超対称ゲージ理論の数値シミュレーションとゲージ/重力対応

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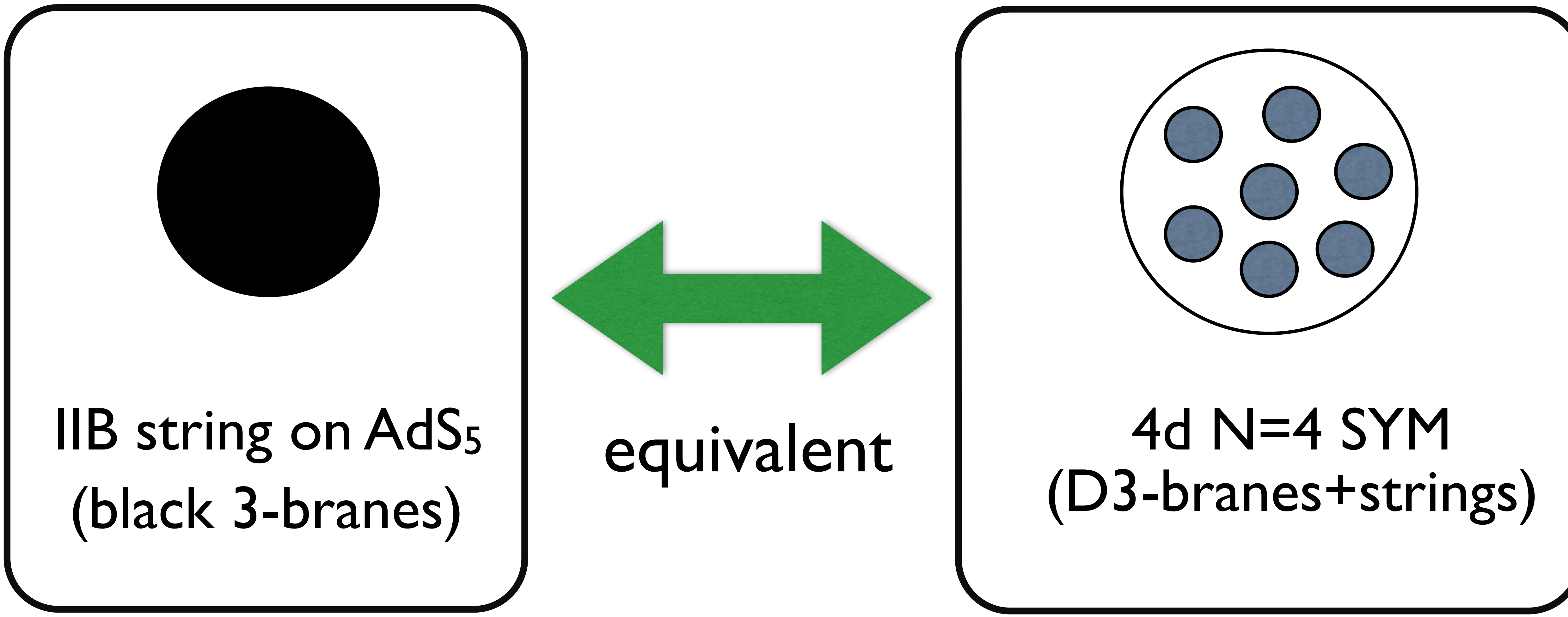
Based on papers with
Anagnostopoulos, Hyakutake, Ishiki, Kanamori, Mannelli, Matsuo,
Matsuura, Miwa, Nishimura, Sekino, Sugino, Takeuchi, Yoneya

@益川塾

Motivation

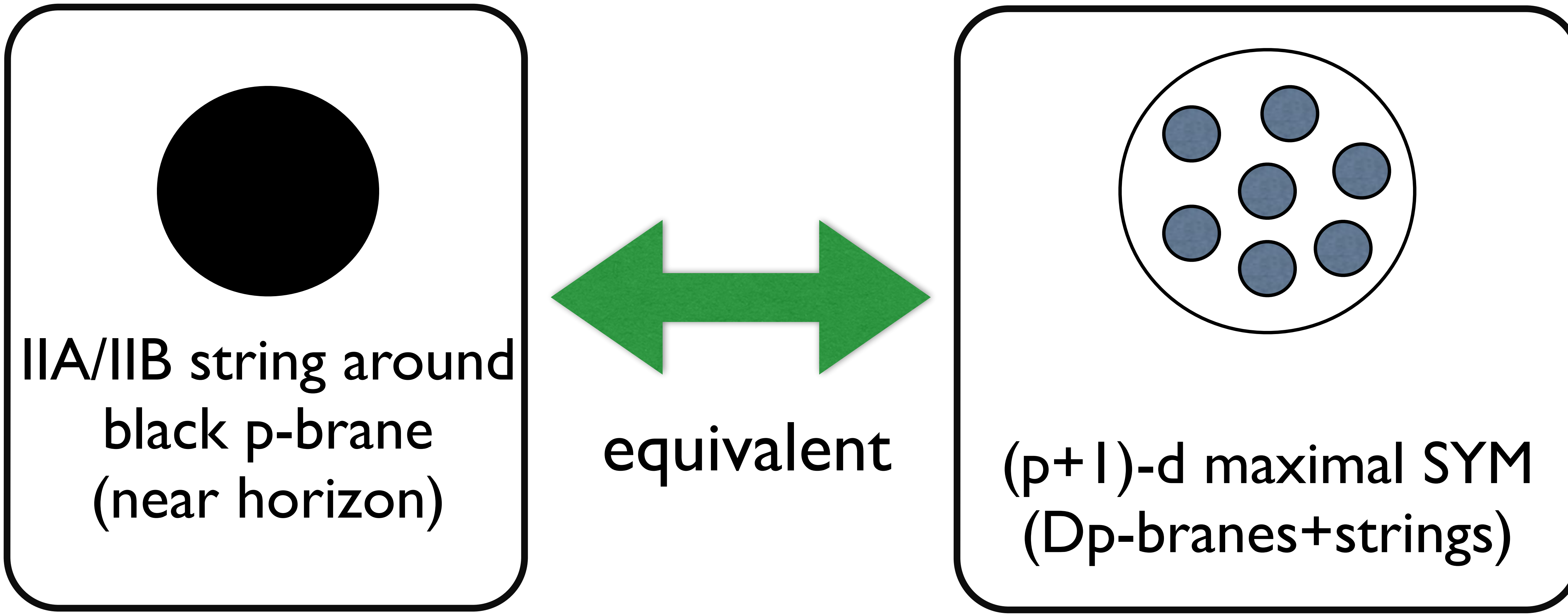
Combine the gauge/gravity duality and numerical techniques (e.g. lattice gauge theory) in order to study *quantum gravity*.

Super Yang-Mills —(Monte Carlo simulation of SYM)→ Quantum Gravity



(Maldacena1997)

Monte Carlo study is possible
but computationally demanding

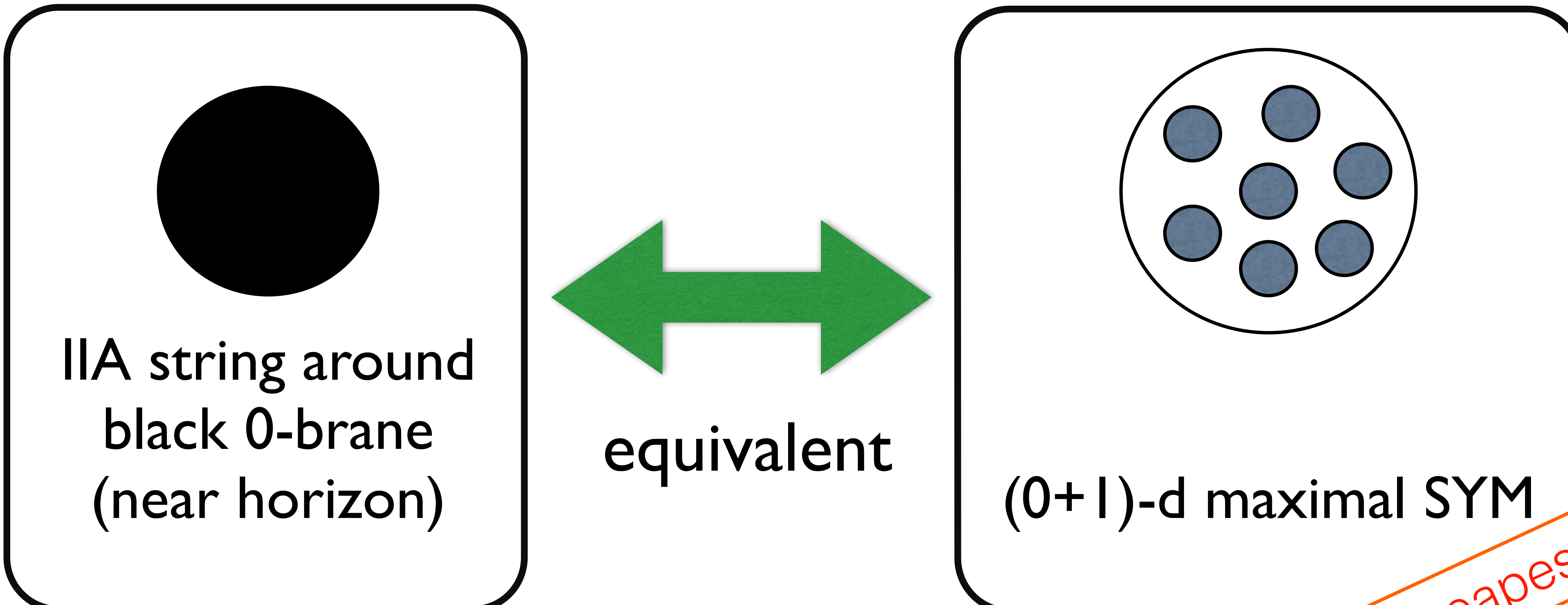


(Maldacena1997, Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

smaller p is easier to simulate on computer.

we study this case

Black hole = matrix model



numerically cheapest

simulation cost $\sim N^6T^{-3}$

high temperature is cheap, low temperature is expensive.

SYM ⟷ STRING

SYM		STRING
$1/\lambda$	⟷	α'/R_{BH}^2
$g_{YM}^2 \sim 1/N$		g_s

$\lambda=\infty, N=\infty$ corresponds to supergravity.

$1/\lambda$ and $1/N$ corrections are interesting.

Which SYM can be simulated?

Possible/Impossible

(without fine tuning; not necessarily lattice)

smaller simulation cost ↑ … ↓ larger simulation cost

	Possible	Impossible
(0+1)-d, (1+1)-d	any number of SUSY, various matter contents	
(2+1)-d	maximal SUSY	less SUSY matter fields
(3+1)-d	without matter (pure $\mathcal{N}=1$) maximal SUSY	SUSY QCD (matter fields)

Simulation methods

key words:

Parameter fine tuning
exact symmetry
sign problem

PARAMETER FINE TUNING PROBLEM

warm-up example :

Pure Yang-Mills (bosonic)

Wilson's lattice gauge theory

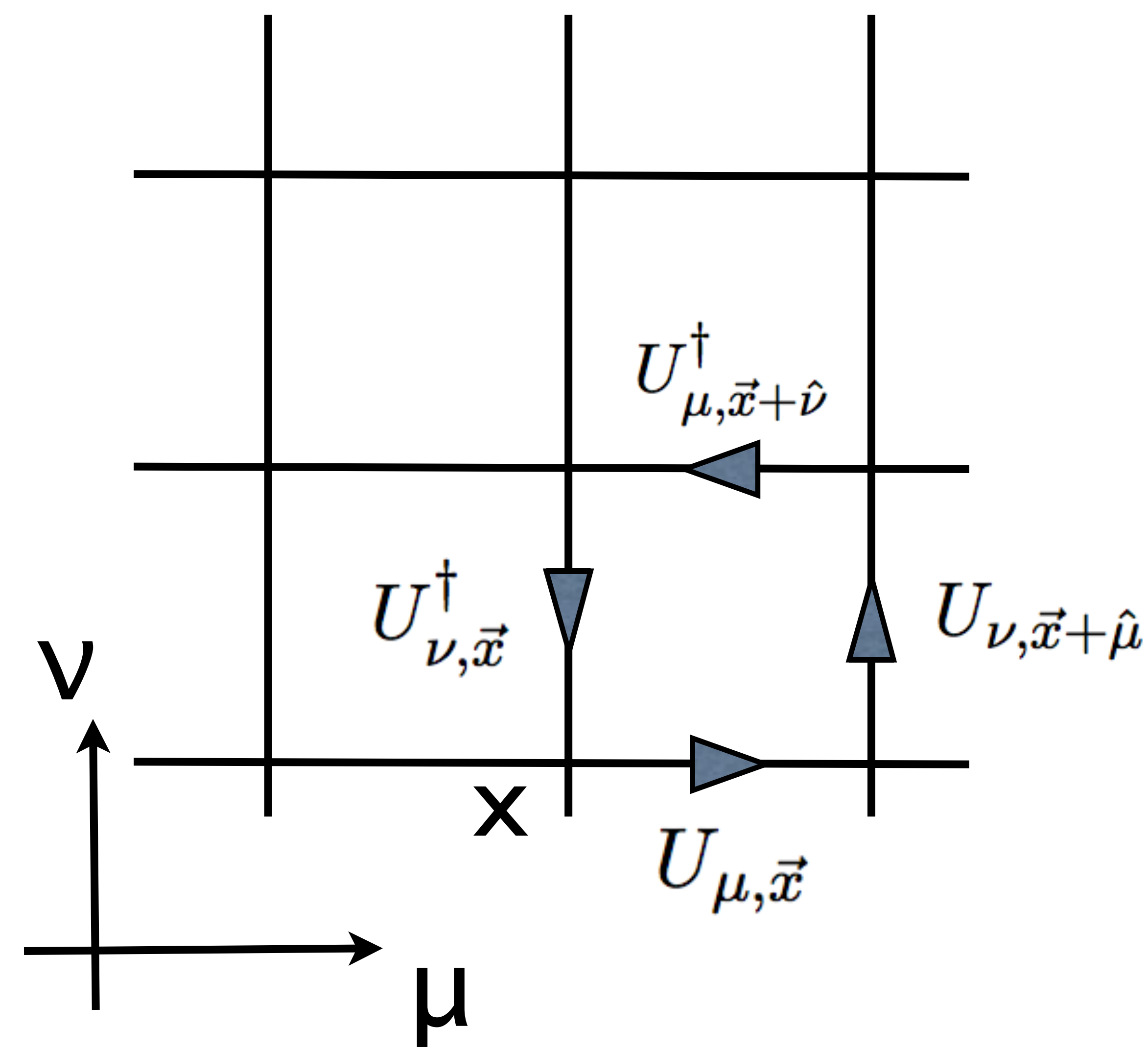
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr\left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}\right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_\mu(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

‘Exact’ symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^\dagger$$

- 90 degree rotation
- discrete translation
- Charge conjugation, parity

These symmetries exist *at discretized level*.

Otherweise raddiative corrections break those symmetries and fine-tuned conuter terms are needed in order to arrive at the correct continuum limit.

