スカラー暗黒物質の対消滅から生じるガンマ線

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T. T., Phys.Rev.Lett. 111 (2013) 091301,

A. Ibarra, T. T., M. Totzauer, S. Wild, Phys.Rev.D. 90 (2014) 043526

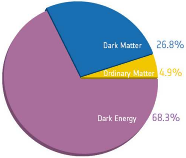
Outline

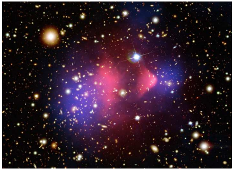
- Introduction
  - DM Production
  - Detectability of DM
- Internal Bremsstrahlung of Majorana DM
- Scalar DM with vector like fermion
  - Gamma-ray Signatures
    - Internal bremsstrahlung $\chi\chi \to f\bar{f}\gamma$
    - Monochromatic gamma-rays $\chi\chi \to \gamma\gamma, \gamma Z$
- Summary

Dark Matter

There are many evidences for DM.

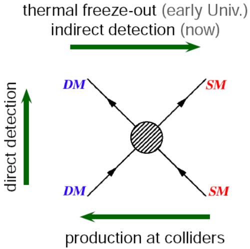
- Rotation curves of spiral galaxies
- CMB observations
- Collision of bullet cluster
- Large scale structure of the universe





WIMP: the most promising DM candidate.
Many experiments focus on WIMP detection.

- Direct detection
- Indirect detection
- Collider search



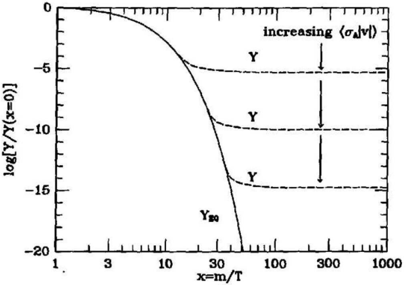
Thermal Production of DM

Evolution of number density is determined by Boltzmann equation.

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\rm eq}^2 \right)$$

$\Updownarrow$ same equation

$$\frac{dY}{dx} = -\frac{\Gamma Y_{\rm eq}}{Hx}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right], \quad x \equiv \frac{m}{T}, \quad \Gamma \equiv \langle \sigma v \rangle n_{\rm eq}, \quad Y \equiv \frac{n}{s}$$



- Relic density is determined by cross section $\langle \sigma v \rangle$.
- σv is expanded by v.
  $\to \sigma v = a + bv^2 + \mathcal{O}(v^4)$
  a: s-wave, b: p-wave
- $\Omega h^2 \approx \dfrac{1.04 \times 10^9 \ [\mathrm{GeV}^{-1}]}{\sqrt{g_*} m_{\rm pl} \langle \sigma v \rangle}$

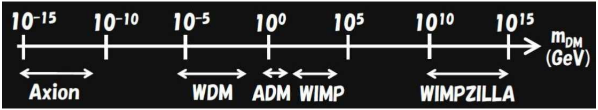
$$\Omega h^2 \approx 0.12 \quad \leftrightarrow \quad \langle \sigma v \rangle \approx 3 \times 10^{-26}\ [\mathrm{cm}^3/\mathrm{s}] = 3 \times 10^{-9}\ [\mathrm{GeV}^{-2}]$$

- If DM is degenerated, co-annihilation effect should be considered.

Other mechanisms

- Asymmetric Dark Matter
  DM mass is almost determined to be a few GeV.
  arXiv:0901.4117, 1308.0338
- Production by decay of metastable particle
  Possible to have DM with rather large ineteractions
  arXiv:0810.4147

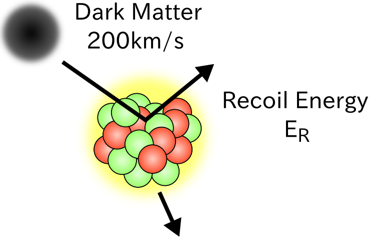
DM mass region



Detectability of Dark Matter

(i) Direct detection

Looking for scattering event with nuclei

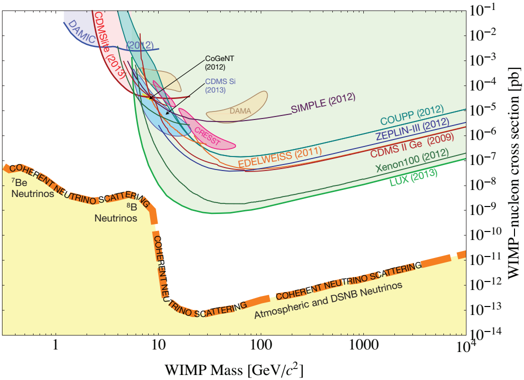


- Nuclei are made from quarks. $\rightarrow$ interactions with quarks are important.
- Many experiments are lauching. LUX, XENON100, CDMSII, DAMA, CoGeNT, CRESST, KIMS.

detection rate:
$$\frac{dR}{dE_R} = \sum_{\text{nuclei}} \frac{\rho_\odot}{m_{DM}} \frac{1}{m_{\text{det}}} \int_{v>v_{\min}} \frac{d\sigma}{dE_R} v f_\odot(\mathbf{v}+\mathbf{v}_e) d^3v$$

- $d\sigma/dE_R$: cross section (Particle physics dependence)
- $\rho_\odot$, v: DM local density, DM velocity (Astrophysics dependence)

