$E_6$ Grand Unified Theory and Family Symmetry with Spontaneous CP Violation

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1. Introduction
2. $E_6$ Grand Unified Theory
3. Family Symmetry
4. Spontaneous CP Violation
5. Predictions (FCNC and Nucleon decay)
6. Impact of LHC
7. Summary
Overview of this talk

- $E_6$ grand unified theory (GUT)
  Various hierarchies of quark and lepton masses and mixings can be explained. ($U_{e3} \sim \lambda$)

- $E_6$ GUT+non-Abelian family symmetry
  The explanation is so natural that realistic Yukawa couplings can be realized after breaking the symmetries.
  Modified universal sfermion masses are predicted.

1. $m_3 \ll m$ is interesting.
   a. light stop... weak scale stability
   b. heavy sfermion... suppress FCNC, CP

2. All FCNC are sufficiently suppressed.

3. Several are within reach of future exp.

Overview of this talk

- $E_6$ GUT+family sym. with sp. CP violation

  (Discrete sym. is introduced.)

Not only old type but also new type of SUSY CP problems can be solved.

Several bonus (light up quark, $V_{ub} \sim \lambda^4$, predictive power)

Examining the neutrino sector

  Same predictions as the usual $E_6$ GUT but non-trivial.
Introduction
Grand Unified Theories

2 Unifications

- Gauge Interactions

\[ SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y \]

- Matter

\[
\begin{array}{cccc}
Q & U^c_R & E^c_R & D^c_R \\
& L & N^c_R \\
\end{array}
\]

\[ 10 + \bar{5} + 1 = 16 \]

Experimental supports for both unifications

GUT is promising
Grand Unified Theories

- Unification of gauge interactions
  
  quantitative evidence:

  Unification of matters
  
  qualitative evidence:

\[(Y_u)_{ij}^{10_i} 10_j 5_H + (Y_d)_{ij}^{10_i} \bar{5}_j 5_H + (Y_\nu)_{ij} \bar{5}_i \bar{5}_j 5_H 5_H\]

\[10_i (Q_i) \text{ have stronger hierarchy than } \bar{5}_i (L)\]

hierarchies of masses and mixings
lepton >> quark (in hierarchies for mixings)
ups >> downs, electrons >> neutrinos (in mass hierarchies)
Masses & Mixings and GUT

These can be naturally realized in SU(5) GUT!!
SU(5) SUSY GUT

\[ 10 = (q, u_R^c, e_R^c) \quad \bar{5} = (d_R^c, l) \quad 1 = \nu_R^c \]

\[ Y_u 10_i 10_j \bar{5}_H + Y_{(d,e)} 10_i \bar{5}_j \bar{5}_H + Y_{\nu_D} \bar{5}_i 1_j 5_H + M_{\nu_R} 1_i 1_j \]

\[ u \gg d, e \gg \nu \]

\[ 10_i \quad \text{have stronger hierarchy than} \quad \bar{5}_i \]

\[ \text{Stronger hierarchy leads to smaller mixings} \]

\[ \text{Quark mixings (CKM)} \quad \ll \quad \text{Lepton mixing (MNS)} \]

\[ 10_i(q_i) \quad \ll \quad \bar{5}_i(l_i) \]
Mass hierarchy and mixings

- Stronger hierarchy leads to smaller mixings

\[
\begin{pmatrix}
\epsilon^2 & \epsilon \\
\epsilon & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & \epsilon \\
-\epsilon & 1
\end{pmatrix}
\rightarrow
1, \epsilon^2
\]

Stronger hierarchy ⇄ Smaller mixings
SU(5) SUSY GUT

\[ 10 = (q, u_R^c, e_R^c) \quad \bar{5} = (d_R^c, l) \quad 1 = \nu_R^c \]

\[ Y_u 10_i 10_j \bar{5}_H + Y_{(d,e)} 10_i \bar{5}_j \bar{5}_H + Y_{\nu_D} 5_i 1_j \bar{5}_H + M_{\nu_R} 1_i 1_j \]

\[ u \gg d, e \gg \nu \]

Stronger hierarchy leads to smaller mixings

Quark mixings (CKM) \( \ll \) Lepton mixing (MNS)

Good agreement with masses & mixings
$E_6$ Grand Unified Theory

The assumption in SU(5) GUT

$10_i$ have stronger hierarchy than $\bar{5}_i$

can be derived.

Various Yukawa hierarchies can be induced from one
Yukawa hierarchy in $E_6$ GUT.
Unification

\[ 27_i = 16_i [10_i + \bar{5}_i + 1_i] + 10_i [\bar{5}_i + \bar{5}_i] + 1_i [1_i] \]

(i = 1, 2, 3)

Three of six \( \bar{5} \) become superheavy after the breaking

\[ E_6 \rightarrow SO(10) \rightarrow SU(5) \]

\[ \langle 1_H \rangle \geq \langle 16_C \rangle \]

\[ W = Y^H 27_i 27_j \langle 27_H \rangle + Y^C 27_i 27_j \langle 27_C \rangle \]

Once we fix \( Y^H, Y^C, \langle 27_H \rangle, \langle 27_C \rangle \),
three light modes of six \( \bar{5} \) are determined.

We assume all Yukawa matrices
Milder hierarchy for $\bar{5}_i(l)$

- $\bar{5}$ fields from $27_3$ become superheavy.

- Light modes $(\bar{5}_1, \bar{5}_1, \bar{5}_2)$ have smaller Yukawa couplings and milder hierarchy than $(10_1, 10_2, 10_3)$

- Larger mixings in lepton sector than in quark sector.

- Small $\tan \beta$

- Small neutrino Dirac masses

Suppressed radiative LFV

Bando-N.M. 01
N.M, T. Yamashita 02
How to obtain various Yukawas?

\[ Y_u 10_i 10_j \bar{5}_H + Y_{(d,e)} 10_i \bar{5}_j \bar{5}_H + Y_\nu \bar{5}_i \bar{5}_j \bar{5}_H \bar{5}_H \]

\[ Y_u \sim \quad Y_{(d,e)} \sim \quad Y_\nu \sim \quad \]

\[(\bar{5}_1, \bar{5}_2, \bar{5}_3) \rightarrow (\bar{5}_1, \bar{5}'_1, \bar{5}_2)\]
SO(10) GUT relations

\[ Y_d = Y_e^T = Y_u = Y_{\nu_D} \]

\[
Y_u \sim \begin{pmatrix}
10_1 & 10_2 & 10_3 \\
10_1 & \begin{pmatrix}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix}
\end{pmatrix}\lambda
\]

\[
Y_{\nu_D} \sim \begin{pmatrix}
\bar{5}_1 & \bar{5}_1 & \bar{5}_2 \\
\bar{5}_1 & \begin{pmatrix}
\lambda^{5.5} & \lambda^{4.5} & \lambda^{2.5} \\
\lambda^5 & \lambda^4 & \lambda^2
\end{pmatrix}
\end{pmatrix}\lambda^{0.5}
\]

\[
Y_d \sim Y_e^T \sim \begin{pmatrix}
10_1 & \begin{pmatrix}
\lambda^6 & \lambda^{5.5} & \lambda^3 \\
\lambda^5 & \lambda^{4.5} & \lambda^2 \\
\lambda^3 & \lambda^{2.5} & 1
\end{pmatrix} & \begin{pmatrix}
\lambda^0.5 & \lambda^0.5 \\
\lambda^0.5 & \lambda^0.5
\end{pmatrix}
\end{pmatrix}\lambda
\]

\[
\bar{5}_1 + \lambda \Delta \bar{5}_3 \quad (\Delta = 3 - r)
\]

\[
\lambda^r \equiv \frac{\langle 27_C \rangle}{\langle 27_\Phi \rangle} \sim \lambda^{0.5}
\]

