x E6 x SU(9) x U(1)

Gauge symmetry breaking by Z3 action

- Z3 asymmetric orbifold compactification

  Z3 action :

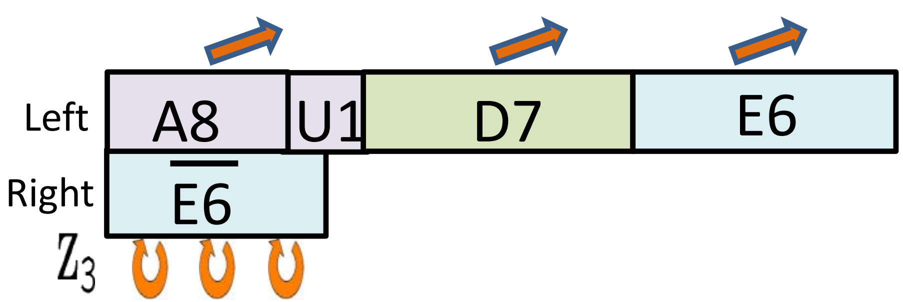
  Right mover → twist action → N=1 SUSY

  Left mover → shift action → Gauge symmetry breaking

- SO(14) x E6 x SU(9) x U(1) Gauge group breaks to Several gauge symmetries.

- SM group, Flipped SO(10)xU(1), Flipped SU(5)xU(1), Trinification SU(3)^3 group can be realized.

- Important data for model building.



Group	Group breaking patterns	Group breaking patterns
Shift	$(0,0,0,0,0)$	$(s,1,1,1/36,0)$
D_7	D_7 $A_6\times U(1)$ $D_6\times U(1)$ $A_1\times D_5\times U(1)$ $A_2\times D_4\times U(1)$ $A_3^2\times U(1)$ $A_5\times U(1)^2$ $A_1^2\times A_4\times U(1)$	D_7 $A_6\times U(1)$ $D_6\times U(1)$ $A_1\times D_5\times U(1)$ $A_2\times D_4\times U(1)$ $A_3^2\times U(1)$ $A_5\times U(1)^2$ $A_1^2\times A_4\times U(1)$
E_6	E_6 $A_5\times U(1)$ $A_2\times A_2\times A_2$ $D_4\times U(1)^2$ $D_5\times U(1)$ $A_4\times A_1\times U(1)$	$D_5\times U(1)$ $A_4\times A_1\times U(1)$
A_8	A_8 $A_6\times U(1)^2$ $A_5\times A_2\times U(1)$ $A_4\times A_1^2\times U(1)^2$ $A_3^2\times U(1)^2$ $A_2^3\times U(1)^2$	$A_7\times U(1)$ $A_6\times A_1\times U(1)$ $A_5\times A_1\times U(1)^2$ $A_4\times A_3\times U(1)$ $A_4\times A_2\times U(1)^2$ $A_3\times A_2\times A_1\times U(1)^2$
$U(1)$	$U(1)$	$U(1)$

Result: Lattice and gauge symmetry

- Our starting point → Narain lattice

Beye, Kobayashi, Kuwakino
arXiv:1304.5621 [hep-th]

Symmetric orbifolds

Asymmetric orbifolds

Lattice

E8 x E8, SO(32)

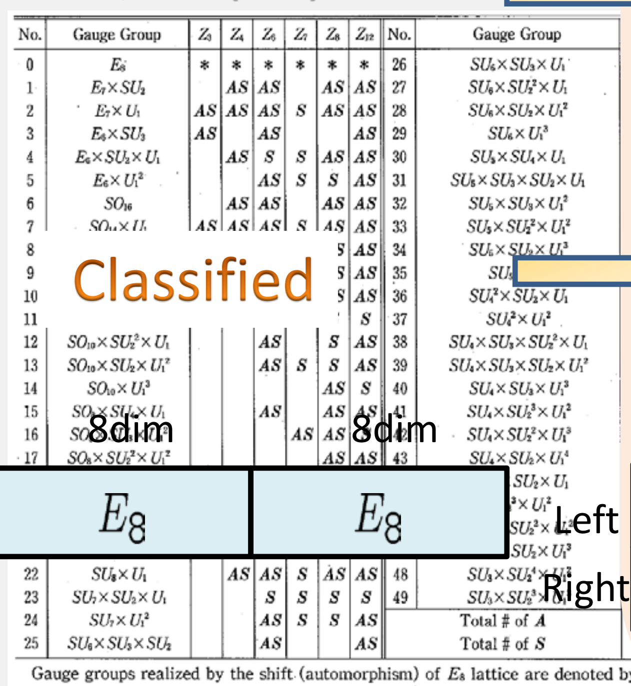
90 lattices

(with right-moving non-Abelian factor, from 24 dimensional lattices)

Gauge symmetry breaking pattern

Classified

Classified



No.	Gauge Group	Z_3	Z_4	Z_6	Z_7	Z_8	Z_{12}	No.	Gauge Group
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2	$E_7\times U_1$	AS	AS	AS	S	AS	AS	28	$SU_6\times SU_2\times U_1^2$
3	$E_6\times SU_3$	AS		AS			AS	29	$SU_6\times U_1^3$
4	$E_6\times SU_2\times U_1$		AS	S	S	AS	AS	30	$SU_5\times SU_4\times U_1$
5	$E_6\times U_1^2$			AS	S	S	AS	31	$SU_5\times SU_3\times SU_2\times U_1$
6	SO_{16}		AS	AS		AS	AS	32	$SU_5\times SU_3\times U_1^2$
7	$SO_{14}\times U_1$	AS	AS	AS	S	AS	AS	33	$SU_5\times SU_2^2\times U_1^2$
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	S	AS	34	$SU_5\times SU_2\times U_1^3$
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	S	AS	35	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	S	AS	36	$SU_4^2\times SU_2\times U_1$
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	$SU_8\times U_1$		AS	AS	S	AS	AS	48	[illegible]
23	$SU_7\times SU_2\times U_1$			S	S	S	S	49	[illegible]
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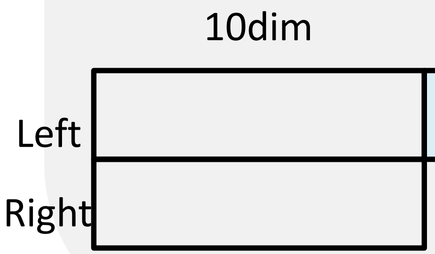
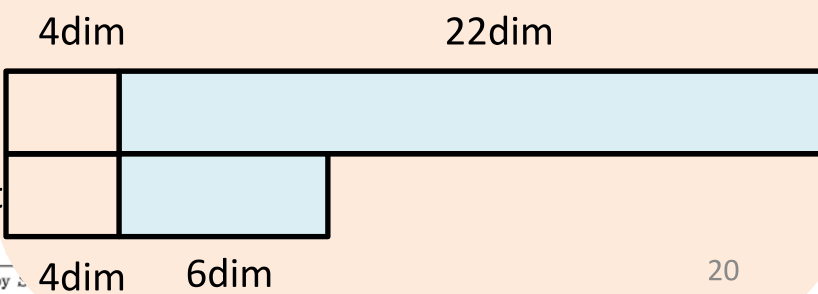
25	$SU_6\times SU_3\times SU_2$			AS			AS	Total # of S	

Gauge groups realized by the shift (automorphism) of E_8 lattice are denoted by





Gauge group patterns of models

SM or GUT group patterns of Z3 asymmetric orbifold models from 90 Narain lattices

Group	SM	Flipped $SO(10)$	Flipped $SU(5)$	Pati-Salam	Left-right symmetric
#1		✓	✓		
#2	✓	✓	✓		✓
#3	✓	✓	✓		✓
#4					
#5	✓		✓		
#6	✓	✓	✓	✓	✓
#7	✓	✓	✓		✓
#8	✓		✓	✓	✓
#9	✓	✓	✓	✓	✓
#10	✓	✓	✓	✓	✓
#11	✓	✓	✓	✓	✓
#12	✓	✓	✓	✓	✓
#13	✓	✓	✓	✓	✓
#14	✓		✓	✓	✓
#15	✓	✓	✓	✓	✓
#16	✓	✓	✓	✓	✓
#17	✓	✓	✓	✓	✓
#18	✓	✓	✓		✓

+ also for the other lattices.