Current limits arXiv:1307.5458

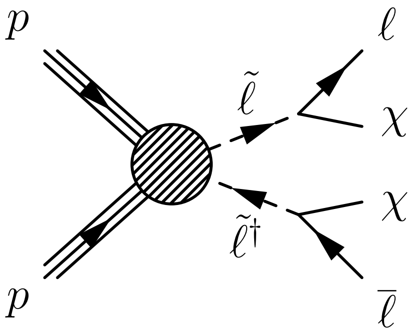


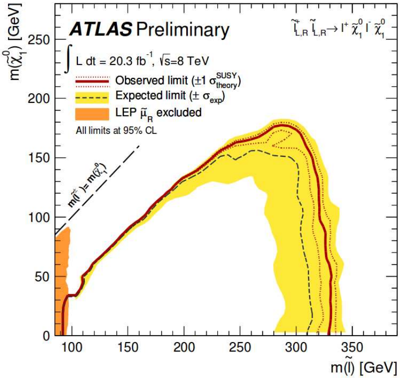
- $\sigma_{SI}^N \lesssim 10^{-9}$ [pb] $\sim 10^{-45}$ [cm^2] at $m_{\rm DM} \sim 30$ GeV.
- Results are inconsistent between LUX and DAMA, CoGeNT, CDMS-Si.
- Neutrino induced background set lower bound.
  (solar, atmospheric, diffuse supernova neutrinos)

(ii) Collider search

Ex. neutralino in SUSY

LHC limit ATLAS-CONF-2013–049





- $pp \to \tilde{\ell}^{\dagger}\tilde{\ell} \to \bar{\ell}\ell +$ missing energy
- DM mass region $m_{\chi} \lesssim 180\ \mathrm{GeV}$ is excluded for $m_{\tilde{\ell}} \approx 300\ \mathrm{GeV}$.

(iii) Indirect detection

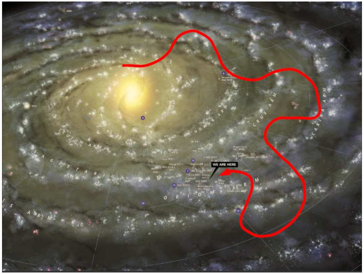
Propagation of charged particle
Propagation equation:

$$\nabla\left(K(E,x)\nabla f\right) + \frac{\partial}{\partial E}\left[b(E,x)f\right] + Q(E,x) = 0$$

$K(E,x)$: diffusion coefficient $\rightarrow$ effect due to magnetic field
$b(E,x)$: energy loss coefficient $\rightarrow$ synchrotron radiation, ICS

$Q(E,x)$: source term of DM
For DM annihilation

$$\rightarrow Q(E,x) = \frac{n_\chi^2}{2}\langle\sigma v\rangle\frac{dN}{dE}$$



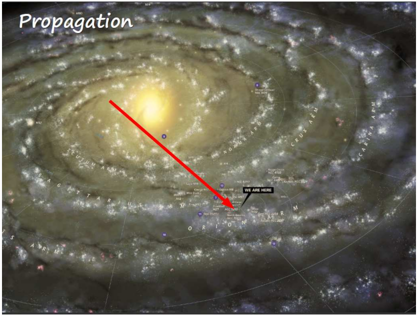
A. Ibarra, ICTP Summer School 2013

Propagation of gamma-ray

prompt γ: $$\frac{d\Phi_\gamma}{dE_\gamma} = \frac{r_\odot \rho_\odot^2}{8\pi m_\chi^2} \overline{J} \langle \sigma v_\gamma \rangle \frac{dN}{dE_\gamma}$$

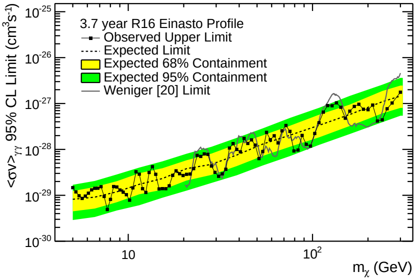
$r_\odot$, $\rho_\odot$, $\overline{J}$: astrophysics dependence

m_χ, $\langle \sigma v_\gamma \rangle$, dN/dE_γ: particle physics dependence

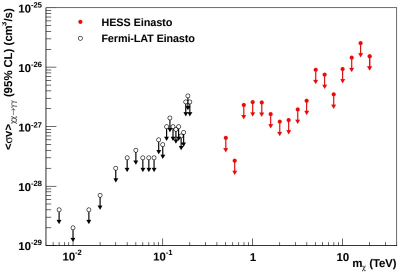


A. Ibarra, ICTP Summer School 2013

Upperbounds for cross section into $\gamma\gamma$



Fermi Coll., arXiv:1305.5597

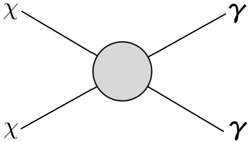


H.E.S.S. Coll., arXiv:1301.1173

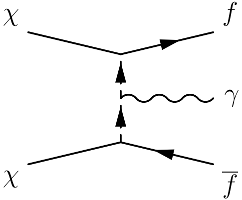
- $\sigma v \lesssim 10^{-29} \sim 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$ in DM mass region $10\ \mathrm{GeV} \lesssim m_{\mathrm{DM}} \lesssim 10\ \mathrm{TeV}$