Small \tan \beta
Small \ Y_{\nu_D}

Large \ u_{e3} \sim \lambda

\[
V_{CKM} \sim \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix}
\]

\[
V_{MNS} \sim \begin{pmatrix}
1 & \lambda^{0.5} & \lambda \\
\lambda^{0.5} & 1 & \lambda^{0.5} \\
\lambda & \lambda^{0.5} & 1
\end{pmatrix}
\]
Right-handed neutrinos

\[ W = \frac{Y^{XY}}{\Lambda} 27_i 27_j \langle \overline{27}_X \rangle \langle \overline{27}_Y \rangle \]

\[ X, Y = \bar{H}, \bar{C} \]

\[ M_R = Y^{XY} \langle \overline{27}_X \rangle \langle \overline{27}_Y \rangle \frac{1}{\Lambda} \]

- The same hierarchy

\[ Y^{XY} \sim Y^H \sim Y^C \]

\[ M_\nu = Y_{\nu_D} M_R^{-1} Y_{\nu_D}^T \sim \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \frac{\langle H_u \rangle^2 \Lambda}{\langle 1_H \rangle^2} \]

\[ \frac{\Delta m^2_{\text{solar}}}{\Delta m^2_{\text{atm}}} \sim \frac{m_{\nu_\mu}^2}{m_{\nu_\tau}^2} \sim \lambda^2 \quad \text{LMA for solar neutrino problem} \]
1st Summary

- $E_6$ unification explains why the lepton sector has larger mixings than the quark sector. (Large $U_{e3} \sim \lambda$)
- Suppressed radiative LFV

Small $Y_{\nu_D}$ Small $\tan \beta$

- A basic Yukawa hierarchy $Y \sim Y_u$ → The other Yukawa hierarchies

$Y_u \sim \bar{5}_i, \ Y_{(d,e)} \sim \bar{5}_i, \ Y_\nu \sim 27_1, 27_2$

Hierarchy of $10_i$ is stronger than that of $\bar{5}_i$

Three $\bar{5}_i$ come from the first 2 generation of
Family symmetry

All three generation quark and leptons can be unified into a single (or two) field(s)
By breaking the horizontal symmetry, realistic quark and lepton masses and mixings can be obtained.
Peculiar sfermion mass spectrum is predicted.

\[ \tilde{m}_{10}^2 = \begin{pmatrix} m_1^2 & \cdot & \cdot \\ \cdot & m_2^2 & \cdot \\ \cdot & \cdot & m_3^2 \end{pmatrix}, \quad \tilde{m}_5^2 = \begin{pmatrix} m_1^2 & \cdot & \cdot \\ \cdot & m_2^2 & \cdot \\ \cdot & \cdot & m_2^2 \end{pmatrix} \]
Family Symmetry

• Origin of Yukawa hierarchy
• Semi-universal sfermion masses to suppress FCNC
  \( \Phi_a, \Phi_3, H_u, H_d (\Phi = Q, U, D, L, E, N) \)

• The 1st 2 generation have universal sfermion masses.
• Large top Yukawa coupling

\[
\begin{align*}
U(2)_H & \longrightarrow U(1)_H \longrightarrow X \\
\langle \tilde{F}^a \rangle / \Lambda & \sim \epsilon \\
\langle A^{ab} \rangle / \Lambda & \sim \epsilon'
\end{align*}
\]

\[
Y \sim \begin{pmatrix}
0 & \epsilon' & 0 \\
\epsilon' & 0 & \epsilon \\
0 & \epsilon & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 + \epsilon^2 & \epsilon \\
0 & \epsilon & O(1)
\end{pmatrix} \tilde{m}^2
\]

\[
Y_u \sim Y_d \sim Y_e \sim Y_\nu?
\]

Not sufficient to suppress FCNC
Large neutrino mixings and FCNC

- The universal sfermion masses only for the 1st 2 generation do not suppress FCNC sufficiently if.

\[
\delta_5 - 1 \sim V_5^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a \end{pmatrix} V_5 \sim a \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}
\]

if \( V_5 \sim V_{MNS} \).

\[
|\text{Im}(\delta_D^c)_{12}| \leq 1.5 \times 10^{-3} \left( \frac{\tilde{m}_Q}{500\text{GeV}} \right)
\]

\[
|\delta_L_{12}| \leq 4 \times 10^{-3} \left( \frac{\tilde{m}_L}{100\text{GeV}} \right)^2
\]

Universality for all three generations is required!
$E_6$ unification solves these problems

• Various Yukawa hierarchies can be obtained from one basic hierarchy.
  
  Breaking $U(2)_H \rightarrow SU(2)_H \rightarrow X$
  
  $\lambda \wedge$ $\lambda^2 \wedge$

  gives the basic hierarchical structure.

• All the three light $\bar{5}$ fields come from $\Psi(27, 2)$

  and therefore have universal sfermion masses.

  Important for suppressing FCNC sufficiently, because the mixings of $V_{\bar{5}}$ are large.
Discussion

• Extension to $U(3)_H$ is straightforward. All three generation quarks and leptons are unified into a single multiplet $(27, 3)$
• If it is local, D term must be cared.
• Peculiar sfermion spectrum

\[ \Psi(27, 2) \]

- $10_1, 10_2$
- $\bar{5}_1, \bar{5}_1, \bar{5}_2$
  
Universal

\[ \Psi(27, 1) \]

- $10_3$

Different

Experiments
• Any mechanisms for the basic hierarchy.
  Extra dimension
  Stringy calculation
  Froggatt-Nielsen mechanism
    (Anomalous U(1))
• $E_6$ Higgs sector (Doublet-triplet splitting)
  Anomalous U(1)  N.M.-Yamashita 02,03

  Generic interactions with O(1) coefficients.
  Orbifold breaking

  $E_6 \rightarrow SU(3)^3$
2nd Summary

- In $E_6$ GUT, one basic hierarchy for Yukawa couplings results in various hierarchical structures for quarks and leptons including larger neutrino mixings.
- Family symmetry can easily reproduce the basic hierarchy, and suppress FCNC naturally.
- The simpler unification of quarks and leptons explains the more questions.

\[
E_6 \times \begin{cases} 
U(2)_H & 2(27, 2 + 1) \\
U(3)_H & 1(27, 3)
\end{cases} \quad \text{larger neutrino mixings} \]

\[
E_6 \quad 3 \times 27 \quad \text{SUSY flavor problem}
\]
Spontaneous CP violation

Old and new type SUSY CP problems can be solved and several bonuses
SUSY CP Problem

- EDM constraints $\Rightarrow$ Real SUSY parameters.
  \[ \phi_{M_{1/2}}, \phi_{\mu}, \phi_{B}, \phi_{A} < 10^{-2-3} \]

- $\mu$ problem is solved by anomalous $U(1)$

- Complex Yukawa couplings $\Rightarrow$ CEDM

  $\text{Im}((\delta_{LL}^{u})_{13}(\delta_{RR}^{u})_{31})) < 3 \times 10^{-7}$
  
  $(\delta_{LL}^{u})_{13}(\delta_{RR}^{u})_{31} \sim \lambda^6 \sim 10^{-4}$ in E6 GUT with $SU(2)$