Three-generation models

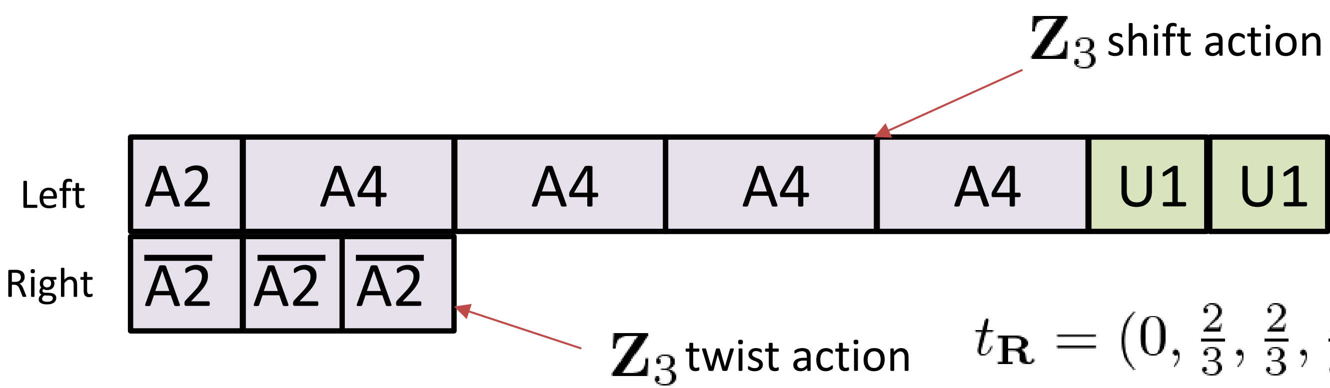
Z3 three generation left-right symmetric model

Z3 asymmetric orbifold compactification

Beye, Kobayashi, Kuwakino
arXiv: 1311.4687 [hep-th]

- Narain lattice: $A_1^2 \times A_4^4 \times U(1)^2 \times \overline{A}_2^2$ lattice $\bigoplus$ $A_2 \times \overline{A}_2$ lattice
- LET: $A_4^6 \xrightarrow[\text{decompose}]{} (A_2 \times A_1 \times U(1))^2 \times A_4^4 \xrightarrow[\text{replace}]{} A_1^2 \times A_4^4 \times U(1)^2 \times \overline{A}_2^2$

  $E_8 \xrightarrow[\text{decompose}]{} E_6 \times A_2 \xrightarrow[\text{replace}]{} A_2 \times \overline{A}_2$
- Z3 shift vector: $$V = (0, \omega_1^{A_1}, 2\omega_1^{A_4} + \omega_3^{A_4} - 3\alpha_1^{A_4} - 4\alpha_2^{A_4} - 2\alpha_3^{A_4} - \alpha_4^{A_4}, -\omega_1^{A_4} + \alpha_1^{A_4} + \alpha_2^{A_4} + \alpha_3^{A_4} + \alpha_4^{A_4}, - \omega_3^{A_4} - 2\omega_4^{A_4} + 2\alpha_4^{A_4}, \omega_2^{A_4} + 2\omega_4^{A_4} - 2\alpha_3^{A_4} - 2\alpha_4^{A_4}, \frac{\sqrt{30}}{5}, \frac{3\sqrt{30}}{10}, 0, 0, 0, 0)/3$$
- Group breaking: $SU(5)^4 \times SU(3) \times SU(2)^2 \times U(1)^2 \rightarrow SU(4)^2 \times SU(3)^3 \times SU(2)^3 \times U(1)^7$



$t_{\mathbf{R}} = (0, \frac{2}{3}, \frac{2}{3}, \frac{2}{3})$

Z3 three generation left-right symmetric model

Massless spectrum ($SU(3)_C \times SU(2)_L \times SU(2)_R \times SU(2)_F \times SU(3)^2 \times SU(4)^2$)

U/T		Irrep.	Q_{B-L}	Deg.
U	Q_R	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	3
U		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	3
T	$\overline{Q}_R$	$(\mathbf{3},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	Q_R	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	1
T	Q_{L2}	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	Q_{L1}	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	H	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T	H	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{4})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\overline{\mathbf{4}})$	$-\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{1})$	$-\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})$	$-\frac{1}{2}$	1

U/T		Irrep.	Q_{B-L}	Deg.
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{4})$	0	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{4}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{2}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\overline{\mathbf{4}})$	$-\frac{1}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{4}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})$	$\frac{1}{3}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	1	1
T		$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})$	-1	1

+ other fields

- Three-generation $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model

Z3 three generation left-right symmetric model

Massless spectrum ($SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times SU(2)_{\mathrm{F}} \times SU(3)^2 \times SU(4)^2$)

+ other fields

U/T		Irrep.	Q_{B-L}	Deg.
U	Q_{R}	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	3
U		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	3
T	$\overline{Q}_{\mathrm{R}}$	$(\mathbf{3},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	Q_{R}	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	1
T	Q_{L2}	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	Q_{L1}	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	H	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T	H	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{4})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\overline{\mathbf{4}})$	$-\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{1})$	$-\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})$	$-\frac{1}{2}$	1

Three-generation fields of LR symmetric model
+
Vector-like fields

Higgs fields for $SU(2)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}} \rightarrow U(1)_Y$

- Three-generation $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}}$ model
- Additional fields are vector-like

Z3 three generation left-right symmetric model

Massless spectrum ($SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times SU(2)_{\mathrm{F}} \times SU(3)^2 \times SU(4)^2$)

U/T		Irrep.	Q_{B-L}	Deg.
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{4})$	0	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{4}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{2}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\overline{\mathbf{4}})$	$-\frac{1}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{4}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})$	$\frac{1}{3}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	1	1
T		$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})$	-1	1

+ other fields

Vector-like fields

- Three-generation $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}}$ model
- Additional fields are vector-like

Z3 three generation left-right symmetric model

Massless spectrum ($SU(3)_{\rm C} \times SU(2)_{\rm L} \times SU(2)_{\rm R} \times SU(2)_{\rm F} \times SU(3)^2 \times SU(4)^2$)

U/T		Irrep.	Q_{B-L}	Deg.
U	$Q_{\rm R}$	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	3
U		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	3
T	$\overline{Q}_{\rm R}$	$(\mathbf{3},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	$Q_{\rm R}$	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	1
T	$Q_{\rm L2}$	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	$Q_{\rm L1}$	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	H	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T	H	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1

The first two-generation is unified into SU(2)F doublet.

U/T		Irrep.	Q_{B-L}	Deg.
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{4})$	0	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{4}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{2}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\overline{\mathbf{4}})$	$-\frac{1}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{4}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	1
T		$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})$	$\frac{1}{3}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	1	1
T		$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})$	-1	1

+ other fields

SU(2)F flavon

- Three-generation $SU(3)_{\rm C} \times SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{\rm B-L}$ model
- Additional fields are vector-like
- Gauge flavor symmetry $SU(2)_F$

Z3 three generation left-right symmetric model

Massless spectrum ($SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times SU(2)_{\mathrm{F}} \times SU(3)^2 \times SU(4)^2$)

U/T		Irrep.	Q_{B-L}	Deg.
U	Q_{R}	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	3
U		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	3
T	$\overline{Q}_{\mathrm{R}}$	$(\mathbf{3},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	Q_{R}	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	1
T	Q_{L2}	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	Q_{L1}	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	1
T	H	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T	H	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{4})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\bar{\mathbf{4}})$	$-\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{1})$	$-\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\bar{\mathbf{4}},\mathbf{1})$	$\frac{1}{2}$	1
T		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})$	$-\frac{1}{2}$	1