Gamma-ray spectra from DM annihilation

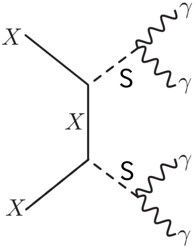
- Line spectrum
  ex. $\chi\chi \to \gamma\gamma$, $\chi\chi \to Z\gamma$
  suppression factor $\sim \alpha_{\rm em}^2$



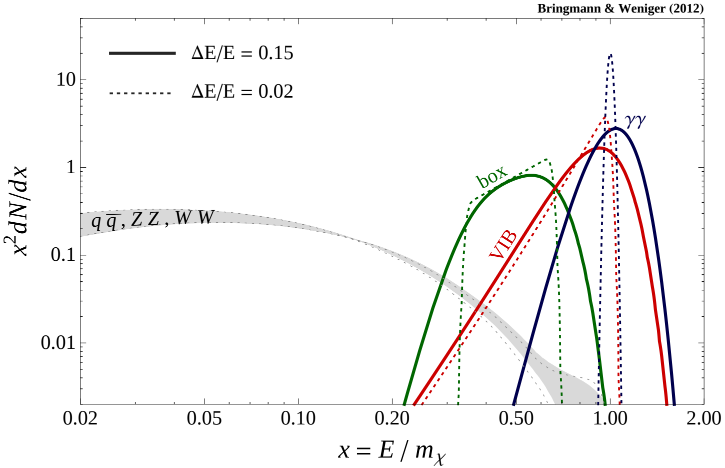
- Internal bremsstrahlung
  $\chi\chi \to f\bar{f}\gamma$
  When chiral suppression is effective,
  it becomes important.
  suppression factor $\sim \alpha_{\rm em}$



- Box type spectrum
  ex. $\chi\chi \to SS \to 4\gamma$
  (S: light mediator)
  if $m_\chi \gg m_S$, box type
  if $m_\chi \approx m_S$, $E_\gamma \sim m_\chi/2$



Gamma-ray spectra from DM annihilation

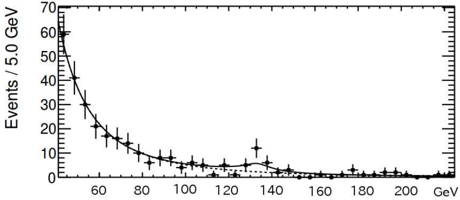


T. Bringbann, C. Weniger arXiv:1208.5481

Sharp gamma-ray spectrum is important for DM signal.

Gamma-ray excesses

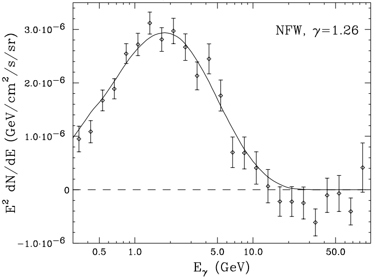
Significance is 3.3σ at 133 GeV.



Fermi Collaboration, arXiv: 1305.5597

- Lower significance by Fermi Collaboration.
- This peak could be a fake.
- Better instruments are needed.

A few GeV γ-ray excess



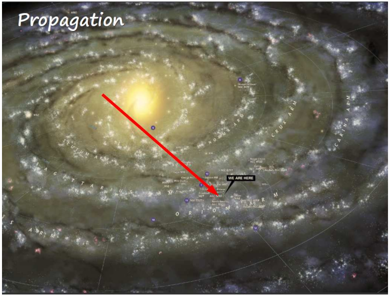
D. Hooper et al, arXiv:1402.6703

- γ-ray excess around a few GeV
- $\langle \sigma v \rangle_{b\bar{b}} \sim 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$ which is same order with that needed for thermal relic.

Gamma-ray Background from Galactic Center

- Acceleration of proton and electron by supermassive black hole
  Scattering with interstellar medium $\rightarrow$ producing pion
  Pion decay $\pi^0 \rightarrow 2\gamma$
- Inverse Compton Scattering ($e^{\pm}\gamma \rightarrow e^{\pm}\gamma$)
  gamma-ray source: CMB, starlight
- Millisecond pulsars

$\rightarrow$ Background modeling

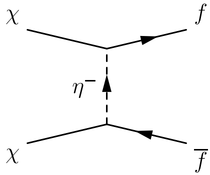


Internal Bremsstrahlung

Internal Bremsstrahlung of Majorana Dark Matter

Consider Majorana DM χ

$$\mathcal{L} = y\eta^{+}\overline{\chi}P_L f + \text{h.c.}$$

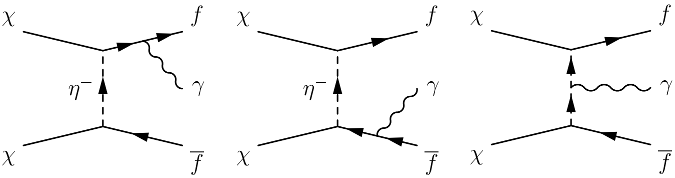


- Cross section for $\chi\chi \to f\overline{f}$ is expanded by v: $\sigma v_{f\overline{f}} \approx a + bv^2$

$$\sigma v_{f\overline{f}} \approx \frac{y^4}{32\pi m_\chi^2}\frac{m_f^2}{m_\chi^2}\frac{1}{(1+\mu)^2} + \frac{y^4}{48\pi m_\chi^2}\frac{1+\mu^2}{(1+\mu)^2}v^2, \quad \mu \equiv \frac{m_\eta^2}{m_\chi^2} > 1$$

- When $m_f \ll m_\chi$, s-wave can be negligible. $\to$ chiral suppression
- Relative velocity v in the present universe is $v \sim 10^{-3}$
- Relic density of DM is determined by p-wave $\to$ y is fixed.