  $\tilde{m}_{10}^2 = \begin{pmatrix} m^2 & m^2 & m_3^2 \end{pmatrix}$
  
  $\delta_{LL}^{u} \equiv V_{10} \left( \frac{\tilde{m}_{10}^2}{m^2} \right) V_{10}^\dagger \sim \begin{pmatrix} 1 & \lambda^5 & \lambda^3 \\ \lambda^5 & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & O(1) \end{pmatrix}$ \[ \lambda \sim 0.22 \]

Complex Yukawa $\Rightarrow V_{10} \sim V_{CKM}$ is complex generically

Additional (discrete) symmetry solves both problems
Decoupling features of SUSY CP problem

- EDM constraints from 1 loop
  \[ \mu = |\mu|e^{i\delta_\mu}, \quad A = |A|e^{i\delta_A} \]
  \[ \delta_\mu, A < 10^{-2(2-3)} \left( \frac{M_{\text{SUSY}}}{100\text{GeV}} \right)^2 \]

- CEDM from Hg(neutron) even if \( \delta_\mu, A = 0 \)

  \[ \text{Im} \left( \delta_{d_L} \right)_{13} (\delta_{d_R})_{31} < 8(24) \times 10^{-6} \]
  \[ \text{Im} \left( \delta_{u_L} \right)_{13} (\delta_{u_R})_{31} < 3(9) \times 10^{-7} \]

Contributions through stop loop are not decoupled. Complex Yukawa couplings induce them generically.
Spontaneous CP violation in SU(2) model

- Doublets under SU(2) family symmetry

\[ 27_a, F_a, \bar{F}^a \]

\[ \langle \bar{F} \rangle \sim \begin{pmatrix} 0 \\ \bar{v} \end{pmatrix}, \quad \langle F \rangle \sim \begin{pmatrix} 0 \\ v e^{i\rho} \end{pmatrix} \]

\[ 27_1 \sim \epsilon^{ab} 27_a \langle F_b \rangle \quad 27_2 \sim 27_a \langle \bar{F}^a \rangle \quad 27_3 \]

\[ W = (Y_H)_{ij} 27_i 27_j 27_H + (Y_C)_{ij} 27_i 27_j 27_C \]

\[ \epsilon^{ab} 27_a 78_A 27_b 27_H (\epsilon^{ab} 27_a 27_b 27_H) \langle 78_A \rangle = Q_{B-L} \]

\[ 27_H = 16_H + 10_H + \langle 1_H \rangle \quad E_6 \rightarrow SO(10) \]

\[ 27_C = \langle 16_H \rangle + 10_C + 1_C \quad SO(10) \rightarrow SU(5) \]

\[ E_6 \text{ Higgs sector} \Rightarrow \begin{cases} H_u \sim 10_H \rightarrow Y_u = Y_H \\ (H_d \sim 10_H + 16_C) \end{cases} \]

Real \( \mu \) and \( B \) require non-trivial discrete charge for \( F \) and \( 27_C \).
A solution for $\mu$ problem

- Negative Higgs charges $\Rightarrow$ massless Higgs
- SUSY (holomorphic) zero mechanism
- SUSY breaking induces the non-vanishing VEV of superheavy positive charged singlet

$$W = SH_u H_d + \Lambda^2 S + \Lambda SZ$$

$$V_{SB} = A_{SHH} S H_u H_d + A_S \Lambda^2 S + \ldots$$

$$\langle S \rangle \sim \frac{A_S \Lambda^2}{M_S^2} \sim A_S$$

$$\mu \sim A_S \sim O(\tilde{m})$$

$$B\mu \sim A_{SHH} \langle S \rangle + F_S \sim O(\tilde{m}^2)$$
Additional discrete symmetry

\[ W = S H_u H_d + \Lambda^2 (1 + \bar{F}F)S + \Lambda S Z \]

\[ V_{SB} = A_{SHH} S H_u H_d + A_S \Lambda^2 S + \ldots \]

\[ \langle S \rangle \sim \frac{A_S \Lambda^2}{M_S^2} \sim A_S \]

\[ \mu \sim A_S \sim O(\tilde{m}) \]

\[ B \mu \sim A_{SHH} \langle S \rangle + F_S \sim O(\tilde{m}^2) \]

Complex

Non trivial discrete charge for \( \bar{F}F \) to forbid \( S \bar{F}F \bar{F}F \)
What happens by the discrete symmetry?

\[ W = (Y_{H})_{ij} 27_i 27_j 27_H + (Y_{C})_{ij} 27_i 27_j 27_C \]

\[ Y_H = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & Q_{B-L} \lambda^5 & 0 \\ Q_{B-L} \lambda^5 & \lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} \text{ real} \]

\[ \epsilon^{ab} 27_a 78_A 27_b 27_H \quad \langle 78_A \rangle = Q_{B-L} V \]

\[ Y_C = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & 0 & 0 \\ \lambda^3 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & \lambda^5 & \lambda^3 \\ \lambda^5 & 0 & 0 \\ \lambda^3 & 0 & 0 \end{pmatrix} \text{ complex} \]

\[ \langle \bar{F} \rangle \sim \begin{pmatrix} 0 \\ \bar{u} \end{pmatrix}, \quad \langle F \rangle \sim \begin{pmatrix} 0 \\ u e^{i\rho} \end{pmatrix} \]
A model with a discrete symmetry $\mathbb{Z}_6$ (Ishiduki-Kim-N.M.-Sakurai09)

- Real up-type Yukawa couplings $\delta u_L, \delta u_R$ can be satisfied.
- Complex down-type Yukawa couplings KM phase can be induced.
- The point

\[
\langle F_a \rangle \sim \begin{pmatrix} 0 \\ v e^{i \delta} \end{pmatrix}
\]

\[
Y^H (F, \bar{F}) : \text{real,} \quad H_u \sim 10_H \quad Y_u = Y^H
\]

\[
Y^C (F, \bar{F}) : \text{complex} \quad H_d \sim 10_H + 16_C
\]

\[
W = Y^H 27_i 27_j \langle 27_H \rangle + Y^C 27_i 27_j \langle 27_C \rangle
\]
A model with a discrete symmetry

- **Bonus 2:** small up quark mass is realized.

Usually, to obtain the CKM matrix

\[ Y_u = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \implies \begin{pmatrix} 0 & Q_{B-L} \lambda^3 \\ Q_{B-L} \lambda^5 & 0 \\ 0 & 0 \end{pmatrix} \]

\[ \epsilon^{ab} \psi_a \psi_b H, \quad \epsilon^{ab} \psi_a \langle A \rangle \psi_b H \]

\[ \langle \epsilon^{ab} F_a \rangle \psi_b \sim \psi_1 \quad \langle A \rangle \propto Q_{B-L} \]

\[ y_u \sim \lambda^6 \implies \left( \frac{1}{3} \right)^2 \lambda^6 \]

Too large \(\Rightarrow\) good value!
A model with a discrete symmetry

- Bonus 3?: # of O(1) parameters = 9-12
  13 physical parameters
  \[ \iff \quad m_u, m_d, m_e, V_{CKM} \]
- One of the relations

\[ m_b = m_\tau (1 + O(\lambda)) \]

\[
\begin{pmatrix}
  m_s = O(1)m_\mu \\
  m_d = O(1)m_e
\end{pmatrix}
\]
Interesting result (perturbation in $\lambda$)

- $V_{ub} \sim \lambda^4$ is obtained

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$V_{CKM} = \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda & 1 \end{pmatrix} \sim \lambda^4$$

$A \sim 0.8$, $\rho, \eta \sim 0.2 - 0.4$

This cancellation depends on the adjoint VEV. (B factory measured the direction of GUT breaking?)