U/T		Irrep.	Q_{B-L}	Deg.
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	0	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{4})$	0	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{4}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{2}{3}$	1
T		$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\bar{\mathbf{4}})$	$-\frac{1}{3}$	1
T		$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{4}{3}$	1
T		$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	1
T		$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	1
T		$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\bar{\mathbf{4}},\mathbf{1})$	$\frac{1}{3}$	1
T		$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	1	1
T		$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	-1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	1	1
T		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\bar{\mathbf{4}},\mathbf{1})$	-1	1

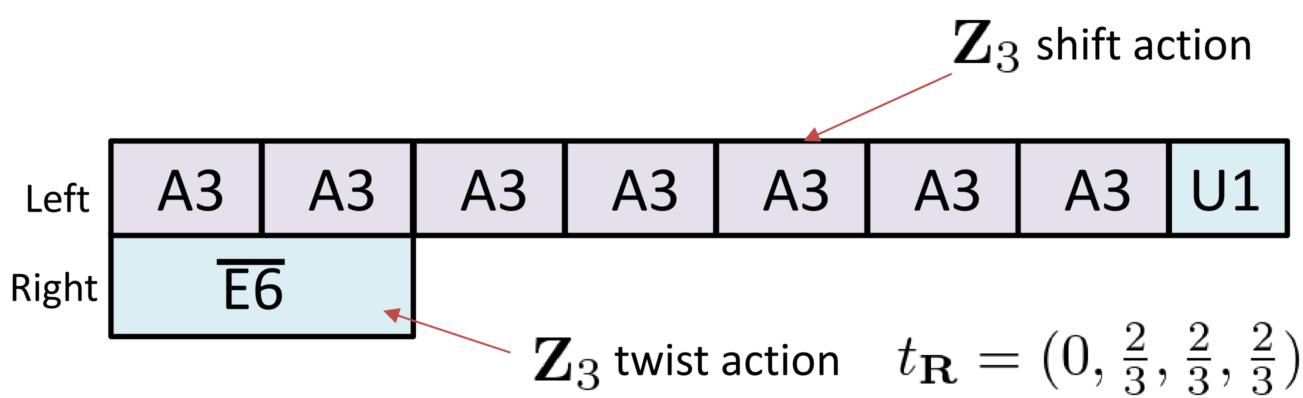
+ other fields

- Three-generation $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}}$ model
- Additional fields are vector-like
- Gauge flavor symmetry $SU(2)_F$
- No Top Yukawa by three point coupling($HQ_{\mathrm{L1}}Q_{\mathrm{R}}$) → higher dim. coupling

Z3 three generation SU(3)xSU(2)xU(1) model

Z3 asymmetric orbifold compactification

- Narain lattice: $A_3^7 \times \overline{E}_6 \times U(1)$ lattice
- LET: $A_3^8 \xrightarrow[\text{decompose}]{} A_3^7 \times A_2 \times U(1) \xrightarrow[\text{replace}]{} A_3^7 \times \overline{E}_6 \times U(1)$
- Z3 shift vector: $V = (\alpha_1^{A_3} + 2\alpha_2^{A_3}, \alpha_1^{A_3} + 2\alpha_2^{A_3}, -\alpha_1^{A_3} - 2\alpha_2^{A_3}, \alpha_3^{A_3}, 0, \alpha_3^{A_3}, \alpha_3^{A_3}, 0, 0)/3$
- Group breaking: $SU(4)^7 \times U(1) \to SU(4) \times SU(3)^3 \times SU(2)^3 \times U(1)^{10}$



Z3 three generation SU(3)xSU(2)xU(1) model

Massless spectrum ($SU(3)_\mathrm{C} \times SU(2)_\mathrm{L} \times SU(2)^2 \times SU(3)^2 \times SU(4)$)

U/T		Irrep.	Q_Y	Deg.
U	l^u	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	3
U	$\bar{l}^u$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	3
U	$\bar{d}$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	3
T	c_1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	3
T	c_2	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{2}{3}$	3
T	$\bar{c}_1$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	3
T	$\bar{c}_2$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	3
T		$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	3
T		$(\mathbf{1},\mathbf{2};\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	3
T		$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	3
T	q	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	3
T	$\bar{u}$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	3
T	h_u	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	3

+ other fields

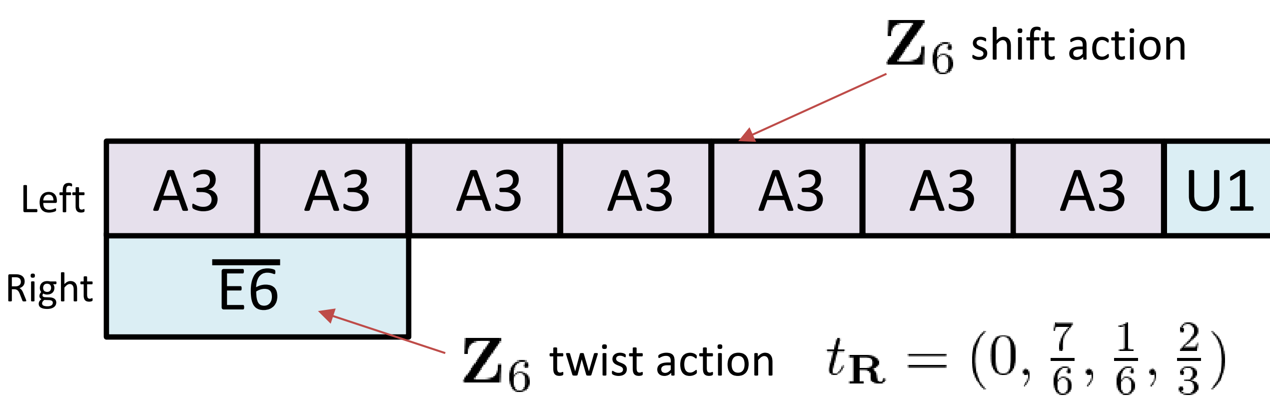
Three-generation fields of SUSY SM model
+
Vector-like fields

- Three-generation $SU(3)_\mathrm{C} \times SU(2)_\mathrm{L} \times U(1)_\mathrm{Y}$ model
- "3"-generation comes from a degeneracy "3“
- Additional fields are vector-like
- Top Yukawa from twisted sector

Z6 three generation SU(3)xSU(2)xU(1) model

Z6 asymmetric orbifold compactification

- Narain lattice: $A_3^7 \times \overline{E}_6 \times U(1)$ lattice
- LET: $A_3^8 \xrightarrow[\text{decompose}]{} A_3^7 \times A_2 \times U(1) \xrightarrow[\text{replace}]{} A_3^7 \times \overline{E}_6 \times U(1)$
- Z6 shift vector: $V = (0, 0, -4\omega_3^{A_3}, 4\omega_3^{A_3}, 2\omega_2^{A_3}, \omega_1^{A_3} + 5\omega_3^{A_3} - \alpha_1^{A_3} - \alpha_2^{A_3}, \omega_1^{A_3} + 2\omega_2^{A_3} + \omega_3^{A_3}, 0, 0)/6$
- Group breaking: $SU(4)^7 \times U(1) \rightarrow SU(4)^2 \times SU(3)^2 \times SU(2)^4 \times U(1)^8$



Z6 three generation SU(3)xSU(2)xU(1) model

Massless spectrum ($SU(3)_C \times SU(2)_L \times SU(4)^2 \times SU(3) \times SU(2)^3 \times U(1)^8$)

$$(\mathbf{3},\mathbf{2}) : 3$$
$$(\mathbf{3},\mathbf{1}) : 7$$
$$(\bar{\mathbf{3}},\mathbf{1}) : 13$$
$$(\mathbf{1},\mathbf{2}) : 13$$
$$\text{others} : 39$$
$$\text{all} : 75$$

cf : Z3 model

$$(\mathbf{3},\mathbf{2}) : 3$$
$$(\mathbf{3},\mathbf{1}) : 6$$
$$(\bar{\mathbf{3}},\mathbf{1}) : 12$$
$$(\mathbf{1},\mathbf{2}) : 27$$
$$\text{others} : 99$$
$$\text{all} : 147$$

- Three-generation $SU(3)_C \times SU(2)_L \times U(1)$ model
- Number of massless states : fewer than Z3 cases

SUSY SM in asymmetric orbifold vacua

・At this stage, we performed model buildings from several lattices of 90 lattices, and get models with

Four-dimensions,
N=1 supersymmetry,
Standard model group(SU(3)*SU(2)*U(1)), LR symmetric group
Three generations,
Quarks, Leptons and Higgs,
No exotics (vector-like)
Top quark mass

} Realized

Other quark masses (Charm quark mass)
Proton stability,
R-parity,
Doublet-triplet splitting,
Moduli stabilization,
...