- Internal Bremsstrahlung



The total amplitude is separated by two parts.

$$i\mathcal{M} = i\mathcal{M}_{\rm FSR} + i\mathcal{M}_{\rm VIB}$$

- Differential cross section (interference term is neglected)

$$\frac{d\sigma v_{f\bar{f}\gamma}}{dx} = \frac{d\sigma v^{\rm FSR}_{f\bar{f}\gamma}}{dx} + \frac{d\sigma v^{\rm VIB}_{f\bar{f}\gamma}}{dx}, \quad x \equiv \frac{E_\gamma}{m_\chi},$$

FSR : broad spectrum

VIB : $E_\gamma \sim m_\chi$ sharp peak spectrum

Concrete formula for the differential cross section

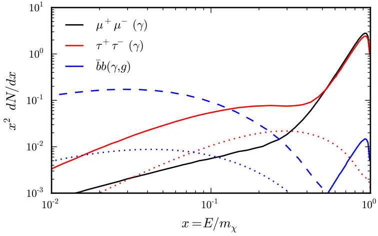
$$\text{FSR}: \frac{d\sigma v^{\rm FSR}_{f\bar{f}\gamma}}{dx} = \sigma v_{f\bar{f}} \frac{\alpha_{\rm em}}{\pi} \frac{1+(1-x)^2}{x} \log\left(\frac{4m_\chi^2(1-x)}{m_f^2}\right) + (\text{Hadronization})$$

$$\text{VIB}: \frac{d\sigma v^{\rm VIB}_{f\bar{f}\gamma}}{dx} = \frac{\alpha_{\rm em} y^4}{32\pi^2 m_\chi^2}(1-x)\left[\frac{2x}{(\mu+1)(\mu+1-2x)} - \frac{x}{(\mu+1-x)^2} - \frac{(\mu+1)(\mu+1-2x)}{2(\mu+1-x)^3}\log\left(\frac{\mu+1}{\mu+1-2x}\right)\right]$$

FSR : model independent

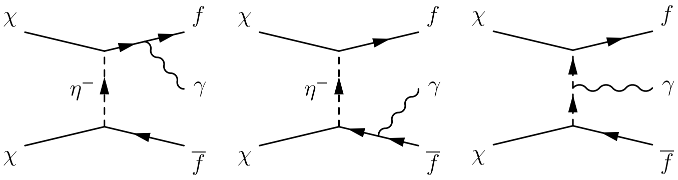
Energy spectra

- If FSR ≪ VIB, characteristic signal.
- Majorana DM
  → chiral suppression



arXiv:1203.1312

Why hard gamma is emitted?



Momentum notation:

Initial state: $\chi(p_1)$, $\chi(p_2)$, Final state: $f(k_1)$, $\overline{f}(k_2)$, $\gamma(k_3)$

When $m_f/m_\chi \ll 1$ and $m_\eta/m_\chi \approx 1$,

$$i\mathcal{M} \sim \frac{i}{(p_1 - k_1)^2 - m_\eta^2} \approx \frac{i}{m_\chi^2 - m_\eta^2 - 2m_\chi E_f} \approx \frac{i}{-2m_\chi E_f}$$

- Emitted f has soft energy.
- $\chi\chi \to f\overline{f}\gamma$ is understood as almost 2-body process $\chi\chi \to (f)\overline{f}\gamma$.
- Energy is taken by $\overline{f}\gamma$ ($E_\gamma \approx E_{\overline{f}} \approx m_\chi$).
- $\to$ Hard gamma emission.

How about Dirac DM?

$$\sigma v_{f\bar{f}} = \frac{y^4}{\pi m_\chi^2} \frac{1}{(1+\mu)^2} + \mathcal{O}(v^2)$$

when s-wave exists, FSR is always dominant.
For complex scalar DM, p-wave dominant (same as Majorana DM).

How about real scalar DM?

$$\sigma v_{f\bar{f}} = \frac{y^4}{4\pi m_\chi^2} \frac{m_f^2}{m_\chi^2} \frac{1}{(1+\mu)^2} - \frac{y^4}{6\pi m_\chi^2} \frac{m_f^2}{m_\chi^2} \frac{1+2\mu}{(1+\mu)^4} v^2 + \frac{y^4}{60\pi m_\chi^2} \frac{1}{(1+\mu)^4} v^4 + \mathcal{O}(v^6)$$

Summary

	Majorana	Dirac	real scalar	compelx scalar
dominant term	p-wave	s-wave	d-wave	p-wave

Scalar Dark Matter Model

The model with scalar Dark Matter

- New particles
  Real singlet scalar χ (DM), $\mathbb{Z}_2 = -1$
  Vector like charged fermion ψ (mediator), $\mathbb{Z}_2 = -1$, $Y = -1$