Kawase, N.M 10

$E_6 \rightarrow SO(10) \times U(1)_{V'}$

$E_6 \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{V'}$

$E_6 \rightarrow SU(4) \times SU(2)_L \times U(1)_R \times U(1)_{V'}$

$|Y_b V_{cb}| = |Y_c| \rightarrow \tan \beta \sim 6$
**Numerical calculation**

- **O(1) coefficients (10 parameters)**
  
  \[
  \begin{align*}
  a &= 0.6, \quad b = -0.5, \quad c = -0.7, \quad d_5 = -0.9 \\
  d_q &= 0.4, \quad d_l = -0.5, \quad f = 1.5, \quad g = -0.9 \\
  \beta_H &= 0.9, \quad \delta = 1.4
  \end{align*}
  \]

  \[
  \begin{align*}
  Y_t &= 6(5) \times 10^{-1} \quad Y_b = 2(3) \times 10^{-2} \quad Y_T = 3(4) \times 10^{-2} \\
  Y_c &= 3(1) \times 10^{-3} \quad Y_s = 5(6) \times 10^{-4} \quad Y_\mu = 1(3) \times 10^{-3} \\
  Y_u &= 4(3) \times 10^{-6} \quad Y_d = 8(3) \times 10^{-5} \quad Y_e = 3(1) \times 10^{-5}
  \end{align*}
  \]

- **\( |V_{CKM}| \)**

  \[
  |V_{CKM}| = \begin{pmatrix}
  1 & 2(2) \times 10^{-1} & 2(4) \times 10^{-3} \\
  2(2) \times 10^{-1} & 1 & 10(4) \times 10^{-2} \\
  20(7) \times 10^{-3} & 10(4) \times 10^{-2} & 1
  \end{pmatrix}
  \]

- **\( J_{CP} \)**

  \[ J_{CP} = 1(3) \times 10^{-5} \]

Ref: Ishiduki-Kim-N.M.-Sakurai09

Heavy fields’ contribution must be taken into account.
E6 Higgs sector

- E6 can be broken into the SM gauge group.
  1. Natural doublet-triplet splitting
  2. \( H_u \sim 10_H \), \( H_d \sim 10_H + 16_C \) is realized
  3. consistent with the discrete symmetry

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<th>( SU(2)_H )</th>
<th>( U(1)_A )</th>
<th>( Z_6 )</th>
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<td>3</td>
</tr>
<tr>
<td>( \bar{C} )</td>
<td>27</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>( C' )</td>
<td>27</td>
<td>1</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>( \bar{C}' )</td>
<td>27</td>
<td>1</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>( A )</td>
<td>78</td>
<td>1</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>( A' )</td>
<td>78</td>
<td>1</td>
<td>-1</td>
<td>3</td>
</tr>
</tbody>
</table>
Neutrino sector

\( \bar{5} \) fields at the low energy are determined as 3 massless modes of this \( 3 \times 6 \) matrix.

\[
\begin{pmatrix}
  \bar{5}_1' & \bar{5}_2' & \bar{5}_3' & \bar{5}_1 & \bar{5}_2 & \bar{5}_3 \\
 0 & -Q_{B-L} \lambda^5 & 0 & 0 & \lambda^{5+r} & \lambda^{3+r} \\
 Q_{B-L} \lambda^5 & \lambda^4 & \lambda^2 & 0 & 0 & 0 \\
 0 & \lambda^2 & 1 & \lambda^{5+r} & 0 & 0 \\
\end{pmatrix} \langle H \rangle \\
\lambda^r \equiv \frac{\langle c \rangle}{\langle H \rangle}
\]

- For lepton doublets, \( \bar{5}_1' \) of three massless modes, \( (\bar{5}_1, \bar{5}_1', \bar{5}_2) \), has no mixing with \( \bar{5}_3 \), because \( Q_{B-L} = 0 \) for lepton doublets.

- Yukawa couplings of charged lepton in \( \bar{5}_1' \) can be obtained through \( Y_C 16_1 10_1 16_C \).

- Dirac neutrino Yukawa couplings of \( \bar{5}_1' \) through \( Y_H 1_i 10_1 10_H \) is vanishing because \( Q_{B-L} = 0 \). \( \rightarrow m_{\nu_\mu} = 0 \) That’s a problem.

- Fortunately, we have higher dimensional interactions

\[
\epsilon^{ab}(\bar{27}_H 27_a)(27_b 27_H 27_H) \rightarrow 1_2 10_1 10_H
\]

We have many parameters on the right-handed neutrino masses. Predictions on neutrino masses and mixings are the same as the usual \( E_6 \) GUT.

- \( (\bar{27}_H 27_H) \) can couple with many interactions, but the effects are only for the neutrino sector in the model.
3rd Summary

- KM theory is naturally realized by spontaneous CP violation in E6 GUT with family symmetry.
  1. Real $\mu$ and $B$ parameters are realized by introducing a discrete symmetry.
  2. The symmetry solves CEDM problem.
  3. Predicted EDM induced by RGE effect is sizable.
  4. $V_{ub} \sim \lambda^3 \rightarrow \lambda^4$, $|Y_c V_{bc}| = |Y_b| \rightarrow \tan \beta \sim 6$
  5. $Y_u \sim \lambda^6 \rightarrow \left(\frac{1}{3}\right)^2 \lambda^6$
  6. E6 Higgs sector consistent with the above scenario with natural realization of D-T splitting.
D term problem

- Non vanishing D terms spoil the universality

\[
m_{10}^2 = \begin{pmatrix} m^2 & 0 & 0 \\ 0 & m^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} + \frac{D_{SU(2)_H}}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + D_{V'} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + D_V \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

\[
m_{5}^2 = \begin{pmatrix} m^2 & 0 & 0 \\ 0 & m^2 & 0 \\ 0 & 0 & m^2 \end{pmatrix} + \frac{D_{SU(2)_H}}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + D_{V'} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + D_V \begin{pmatrix} -3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix}
\]

This may induce too large FCNC.

This problem looks to be serious before LHC.