} Need further model building from other Narain lattices and effective theory analysis
(Z6, Z12, Z2xZ2, …)

SUSY SM in asymmetric orbifold vacua

- Toward moduli stabilization in heterotic string theory

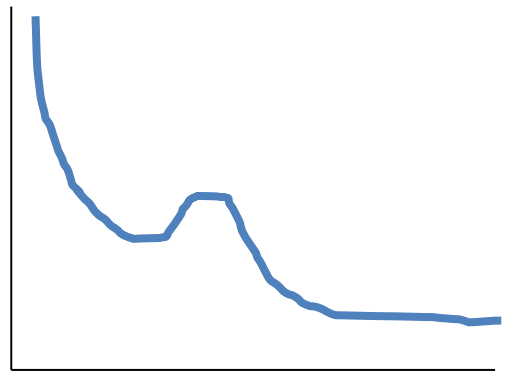
▪In asymmetric orbifolds, number of geometrical moduli is small

▪3-generation model with a dilaton field

▪Strong dynamics in hidden sector (enhancement point)

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)^n \times \underline{SU(N_1) \times SU(N_2)}$$

→ Potential for a dilaton field



Flavor symmetry at symmetry enhanced point

Discrete flavor symmetry in string model

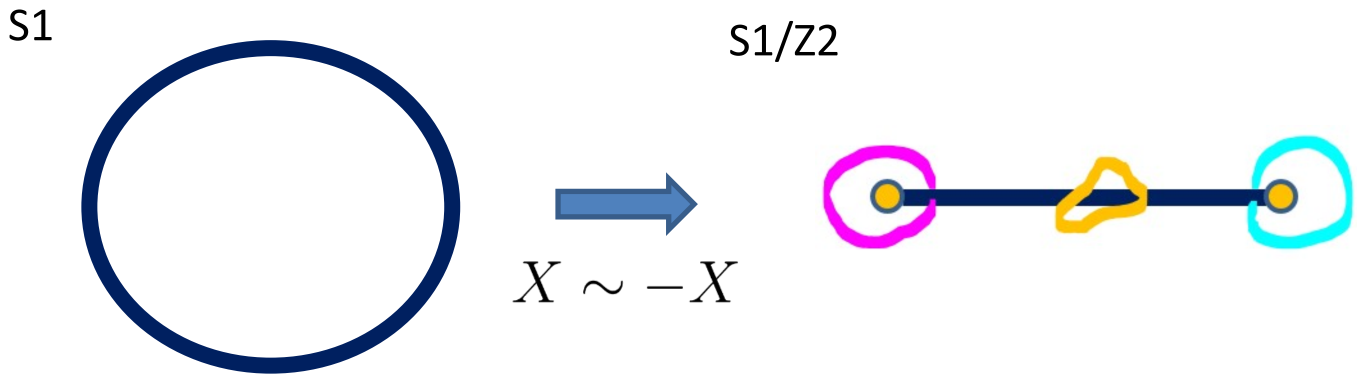
・In heterotic orbifold models, non-Abelian discrete symmetries arise from extra-dimensional spaces.

Kobayashi, Nilles, Plöger, Raby, Ratz '07

$$\begin{cases} S^1/Z_2 \rightarrow D_4 \text{ symmetry} \\ T^2/Z_3 \rightarrow \Delta(54) \text{ symmetry} \end{cases}$$

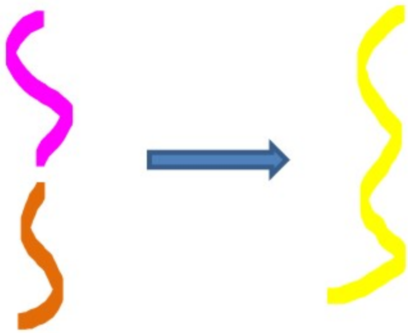
・Closed string on orbifold is specified by boundary condition

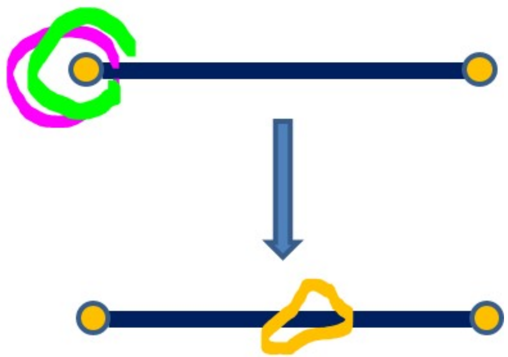
-- Untwisted string (Bulk modes)
-- Twisted string (localized modes on brane)



Discrete flavor symmetry in string model

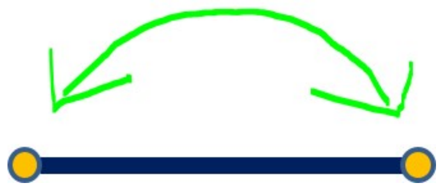
・Two strings are connected and become a string if boundary conditions fit each other.





・String selection rule can be described by Z4 symmetry

・Fixed points of S1/Z2 are equivalent. These is a permutation symmetry (Z2) of fixed points



・String model has Z4 symmetry from interaction, and S1/Z2 orbifold has geometrical Z2 symmetry, which is a permutation symmetry of fixed points.

→ Non-Abelian discrete symmetry $D_4 \cong Z_4 \rtimes Z_2$

Discrete flavor symmetry in string model

- 1 dimensional orbifold : S1/Z2

$$\longrightarrow \quad D_4 \cong Z_4 \rtimes Z_2$$

- 2 dimensional orbifold : T2/Z3

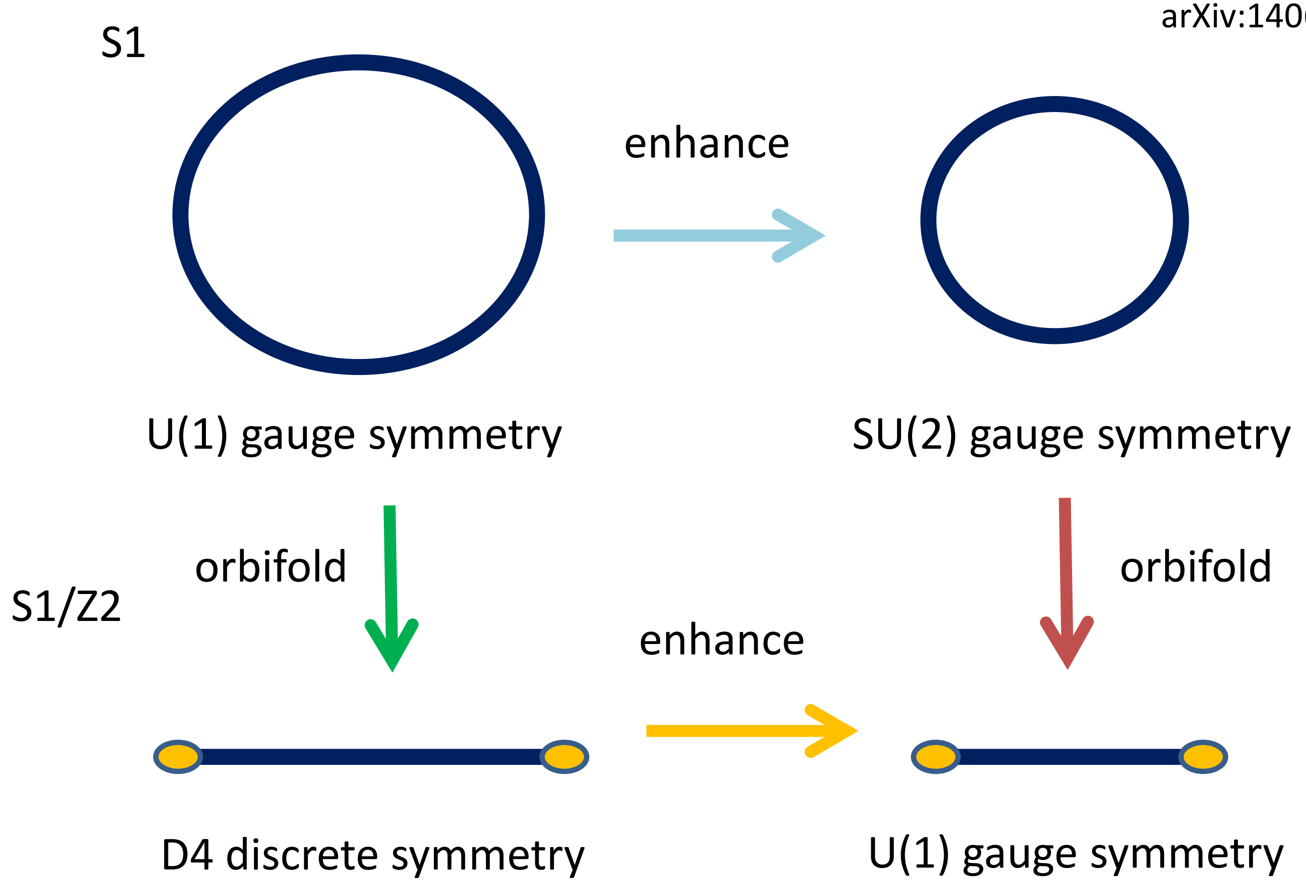
$$\longrightarrow \quad \Delta(54) \cong (Z_3 \times Z_3) \rtimes S_3$$