‘parameter fine tuning’

Super Yang-Mills

‘No-Go’ for lattice SYM

- SUSY algebra contains infinitesimal translation.

$$\{Q, \bar{Q}\} \sim \partial$$

- Infinitesimal translation is broken on lattice by construction.
- So it is impossible to keep all supercharges exactly on lattice.
- Still it is possible to preserve a part of supercharges. (subalgebra which does not contain ∂)

$$S = \frac{N}{\lambda}\int dt\, Tr\Big\{\frac{1}{2}(D_tX_i)^2 - \frac{1}{4}[X_i,X_j]^2 + \frac{1}{2}\bar{\psi}D_t\psi - \frac{1}{2}\bar{\psi}\gamma^i[X_i,\psi]\Big\}$$

- Matrix quantum mechanics is UV finite.

No fine tuning!

(4d N=4 is also UV finite, but it is because of a cancellation of UV divergences.)

- We don't even have to use lattice. Just fix the gauge & introduce momentum cutoff!
(M.H.-Nishimura-Takeuchi, 2007)

- Take the static diagonal gauge

$$A_0(t) = diag(\alpha_1, \cdots, \alpha_N)/\beta$$

$$\alpha_1, \cdots, \alpha_N \in (-\pi, \pi]$$

- Add Faddeev-Popov term

$$S_{FP} = -\sum_{a \neq b} \log \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|$$

- Introduce momentum cutoff Λ

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_i(n) e^{2\pi i n t/\beta}$$

- continuum limit is $\Lambda \to \infty$.

2 dimensions

lattice can work!

Use other exact symmetries and/or a few exact SUSY to forbid SUSY breaking radiative correction.

- lattice with a few exact SUSY+R-symmetry
  - no fine tuning at perturbative level (Cohen-Kaplan-Katz-Unsal 2003, Sugino 2003, Catterall 2003, D'Adda et al 2005, ...)
  - works even nonperturbatively (←simulation) (Kanamori-Suzuki 2008, M.H.-Kanamori 2009, 2010)

more simulations are going on.

3d & 4d

lattice or non-lattice, depending on theories.

- 3d N=8 : "Hybrid" formulation:

  BMN matrix model + fuzzy sphere

  (Maldacena-Seikh Jabbari-Van Raamsdonk 2002)

- 4d N=1 *pure* SYM : lattice chiral fermion assures SUSY
  (Kaplan 1984, Curci-Veneziano 1986)

- 4d N=4 :
  - again "Hybrid" formulation:Lattice + fuzzy sphere
    (M.H.-Matsuura-Sugino 2010, M.H. 2010)
  - Large-N Eguchi-Kawai reduction(Ishii-Ishiki-Shimasaki-Tsuchiya, 2008)
  - Another Matrix model approach(Heckmann-Verlinde, 2011)
  - recent analysis of 4d lattice:
    Fine tuning is needed, but only for 3 bare lattice couplings.
    (Catterall-Dzienkowski-Giedt-Joseph-Wells, 2011)

simulations of 4d N=4 are ongoing by several groups.

MONTE CARLO METHOD AND SIGN PROBLEM

The principle of Monte-Carlo

- Consider field theory on Euclidean spacetime with the action $S[\phi]$.

- Generate field configurations with <u>probability</u> $e^{-S[\phi]}$
Then,

$$\langle \mathcal{O} \rangle = \frac{\int [d\phi] \mathcal{O}[\phi] e^{-S[\phi]}}{\int [d\phi] e^{-S[\phi]}} \simeq \frac{1}{n} \sum_{i=1}^{n} \mathcal{O}[\phi_i]$$

- <u>Crucial assumption</u>: $e^{-S[\phi]} > 0$

Metropolis algorithm

(Metropolis-Rosenbluth-et al, 1953)

- Consider the Gaussian integral,

$$S[x] = \frac{x^2}{2}, \qquad Z = \int_{-\infty}^{\infty} dx e^{-S[x]}.$$

(1) vary the 'field' x randomly:

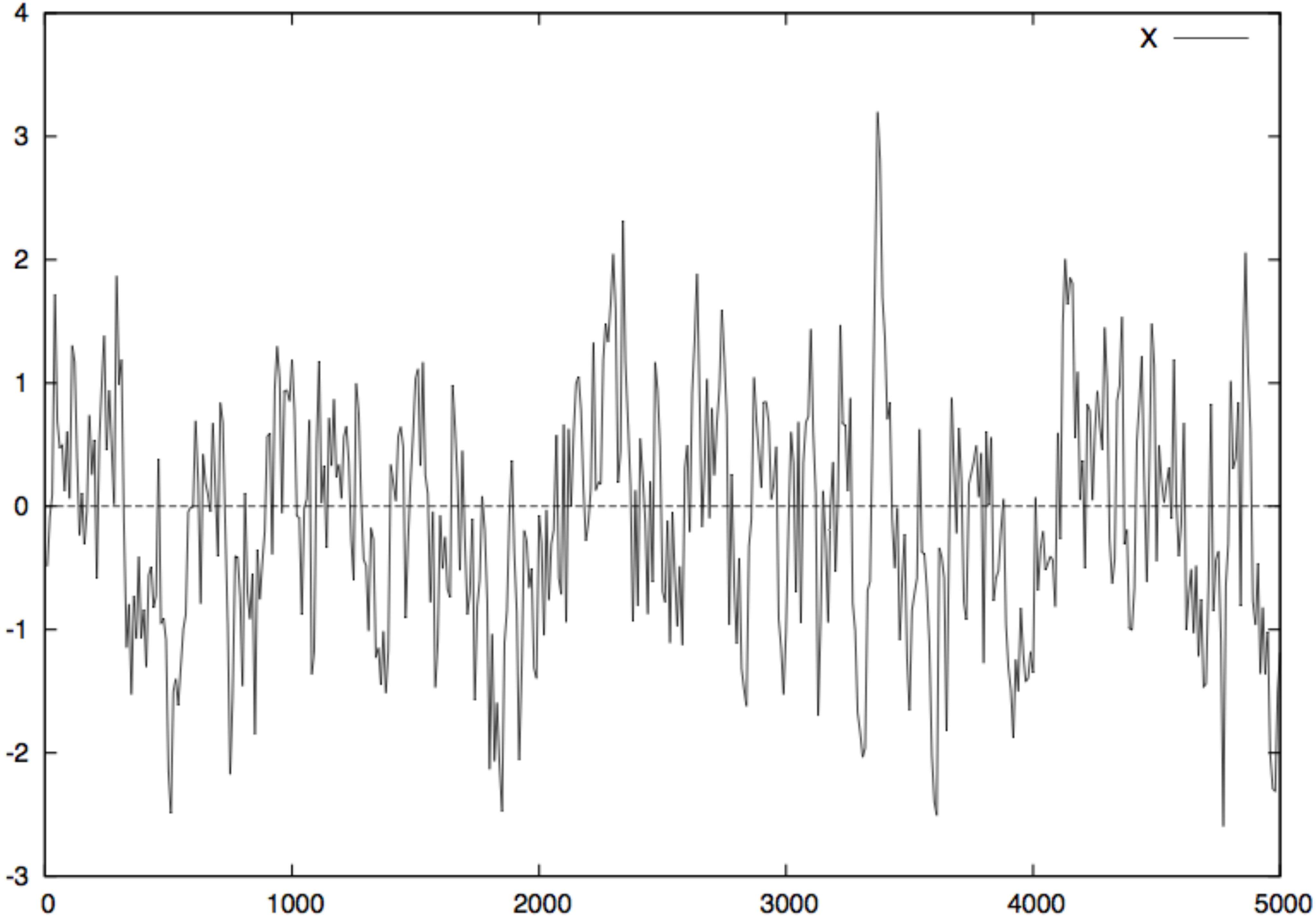
$$x \to x + \Delta x, \qquad -0.5 < \Delta x < 0.5$$

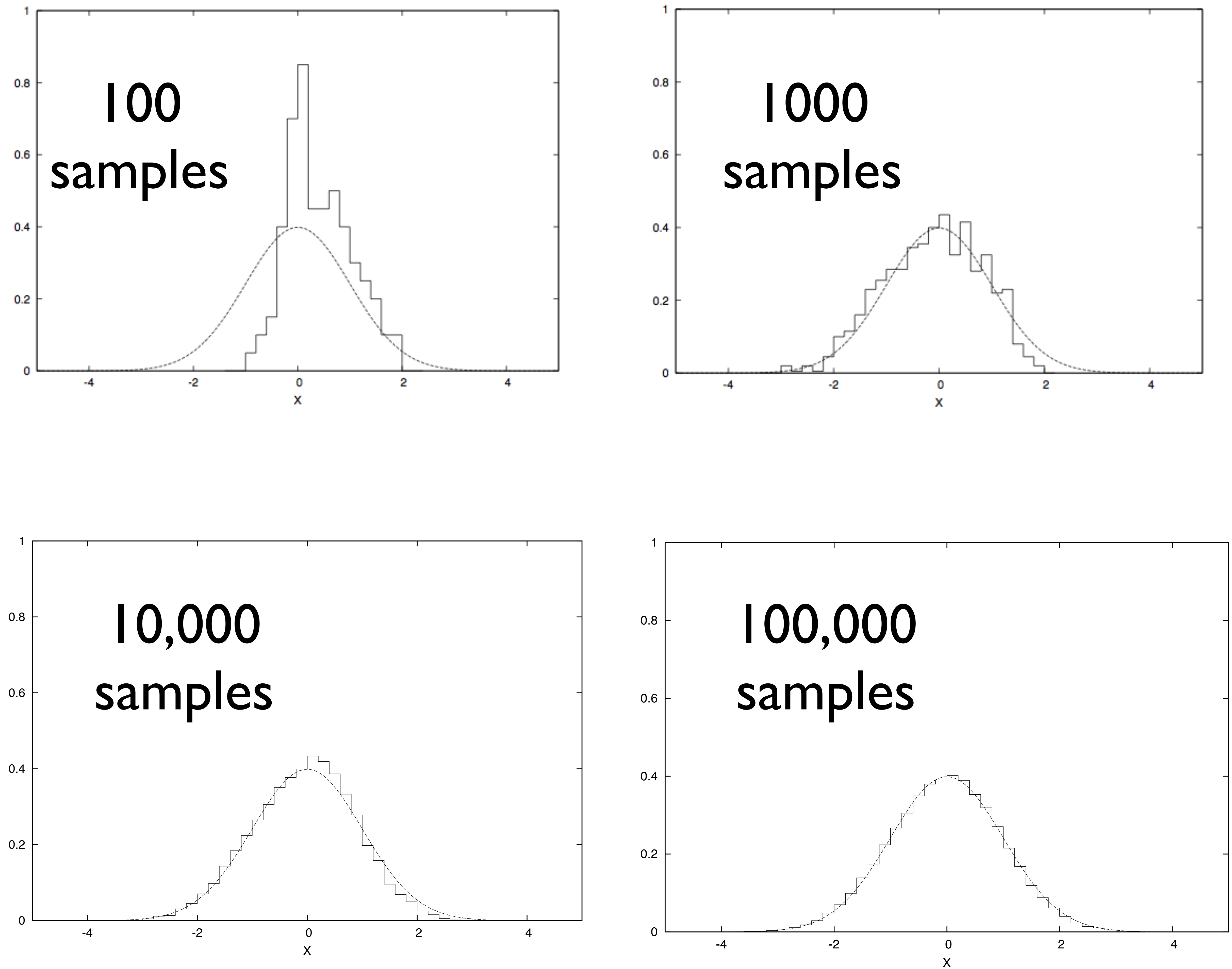
(2) accept the new 'configuration' with a probability

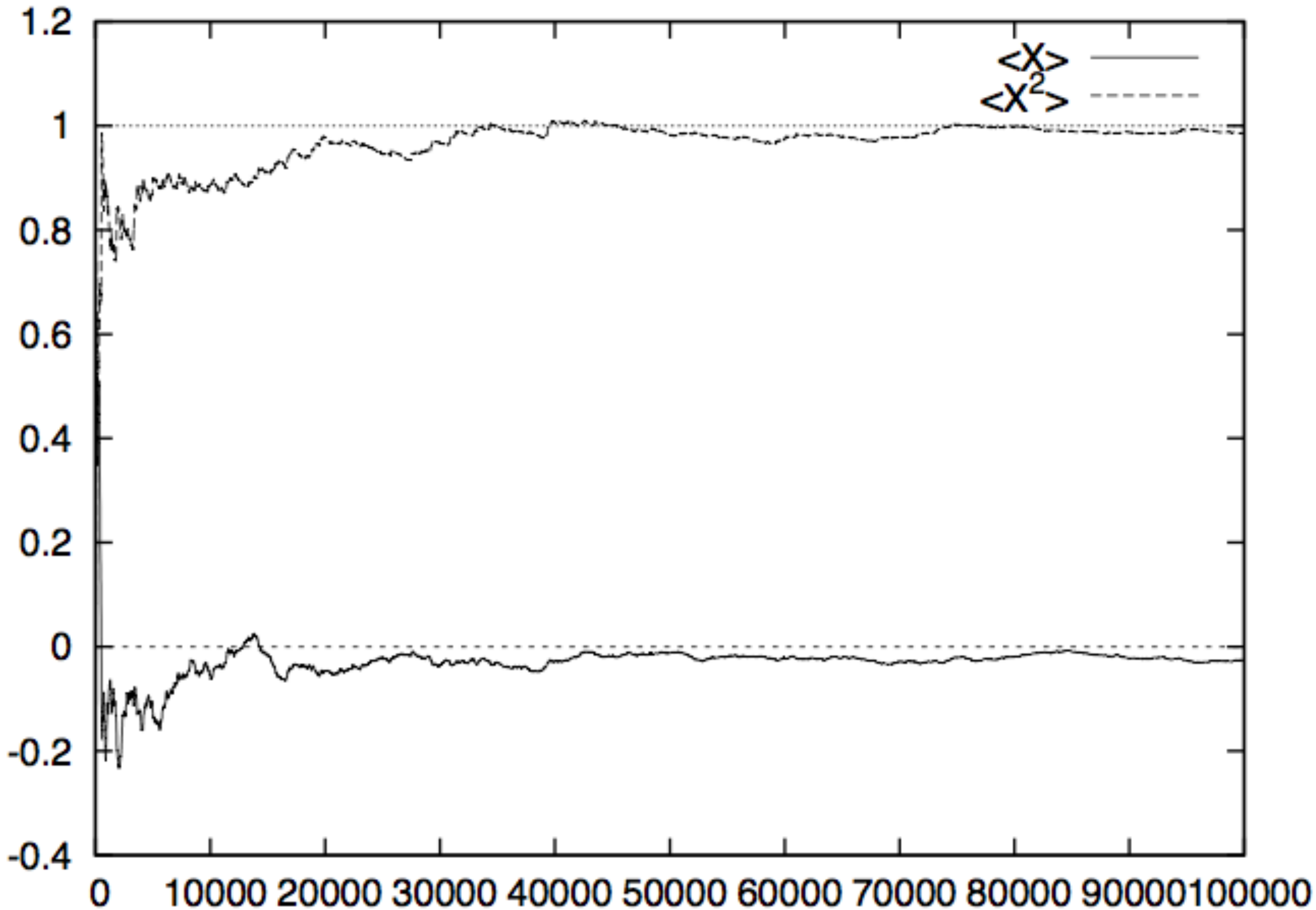
$\min\{1, e^{-\Delta S}\}$ where $\Delta S = S[x + \Delta x] - S[x]$

'Metropolis test'

Initial condition : x=0




100
samples
1000
samples
10,000
samples
100,000
samples
1
0.8
0.6
0.4
0.2
0
-4
-2
0
2
4
x



$$\langle \mathcal{O} \rangle = \frac{\int [d\phi] \mathcal{O}[\phi] e^{-S[\phi]}}{\int [d\phi] e^{-S[\phi]}} \simeq \frac{1}{n} \sum_{i=1}^{n} \mathcal{O}[\phi_i]$$

Fermions

$$S = S_B + S_F, \qquad S_F = \int d^4x \bar{\psi} D \psi$$

$$D = \gamma^\mu (\partial_\mu - i A_\mu)$$

Fermions appear in a bilinear form.