- Interactions

$$\mathcal{L}_Y = y\chi\overline{\psi}P_R f + \mathrm{h.c.}$$

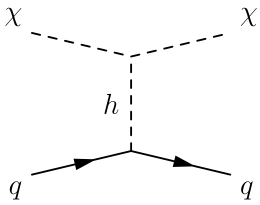
$$\mathcal{V} = m_\phi^2 \phi^\dagger \phi + \frac{m_\chi^2}{2}\chi^2 + \frac{\lambda_\phi}{2}\left(\phi^\dagger\phi\right)^2 + \frac{\lambda_\chi}{4!}\chi^4 + \frac{\lambda}{2}\chi^2\left(\phi^\dagger\phi\right)$$

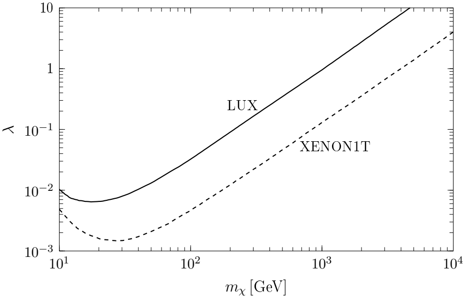
  where ϕ is the SM Higgs doublet.

- DM χ interacts with SM particles through y and λ.
  (The other parameters: m_χ and m_ψ)

$$\text{After } \phi \text{ gets VEV} \rightarrow \quad \phi(x) = \langle\phi\rangle + \frac{h(x)}{\sqrt{2}}$$

Constraint on coupling λ





- The coupling λ should be suppressed from the constraint of direct detection.

$$\sigma_p = \frac{c\lambda^2 m_p^4}{4\pi m_h^4}\frac{1}{(m_\chi + m_p)^2} \lesssim 7.6 \times 10^{-46}\ [\mathrm{cm}^2] \text{ at } m_\chi \sim 30 \text{ GeV}$$

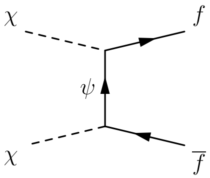
LUX Collaboration, arXiv: 1310.8214

where $c = 0.345$ and m_p is proton mass.

- The coupling is limited as $\lambda \lesssim 10^{-2}$ in all DM mass region.

Thermal relic density of DM

- $\chi\chi \to hh$, $\chi\chi \to h \to f\bar{f}$ are subdominant.
- The most important channel is $\chi\chi \to f\bar{f}$ mediated by ψ.



- The cross section is expanded as $\sigma v = a + bv^2 + cv^4 + \mathcal{O}(v^6)$

$$\sigma v_{f\bar{f}} = \frac{y^4}{4\pi m_\chi^2}\frac{m_f^2}{m_\chi^2}\frac{1}{(1+\mu)^2} - \frac{y^4}{6\pi m_\chi^2}\frac{m_f^2}{m_\chi^2}\frac{1+2\mu}{(1+\mu)^4}v^2 + \frac{y^4}{60\pi m_\chi^2}\frac{1}{(1+\mu)^4}v^4 + \mathcal{O}(v^6), \qquad \mu \equiv \frac{m_\psi^2}{m_\chi^2}$$

· when $m_f \ll m_\chi$, s-wave and p-wave are negligible.
$\to$ chiral suppression
· This can be interpreted from J and CP conservation.

Interpretation of d-wave

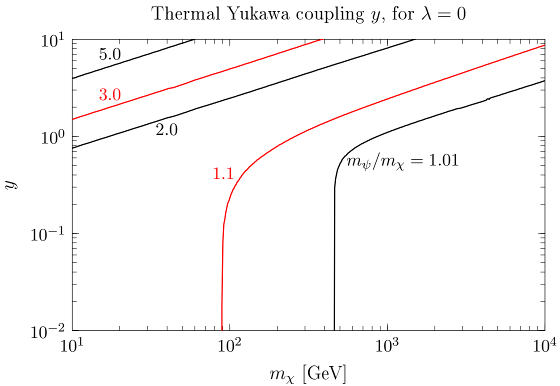
CP and total angular momentum J should be conserved between initial and final states.

- s-wave
  initial state: CP=even, $J=0$ ($J^{PC}=0^{++}$)
  $\rightarrow$ possible effective operator: $\mathcal{O}_S \sim \chi\chi\overline{f}f$
  But suppressed by m_f since $f\overline{f}$ corresponds to mass term.
- p-wave
  initial state: CP=odd, $J=1$ ($J^{PC}=1^{-+}$)
  Any $J^{PC}=1^{-+}$ bi-linear operator cannnot be constructed for final state.

$$\mathcal{O}_P \sim \left(\chi \overleftrightarrow{\partial_i} \chi\right)\left(\overline{f}\gamma^i f\right) = 0$$

Note: p-wave exists for complex scalar DM.

Thermal relic density of DM



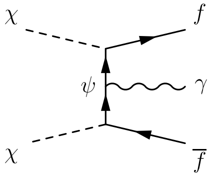
- Contours for several mass splittings $m_\psi/m_\chi = 1.01$, 1.1, 2, 3, 5 ($\lambda = 0$).
- micrOmegas is used (co-annihilations are included).

- When masses are degenerated, co-annihilation effect is important.
- DM mass is bounded ($m_\chi \lesssim 2$ TeV) by perturbativity ($y \lesssim \sqrt{4\pi}$).