・Non-Abelian discrete symmetries have a stringy origin, which are determined by the geometrical structure of the extra dimension space

Gauge origin of discrete flavor symmetry

◆ symmetry enhance point in moduli space

Beye, Kobayashi, Kuwakino
arXiv:1406.4660 [hep-th]



・D4 non-Abelian discrete symmetry is enhanced to U(1) continuous gauge symmetry

Gauge origin of discrete flavor symmetry

- 1-dimensional orbifold model at symmetry enhance point

  -- Massless spectrum of U(1) orbifold theory

Sector	Field	$U(1)$ charge	Z_4 charge
U	U	0	1
U	U_1	1	1
U	U_2	-1	1
T	M_1	$\frac{1}{4}$	i
T	M_2	$-\frac{1}{4}$	$-i$

  -- This model has symmetry : $U(1) \rtimes Z_2$

  Z2 symmetry can be described by $q \to -q$ or $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

  -- Non zero VEV of Kahler moduli field (radion) T breaks the U(1) symmetry to Z4 Abelian discrete symmetry

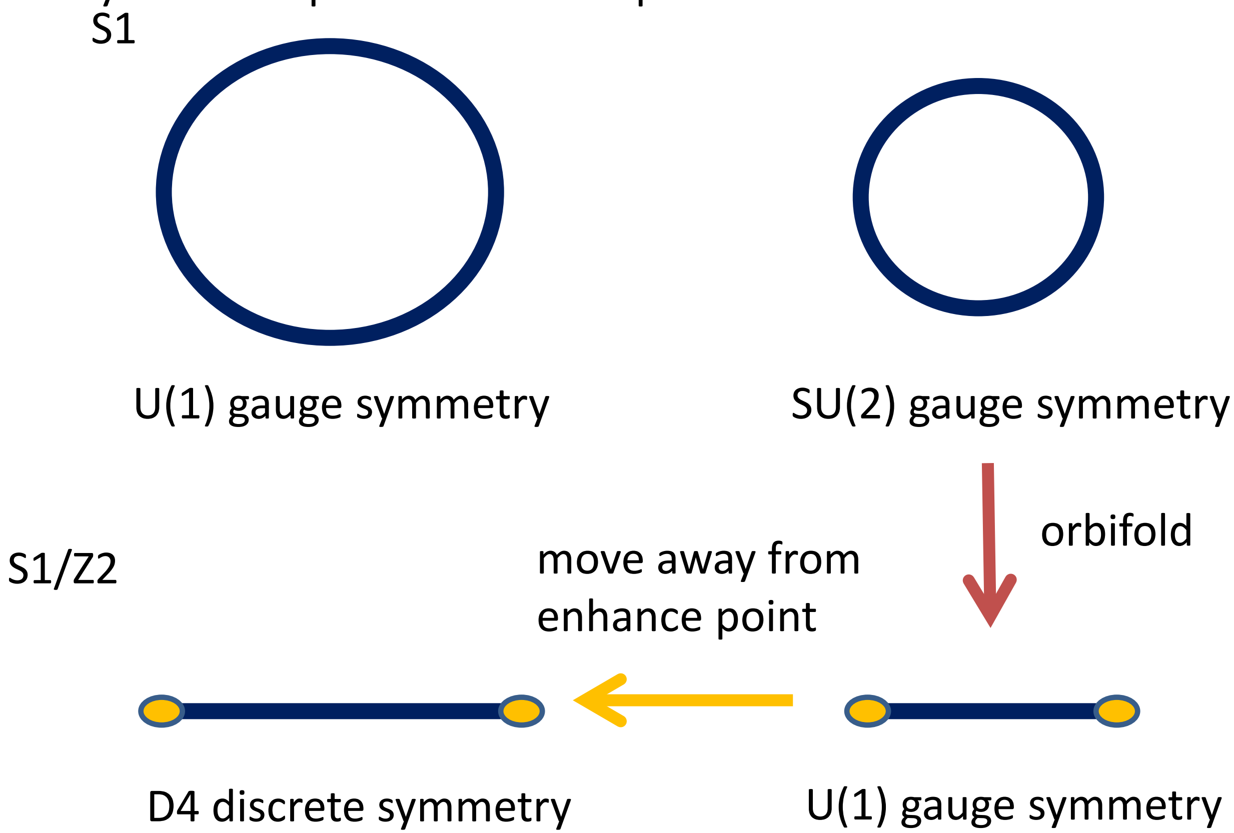
$$T = \frac{1}{\sqrt{2}}(U_1 + U_2) \qquad \langle U_1 \rangle = \langle U_2 \rangle \longrightarrow \langle T \rangle \neq 0$$

  Z4 symmetry can be described by $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