(if not, make them bilinear by introducing auxiliary fields.)

→ can be integrated out *by hand*.

$$\int [dA][d\psi] e^{-S_B[A] - S_F[A,\psi]} = \int [dA] \det D[A] \cdot e^{-S_B[A]}$$

※ Pfaffian in the case of Maximal SYM

So, simply use the 'effective action',

$$S_{eff}[A] = S_B[A] - \log\det D[A]$$

Crucial assumption: det D > 0

Sign problem (phase problem)

- 'Probability' must be real positive.
- Life is sometimes hard... path integral weight e^{-S} can be *complex*! (after the Wick rotation)
    - Chern-Simons term (pure imaginary!)
    - Finite baryon chemical potential
    - Yukawa coupling
    - Super Yang-Mills

det D is complex (→ Finite baryon chemical potential, Yukawa coupling, Super Yang-Mills)

Such path integral measures cannot be generated by the Monte-Carlo method :(

reweighting method

- Use the 'phase-quenched' effective action

$$S_{eff}[A] = S_B[A] - \log|\det D[A]|$$

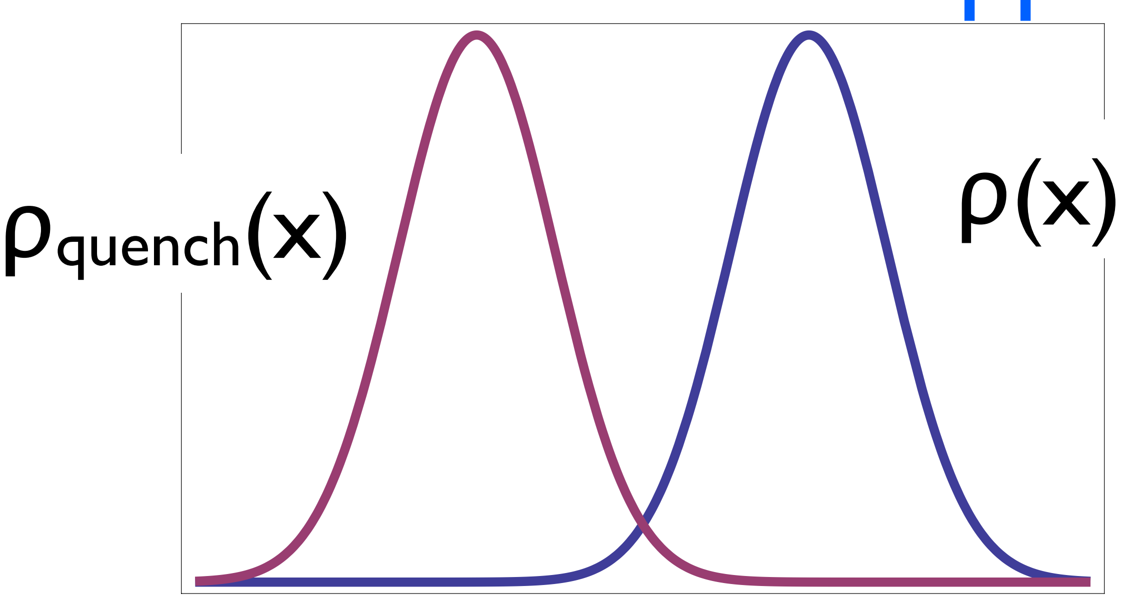
- Phase can be taken into account by the 'phase reweighting' :

$$\begin{aligned}\langle \mathcal{O} \rangle &= \frac{\int [dA] \det D \cdot e^{-S_B} \cdot \mathcal{O}}{\int [dA] \det D \cdot e^{-S_B}} \\ &= \frac{\int [dA] (phase) \cdot |\det D| \cdot e^{-S_B} \cdot \mathcal{O} / \int [dA] |\det D| \cdot e^{-S_B}}{\int [dA] (phase) \cdot |\det D| \cdot e^{-S_B} / \int [dA] |\det D| \cdot e^{-S_B}} \\ &= \frac{\langle (phase) \cdot \mathcal{O} \rangle_{phase\ quench}}{\langle (phase) \rangle_{phase\ quench}}\end{aligned}$$

usually the reweighting does not work in practice...

- violent phase fluctuation
  → both numerator and denominator becomes almost zero.   0/0 = ??
- vacua of full and phase-quenched model can be completely different.

'overlapping problem'



$$\rho(x) \propto \rho_{quench} \cdot \langle phase \rangle_x$$

Miracles happen in SYM!

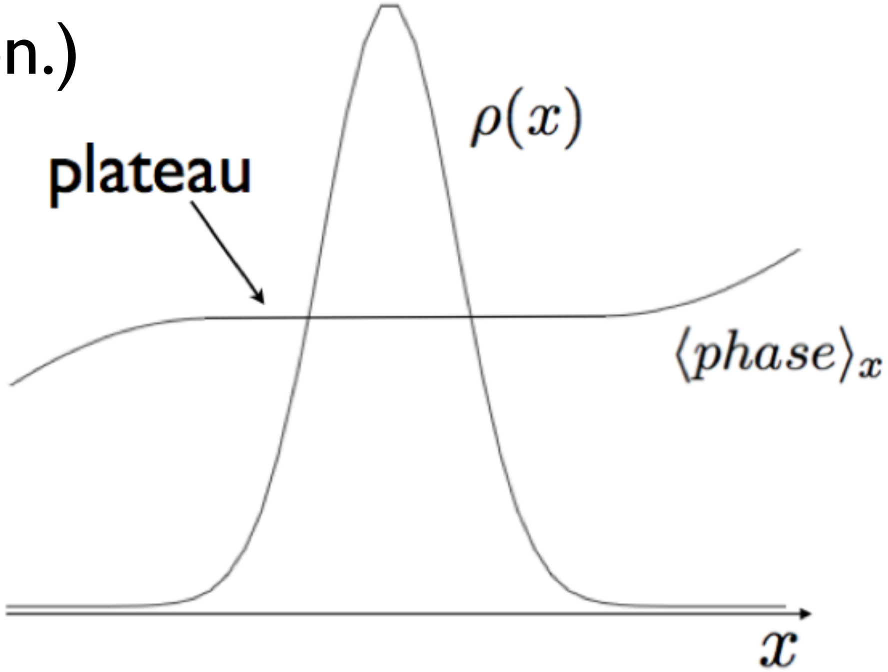
- Almost no phase except for very low temperature.

  (Anagnostopoulos-M.H.-Nishimura-Takeuchi 2007,
  Catterall-Wiseman 2008, Catterall et al 2011.)

- Even when the phase fluctuates,
  phase quench gives right answer.
  ('right' in the sense it reproduces gravity prediction.)

- Can be justified numerically.

  (M.H.-Nishimura-Sekino-Yoneya 2011,
  Buchoff-M.H.-Matsuura, in progress)

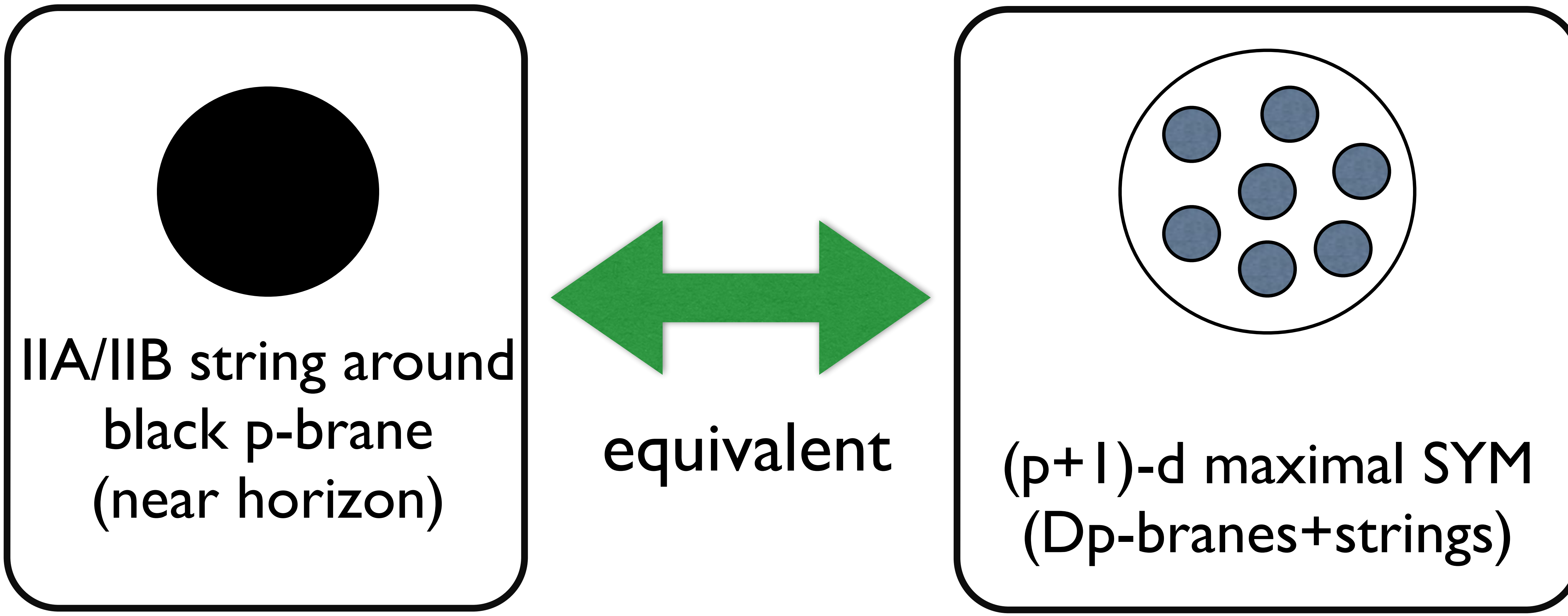


Summary of this part

- Maximal SYM in 1,2,3,4-dimensions can be studied by Monte Carlo.
- For 1,2 and 4-d, simulations are ongoing.
- Sign Problem? No Problem.

  (But no theoretical justification for the moment)

GAUGE/GRAVITY DUALITY



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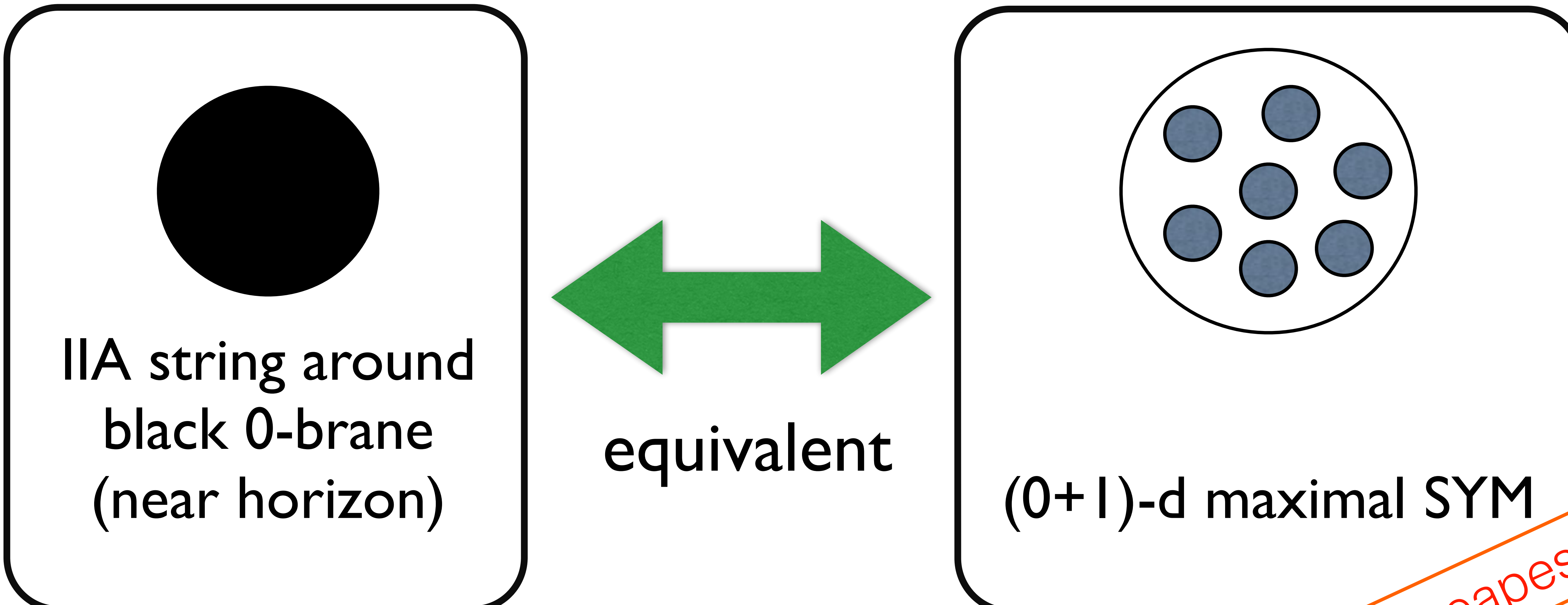

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simulation cost $\sim N^6T^{-3}$

high temperature is cheap, low temperature is expensive.

SYM		STRING
$1/\lambda$	$\Longleftrightarrow$	α'/R_{BH}^2
$g_{YM}^2 \sim 1/N$		g_s

$\lambda=\infty$, $N=\infty$ corresponds to supergravity.

$1/\lambda$ and $1/N$ corrections are interesting.

But first of all, we have to test this conjecture.

