Massive (bi)gravity, and modified gravity models

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Introduction

- Einstein theory: great, beautiful achievement
- Fantastic success
- Phenomenology: black holes, neutron stars, gravitational waves
- Sorry Einstein, but what if…

Revolution

- 2011 Nobel Prize: discovery of acceleration at large scales
- What drives it accounts for 68% of the total matter distribution
- What is it?

Gravity is changing?

- Maybe not
- Simply a cosmological constant
- But if it does, how?
- The theory must be a sensible one
- No ghosts, viable, and phenomenologically interesting

Modified gravity models

- f(R) models [Capozziello: IJMP 2002; ADF, Tsujikawa: LRR 2010]
- Extra dim.: DGP [Dvali, Gabadadze, Porrati: PLB 2000]
- Effective theories of ED (Galileons)
  [Nicolis, Rattazzi, Trincherini: PRD 2009]
- Horndeski (generalized Galileon) [Horndeski: IJTP 1974]
- Massive gravity

dRGT Massive gravity

[de Rham, Gabadadze, Tolley: PRL 2011]

- What if the graviton has a mass?
- Boulware-Deser theorem: in general a ghost is present
- Can this ghost be removed?
- dRGT showed that it is possible
- Formal proofs [Hassen, Rosen: JHEP 2012; Kugo, Ohta: PTEP 2014]
- If so, what kind of theory is this?

Lagrangian of Massive gravity

- Introduce the Lagrangian

$$\mathscr{L} = \frac{M_P^2}{2}\sqrt{-g}\left[R - 2\Lambda + 2m_g^2 \mathscr{L}_{MG}\right], \qquad \mathscr{L}_{MG} = \mathscr{L}_2 + \alpha_3 \mathscr{L}_3 + \alpha_4 \mathscr{L}_4,$$

$$K^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu},$$

$$\mathscr{L}_2 = \frac{1}{2}\left([K^2] - [K]^2\right),$$

$$\mathscr{L}_3 = \frac{1}{6}\left([K]^3 - 3[K][K^2] + 2[K^3]\right),$$

$$\mathscr{L}_4 = \frac{1}{24}\left([K]^4 - 6[K]^2[K^2] + 3[K^2]^2 + 8[K][K^3] - 6[K^4]\right).$$

New ingredient: fiducial metric

- Non-dynamical object: fiducial metric
- In terms of 4 new scalars, it can be written as

$$f_{\mu\nu} = f_{ab}(\varphi^c)\,\partial_\mu \varphi^a \partial_\nu \varphi^b$$

- The explicit form f_{ab} must be given
- What is this theory? How to fix the fiducial metric?

dRGT gravity: degrees of freedom

- Introducing 4 new scalar fields: Stuckelberg fields
- Then 4 sc dof, 4 vct dof, 2 Gws dof + 4 SF dof
- Unitary gauge (remove 4 SF dof): 4 sc , 4 vct, 2 Gws dof
- Constraints kill 2 sc dof and 2 vect dof: 2+2+2=6 dof
- dRGT kills one mode, the BD ghost. Finally only 5 dof.

No stable FLRW solutions

- FLRW background allowed
  [E. Gumrukcuoglu, C. Lin, S. Mukohyama: JCAP 2011][Langlois, Naruko: CQG 12/13]
- de Sitter solutions exist: but less propagating modes than expected
- But no stable FLRW exists: one of the 5 dof is ghost
  [ADF, E. Gumrukcuoglu, S. Mukohyama: PRL 2012]
- Inhomogeneity? Anisotropies? [D'Amico et al: PRD 2011]
  [E. Gumrukcuoglu, C. Lin, S. Mukohyama: JCAP 2011. ADF, EG, SM: JCAP 2012]
- Something else?

Why FLRW is unstable?

[ADF, E. Gumrukcuoglu, S. Mukohyama: PRL 2012]

- Study Bianchi-I metric (with FLRW limit, σ → 0)

$$ds^2 = -N^2 dt^2 + a^2 \left(e^{4\sigma} dx^2 + e^{-2\sigma} \delta_{ij} dx^i dx^j \right)$$

- Fix a FLRW fiducial metric

$$f_{\mu\nu} = -n^2 \partial_\mu \varphi^0 \partial_\nu \varphi^0 + \alpha^2 \left(\partial_\mu \varphi^1 \partial_\nu \varphi^1 + \delta_{ij} \partial_\mu \varphi^i \partial_\nu \varphi^j \right)$$

- 5 modes propagate but 1 light ghost mode (not BD)

$$\kappa_1 \simeq \frac{p_T^4}{8 p^4}, \quad \kappa_2 \simeq -\frac{2 a^4 M_{GW}^2 p_L^2}{1 - r^2} \sigma, \quad \kappa_3 \simeq -\frac{p_T^2}{2 p_L^2} \kappa_2, \quad r = an/(\alpha N)$$

Can we get rid of this extra ghost?