$$U(1) \rtimes Z_2 \to D_4 \cong Z_4 \rtimes Z_2$$

Gauge origin of discrete flavor symmetry

- symmetry enhance point in moduli space

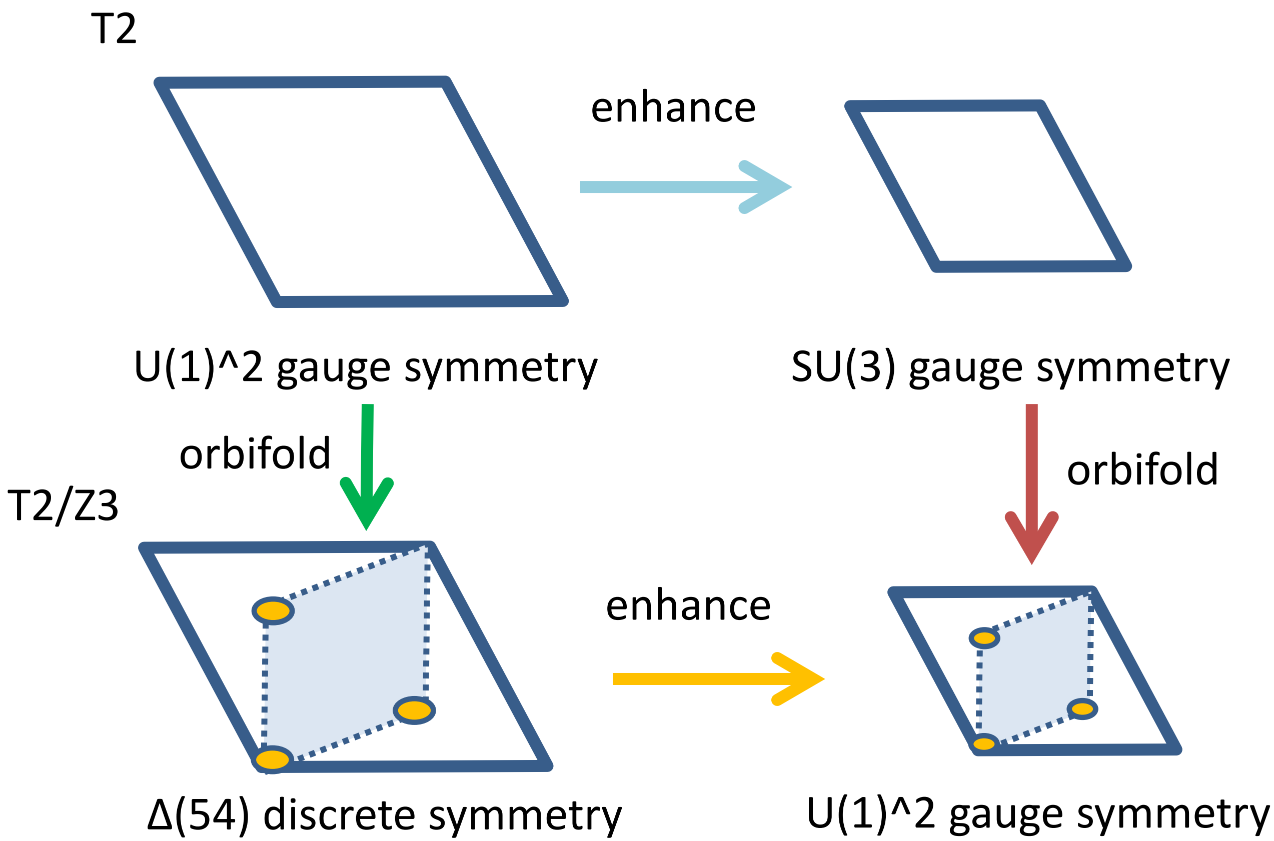


-- Symmetry breaking patterns are summarized as

$$SU(2) \xrightarrow[\text{orbifolding}]{} U(1) \rtimes Z_2 \xrightarrow[\langle T \rangle]{} D_4 \cong Z_4 \rtimes Z_2$$

Gauge origin of discrete flavor symmetry

◆ symmetry enhance point in moduli space (2-dim)



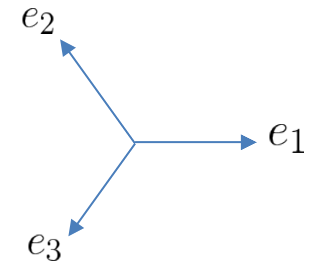
・Δ(54) non-Abelian discrete symmetry is enhanced to U(1)^2 continuous gauge symmetry

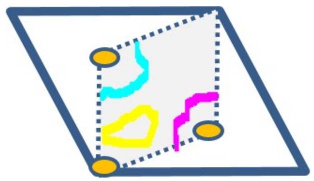
Gauge origin of discrete flavor symmetry

- 2-dimensional orbifold model at symmetry enhance point
  - -- Massless spectrum of U(1)^2 orbifold theory

Sector	Field	$U(1)^2$ charge	Z_3^2 charge
U	U	0	$(0,0)$
U	U_1	$-e_1$	$(0,0)$
U	U_2	$-e_2$	$(0,0)$
U	U_3	$-e_3$	$(0,0)$
T	M_1	$\frac{e_1}{3}$	$(1,1)$
T	M_2	$\frac{e_2}{3}$	$(2,0)$
T	M_3	$\frac{e_3}{3}$	$(0,2)$

e_i : Simple roots of SU(3)





-- This model has symmetry : $U(1)^2 \rtimes S_3$

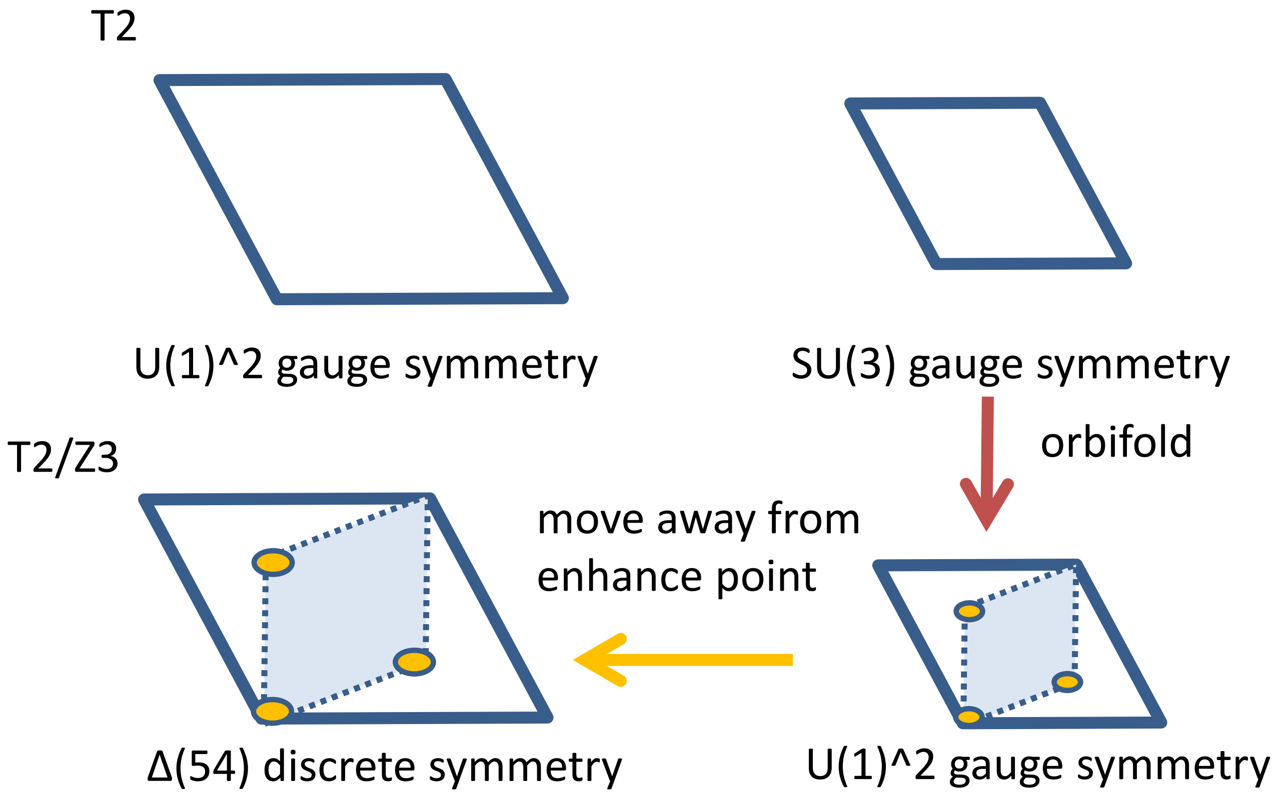
-- Non zero VEV of Kahler moduli field (radion) T breaks the U(1)^2 symmetry to Z3 x Z3 Abelian discrete symmetry

$$T = \frac{1}{\sqrt{3}}(U_1 + U_2 + U_3) \qquad \langle U_1 \rangle = \langle U_2 \rangle = \langle U_3 \rangle \longrightarrow \langle T \rangle \neq 0$$

$$U(1)^2 \rtimes S_3 \to \Delta(54) \cong (Z_3 \times Z_3) \rtimes S_3$$

Gauge origin of discrete flavor symmetry

◆ symmetry enhance point in moduli space



-- Symmetry breaking patterns are summarized as

$$SU(3) \xrightarrow[\text{orbifolding}]{} U(1)^2 \rtimes S_3 \xrightarrow[\langle T \rangle]{} \Delta(54) \cong (Z_3 \times Z_3) \rtimes S_3$$