D0-brane quantum mechanics

$$S = \frac{N}{\lambda}\int dt\, Tr\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i[X_i, \psi]\Big\}$$

- Matrix model of M-theory (Banks-Fishler-Shenker-Susskind, 1996 de Wit-Hoppe-Nicolai, 1988)
- gauge/gravity duality → dual to black 0-brane

It should reproduce thermodynamics of black 0-brane.

effective dimensionless temperature $T_{eff} = \lambda^{-1/3} T$

strong coupling = low temperature → more simulation cost

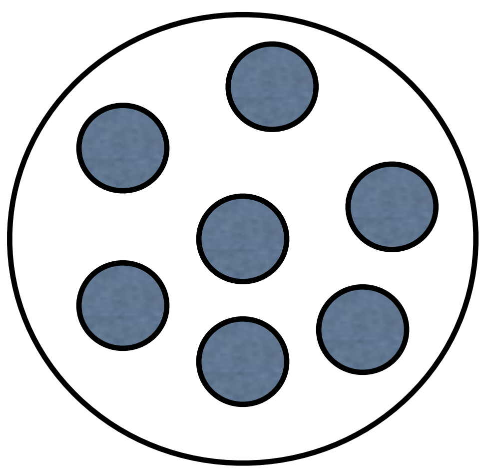
problem with flat direction

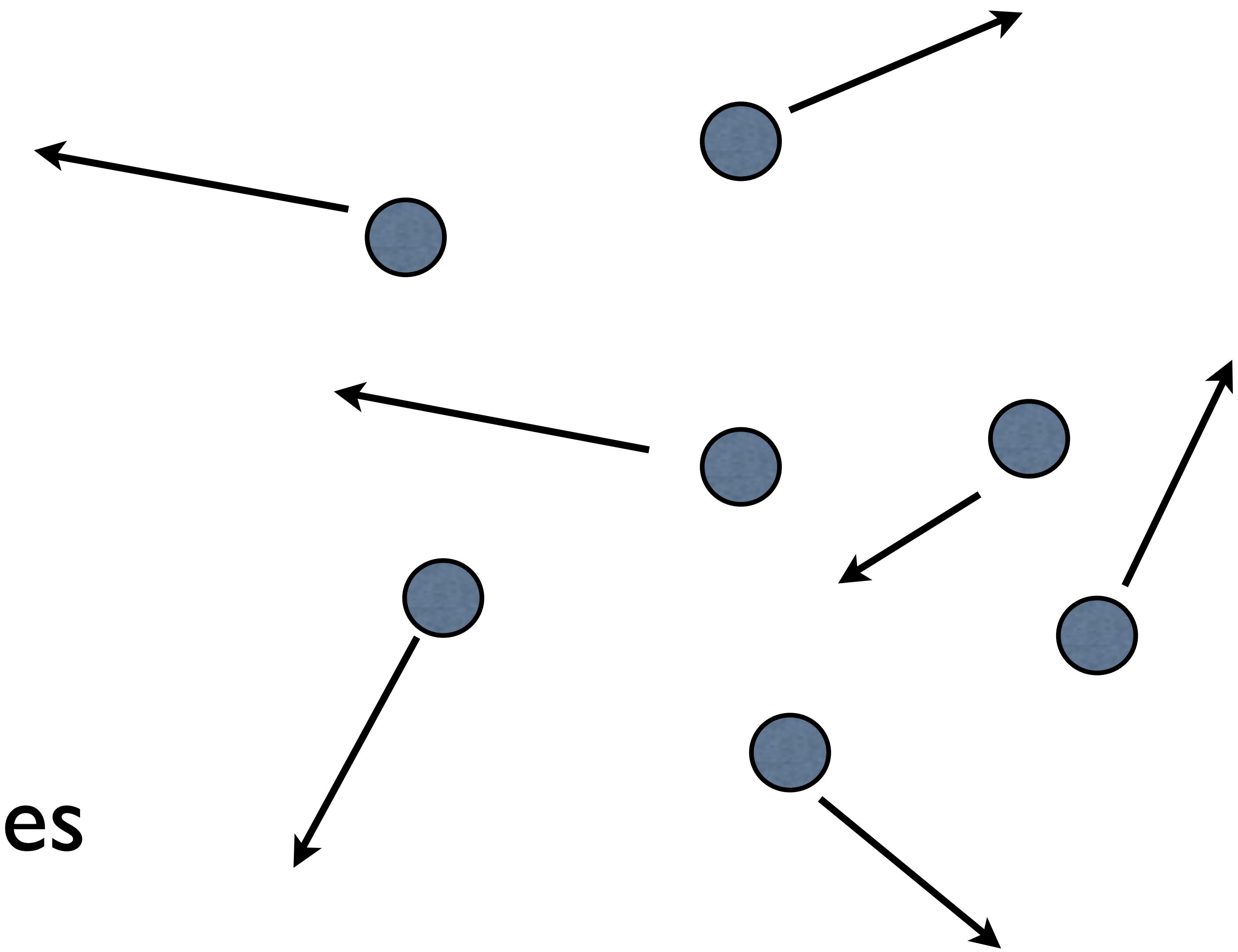
$$S = \frac{N}{\lambda}\int dt\, Tr\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i[X_i, \psi]\Big\}$$

There is a flat direction even at quantum level.

$$[X_i, X_j] = 0$$

‘eigenvalues’ = position of D0-branes





bound state of eigenvalues
= black hole

flat direction
$\sim$ gas of D0-branes

One has to restrict the path integral
in order to extract the black hole.

Confirmation at *classical* string level

($N=\infty$, $g_s=0$)

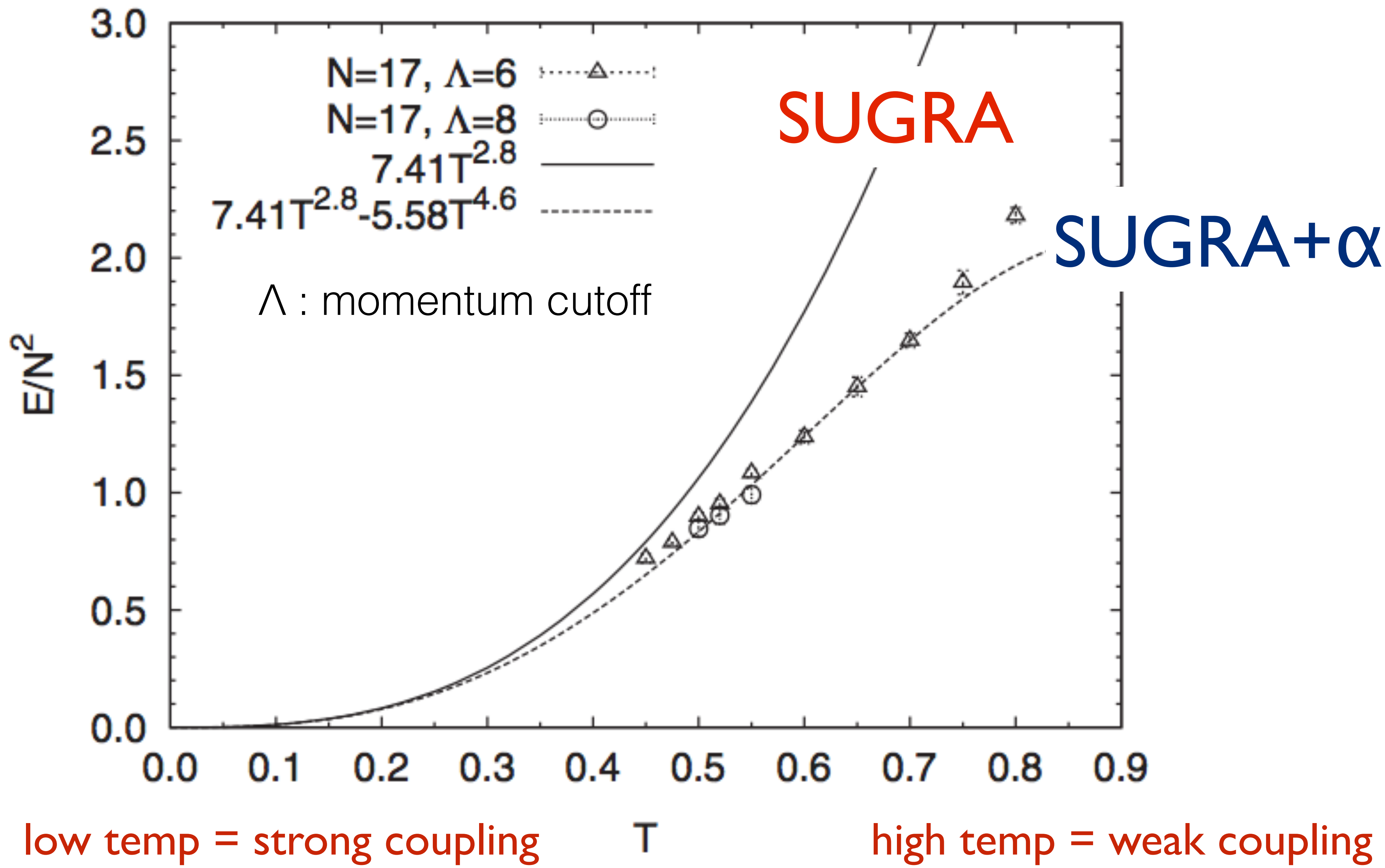
How to tame the flat direction

In string theory, this BH is stable at $g_s=0$.

In the gauge theory, bound state should become stabler as N becomes larger

We can confirm this expectation numerically.

solution: take N large enough.



Anagnostopoulos-M.H.-Nishimura-Takeuchi, PRL 2008
M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

($\lambda^{-1/3}T$: dimensionless effective temperature)

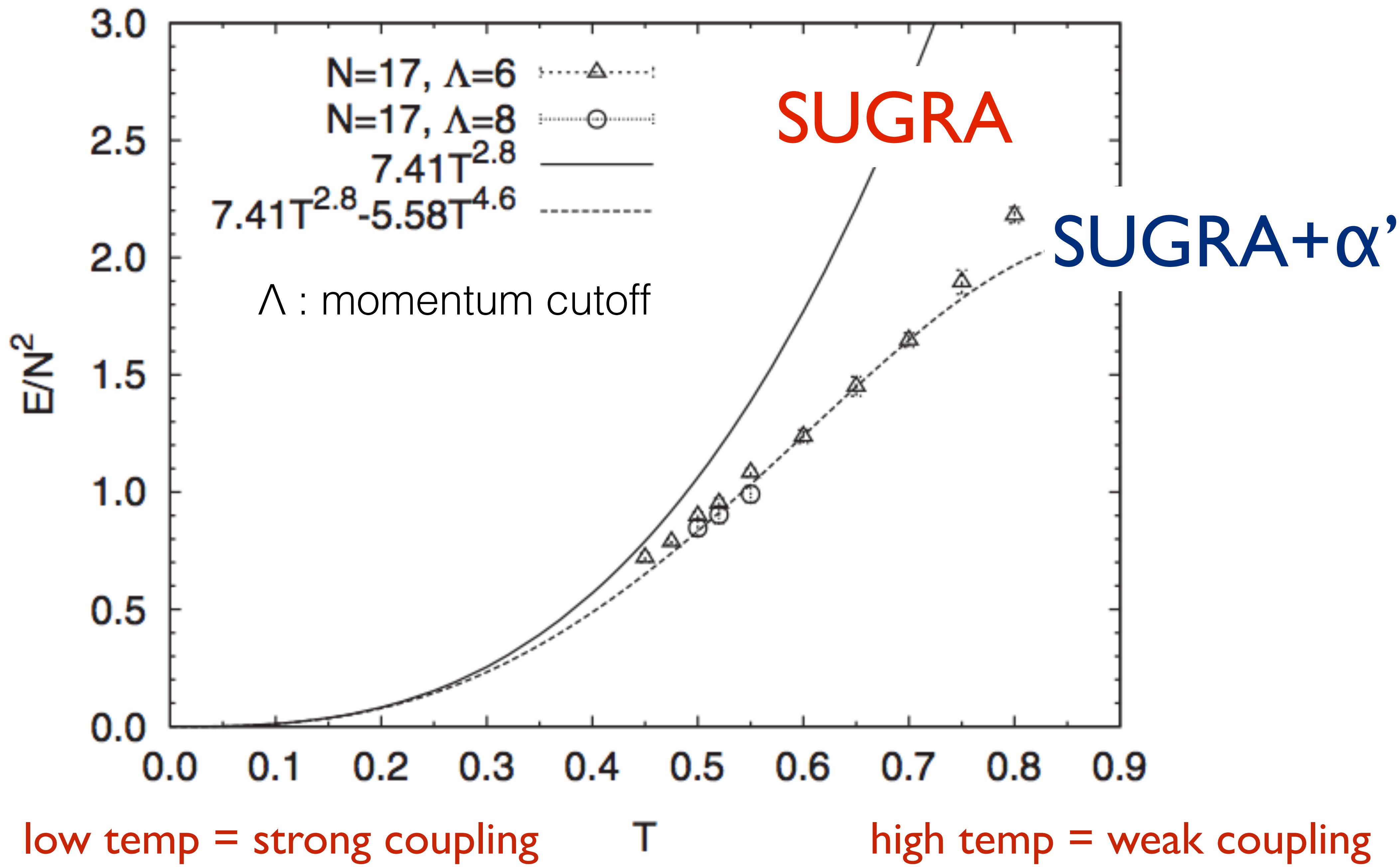
(see also papers by Catterall-Wiseman and by Kadoh)

α' correction

- deviation from the strong coupling (low temperature) corresponds to the α' correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2 = 7.41T^{2.8} - 5.58T^{4.6}$

(4.6 = 2.8 + 1.8)
prediction by string

'prediction' by SYM simulation



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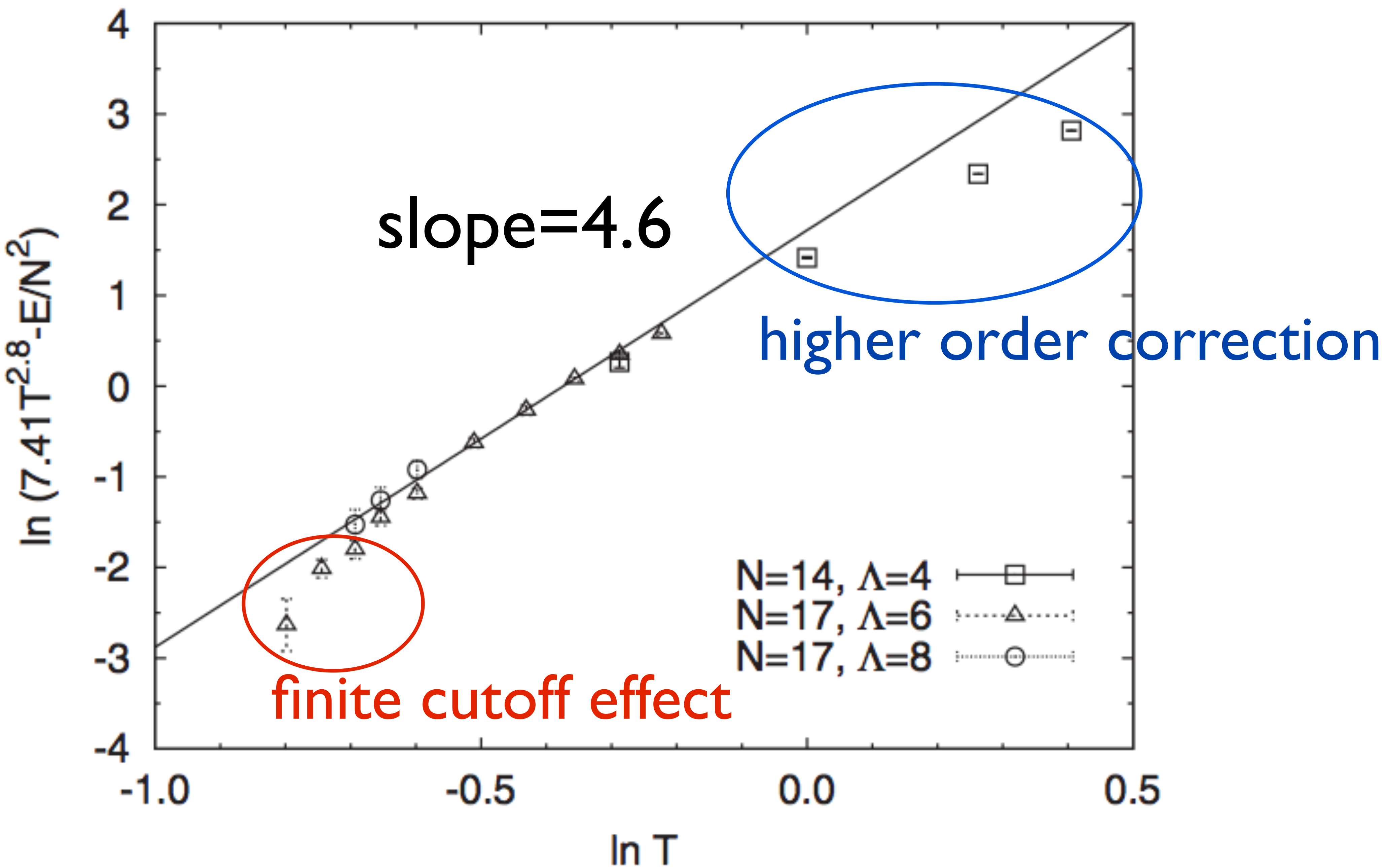
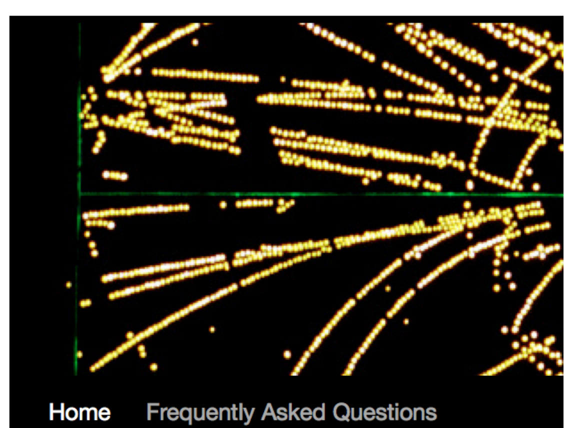


M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

Confirmation at *quantum* string level

(finite-N)

Not Even Wrong



This Week's Hype

Posted on May 25, 2014 by woit

Peter Woit's "This week's Hype"
on May 25, 2014

g_s correction in the gravity side (Y. Hyakutake, PTEP 2013)

$$
\begin{aligned}
E/N^2 = {} & 7.41T^{2.8} - 5.58T^{4.6} + \ldots. \\
& +(1/N^2)(-5.77T^{0.4} + aT^{2.2} + \ldots) \\
& +(1/N^4)(bT^{-2.6} + cT^{-2.0} + \ldots) \\
& + \ldots..
\end{aligned}
$$

- We study $T \sim 0.1$, so that unknown part is negligible.