We expected some hints from LHC.
Predictions of $E_6 \times SU(2)_F \times U(1)_A$

Flavor physics: Kim-N.M.-Matsuzaki-Sakurai-Yoshikawa 06,08

$m_3$ must be around the weak scale, because of the stability of the weak scale, while $m$ can be taken larger.

$$
\begin{pmatrix}
m^2 & m^2 \\
m^2 & m_3^2
\end{pmatrix}
$$

Nucleon decay: Y. Muramatsu-N.M. 13,14

Important prediction of Natural (Anomalous U(1)) GUT
Nucleon decay via dim. 6 operators is enhanced, while nucleon decay via dim. 5 is suppressed.
Non universal SUSY breaking

- Universal sfermion masses for $\tilde{5}$ fields
  \[ \delta_{\tilde{5}} \sim \delta_{\tilde{d}_R^c} \sim \delta_{\tilde{l}} \sim 0 \]

- Non universality for $\bf{10}$ fields
  \[ \delta_{\bf{10}} \sim \delta_{\tilde{q}} \sim \delta_{\tilde{u}_R} \sim \delta_{\tilde{e}_R} \]
  \[ \sim V^\dagger_{\text{CKM}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_{\text{CKM}} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \]
  \[ V_{\tilde{q}} \sim V_{\tilde{u}_R} \sim V_{\tilde{e}_R} \sim V_{\text{CKM}} \]

- Weak scale stability requires $m^2_3 \sim O((100\text{GeV})^2)$
  but almost no constraint for $m_0$
Structures suppressing FCNC for $\tilde{5}_i$

- Small Yukawa couplings
- Small $\tan \beta$
- Universal sfermion masses for $\tilde{5}_i$
- $m$ can increase without destabilizing the weak scale. (Effective SUSY)

$\Psi_a(27, 2)$

$m$

$10_1, 10_2$

$\bar{5}_1, \bar{5}_2, \bar{5}_3$

$\Psi_3(27, 1)$

$10_3$

$m_3$

$m >> m_3 \sim O(m_W)$
How does FCNC processes take place in this model?

Since 10 contains Q, the form of unitary matrix $V$ is CKM-like. We can parametrize it with Cabibbo angle $\lambda$.

$$V \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad \lambda = 0.22 \quad \Delta m^2 = (m_3^2 - m^2)$$

$$\tilde{m}_{e_R}^2 = \tilde{m}_{\tilde{q}_L}^2 = \tilde{m}_{\tilde{u}_R}^2 = V^\dagger \begin{pmatrix} m^2 \\ m^2 \\ m_3^2 \end{pmatrix} V \sim \begin{pmatrix} m^2 & \Delta m^2 \lambda^5 & \Delta m^2 \lambda^3 \\ \Delta m^3 \lambda^3 & m^2 & \Delta m^2 \lambda^2 \\ \Delta m^3 \lambda^2 & \Delta m^2 \lambda^2 & m_3^2 \end{pmatrix}$$
Non decoupling feature of this model (in lepton flavor violation)

\[
\tilde{m}_{e_R}^2 \sim \begin{pmatrix}
\frac{m^2}{\Delta m^2 \lambda^5} & \frac{\Delta m^2 \lambda^3}{m^2} \\
\frac{\Delta m^2 \lambda^3}{\Delta m^3 \lambda^3} & \frac{\Delta m^2 \lambda^2}{m_3^2}
\end{pmatrix}
\quad \lambda = 0.22
\Delta m^2 = (m_3^2 - m^2)
\]

- By picking up the 3-2 element, the size of $\tau \rightarrow \mu$ transition rate is order $\lambda^2$.

\[
\tau \rightarrow \mu \gamma \quad \Delta m_{\tau\mu} \approx \frac{1}{m_3^2} \Delta m^2 \lambda^2 \frac{1}{m^2} \quad \rightarrow \frac{\lambda^2}{m_3^2}
\]

- For $\mu \rightarrow e \gamma$, there are two passes to change the flavor $\mu \rightarrow e$. Both they are order $\lambda^5$.

\[
\mu \rightarrow e \gamma \quad \Delta m_{\mu e} \approx \frac{1}{m^2} \Delta m^2 \lambda^5 \frac{1}{m^2} \quad \rightarrow 0
\]

If we raise overall SUSY scale $m$ ...

\[
m^2 \rightarrow \infty
\]

Propagator suppression from 1 or 2 generation becomes stronger, but mass difference $\Delta m^2$ increase. As a result, both transition rate remain finite, and don’t decouple!
Can we discover the LFV at the future experiments?

- $\tau \rightarrow \mu \gamma$
  Detectable, when $\tan \beta$ is large and $m_{\tilde{e}_{R3}} < 250 \text{GeV}$

- $\mu \rightarrow e \gamma$
  Detectable if $m_{\tilde{e}_{R3}} < 400 \text{GeV}$

**Data from future experiments:**

- At (super-)KEKB with $\tau \rightarrow \mu \gamma$
  - $> 7.0 \times 10^{-8}$ (exclude)

- At MEG experiment with $\mu \rightarrow e \gamma$
  - $> 1.0 \times 10^{-11}$ (exclude)
  - $\mathcal{O}(10^{-12})$
  - $\mathcal{O}(10^{-13})$
  - $\mathcal{O}(10^{-14})$
  - $< 1.0 \times 10^{-14}$
This model says that final state lepton tends to be right-handed.

- Final state lepton has different chirality from initial one.

\[ q = p - p' \]

\[ \epsilon^\mu \bar{u}_i(p) i \sigma_{\mu\nu} q^\nu (A_L^{ij} P_L + A_R^{ij} P_R) u_j(p') \]

- Intermediate state must be right-handed to pick up the \( \tilde{m}_R^2 \).

How can we see this feature experimentally?

It is possible to check this feature experimentally by measuring the angular distribution of final state lepton.
Predictions (Quark sector)

- The magnitudes are the same order as of the RGE effects in the universal mass case.
- New CP phases!!
  The CP violation in B meson system may be detectable


## CP violation in B meson

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>E6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{B_s \rightarrow J/\psi \phi}$</td>
<td>O(0.04)</td>
<td>O(0.006)</td>
</tr>
<tr>
<td>$\Delta S_{K\phi}$, $\Delta S_{K \eta'}$</td>
<td>&lt;0.15</td>
<td></td>
</tr>
<tr>
<td>$A_{CP}(b \rightarrow s \gamma)$</td>
<td>0.006</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>$A_{CP}(B \rightarrow V \gamma)$</td>
<td>very small</td>
<td></td>
</tr>
<tr>
<td>$S_{B \rightarrow K_s \pi^0 \gamma}$</td>
<td>very small</td>
<td></td>
</tr>
</tbody>
</table>

For $\tan \beta \sim 10$
$B_d \rightarrow \phi Ks, \eta' Ks$

$\Delta S_{\phi Ks}^{SUSSY}, \Delta S_{\eta' Ks}^{SUSSY} \sim O(0.1)$ is possible.

Gluino contribution is decoupled. Chargino contribution is not decoupled.

in the limit $m >> m_3$

$O(0.1)$ deviation in B factory may be confirmed in SuperB factory.

$\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}})$

<table>
<thead>
<tr>
<th>Process</th>
<th>World Average</th>
<th>BaBar</th>
<th>Belle</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \rightarrow c\bar{c}s$</td>
<td>$0.68 \pm 0.03$</td>
<td>$0.12 \pm 0.31 \pm 0.10$</td>
<td>$0.50 \pm 0.21 \pm 0.06$</td>
<td>$0.39 \pm 0.18$</td>
</tr>
<tr>
<td>$\phi K^0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BaBar</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Belle</td>
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<tr>
<td>Average</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\eta' K^0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BaBar</td>
<td></td>
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<tr>
<td>Belle</td>
<td></td>
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</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
### Summary table of E6 predictions

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>E6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Br}(\mu \to e\gamma)$</td>
<td>$\approx 0$</td>
<td>$10^{-11} - 10^{-14}$</td>
</tr>
<tr>
<td>$\text{Br}(\tau \to \mu\gamma)$</td>
<td>$\approx 0$</td>
<td>$10^{-8} - 10^{-10}$</td>
</tr>
<tr>
<td>$S_{B_s \to J/\psi \phi}$</td>
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<td>very small</td>
<td></td>
</tr>
</tbody>
</table>
Strictly speaking, $\delta_5 \neq 0$, $\delta_{LR} \neq 0$

when $U(2)_H$, e.g.

$$\delta_5 \sim \left( \begin{array}{ccc} \lambda^5 & \lambda^4 & \lambda^{3.5} \\ \lambda^4 & \lambda^3 & \lambda^{2.5} \\ \lambda^{3.5} & \lambda^{2.5} & \lambda^2 \end{array} \right)$$

This can be consistent with the experiments, but the predictions can be changed.