Field-theoretical application

- The previous result in string models

Beye, Kobayashi, Kuwakino
arXiv: 1502.00789 [hep-ph]

$$U(1) \rtimes Z_2 \longrightarrow D_4 \cong Z_4 \rtimes Z_2$$
$$U(1)^2 \rtimes S_3 \longrightarrow \Delta(54) \cong (Z_3 \times Z_3) \rtimes S_3$$

suggests that $U(1)^n \rtimes S_m$ theory or $U(1)^n \rtimes Z_m$ theory can be an origin of non-Abelian discrete symmetries

-- Generalization of denominator of U(1) charge to N

$$\begin{cases} q = \frac{1}{4} \rightarrow q = \frac{1}{N} \\ q = \frac{e_i}{3} \rightarrow q = \frac{e_i}{N} \end{cases} \Longrightarrow \begin{cases} Z_4 \rightarrow Z_N \\ Z_3 \times Z_3 \rightarrow Z_N \times Z_N \end{cases}$$

- Gauge extensions of phenomenologically interesting non-Abelian discrete symmetries

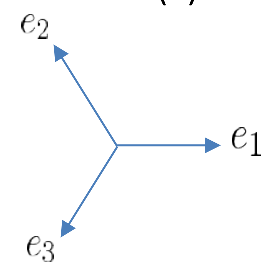
$$S_3, S_4, D_4, A_4, \Delta(27), \Delta(54), \cdots$$

S_4 non-Abelian discrete symmetry

- $U(1)^2 \rtimes S_3$ model ($N=2$)

Field	$U(1)^2$ charge	Z_2^2 charge	S_4 rep.
U_1, U_2, U_3	$-e_1, -e_2, -e_3$	$(0,0),(0,0),(0,0)$	—
M_1, M_2, M_3	$\frac{e_1}{2}, \frac{e_2}{2}, \frac{e_3}{2}$	$(1,1),(1,0),(0,1)$	$\mathbf{3}$
M	0	$(0,0)$	$\mathbf{1}$
N_1, N_2, N_3	e_1, e_2, e_3	$(0,0),(0,0),(0,0)$	$\mathbf{1} \oplus \mathbf{2}$

e_i : Simple roots of SU(3)



- VEV relation $\langle U_1 \rangle = \langle U_2 \rangle = \langle U_3 \rangle$ maintains S3, but breaks U(1)^2 → Z2^2.
- Resulting symmetry is $U(1)^2 \rtimes S_3 \to S_4 \cong (Z_2 \times Z_2) \rtimes S_3$

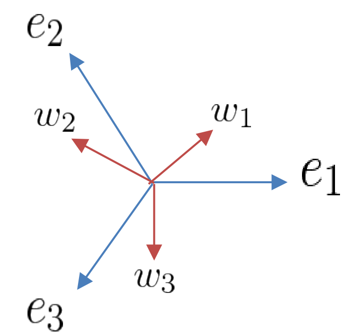
A_4 non-Abelian discrete symmetry

- $U(1)^2 \rtimes Z_3$ model ($N=2$)

Field	$U(1)^2$ charge	Z_2^2 charge	A_4 rep.
U_1, U_2, U_3	$-e_1, -e_2, -e_3$	$(0,0), (0,0), (0,0)$	—
M_1, M_2, M_3	$\frac{e_1}{2}, \frac{e_2}{2}, \frac{e_3}{2}$	$(1,1), (1,0), (0,1)$	$\mathbf{3}$
M	0	$(0,0)$	$\mathbf{1}$
N_1, N_2, N_3	e_1, e_2, e_3	$(0,0), (0,0), (0,0)$	$\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$
A_1, A_2, A_3	$\frac{3w_1}{2}, \frac{3w_2}{2}, \frac{3w_3}{2}$	$(1,0), (0,1), (1,1)$	$\mathbf{3}$

e_i : Simple roots of SU(3)

w_i : Fundamental weights of SU(3)



- Field A_i breaks $S_3 \to Z_3$
- VEV relation $\langle U_1 \rangle = \langle U_2 \rangle = \langle U_3 \rangle$ maintains Z3, but breaks U(1)^2 → Z2^2.
- Resulting symmetry is $U(1)^2 \rtimes Z_3 \to A_4 \cong (Z_2 \times Z_2) \rtimes Z_3$

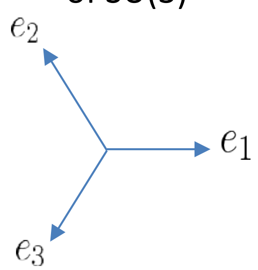
U(1) flavor model

$U(1)^2 \rtimes S_3$ lepton flavor model

- $U(1)^2 \rtimes S_3 \times Z_2$ model
  - Gauge extension of Δ(54) discrete lepton flavor model

Field	$U(1)^2$ charge	Z_2 charge	$\Delta(54)$ rep.
(L_e, L_μ, L_τ)	$(\frac{2e_1}{3}, \frac{2e_2}{3}, \frac{2e_3}{3})$	0	$\mathbf{3}_{1(2)}$
(e^c, μ^c, τ^c)	$(-3e_1, -3e_2, -3e_3)$	1	$\mathbf{1}_+ \oplus \mathbf{2}_1$
H_u	0	0	$\mathbf{1}_+$
H_d	0	0	$\mathbf{1}_+$
(A_1, A_2, A_3)	$(\frac{2e_1}{3}, \frac{2e_2}{3}, \frac{2e_3}{3})$	0	$\mathbf{3}_{1(2)}$
(B_1, B_2, B_3)	$(-\frac{4e_1}{3}, -\frac{4e_2}{3}, -\frac{4e_3}{3})$	0	$\mathbf{3}_{1(2)}$
(C_1, C_2, C_3)	$(\frac{e_1}{3}, \frac{e_2}{3}, \frac{e_3}{3})$	0	$\mathbf{3}_{1(1)}$
(D_1, D_2, D_3)	$(\frac{7e_1}{3}, \frac{7e_2}{3}, \frac{7e_3}{3})$	1	$\mathbf{3}_{1(1)}$

e_i : Simple roots of SU(3)



$U(1)^2 \rtimes S_3$ lepton flavor model

- Superpotential for neutrinos and charged leptons ($U(1)^2 \rtimes S_3 \times Z_2$ invariance)

$$
\begin{aligned}
W_\nu &= y_1^\nu \left(B_1 L_e L_e + B_2 L_\mu L_\mu + B_3 L_\tau L_\tau\right) H_u H_u/\Lambda^2 \\
&\quad + y_2^\nu \left(A_1(L_\mu L_\tau + L_\tau L_\mu) + A_2(L_e L_\tau + L_\tau L_e) + A_3(L_e L_\mu + L_\mu L_e)\right) H_u H_u/\Lambda^2 \\
&\quad + y_3^\nu \left(C_1^2(L_\mu L_\tau + L_\tau L_\mu) + C_2^2(L_e L_\tau + L_\tau L_e) + C_3^2(L_e L_\mu + L_\mu L_e)\right) H_u H_u/\Lambda^3
\end{aligned}
$$

$$
W_e = y_1^e \left(D_1 L_e e^c + D_2 L_\mu \mu^c + D_3 L_\tau \tau^c\right) H_d/\Lambda
$$

- Mass matrices

$$
M_\nu = \frac{v_u^2}{\Lambda^2} \begin{pmatrix} y_1^\nu b_1 & y_2^\nu a_3 & y_2^\nu a_2 \\ y_2^\nu a_3 & y_1^\nu b_2 & y_2^\nu a_1 \\ y_2^\nu a_2 & y_2^\nu a_1 & y_1^\nu b_3 \end{pmatrix} + \frac{y_3^\nu v_u^2}{\Lambda^3} \begin{pmatrix} 0 & c_3^2 & c_2^2 \\ c_3^2 & 0 & c_1^2 \\ c_2^2 & c_1^2 & 0 \end{pmatrix}
$$

$$
M_e = \frac{y_1^e v_d}{\Lambda} \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix}
$$

- Flavon superpotential

$$
W_f = \lambda_1 A_1 A_2 A_3 + \lambda_2 B_1 B_2 B_3 + \lambda_3 C_1 C_2 C_3 + \lambda_4 \left(A_1^2 B_1 + A_2^2 B_2 + A_3^2 B_3\right)
$$

