Supersymmetric dynamics with non-Lagrangian sectors

Y. Tachikawa

- Non-Lagrangian sector を何故考えるか？
- Non-Lagrangian sector の一つの例
- Non-Lagrangian sector を使った系の解析

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$$\sum_{i=1,2,3} \frac{1}{\alpha_i} F^{(i)}_{\mu\nu} F^{(i)}_{\mu\nu} + D_\mu \phi^\dagger D_\mu \phi + \sum_{\psi=Q,\ell,u,d,e} \bar{\psi}_{\dot{\alpha}} \sigma^{\mu\dot{\alpha}\beta} D_\mu \psi_\beta$$

$$+V(\phi) + y_{ij}(Q^i_L \phi)\bar{d}^j_R + \tilde{y}_{ij}(Q^i_L \phi^\dagger)\bar{u}^j_R + \hat{y}_{ij}(\ell^i_L \phi)\bar{e}^j_R + c.c.$$

Gauge fields, SU(3)xSU(2)xU(1)

$$\sum_{i=1,2,3} \frac{1}{\alpha_i} F^{(i)}_{\mu\nu} F^{(i)}_{\mu\nu} + D_\mu \phi^\dagger D_\mu \phi + \sum_{\psi=Q,\ell,u,d,e} \bar{\psi}_{\dot{\alpha}} \sigma^{\mu\dot{\alpha}\beta} D_\mu \psi_\beta$$

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Gauge fields, SU(3)xSU(2)xU(1)

Higgs kin.

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Gauge fields, SU(3)xSU(2)xU(1)

Higgs kin.

fermions kin.

$$\sum_{i=1,2,3} \frac{1}{\alpha_i} F^{(i)}_{\mu\nu} F^{(i)}_{\mu\nu} + D_\mu \phi^\dagger D_\mu \phi + \sum_{\psi=Q,\ell,u,d,e} \bar{\psi}_{\dot{\alpha}} \sigma^{\mu\dot{\alpha}\beta} D_\mu \psi_\beta$$

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Higgs pot.

Gauge fields, SU(3)xSU(2)xU(1)

Higgs kin.

fermions kin.

$$\sum_{i=1,2,3} \frac{1}{\alpha_i} F^{(i)}_{\mu\nu} F^{(i)}_{\mu\nu} + D_\mu\phi^\dagger D_\mu\phi + \sum_{\psi=Q,\ell,u,d,e} \bar{\psi}_{\dot{\alpha}}\sigma^{\mu\dot{\alpha}\beta} D_\mu\psi_\beta$$