If we take $m_0 >> m_3$, this model dependent parts can be neglected.

No weak scale instability!!
Nucleon decay

N.M.-Y. Muramatsu 13, 14
Natural(Anomalous U(1)) GUT

- Natural: All the interactions which are allowed by the symmetry are introduced with O(1) coefficients. (incl. higher dimensional int.)
- We can define the model only by fixing the symmetry of the model (except O(1) coefficients). The parameters for the definition are mainly about 10 charges for the fields.
- The predictions are expected to be stable under the quantum corrections or gravity effects.
- This assumption is quite natural.
- Infinite number of interactions can be controlled.
- Doublet-triplet splitting problem can be solved.
- Realistic quark and lepton masses and mixings.
- Non trivial explanation for gauge coupling unification.
- Anomalous U(1) gauge symmetry plays an essential role.
Natural Gauge Coupling Unification

New Explanation for the success

- Fix a model
  with \( \Lambda \)
- Calculate \( \alpha_i(\Lambda_W) \)

- Calculate \( \alpha_i(\mu) \)
  with \( \alpha_i(\Lambda_W) \)
- Always meet at a scale \( \Lambda \)

\( \Lambda_A < \Lambda \sim \Lambda_G \sim 2 \times 10^{16} \text{ GeV} \)

N.M. 01, N.M-Yamashita 02
Nucleon decay via dim. 6 is enhanced

- Unification scale becomes lower.
  \[ \Lambda_U \sim \lambda^{-\alpha} \Lambda_G < \Lambda_G \]
  Proton decay via dimension 6 op.

- \( \tau(p \rightarrow e\pi) \sim (2 - 8) \times 10^{34} \) years (\( \Lambda_U \sim \Lambda_G/2 \))

- \( \tau_{\text{exp}}(p \rightarrow e\pi) > 1.4 \times 10^{34} \) years

- Generic interactions \[ \lambda^{-\alpha} < 1 \]
GUT model identification by nucleon decay

two important ratios of partial decay widths to identify GUT model

\[ R_1 \equiv \frac{\Gamma_{n \rightarrow \pi^0 + \nu c}}{\Gamma_{p \rightarrow \pi^0 + e^-}} \quad \text{to identify grand unification group} \]

\[ R_2 \equiv \frac{\Gamma_{p \rightarrow K^0 + \mu^-}}{\Gamma_{p \rightarrow \pi^0 + e^-}} \quad \text{to identify Yukawa structure at GUT scale} \]

\[ \mathcal{L}_{\text{eff}} = \frac{g_{GUT}^2}{M_{X_{SU(5)}}^2} \left\{ \begin{array}{c} (\bar{c}_{R_i}^c u_{R_j}) (\bar{u}_{L_j}^c d_{L_j}) + (\bar{c}_{R_i}^c u_{R_j}) (\bar{u}_{L_j}^c d_{L_j}) \\ + (e_{L_i}^c u_{L_j}) (\bar{u}_{R_j}^c d_{R_i}) + (e_{L_i}^c u_{L_j}) (\bar{u}_{R_j}^c D_{R_i}) \\ - (\bar{\nu}_{L_i}^c d_{L_j}) (\bar{u}_{R_j}^c d_{R_i}) - (\bar{N}_{L_i}^c d_{L_j}) (\bar{u}_{R_j}^c D_{R_i}) \end{array} \right\} \]

\[ + \frac{g_{GUT}^2}{M_{X_{SO(10)}}^2} \left\{ (\bar{E}_{R_i}^c u_{R_j}) (\bar{u}_{R_j}^c d_{R_i}) - (\bar{\nu}_{L_i}^c d_{L_j}) (\bar{u}_{R_j}^c d_{R_i}) \right\} \]

\[ + \frac{g_{GUT}^2}{M_{X_{E_6}}^2} \left\{ (\bar{E}_{L_i}^c u_{L_j}) (\bar{u}_{R_i}^c D_{R_j}) - (\bar{N}_{L_i}^c d_{L_j}) (\bar{u}_{R_i}^c D_{R_j}) \right\} \]

: dimension-6 operators which have anti electron in final state

: dimension-6 operators which have anti neutrino in final state
$R_1 = \frac{\Gamma_{\nu \rightarrow \pi^0 + e^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$
4\textsuperscript{th} Summary

- Observed (or observing)
  \[ V_{13} \sim \lambda, \ (\delta_{\text{lepton}} \sim O(1)) \]
  \[ (V_{ub} \sim \lambda^4) \]

- Not yet
  \textbf{Nucleon decay} \quad \textbf{large} \quad \frac{\Gamma_{n \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}} \quad \frac{\Gamma_{p \rightarrow K^0 + \mu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}

  Unfortunately most of FCNC processes are decoupled when SUSY breaking scale is large.
Impact of LHC

- 125 GeV Higgs
  - Stop mass \( m_3 \) must be larger than 1 TeV
- No SUSY particle

\[ \Rightarrow \] SUSY breaking scale may be larger than we expected.

The little hierarchy problem is more serious. FCNC problem (D term problem) is milder.
The little hierarchy problem .
-Very serious problem must be solved!

N.M.-K.Takayama 14
Little hierarchy problem

\[ \tilde{m}_{10}^2 = \begin{pmatrix} m^2 & \cdot & \cdot & \cdot \\ \cdot & m^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & m_3^2 \\ \cdot & \cdot & \cdot & m^2 \end{pmatrix}, \]

\[ \tilde{m}_{5}^2 = \begin{pmatrix} m^2 & \cdot & \cdot & \cdot \\ \cdot & m^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & m^2 \\ \cdot & \cdot & \cdot & m^2 \end{pmatrix} \]
Low mediation scale scenarios

- **Gauge mediation**
  Mass of the messenger particles can be small. Unfortunately it is difficult to obtain large $A_t$

- **Mirage mediation**
  Choi-Falkowski-Nilles-Olechowski-Pokorski04, Jeong-Kobayashi-Okumura05, Kitano-Nomura05

Due to the cancellation between anomaly mediation and RG effects of moduli contribution, the mediation scale can be lower effectively.

Special boundary conditions are required

$$M_{1/2} = \tilde{A}_t = \sqrt{2}\tilde{m}_\tilde{t}$$
Cosmological Gravitino Problem

SUSY is still promising

Decay of gravitino produced in early universe spoils BBN.

One solution

$O(100\text{TeV})$ gravitino

It decays before BBN!

High scale SUSY but destabilizes the weak scale.

Roughly $m_{3/2} > 50 \text{ TeV}$
What we would like to show

- Gravitino mass $m_{3/2} = \mathcal{O}(100) \text{ TeV}$ to solve the cosmological gravitino problem
- The other SUSY breaking parameters = $\mathcal{O}(1) \text{ TeV}$ for the naturalness

As in mirage mediation

The little hierarchy problem can be less severe. \( \mathcal{O}(\%) \) tuning is realized.