- The Boulware-Deser ghost is absent by construction
- However, among the 5 remaining ones, for a general FLRW, still one is a ghost
- It cannot be integrated out (not massive)
- Either abandon omogeneity and/or isotropy
- Change/extend the theory [also Huang, Piao, Zhou: PRD 2012]

Quasi-dilaton massive gravity

[D'Amico, Gabadadze, Hui, Pirtskhalava: PRD 2013]

- dRGT on FLRW: reduction of dof + ghost
- Avoid this behavior by introducing scalar field
- SF interacts with Stuckelberg fields/fiducial metric
- Non-trivial dynamics / perturbation behavior
- May heal the model? Still 2 GWs but massive: dof = 5 + 1

Symmetries of the model

- Lagrangian invariant under quasidilaton symmetry

$$\sigma \to \sigma_0, \quad \varphi^a \to e^{-\sigma_0/M_P} \varphi^a$$

- SFs satisfy Poincare symmetry

$$\varphi^a \to \varphi^a + c^a, \quad \varphi^a \to \Lambda^a{}_b \varphi^b$$

- Fiducial metric [ADF, Mukohyama: 2013]

$$\tilde{f}_{\mu\nu} = \eta_{ab} \partial_\mu \varphi^a \partial_\nu \varphi^b - \frac{\alpha_\sigma}{M_P^2 m_g^2} e^{-2\sigma/M_P} \partial_\mu \sigma \partial_\mu \sigma$$

Quasidilaton Lagrangian

- Following Lagrangian

$$\mathscr{L} = \frac{M_P^2}{2}\sqrt{-g}\left[R - 2\Lambda - \frac{\omega}{M_P^2}\partial_\mu\sigma\,\partial_\nu\sigma + 2m_g^2\left(\mathscr{L}_2 + \alpha_3\mathscr{L}_3 + \alpha_4\mathscr{L}_4\right)\right],$$

where
$$K^\mu{}_\nu = \delta^\mu{}_\nu - e^{\sigma/M_P}\left(\sqrt{g^{-1}\tilde{f}}\right)^\mu{}_\nu,$$

$$\mathscr{L}_2 = \frac{1}{2}\left([K]^2 - [K^2]\right),$$

$$\mathscr{L}_3 = \frac{1}{6}\left([K]^3 - 3[K][K^2] + 2[K^3]\right),$$

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Background

- Give the ansatz

$$ds^2 = -N^2 dt^2 + a^2 d\vec{x}^2, \quad \phi^0 = \phi^0(t), \quad \phi^i = x^i, \quad \sigma = \bar{\sigma}(t)$$

- Fiducial metric $\tilde{f}_{00} = -n(t)^2, \quad \tilde{f}_{ij} = \delta_{ij}$
- Defining $H = \dot{a}/(aN), \quad X = e^{\bar{\sigma}/M_P}/a, \quad r = an/N$
- de Sitter solution $\left(3 - \frac{\omega}{2}\right) H^2 = \Lambda + \Lambda_X, \quad \omega < 6$

de Sitter solution

- Existence of de Sitter solution
- All expected 5 modes propagate
- Only if $\alpha_\sigma / m_g^2 > 0$ all the modes are well behaved: no ghost, and no classical instabilities.
- This same result can be generalized to general quasi-dilaton field.