Gamma-ray Signatures

Possible processes

- $\chi\chi \to f\overline{f}\gamma$

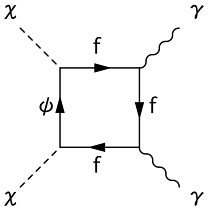


Internal bremsstrahlung

T. T, arXiv:1307.6181

F. Giacchino, L. Lopez-Honorez, M.H.G. Tytgat, arXiv:1307.6480

- $\chi\chi \to \gamma\gamma$, $\chi\chi \to \gamma Z$



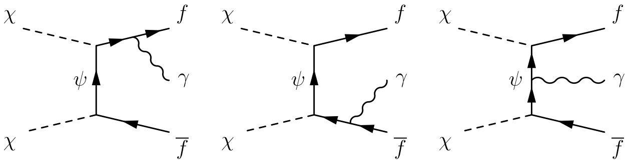
Monochromatic gamma-ray line

A. Ibarra, T. T, M. Totzauer, S. Wild, arXiv:1405.6917

F. Giacchino, L. Lopez-Honorez, M. Tytgat, arXiv:1405.6921

Both gamma-ray emissions are expected to be stronger than Majorana case since y is large enough.

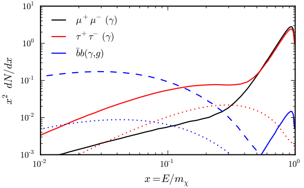
Internal bremsstrahlung



· differential cross section

$$\frac{d\sigma v_{f\bar{f}\gamma}}{dx} = \frac{\alpha_{\rm em} y^4}{4\pi^2 m_\chi^2}(1-x)\left[\frac{2x}{(\mu+1)(\mu+1-2x)} - \frac{x}{(\mu+1-x)^2} - \frac{(\mu+1)(\mu+1-2x)}{2(\mu+1-x)^3}\log\left(\frac{\mu+1}{\mu+1-2x}\right)\right], \quad x \equiv \frac{E_\gamma}{m_\chi}$$

· When $\mu \lesssim 4$, a sharp peak appears around $E_\gamma \sim m_\chi$



T. Bringmann et al., arXiv:1203.1312

Line spectra: $\chi\chi \to \gamma\gamma,\, \gamma Z$

In the limit of $v \to 0$, these analytically can be calculated.
Initial state: $p_1 = p_2 = (m_\chi, \mathbf{0}) \equiv p$, Final state: k_1, k_2

Flow of calculation **G. Bertone et al. arXiv:0904.1442**

1. In general, $i\mathcal{M}$ is decomposed as

$$\mathcal{M}^{\mu\nu} = p^\mu p^\nu A + k_1^\mu k_1^\nu B + k_2^\mu k_2^\nu C + \cdots + g^{\mu\nu} \mathcal{A}_{\gamma\gamma(\gamma Z)}$$

   where $i\mathcal{M} = i\epsilon_\mu^*(k_1)\epsilon_\nu^*(k_2)\mathcal{M}^{\mu\nu}$.
   only $\mathcal{A}_{\gamma\gamma(\gamma Z)}$ remains.
2. Simplify $\mathcal{A}_{\gamma\gamma(\gamma Z)}$ by Passarino-Veltman reduction
3. cross sections:

$$\sigma v_{\gamma\gamma} = \frac{\alpha_{\rm em}^2 y^4}{32\pi^3 m_\chi^2} |\mathcal{A}_{\gamma\gamma}|^2, \quad \sigma v_{\gamma Z} = \frac{\alpha_{\rm em}^2 y^4 \tan^2\theta_W}{16\pi^3 m_\chi^2} \left(1 - \frac{m_Z^2}{4m_\chi^2}\right) |\mathcal{A}_{\gamma Z}|^2$$

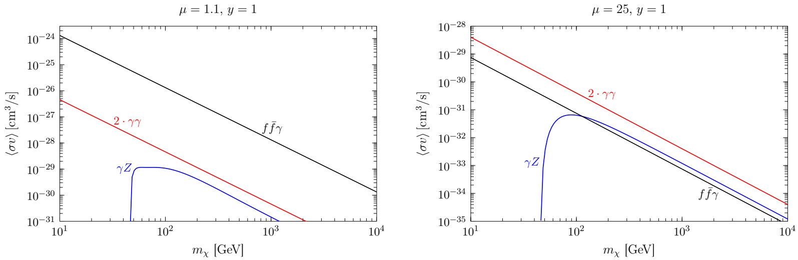
$$\mathcal{A}_{\gamma\gamma} = 2 + \mathrm{Li}_2\left(\frac{1}{\mu}\right) - \mathrm{Li}_2\left(-\frac{1}{\mu}\right) - 2\mu\mathrm{Arcsin}^2\left(\frac{1}{\sqrt{\mu}}\right), \quad \mu = m_\psi^2/m_\chi^2 > 1$$