Point: anomaly mediation cancels the RG contribution.
Cancellation property of anomaly mediation

- Gaugino mass

RGE \[ \frac{d}{dt} M_a = \frac{1}{8\pi^2} b_a g_a^2 M_a \]

\[ M_a(\mu)_{\text{anomaly}} = \frac{b_a g_a^2}{16\pi^2} m_{3/2} \]

\[ M_a(\mu)_{\text{gravity}} = \tilde{M}_a + \frac{b_a g_a^2}{8\pi^2} \tilde{M}_a \ln \frac{\mu}{\Lambda} \]

\[ M_a(\mu) = M_a(\mu)_{\text{anomaly}} + M_a(\mu)_{\text{gravity}} \]

\[ = M_{1/2} + \frac{b_a g_a^2}{8\pi^2} M_{1/2} \ln \frac{\mu}{M_{\text{mir}}} \quad (\tilde{M}_1 = \tilde{M}_2 = \tilde{M}_3 = M_{1/2}) \]

\[ \ln \frac{\Lambda_G}{M_{\text{mir}}} = \frac{m_{3/2}}{2 M_{1/2}} \]

\[ M_{\text{mir}} \sim 1 \text{ TeV} \leftrightarrow \frac{m_{3/2}}{M_{1/2}} \sim 60 \]
Cancellation property (no Yukawa)

\[ A_{ijk} \]

\[ A_{ijk}(\mu) = \tilde{A}_{ijk} - \frac{1}{8\pi^2} (\gamma_i + \gamma_j + \gamma_k) M_{\text{mir}}^{1/2} \ln \frac{\mu}{M_{\text{mir}}} \]

\[ \gamma_i = 2 \sum_a C_{i\alpha}^a g_{\alpha}^2 \]

- **Scalar fermion mass square** \( m_i^2 \)

\[ m_i^2(\mu) = \tilde{m}_i^2 - \frac{1}{4\pi^2} \gamma_i M_{\text{mir}}^{1/2} \ln \frac{\mu}{M_{\text{mir}}} - \frac{1}{8\pi^2} \gamma_i M_{\text{mir}}^{1/2} \left( \ln \frac{\mu}{M_{\text{mir}}} \right)^2 \]

\[ + \frac{3}{40\pi^2} Y_i g_1^2 \tilde{S} \ln \frac{\mu}{\Lambda} \quad (i \neq t_L, t_R, H_u) \]

\[ \tilde{S} = \sum_i Y_i \tilde{m}_i^2 \]

- These parameters at the mirage scale \( M_{\text{mir}} \) become gravity
What happens for sfermions with large top Yukawa?

\[ A_t(\mu) = \tilde{A}_t + 6\rho(\tilde{A}_t - M_{1/2}) - \frac{1}{8\pi^2}(\gamma_{H_u} + \gamma_{t_L} + \gamma_{t_R})M_{1/2} \ln \frac{\mu}{M_{\text{mir}}} \]

\[ m_i^2(\mu) = \tilde{m}_i^2 - k_i\rho \left[ (\tilde{A}_t - M_{1/2})^2 (1 + 6\rho) + \tilde{\Sigma}_t - M_{1/2}^2 \right] \]

\[ - \frac{1}{4\pi^2} \left[ \gamma_i M_{1/2}^2 + k_i(\tilde{A}_t - M_{1/2})(1 + 6\rho)y_t^2 \right] \ln \frac{\mu}{M_{\text{mir}}} \]

\[ - \frac{1}{8\pi^2} \gamma_i M_{1/2}^2 \left( \ln \frac{\mu}{M_{\text{mir}}} \right)^2 \quad (i = t_L, t_R, H_u) \]

Special boundary conditions

\[ M_{1/2} = \tilde{A}_t = \tilde{\Sigma}_t \overset{\text{def}}{=} \tilde{m}_{t_L}^2 + \tilde{m}_{t_R}^2 + \tilde{m}_{H_u}^2 \rightarrow \tilde{A}_t = \sqrt{2}\tilde{m}_t \quad (\tilde{m}_{H_u} = 0) \]

are required to obtain \( m_i^2(M_{\text{mir}}) = \tilde{m}^2 \) and \( A_t(M_{\text{mir}}) = \tilde{A}_t \).
Generalization of mirage mediation

- In the usual mirage mediation, universal sfermion masses are adopted.
- If the conditions $M_{1/2} = \tilde{A}_t = \tilde{\Sigma}_t \overset{\text{def}}{=} \tilde{m}_{t_L}^2 + \tilde{m}_{t_R}^2 + \tilde{m}_{H_u}^2$ are satisfied, any values for the other parameters are OK for mirage phenomena when $\tilde{S} = 0$.
- For example, natural(effective) SUSY type sfermion masses $m_3^2 = \tilde{m}_{t_L}^2 = \tilde{m}_{t_R}^2 = \tilde{m}_{\tau_R}^2 \ll m_0^2 = \tilde{m}_i^2 (i \neq t_R, t_L, \tau_R)$; LHC and FCNC constraints become milder than the usual mirage mediation with fixed $m_3$ [Asano-Higaki12]
- In this talk, we do not address $m_0$ so much because the value is not so important in our arguments
What happens without special boundary conditions?

Without these boundary conditions, the situation becomes just the situation in which the gravity mediation and anomaly mediation contribute at the same time.
Sizable anomaly mediation

It is known that anomaly mediation leads to negative mass square of some of the sfermions. → upper bound of $m_{3/2}$

$$\frac{m_{3/2}}{M_{1/2}} = 60 \leftrightarrow M_{\text{mir}} = 1 \text{TeV}$$

$M_{1/2} = \sqrt{2}\tilde{m}_t$ at mirage point

Wide region where $\tilde{m}_i^2 (\Lambda_G) > 0$
in general setup.
\[ \Delta m_{H_u}^2 = m_{H_u}^2 (m_{SUSY}) - \tilde{m}_{H_u}^2 \]

from \( m_{3/2}, M_{1/2}, \tilde{m}_t, \tilde{A}_t \).

We fixed

\[
\frac{m_{3/2}}{M_{1/2}} = 60 \quad \tilde{m}_t = \sqrt{2} \text{ TeV}
\]

1. O(\%) tuning is realized! (\( \leftrightarrow \) Width of 1\% band is O(TeV))

2. Mild dependence on \( \tilde{A}_t \) (Important to obtain heavy Higgs.)
\[ m_h^2 / 2 \Delta m_{H_u}^2 \quad \text{for} \quad \frac{m_{3/2}}{M_{1/2}} = 50, 60, 70, 80 \quad \tilde{m}_t = \sqrt{2} \, \text{TeV} \]

1. \( \mathcal{O}(\%) \) tuning is realized! (\( \rightarrow \) Width of 1\% band is \( \mathcal{O}(\text{TeV}) \))
2. Mild dependence on \( \tilde{A}_t \) (Important to obtain heavy Higgs.)
Little hierarchy problem

\[ \Delta m_{H_u}^2(1\text{TeV}) = c_0 M_{1/2}^2 + c_1 \tilde{\Sigma}_t + c_2 \tilde{A}_t^2 + c_3 \tilde{A}_t M_{1/2} \]

- In CMSSM, \( c_0 = -1.6, c_1 = -0.40, c_2 = -0.082, c_3 = -0.26 \)

\[ \Delta m_{H_u}^2 = -2.34 M_{1/2}^2 \] if \( M_{1/2} = \tilde{A}_t = \tilde{\Sigma}_t \).