Scalar contribution

- In the unitary gauge, integrating out auxiliary modes
- 2 scalar modes propagate: one with 0 speed, the other with speed equal to 1.
- Ghost conditions

$$0<\omega<6\,,\quad X^2<\frac{\alpha_\sigma H^2}{m_g^2}<r^2X^2\,,\quad r>1$$

Vector and GW contributions

- Vector modes reduced action

$$\mathcal{L}=\frac{M_P^2}{16}a^3 N\left[\frac{T_V}{N^2}|\dot{E}_i^T|^2-k^2 M_{GW}^2|E_i^T|^2\right],\quad T_V>0$$

- Therefore

$$c_V^2=\frac{M_{GW}^2}{H^2}\frac{r^2-1}{2\omega},\quad M_{GW}^2=\frac{(r-1)X^3 m_g^2}{X-1}+\frac{\omega H^2(rX+r-2)}{(X-1)(r-1)},\quad M_{GW}^2>0$$

- GW reduces action

$$\mathcal{L}=\frac{M_P^2}{8}a^3 N\left[\frac{1}{N^2}|\dot{h}_{ij}^{TT}|^2-\left(\frac{k^2}{a^2}+M_{GW}^2\right)|h_{ij}^{TT}|^2\right]$$

Directions

- Late-time stable de Sitter solution does exist
- Is the theory free of ghosts during cosmic history?
- Consistent with the cosmological data?
- Solar system constraints?
- Massive gravity? Role and meaning of the fiducial metric

Bigravity

[Hassan, Rosen: JHEP 2012]

- Promote fiducial metric to a dynamical component
- Introduce for it a new Ricci scalar
- Degrees of freedom in the 3+1 decomposition:
  - Total: (4 sc + 4 vt + 2GW) · 2
  - Gauge: 2 sc + 2 vt
  - Constraints: (2 sc + 2 vt) · 2 + 1 no-BD-ghost
  - Finally: T-G-C => 1 sc + 2 vt + 4 GW

Bimetric Lagrangian

- For the two metrics

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu , \quad d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu ,$$

- Introduce a ghost free action

$$\mathscr{L} = \sqrt{-g}\left[M_G^2 \left(\frac{R}{2} - m^2 \sum_{n=0}^{4} c_n V_n \left(Y^\mu{}_\nu \right) \right) + \mathscr{L}_m \right] + \frac{\kappa M_G^2}{2} \sqrt{-\tilde{g}}\, \tilde{R}$$

where

$$Y^\mu{}_\nu = \sqrt{g^{\mu\alpha} \tilde{g}_{\alpha\nu}} ,$$

$$[Y^n] = Tr\left(Y^n \right), \quad V_0 = 1, \quad V_1 = [Y],$$

$$V_2 = [Y]^2 - [Y^2], \quad V_3 = [Y]^3 - 3[Y][Y^2] + 2[Y^3],$$

$$V_4 = [Y]^4 - 6[Y]^2[Y^2] + 8[Y][Y^3] + 3[Y^2]^2 - 6[Y^4].$$

Background dynamics

- Assume FLRW ansatz

$$ds^2 = a^2(-dt^2 + d\vec{x}^2), \quad d\tilde{s}^2 = \tilde{a}^2(-\tilde{c}^2 dt^2 + d\vec{x}^2)$$

- Define $\xi = \tilde{a}/a, \quad H = \dot{a}/a^2$
- Existence of two branches [Comelli, Crisostomi, Pilo: JHEP 12]

$$\Gamma(\xi)(\tilde{c}\, a H - \dot{\tilde{a}}/\tilde{a}) = 0, \quad \Gamma = c_1 \xi + 4 c_2 \xi^2 + 6 c_3 \xi^3$$

- Physical branch: $\tilde{c} = \dot{\tilde{a}}/(\tilde{a}\, a H)$

GR-like dynamics

[ADF, Nakamura, Tanaka: PTEP 14]

- At low energies, Friedmann equation is recovered

$$3H^2 = \frac{\rho_m}{\tilde{M}_G^2}, \quad \xi \approx \xi_c, \quad \tilde{M}_G^2 = M_G^2 (1 + \kappa \xi_c^2), \quad \tilde{c} \approx 1,$$

where $\xi \to \xi_c$ when $\rho_m \to 0$

- The effective gravitational constant is different form the bare one, time independent
- Low energy cosmological dynamics consistent with data

Solar system constraints?