How to tame the flat direction

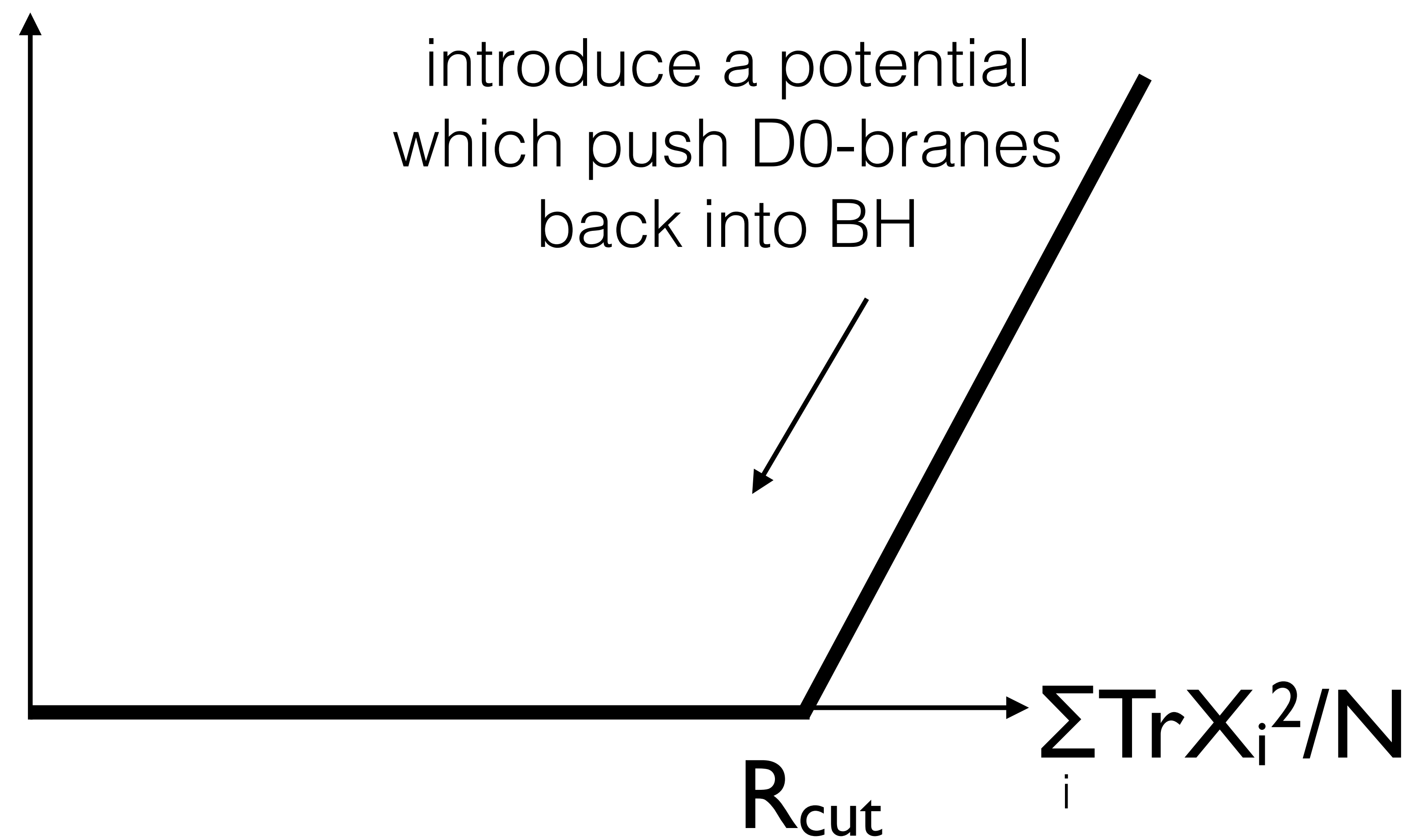
We have to consider small values of N.

$\Rightarrow$ FLAT DIRECTION IS BACK!

It is unavoidable, because we want to study an *unstable* object — evaporating BH.

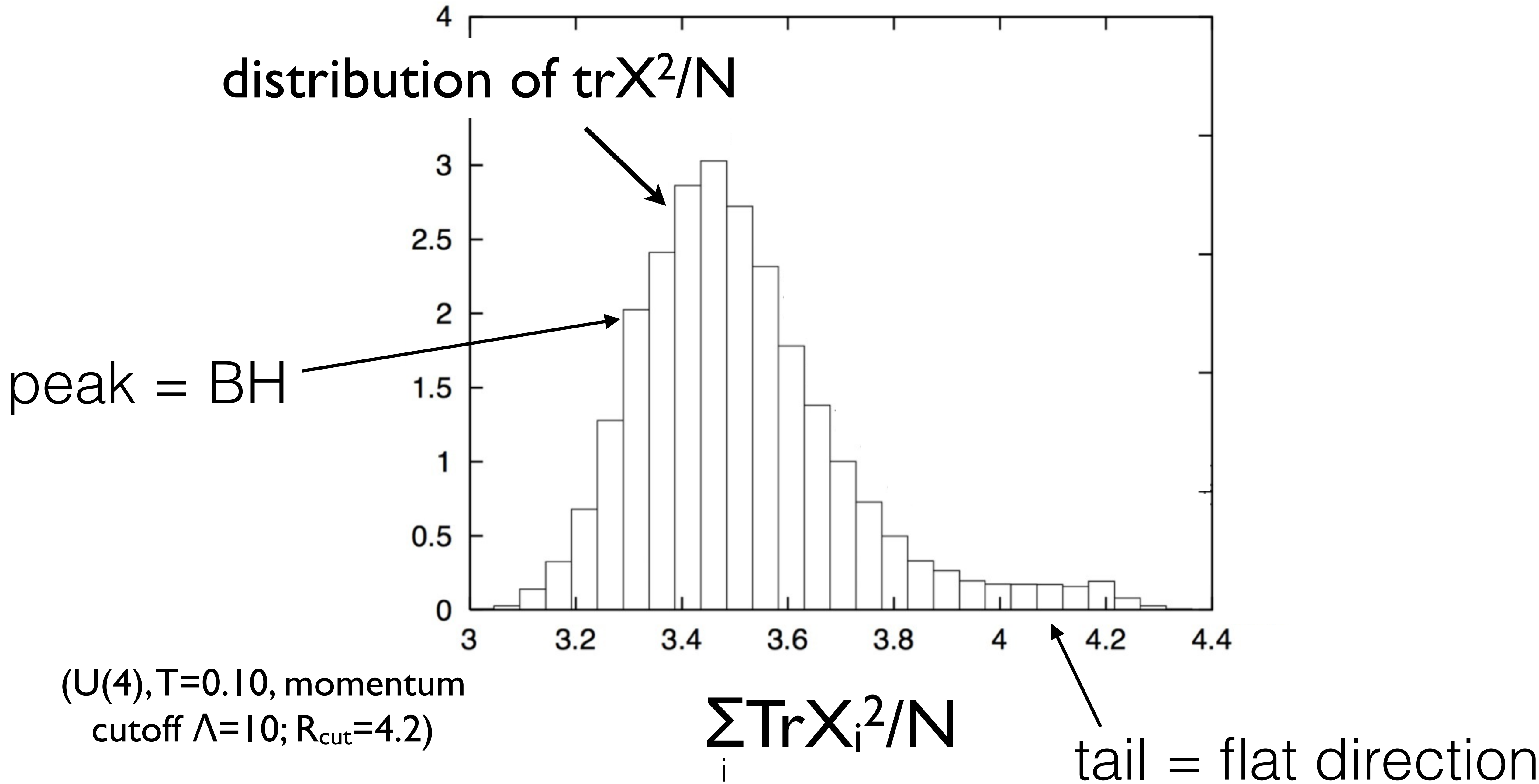
A practical solution (1)

Put the BH in a box.



add potential $\gamma \int dt \, |\mathrm{Tr}X^2/N - R_{cut}|$ at $\mathrm{Tr}X^2/N > R_{cut}$

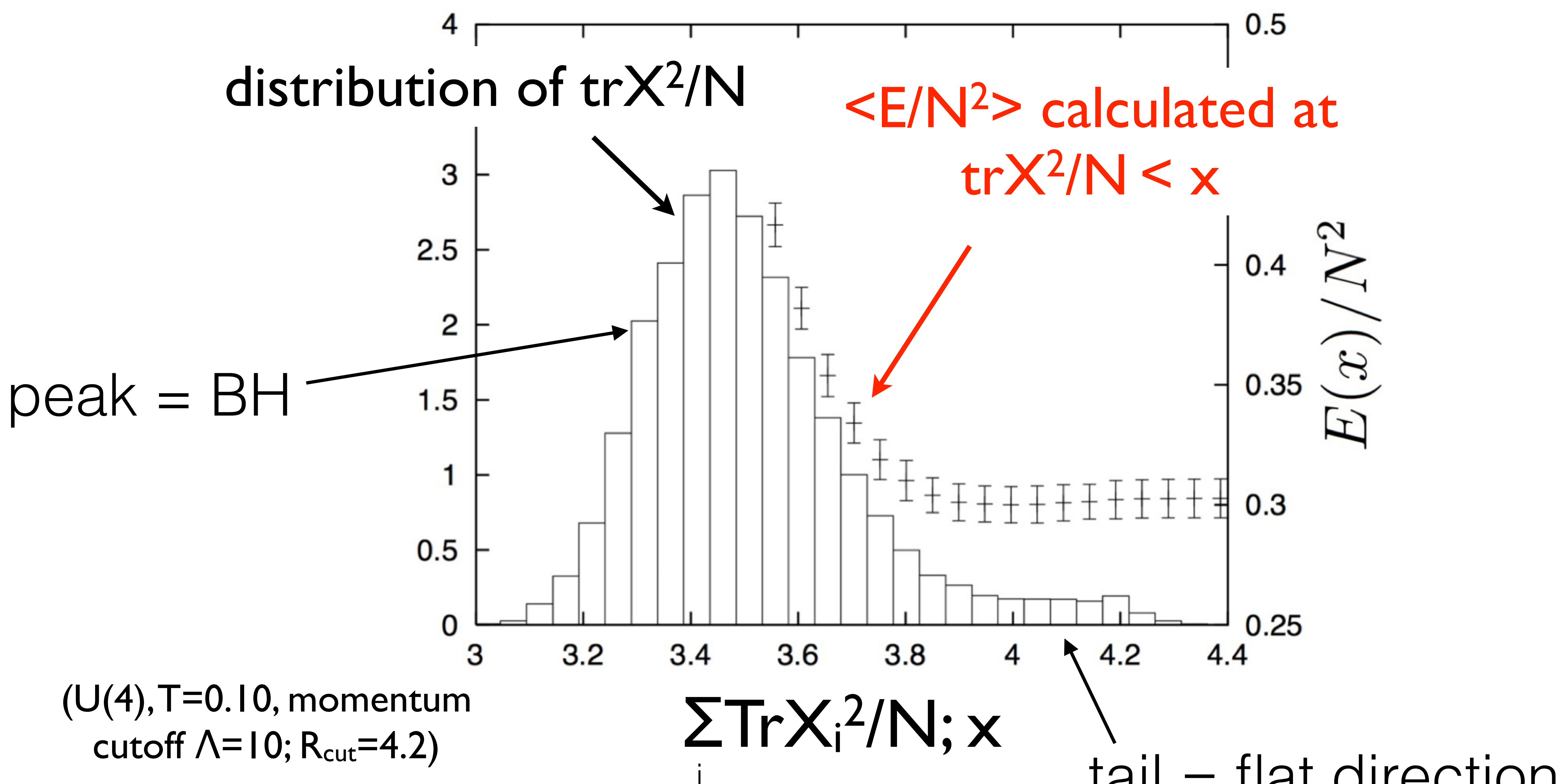
A practical solution (2)



(U(4), T=0.10, momentum cutoff Λ=10; R_{cut}=4.2)

Where is the border of BH?