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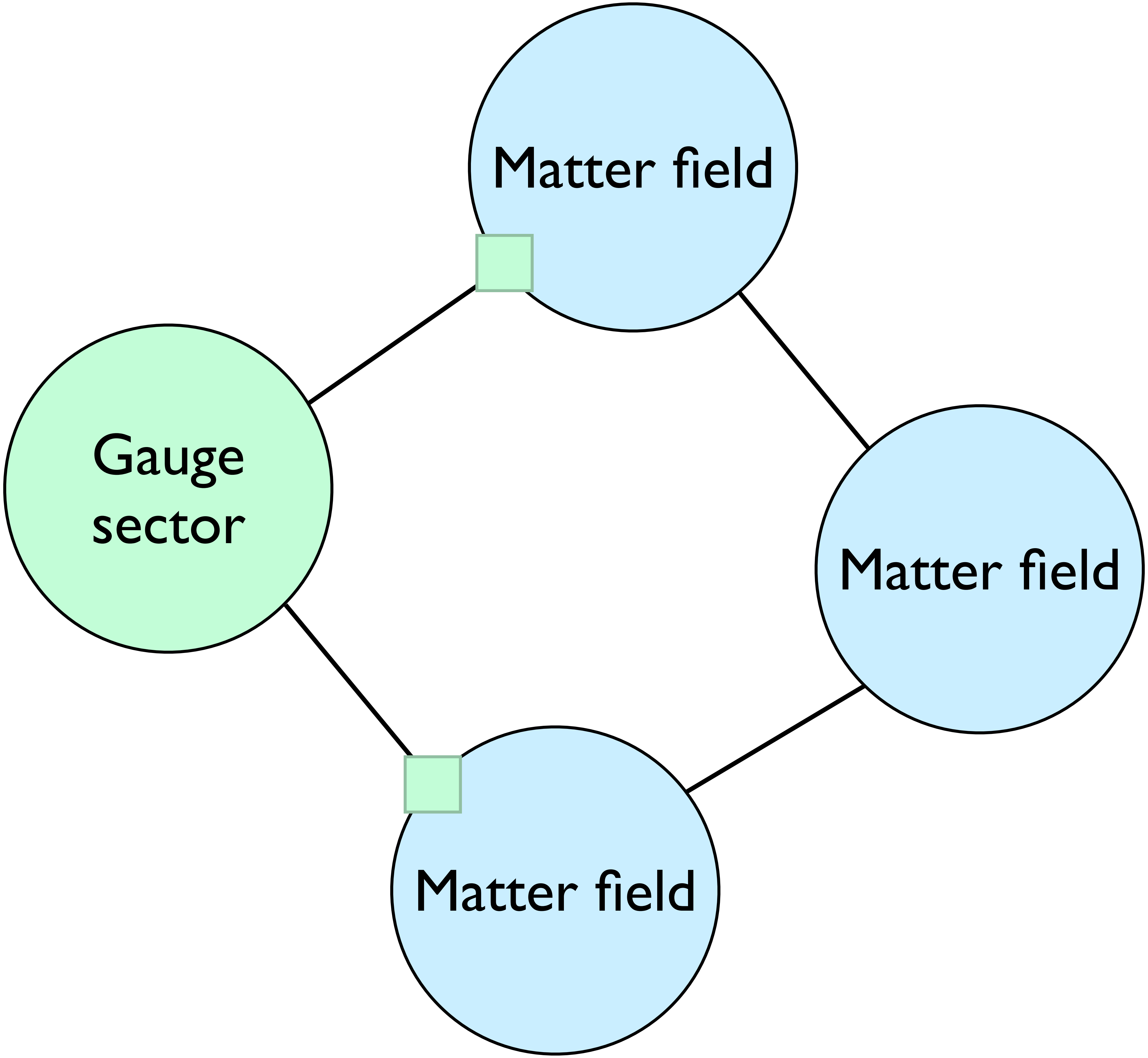
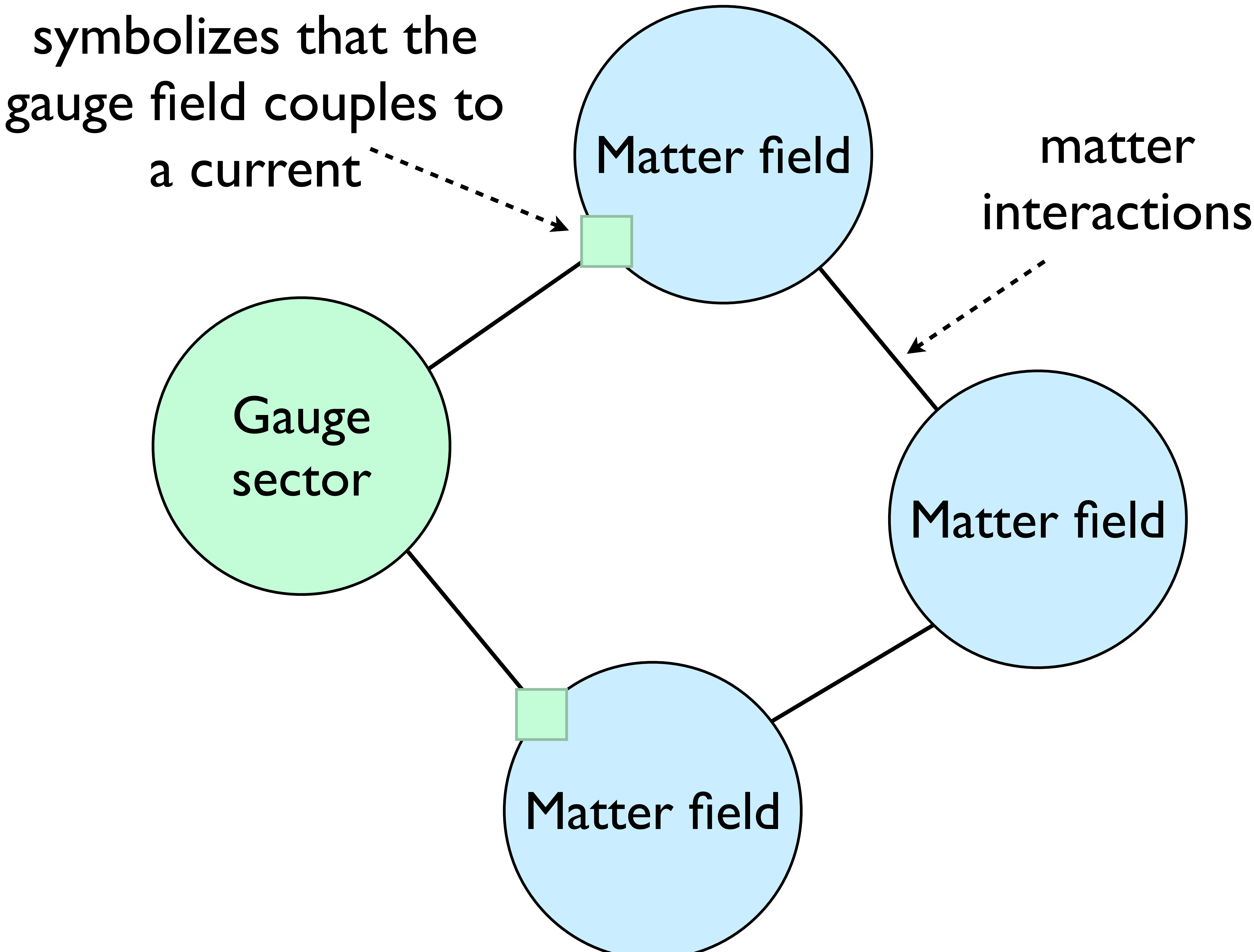
Higgs pot.