$$\begin{aligned}
\mathcal{A}_{\gamma Z} = {} & 2 - \frac{\xi}{4-\xi} B_0\left(m_Z^2; 0, 0\right) - \frac{\xi}{4-\xi} B_0\left(m_Z^2; m_\psi^2, m_\psi^2\right) \\
& + \frac{2\xi}{(4-\xi)(1-\mu)} B_0\left(m_\chi^2; 0, m_\psi^2\right) - \frac{2\mu\xi}{(4-\xi)(1-\mu)} B_0\left(4m_\chi^2; m_\psi^2, m_\psi^2\right) \\
& + m_\psi^2 \frac{4-4\mu-\xi}{1-\mu} C_0\left(m_Z^2, 4m_\chi^2, 0; m_\psi^2, m_\psi^2, m_\psi^2\right) \\
& + \frac{m_\psi^2}{2} \frac{(4+\xi)(-2+2\mu+\xi)}{(1-\mu)(4\mu+\xi)} C_0\left(-m_\chi^2 + \frac{m_Z^2}{2}, m_\chi^2, 0; m_\psi^2, 0, m_\psi^2\right) \\
& + m_\chi^2 \left[\frac{\xi(1+\mu)}{2(1+\mu)} - \frac{4\xi(1+\mu)^2}{(4-\xi)(4\mu+\xi)}\right] C_0\left(-m_\chi^2 + \frac{m_Z^2}{2}, m_\chi^2, m_Z^2; 0, m_\psi^2, 0\right) \\
& + m_\chi^2 \left[\frac{2\mu(1-\mu)+\xi}{2(1-\mu)} - \frac{4(1+\mu)}{4-\xi}\right] C_0\left(-m_\chi^2 + \frac{m_Z^2}{2}, m_\chi^2, m_Z^2; m_\psi^2, 0, m_\psi^2\right)
\end{aligned}$$

where B_0 and C_0 are Passarino-Veltman integrals.
when $\xi \equiv m_Z^2/m_\chi^2 \to 0$, $\mathcal{A}_{\gamma Z} \to \mathcal{A}_{\gamma\gamma}$

Cross sections

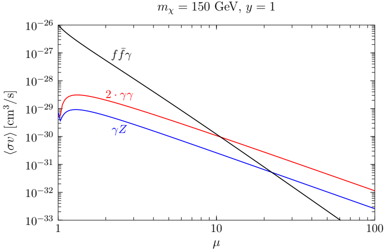
- DM mass dependence

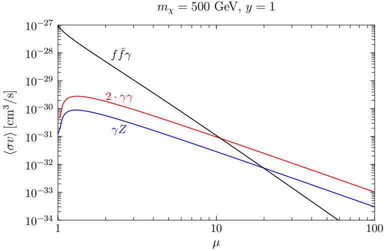


- Mass splitting is fixed to $\mu = 1.1$ (left), $\mu = 25$ (right).
- $\sigma v_{\gamma\gamma}$ and $\sigma v_{\gamma Z}$ are same order.
- Detectability of $\chi\chi \to \gamma\gamma, \gamma Z$ depends on experimental energy resolution.

Cross sections

- $\mu = m_\psi^2 / m_\chi^2$ dependence



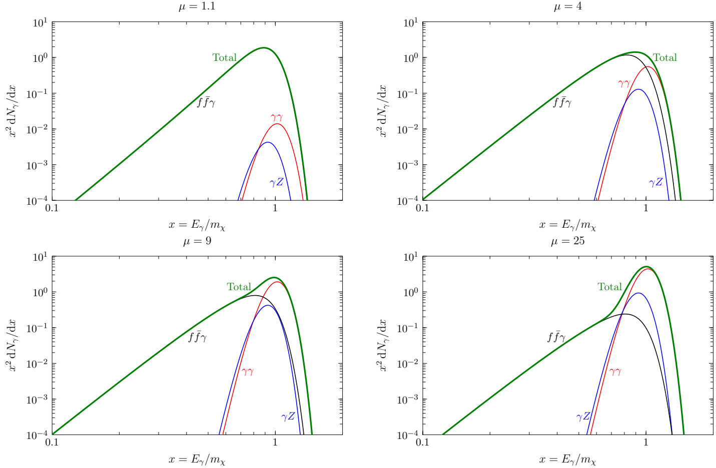


- DM mass is fixed to 150 GeV (left) and 500 GeV (right).
- when $\mu \gtrsim 10$, $\sigma v_{f\bar{f}\gamma} < \sigma v_{\gamma\gamma,(\gamma Z)}$ due to μ dependence of the cross sections.

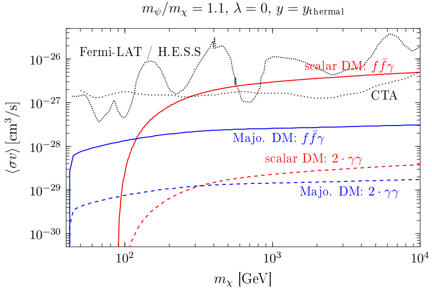
$$\sigma v_{f\bar{f}\gamma} \propto \frac{y^4}{\mu^4} \frac{1}{m_\chi^2}, \qquad \sigma v_{\gamma\gamma},\ \sigma v_{\gamma Z} \propto \frac{y^4}{\mu^2} \frac{1}{m_\chi^2}$$

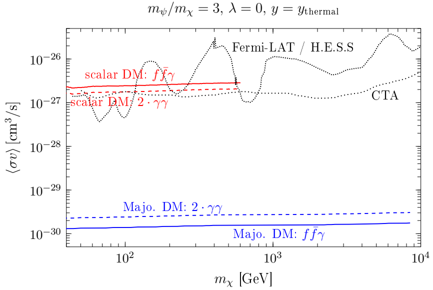
Total Energy spectrum (10% energy resolution)

$$\frac{dN_\gamma}{dx} = \frac{1}{\langle\sigma v\rangle}\left[\frac{d\langle\sigma v\rangle_{f\bar{f}\gamma}}{dx} + 2\frac{d\langle\sigma v\rangle_{\gamma\gamma}}{dx} + \frac{d\langle\sigma v\rangle_{\gamma Z}}{dx}\right]$$



Comparison with Gamma-ray Experiments





- Scalar DM χ is testable by future gamma-ray experiments such as CTA.