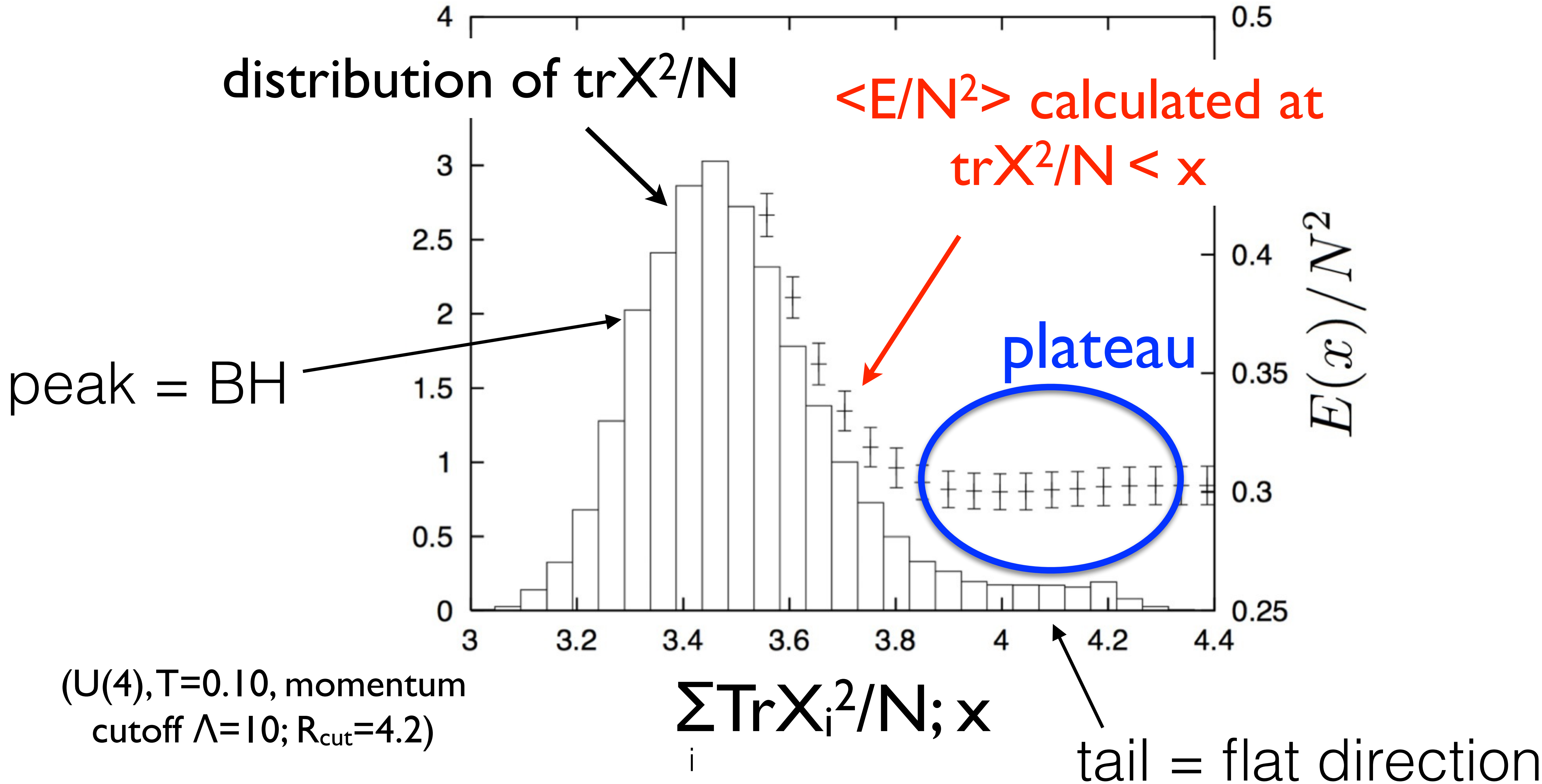
A practical solution (3)



(U(4), T=0.10, momentum cutoff Λ=10; R_{cut}=4.2)

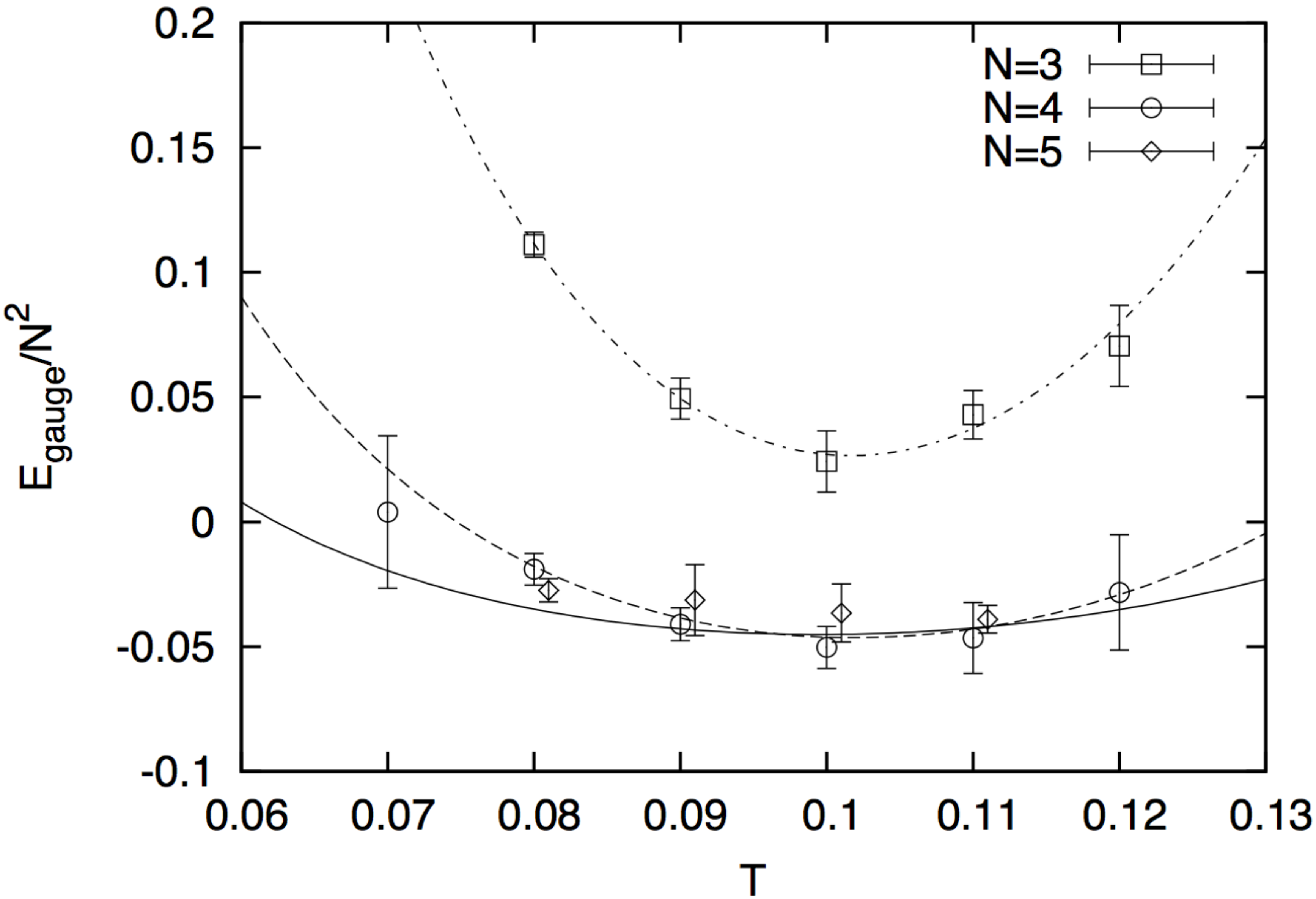
Where is the border of BH?

A practical solution (4)



(U(4), T=0.10, momentum cutoff Λ=10; R_{cut}=4.2)

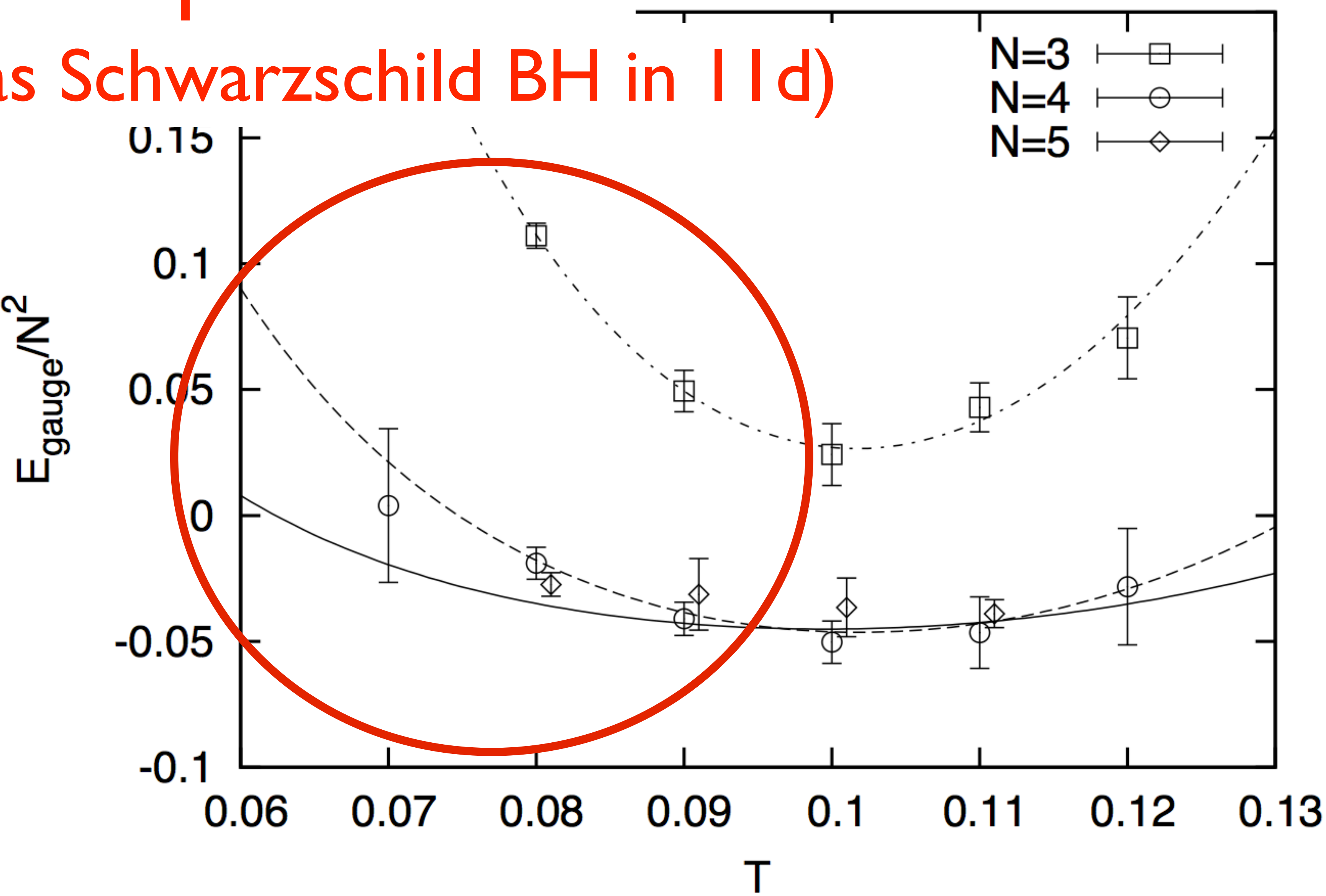
value @ plateau = energy of BH



M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

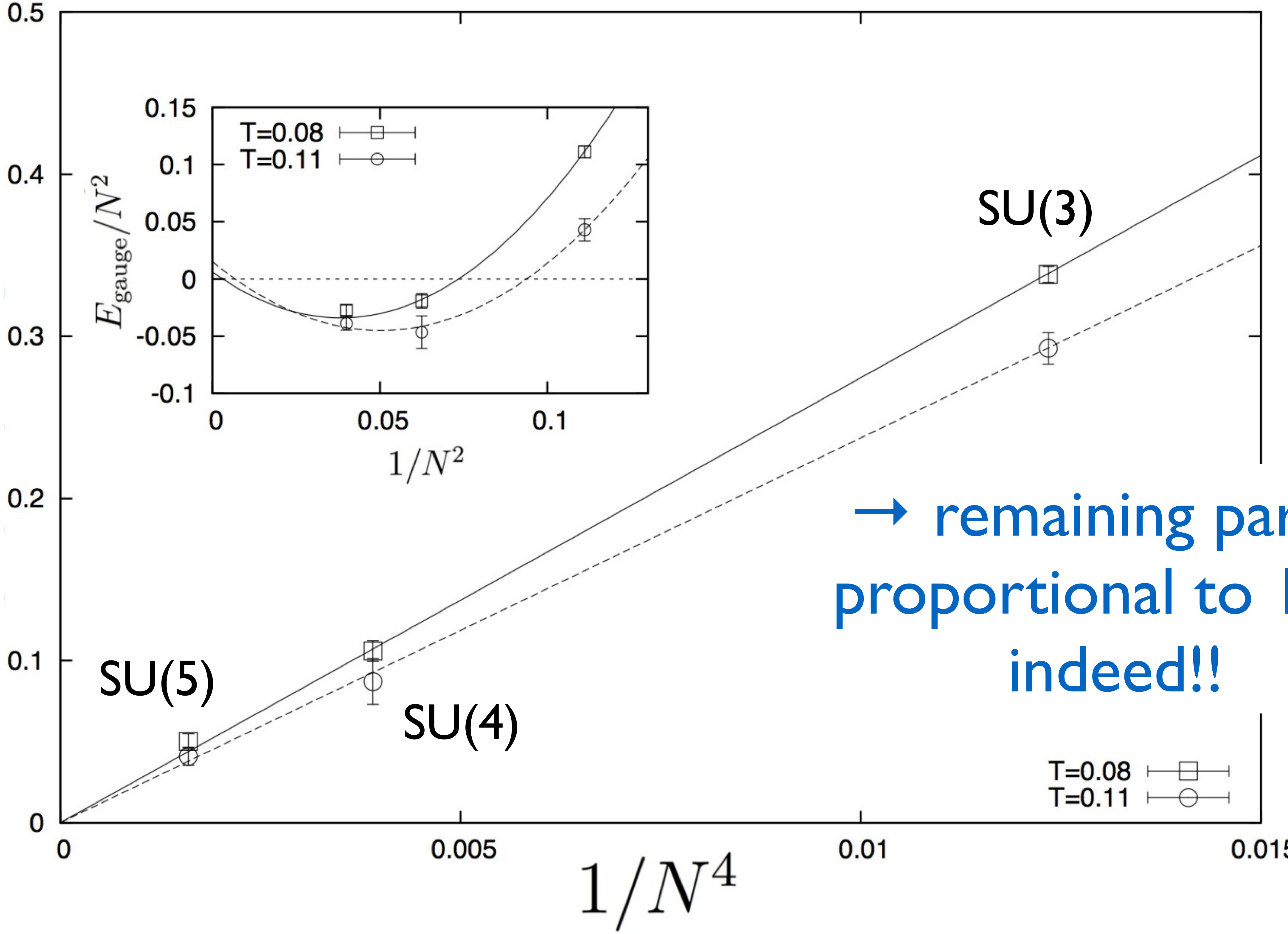
Negative specific heat

(the same as Schwarzschild BH in 11d)