$U(1)^2 \rtimes S_3$ lepton flavor model

- By solving vacuum structure, neutrino mass matrix becomes

$$M_\nu = \frac{v_u^2}{\Lambda^2}\begin{pmatrix} -y_1^\nu \frac{\lambda_1}{2\lambda_4}\frac{a_2 a_3}{a_1} & y_2^\nu a_3 & y_2^\nu a_2 \\ y_2^\nu a_3 & -y_1^\nu \frac{\lambda_1}{2\lambda_4}\frac{a_1 a_3}{a_2} & y_2^\nu a_1 \\ y_2^\nu a_2 & y_2^\nu a_1 & -y_1^\nu \frac{\lambda_1}{2\lambda_4}\frac{a_1 a_2}{a_3} \end{pmatrix} + \frac{y_3^\nu v_u^2}{\Lambda^3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & c_1^2 \\ 0 & c_1^2 & 0 \end{pmatrix}$$

$$= A\begin{pmatrix} B a_2' a_3' & a_3' & a_2' \\ a_3' & B\frac{a_3'}{a_2'} & 1+C \\ a_2' & 1+C & B\frac{a_2'}{a_3'} \end{pmatrix}$$

$$A \equiv \frac{v_u^2 y_2^\nu a_1}{\Lambda^2} \qquad a_2' \equiv \frac{a_2}{a_1}$$

$$B \equiv -\frac{y_1^\nu}{y_2^\nu}\frac{\lambda_1}{2\lambda_4} \qquad a_3' \equiv \frac{a_3}{a_1}$$

$$C \equiv \frac{y_3^\nu}{y_2^\nu}\frac{c_1^2}{a_1 \Lambda}$$

- We consider the case of real mass matrix and inverted hierarchy for simplicity.

5 real parameters: A, B, C, a_2', a_3' —fitting→ Oscillation parameters: $\Delta m_{21}^2, \Delta m_{31}^2, \theta_{12}, \theta_{23}, \theta_{13}$

$\delta = \beta_1 = \beta_2 = 0^\circ$ ← for simplicity

m_3 ← prediction

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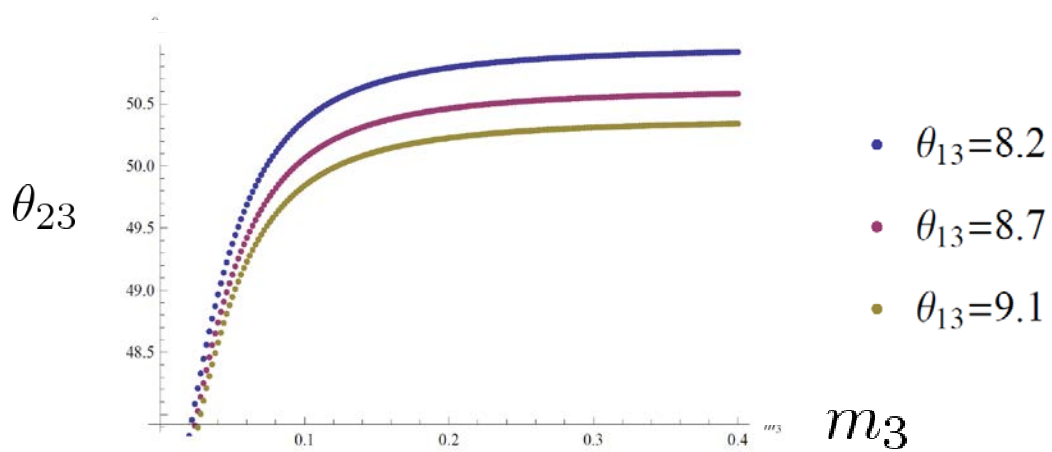
- Choosing suitable parameters, we can fix experimental values

$$\left\{\begin{aligned} A &= 0.00197\,\mathrm{eV}, \\ B &= 30.4, \\ C &= -5.93, \\ a_2' &= -1.08, \\ a_3' &= -1.06, \end{aligned}\right.$$

$$\left\{\begin{aligned} \Delta m^2_{21} &= 7.60 \times 10^{-5}\ \mathrm{eV}^2 \\ \Delta m^2_{31} &= -2.38 \times 10^{-3}\ \mathrm{eV}^2 \\ \theta_{12} &= 35.26^\circ \end{aligned}\right.$$

Several values for angles θ_{23} and θ_{13}

- Prediction of our model : m_3 against angles θ_{23} and θ_{13}



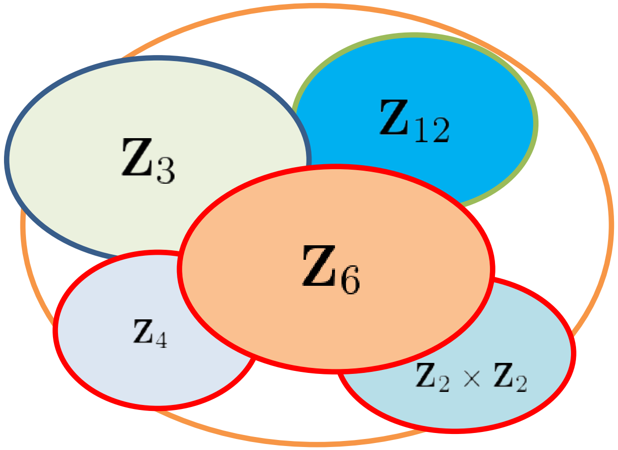
- This solution is consistent with 2σ range of recent fits from neutrinoless double beta decay

Summary

- Z3 asymmetric orbifold compactification of heterotic string
- Our starting point : Narain lattice
- 90 lattices with right-moving non-Abelian factor can be constructed from 24 dimensional lattices
- We calculate group breaking patterns of Z3 models
- Three generation SUSY SM / left-right symmetric model

- Z6 three-generation model

- Outlook: Search for a realistic model
  -- Search for Z3 models from other lattices
  -- Other orbifolds Z6, Z12, Z3xZ3…
  -- Yukawa hierarchy
  -- (Gauge or discrete) Flavor symmetry,
  -- Moduli stabilization, etc.



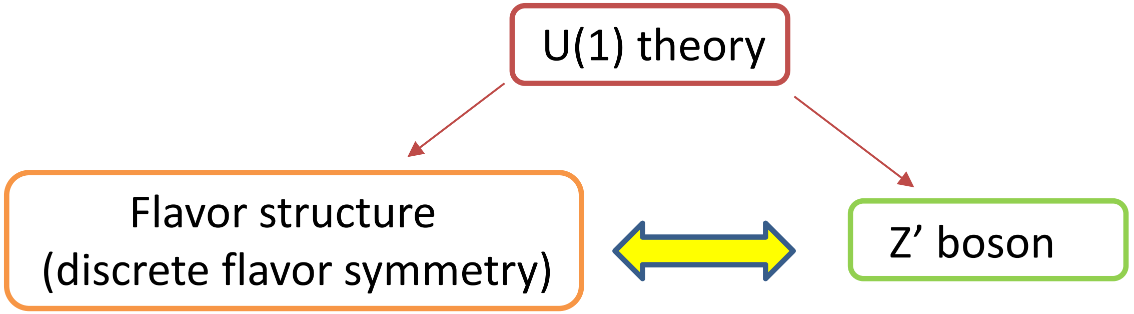
Summary

- (Non-)Abelian gauge origin of non-Abelian discrete flavor symmetry
- This can be understood naturally in orbifold string models
- Phenomenologically interesting non-Abelian discrete symmetries can be realized from U(1) theories with a permutation (rotation) symmetry

$$\begin{cases} U(1) \rtimes Z_2 \to S_3, D_3, \cdots \\ U(1)^2 \rtimes S_3 \to S_4, \Delta(54), \cdots \\ U(1)^2 \rtimes Z_3 \to A_4, \Delta(27), \cdots \end{cases}$$

- We apply this mechanism to lepton flavor model

- Outlook : Realization in string theory
  Higher dimensional gauge theory
  Z' boson(s) from U(1) breaking may relate to origin of Yukawa hierarchy



Z' bosons as a probe of flavor structure ?