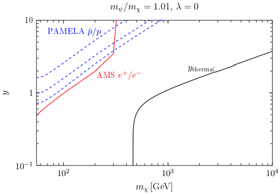
Future experiments

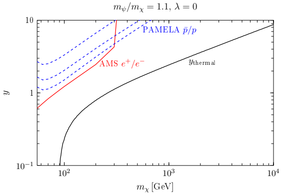
	GAMMA400	DAMPE	CTA
Energy range [GeV]	0.1-3000	5-10000	$>$10
Angular res [deg]	$\sim$0.01	0.1 at 100 GeV	0.1
Energy res [%]	$\sim$1	$\sim$1 at 800 GeV	15

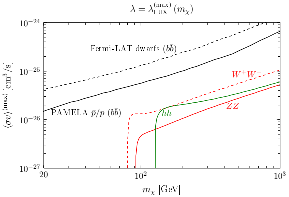
Constraints

- DM relic density ($\Omega h^2 \approx 0.12$)
- Perturbativity ($y \lesssim \sqrt{4\pi},\ 4\pi$)
- Direct detection
- Collider search ($\psi\overline{\psi}$ production)
- Indirect detection (e^+e^-, anti-proton, gamma-ray)
  $\chi\chi \to f\overline{f}\gamma,\ f\overline{f}Z$

For $m_\psi/m_\chi = 1.01$ For $m_\psi/m_\chi = 1.1$





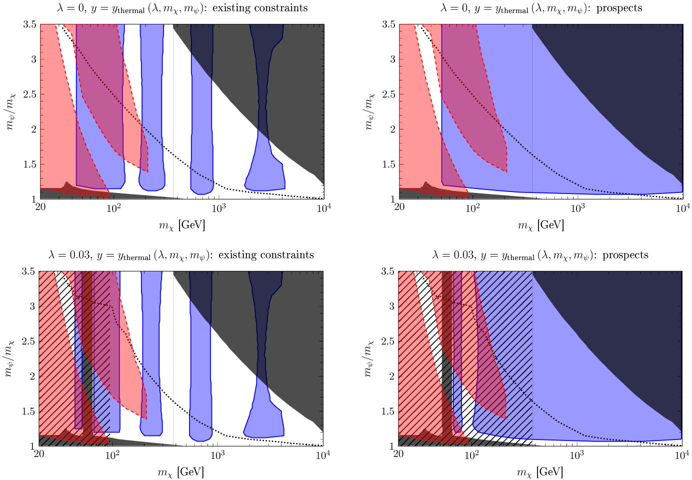


$\to$ weak constraint

Allowed parameter space and future prospects

$\lambda = 0.0$

$\lambda = 0.03$



- Only narrow parameter region is remaining and will be tested by CTA and XENON1T.

Summary

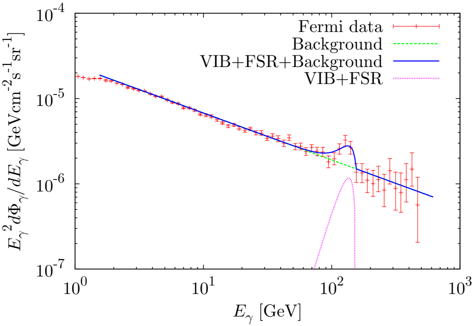
- Gamma-ray signatures from DM annihilations are characteristic.
- In the toy model we considered here, the annihilation cross section is dominated by d-wave.
  $\rightarrow$ Large cross sections for sharp gamma-rays are obtained.
- Only small parameter space is remaining.
- Most of the parameter space is testable by future gamma-ray and direct detection experiments.

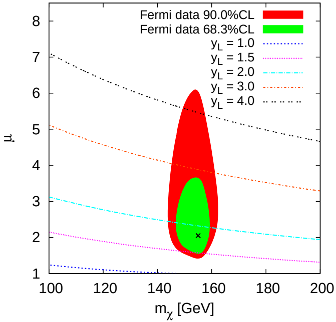
Future work

- Enhancement of internal bremsstrahlung of Majorana DM by cosidering co-annihilation.
- Strong ν flux via electroweak bremsstrahlung?
  $\chi\chi \rightarrow \ell\bar{\ell}Z$, $\chi\chi \rightarrow \ell\overline{\nu}W^+$

Backup slide

Fitting to 130 GeV gamma-ray excess





- $\chi^2_{\rm min} = 65.57$ (51 d.o.f) at $m_\chi = 155$ GeV, $\mu = 2.05$.
- $\langle \sigma v \rangle_{f\bar{f}\gamma} = 4.72 \times 10^{-27}\ {\rm cm}^3/{\rm s}$.