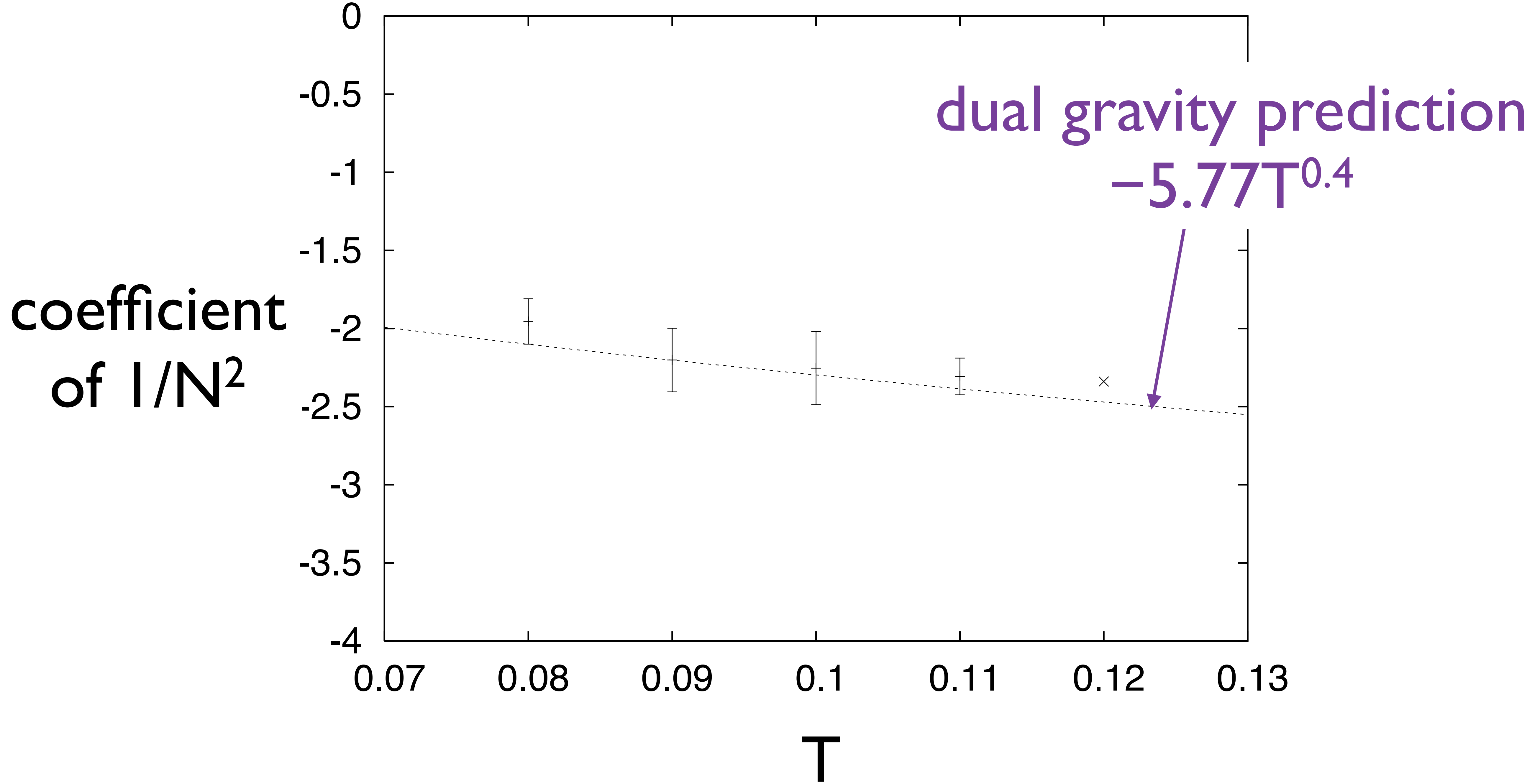
M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

E/N^2 - $(7.41T^{2.8}$-$5.77T^{0.4}/N^2)$ vs. $1/N^4$



→ remaining part is proportional to $1/N^4$ indeed!!

M.H.-Hyakutake-Ishiki-Nishimura, Science 2014



M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

Correlation functions
(GKPW relation)




G K P W

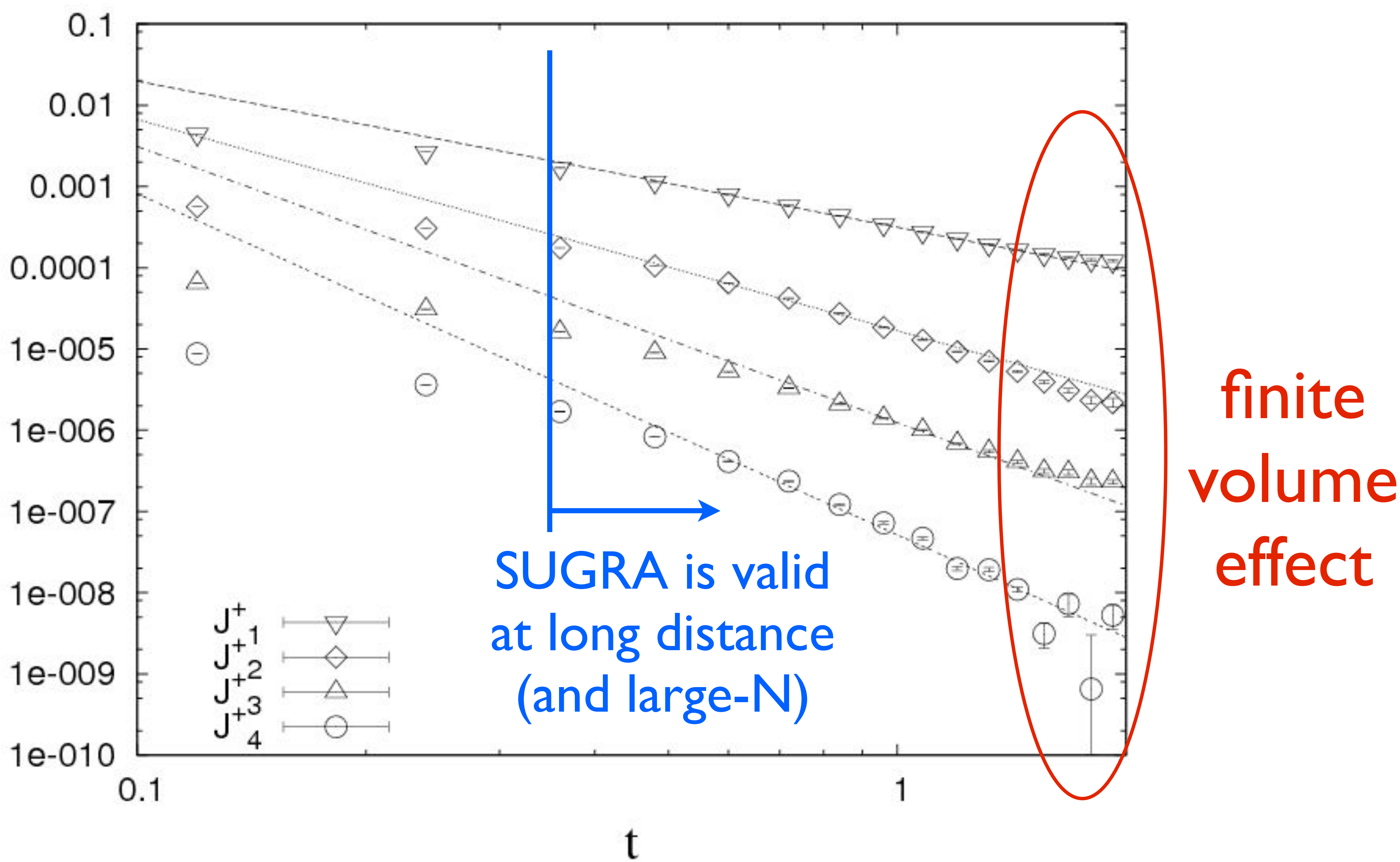
- Recipe to calculate the correlation function at <u>large-N</u> and <u>strong coupling</u> from supergravity
  (Gubser-Klebanov-Polyakov 1998, Witten1998)

- Similar relation holds also in D0-brane theory.

(Sekino-Yoneya 1999)

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \sim t^{-p}$$

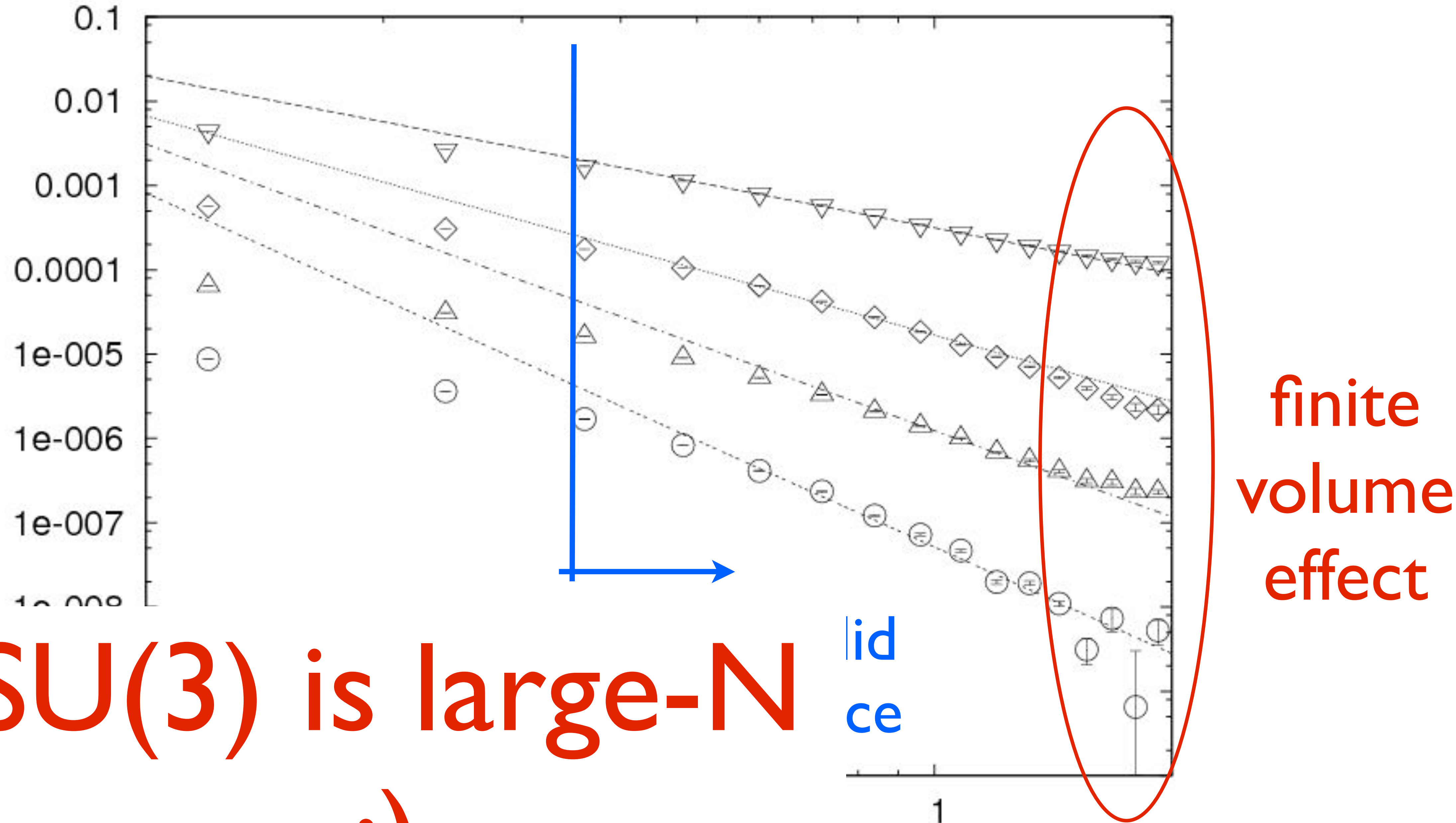
calculable via SUGRA



two-point functions, SU(3), pbc

$$J^{+ij}_{l;i_1\cdots i_l} \equiv \frac{1}{N} \cdot Str\left(F_{ij} X_{i_1} \cdots X_{i_l}\right) \qquad (F_{ij} \equiv -i[X_i, X_j])$$

(M.H.-Nishimuea-Sekino-Yoneya 2009,2011)



SU(3) is large-N
:)

two-point functions, SU(3), pbc

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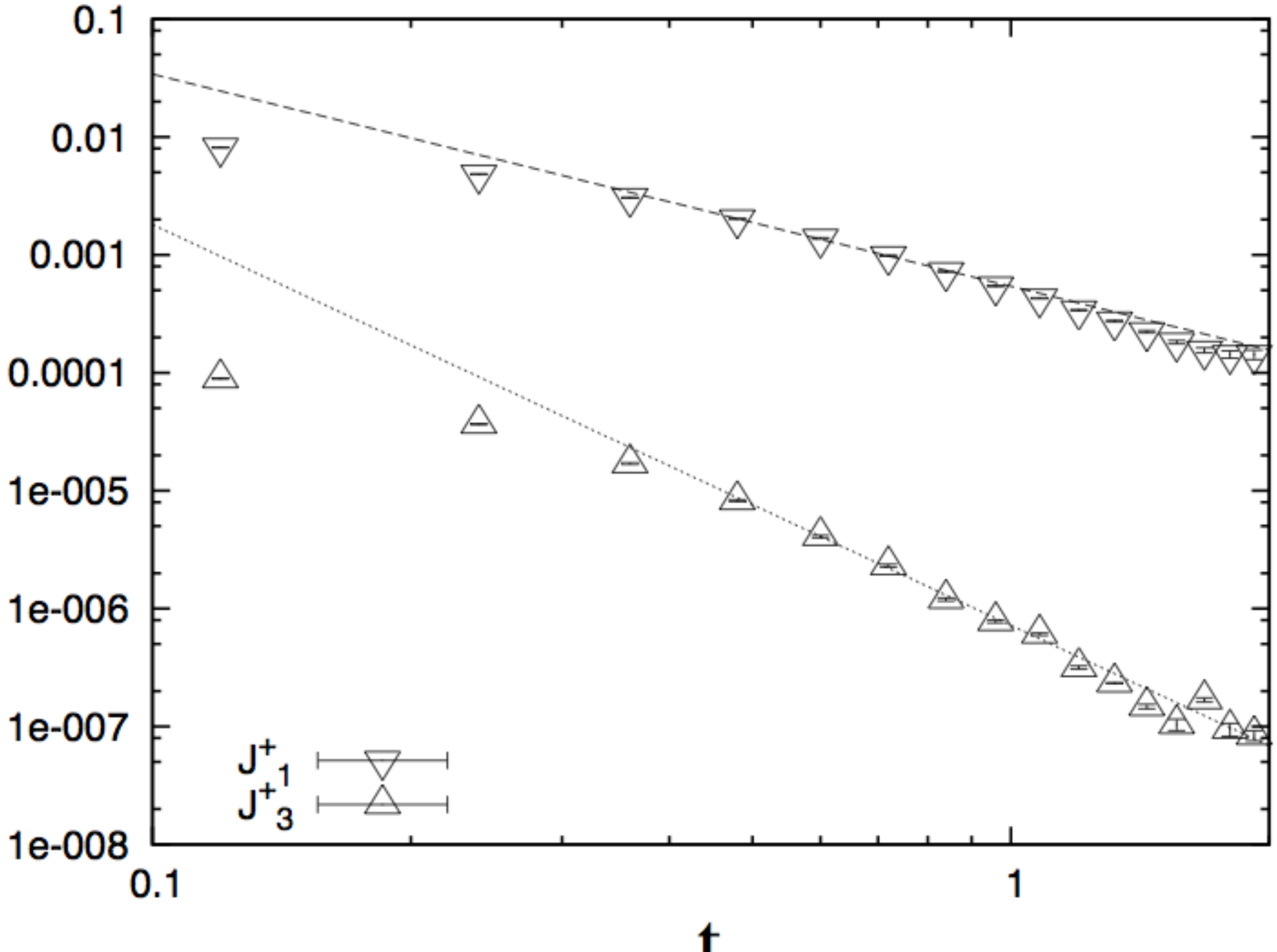
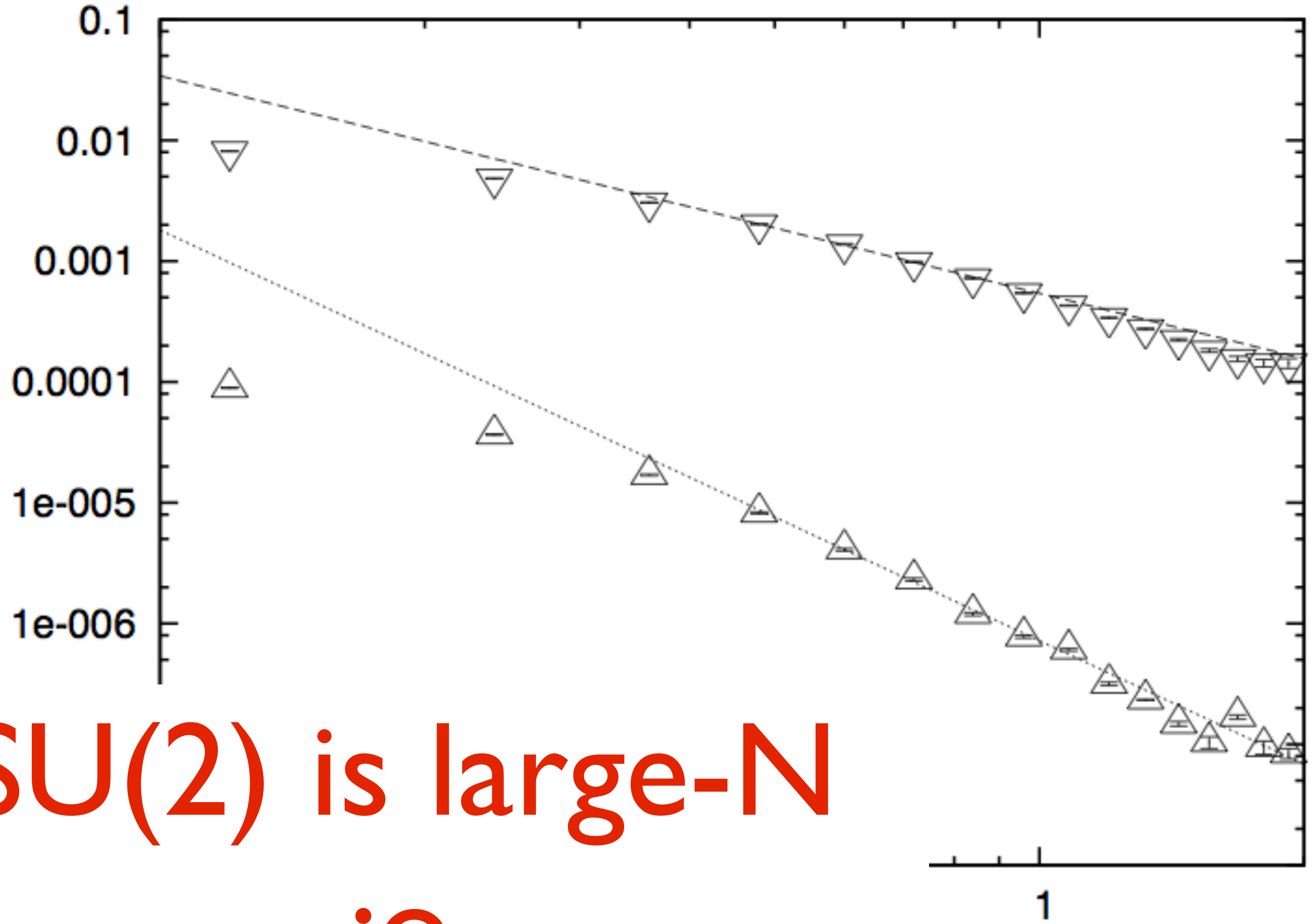