[ADF, Nakamura, Tanaka: PTEP 14]

- Gravitational potential of a star in he Minkowski limit
- Ansatz

$$ds^2 = -e^{u-v} dt^2 + e^{u+v}(dr^2 + r^2 d\Omega^2), \quad d\tilde{s}^2 = -\xi_c^2 e^{\tilde{u}-\tilde{v}} dt^2 + \xi_c^2 e^{\tilde{u}+\tilde{v}}(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2).$$

- Defining $\tilde{r} = e^{R(r)} r, \quad C = \frac{d \ln \Gamma}{d \ln \xi}, \quad C \gg 1,$
- Then at second order, u → 0, (Vainshtein mechanism) with same effective gravitational constant $\nabla^2 v \approx -\frac{\rho_m}{\tilde{M}_G^2}$

[Babichev, Deffayet, Esposito-Farese, PRL 11; Kimura, Kobayashi, Yamamoto PRD 12]

Graviton oscillations

[ADF, Nakamura, Tanaka: PTEP 14]

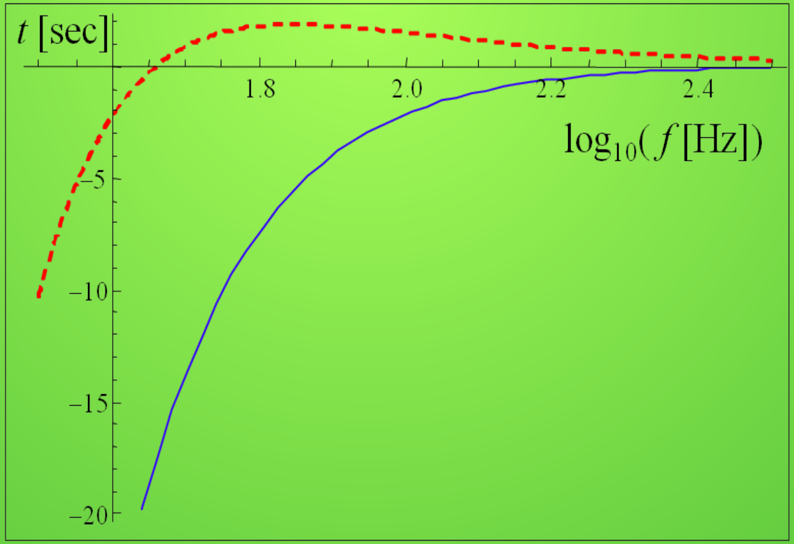
- Study propagation of 4 GW – coupled 2 by 2

$$\ddot{h} - \nabla^2 h + m^2 \Gamma_c (h - \tilde{h}) = 0,$$
$$\ddot{\tilde{h}} - \tilde{c}^2 \nabla^2 \tilde{h} + \frac{m^2 \Gamma_c}{\kappa \xi_c^2} (h - \tilde{h}) = 0.$$

- Define $\mu^2 = \frac{(1 + \kappa \xi_c^2) \Gamma_c m^2}{\kappa \xi_c^2}$
- Eingenmodes: one massless and one massive μ
- Graviton oscillations possible

Inverse chirp signal

- For NS–NS: $h = A(f)e^{i\Phi(f)}[B_1 e^{i\delta\Phi_1(f)} + B_2 e^{i\delta\Phi_2(f)}]$
- Graviton modes have inverse chirp signal:
  arrival time vs frequency reversed for the second (red) mode



Constraints

- Cosmological dynamics similar to GR
- To pass solar system tests: need hierarchy in the graviton mass term
- Weak field approximations + 2nd order perturbations

$$r_V = O\left(\left(C\, r_g \lambda_\mu^2\right)^{1/3}\right), \quad \nabla^2 v = -\tilde{M}_G^{-2} \%rho_m$$

- Black holes? [Babichev, Fabbri CQG 13, 1401.6871]

Matter couplings

[Yamashita, ADF, Tanaka: IJMP 2014]

- What about matter couplings?
- How to couple matter fields to the two metrics?
- Is it safe to introduce general couplings?
- What is the new phenomenology to be observed?

- General couplings introduce BD ghost
- Some possible way out
- Safe prescription: one matter – one metric coupling
- Special cases allowed

$$\mathscr{L} \ni \sqrt{-\tilde{g}} \sum_i c_i f_i\left(\sqrt{\tilde{g}^{\mu\beta}\left(g_{\beta\nu} + \alpha\, \partial_\beta \sigma\, \partial_\nu \sigma\right)}\right)$$

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Directions

- Bigravity, possible way to give the graviton a mass
- Further study is required
- Strong field environments
- Phenomenology

Conclusions

- What is gravity?
- Yet, a field to investigate
- Can the graviton (or one of them) be massive?
- Experimental and theoretical research is needed