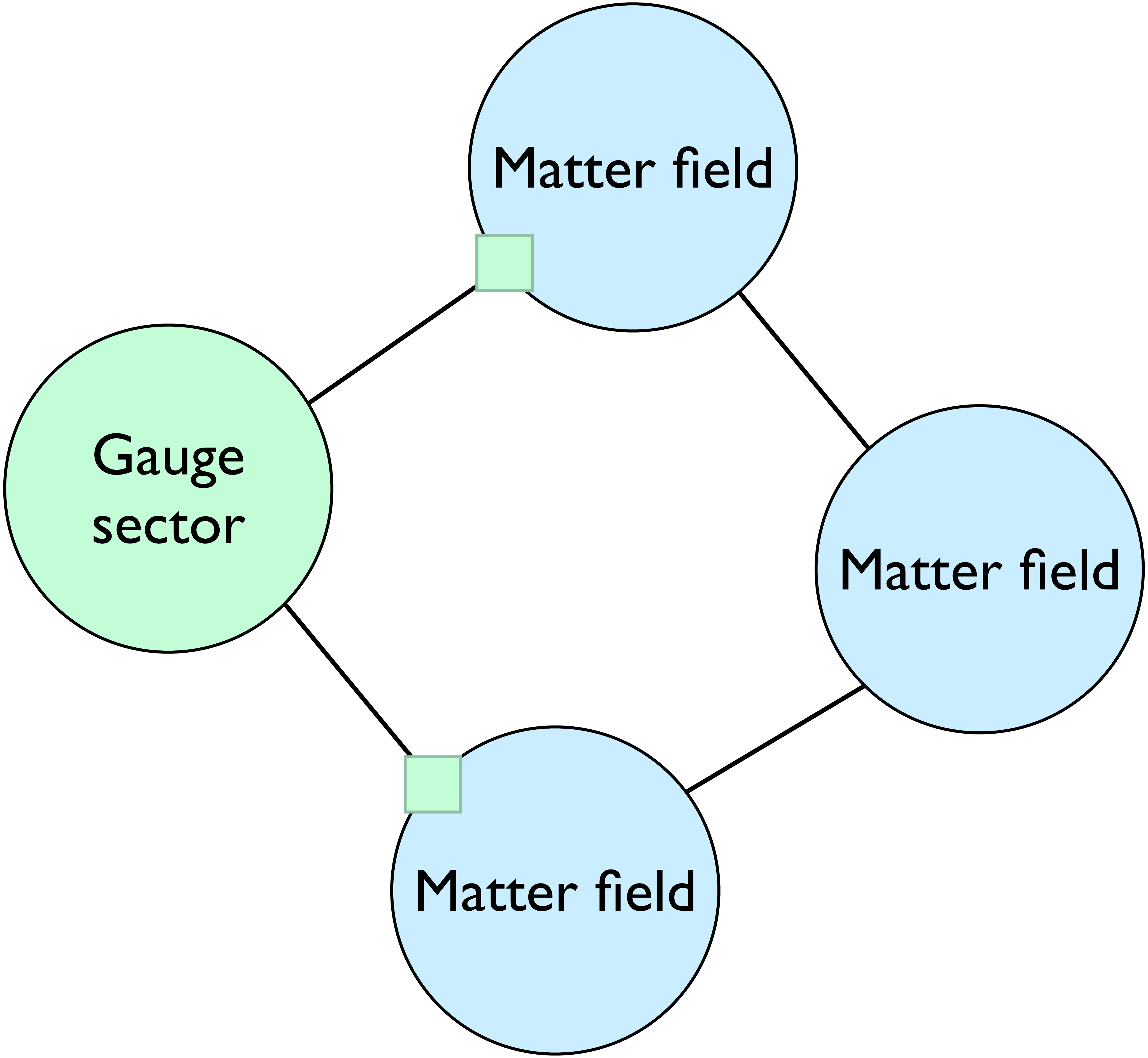
Yukawa interactions

- A conventional QFT problem:
  - Pick elementary fermions and bosons with symmetry currents.
  - Couple gauge fields to currents.
  - Add interaction terms
- We then analyze the system.