two-point functions, SU(2), pbc

(M.H.-Nishimura-Sekino-Yoneya 2011)



SU(2) is large-N

:o

two-point functions, SU(2), pbc

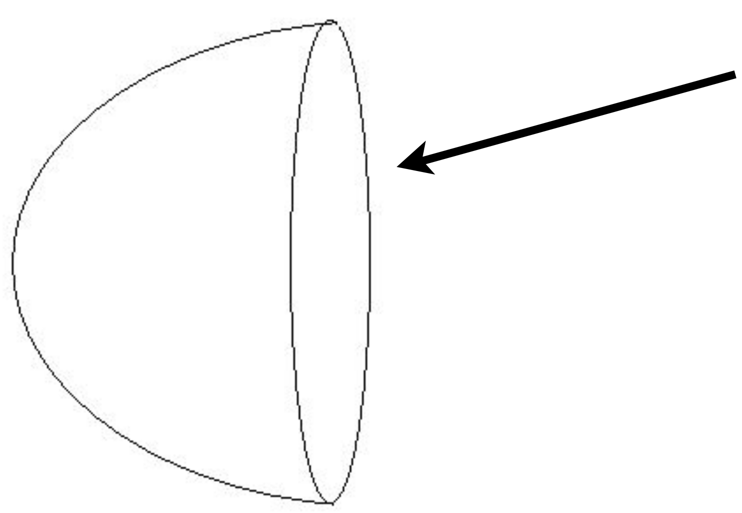
(M.H.-Nishimura-Sekino-Yoneya 2011)

Polyakov loop with scalar

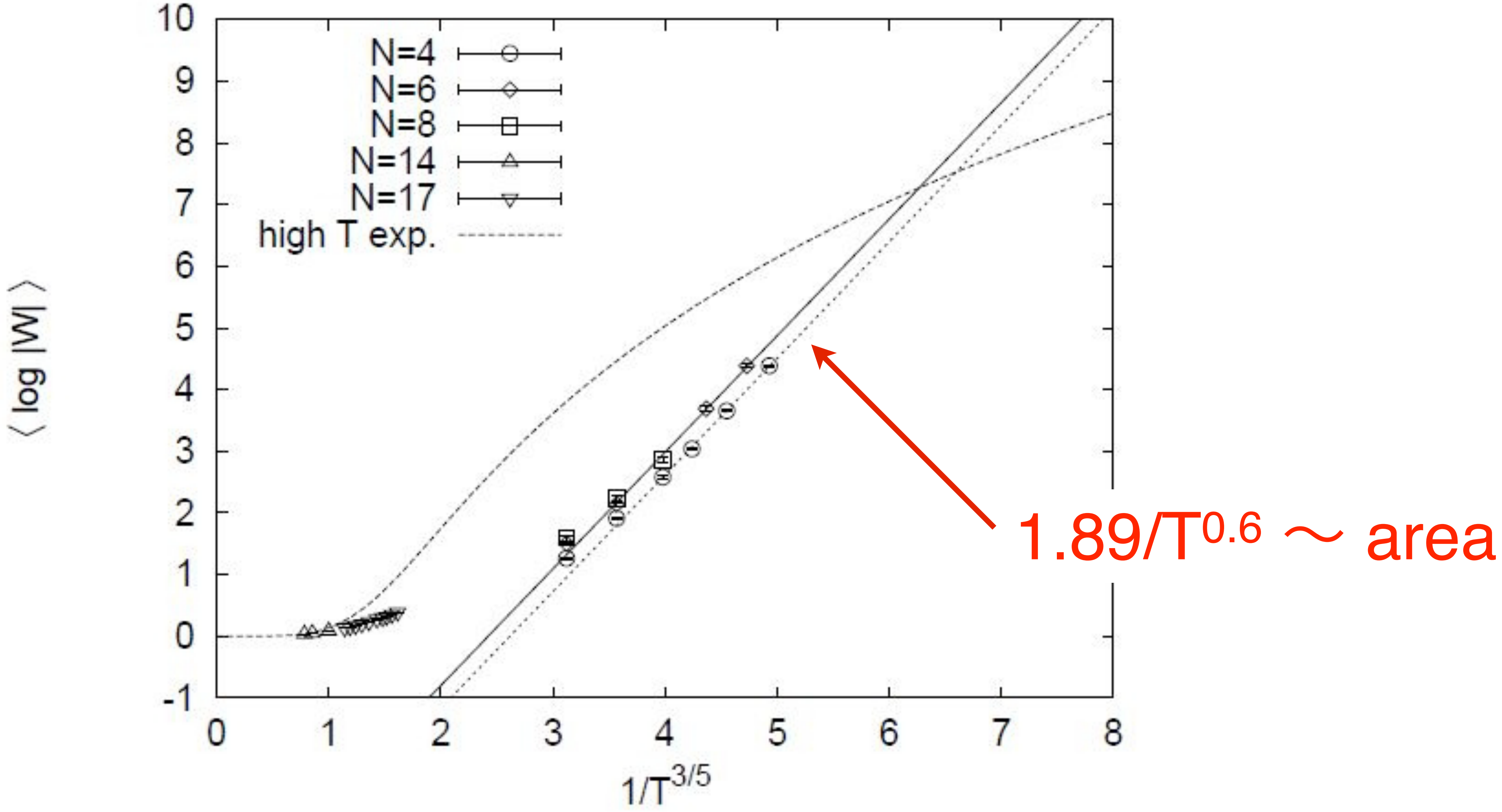
(Maldacena 1998; Rey-Yee 1998)

$$W = Tr\mathrm{P}\exp\left(\int (iA + X)dt\right)$$

$$\log\langle W\rangle \sim \langle \log W\rangle \sim \text{area of minimal surface}$$



boundary=Polyakov loop



M.H.-Miwa-Nishimura-Takeuchi, 2008

conclusion

Maldacena’s conjecture is correct
at finite temperature,
including 1/λ and 1/N corrections,
at least to the next-to-leading order.

Let’s find good numerical problems in SYM
which are useful for learning about quantum gravity!

九後さん: SFTも計算機に載せて
京スーパーコンピューターで調べませんか？

backup slides

black p-brane solution

$$ds^2 = \alpha' \left\{ \frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{d_p N}} \left[-\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right) dt^2 + \sum_{i=1}^{p} dy_i^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{\frac{7-p}{2}}\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right)} dU^2 + g_{YM}\sqrt{d_p N}\, U^{\frac{p-3}{2}} d\Omega_{8-p}^2 \right\},$$

>> 1

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right),$$

<< 1

SUGRA is valid at

$$\lambda^{1/3} N^{-4/21} \ll U \ll \lambda^{1/3} \quad (p = 0)$$

higher dimensions require more computational cost

$$\int [dA][d\psi] e^{-S_B[A]-S_F[A,\psi]} = \int [dA] \det D[A] \cdot e^{-S_B[A]}$$

※ Pfaffian for Majorana fermions

Dirac operator (adjoint repr.) : $N^2L^{p+1} \times N^2L^{p+1}$

cost for calculating determinant is

$(N^2L^{p+1})^3 = N^6L^{3(p+1)}$

(0+1)-d is the best starting point

Wilson's lattice gauge theory

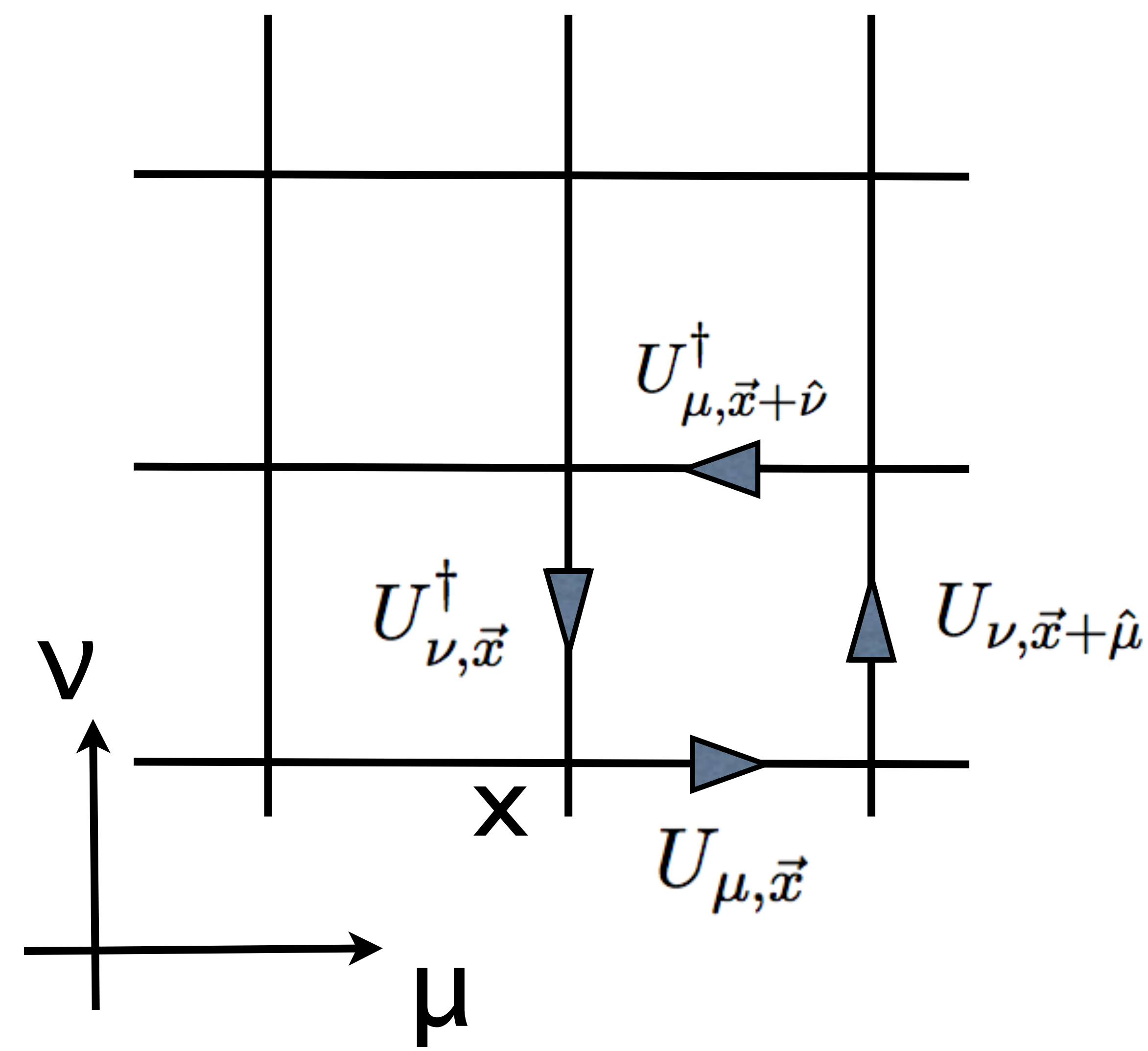
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr\left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}\right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_\mu(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

‘Exact’ symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^\dagger$$

- 90 degree rotation
- discrete translation
- Charge conjugation, parity

These symmetries exist *at discretized level*.

Continuum limit $a \to 0$ respects exact symmetries at discretized level.

Exact symmetries at discretized level

↓

gauge invariance, translational invariance, rotationally invariant,... in the continuum limit.

What happens if the gauge symmetry is explicitly (not spontaneously) broken, (e.g. the sharp momentum cutoff prescription)?

- We are interested in low-energy, long-distance physics (compared to the lattice spacing a).
- So let us integrate out high frequency modes.

Then...

gauge symmetry breaking radiative corrections can appear.

To kill them, one has to add counterterms to lattice action, whose coefficients must be fine-tuned!

'fine tuning problem'

This is the reason why we *must* preserve symmetries exactly.