- In mirage (anomaly mediation with \( m_{3/2}/M_{1/2} = 60 \)),

\[ c_0 = 0.29, c_1 = -0.40, c_2 = -0.082, c_3 = 0.16 \]

\[ \Delta m_{H_u}^2 = -0.031 M_{1/2}^2 \] if \( M_{1/2} = \tilde{A}_t = \tilde{\Sigma}_t \).

- Reasons for this improvement
  1. Smaller coefficients \( c_i \)
  2. Cancellation (Different signature)

(Naturalness arguments strongly depend on high energy physics)

What happens for other values of \( m_{3/2}/M_{1/2} \)?
What happens for other values of $m_{3/2}/M_{1/2}$?

$c_1 = -0.40, c_2 = -0.082$

1. Smaller coefficients $c_i$
2. Different signature

Improvement is generally expected
Important observation

- Gravitino mass $m_{3/2} = O(100)$ TeV
to solve the cosmological gravitino problem
- The other SUSY breaking parameters $= O(1)$ TeV for the naturalness

The little hierarchy problem can be less severe.
$O(\%)$ tuning is realized.
Point: anomaly mediation cancels the RG contribution.
$\tilde{A}_t$ can be larger without changing $\Delta m^2_{H_u}$

If $M_{mir} \sim O(\text{TeV})$, we may observe directly the GUT signatures through the mass spectrum of the other sfermions than stops.
Sizable D-term contribution as a signature of $E_6 \times SU(2)_F \times U(1)_A$

Natural (Effective) SUSY type sfermion masses

\[
\begin{align*}
10: & \quad \begin{pmatrix} m^2 & m^2 \\ m^2 & m^2_3 \end{pmatrix} \\
5: & \quad \begin{pmatrix} m^2 & m^2 \\ m^2 & m^2 \end{pmatrix}
\end{align*}
\]

Most of models which predict natural SUSY sfermion masses are suffering from CEDM constraints. If natural SUSY sfermion masses are observed, this scenario is implied.

Small deviation from natural SUSY sfermion masses can be a signature of $E_6$ GUT with family symmetry.
A signature from sizable D-term contributions

\[ m_{10}^2 = \begin{pmatrix} m^2 & 0 & 0 \\ 0 & m^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} + \frac{D_{SU(2)} F}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + D_{V'} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + D_V \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ m_5^2 = \begin{pmatrix} m^2 & 0 & 0 \\ 0 & m^2 & 0 \\ 0 & 0 & m^2 \end{pmatrix} + \frac{D_{SU(2)} F}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + D_{V'} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + D_V \begin{pmatrix} -3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix} \]

\[ \Delta m_{10,2}^2 \equiv m_{10,2}^2 - m_{10,1}^2 = -D_{SU(2)} F \]

\[ \Delta m_{5,2}^2 \equiv m_{5,2}^2 - m_{5,1}^2 = -3D_{V'} + 5D_V \]

\[ \Delta m_{5,3}^2 \equiv m_{5,3}^2 - m_{5,1}^2 = -D_{SU(2)} F \]

An important prediction \[ \Delta m_{10,2}^2 = \Delta m_{5,3}^2 \]

How large D-term can be allowed?
We consider FCNC constraints
FCNC constraints for mass insertion parameters

mass insertion parameter

\[
(\delta^\psi_{ij})_{\Gamma\Gamma} \equiv \frac{(U^\dagger_{\psi\Gamma} \tilde{m}^2_{\psi\Gamma} U_{\psi\Gamma})_{ij}}{m^2_{\tilde{\psi}}} \quad (\Gamma = L, R)
\]

diagonalizing matrices \( U \)
for 10 matters

\[
L_u \sim L_d \sim R_u \sim R_e \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}
\]

for 5 matters

\[
L_e \sim L_v \sim R_d \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}
\]

Constraints from \( \epsilon_K \) is the strongest because decoupling feature is weak.
FCNC constraints from $K^0 - \bar{K}^0$ mixing ($\varepsilon_K$)

\[ \sqrt{|\text{Im}(\delta^d_{12})^2_{LL}|} < 2.9 \times 10^{-3} \left( \frac{m_\tilde{d}}{500\text{GeV}} \right) \]
\[ \sqrt{|\text{Im}(\delta^d_{12})^2_{RR}|} < 2.9 \times 10^{-3} \left( \frac{m_\tilde{d}}{500\text{GeV}} \right) \]
\[ \sqrt{|\text{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}|} < 1.1 \times 10^{-4} \left( \frac{m_\tilde{d}}{500\text{GeV}} \right) \]

mass insertion parameter in this model

\[ (\delta^d_{12})_{LL} \sim \left( \frac{2}{3} + i\frac{4}{27} \right) \left( \lambda \frac{\Delta m^2_{10,2}}{m^2_\tilde{d}} + \lambda^5 \frac{\Delta m^2_{10,3}}{m^2_\tilde{d}} \right) \]
\[ (\delta^d_{12})_{RR} \sim \frac{2}{3} (1 + i) \left( \lambda^{0.5} \frac{\Delta m^2_{5,2}}{m^2_\tilde{d}} + \lambda^{1.5} \frac{\Delta m^2_{5,3}}{m^2_\tilde{d}} \right) \]

$\lambda \sim 0.22$

Ciuchini et al. (1998)
Result 1

When $m_{\tilde{d}} = 10$ TeV

$x \sim y \sim 0.1$

= D-term can be 1 TeV!

signature of $E_6 \times SU(2)_F \times U(1)_A$ SUSY GUT model in future experiments (100 TeV proton collider or muon...
Gravitino mass $m_{3/2} = O(100)$ TeV
to solve the cosmological gravitino problem
The other SUSY breaking parameters = $O(1)$ TeV
for the naturalness

The little hierarchy problem can be less severe. $O(\%)$ tuning is realized.
Point: anomaly mediation cancels the RG contribution.
$	ilde{A}_t$ can be larger without changing $\Delta m_{H_u}^2$

If $M_{mir} \sim O(\text{TeV})$, we may observe directly the GUT signatures through the mass spectrum of the other sfermions than stops.
(ex. D-term contributions of GUT.)

- Natural SUSY type sfermion masses may be directly observed.
- An important prediction $m_{10,2}^2 - m_{10,1}^2 = m_{5,3}^2 - m_{5,1}^2$
- D term can be 1 TeV! ($\epsilon_K$)
Summary

- GUT is promising
  Experimental supports for two unifications

- $E_6$ GUT is interesting
  An assumption in $SU(5)$ GUT can be derived.
  Various Yukawa hierarchies in SM can be obtained from one hierarchy.

- Family symmetry
  Unification of three generation quark and leptons with realistic Yukawa
  SUSY flavor problem is solved (Natural SUSY type sfermion masses)

- Spontaneous CP violation
  Origin of KM phase can be understand
  SUSY CP problem is solved. (Even CEDM constraints can be satisfied.)
Summary

_predictions

Observed \( V_{13} \sim \lambda, (\delta \sim O(1), V_{ub} \sim \lambda^4) \)
Not yet
- Nucleon decay
- Sfermion mass spectrum (Natural SUSY type)
- D-term contribution can be a smoking gun.

\[ m_{10,2}^2 - m_{10,1}^2 = m_{5,3}^2 - m_{5,1}^2 \]

Signatures for future flavor experiments?

Future works

- Cosmology (Inflation, DM, Baryogenesis etc)
- More predictions