- It often becomes strongly coupled in the IR, interesting!
- Somehow we assume it's asymptotically free in the UV.
- It doesn't have to be.


Matter field
Gauge
sector
Matter field
Matter field


symbolizes that the gauge field couples to a current
Matter field
matter interactions
Gauge sector
Matter field
Matter field


Matter field
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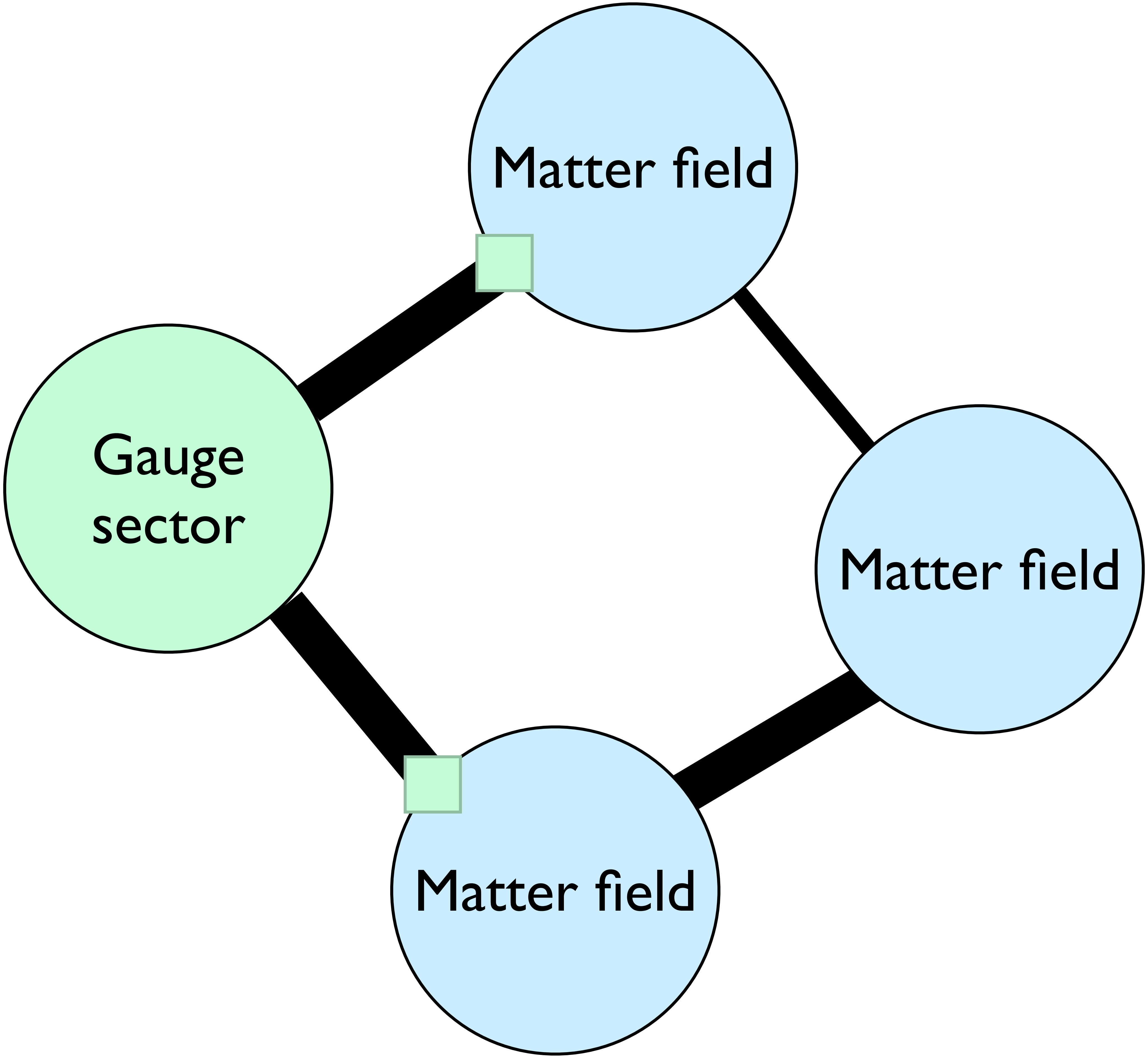
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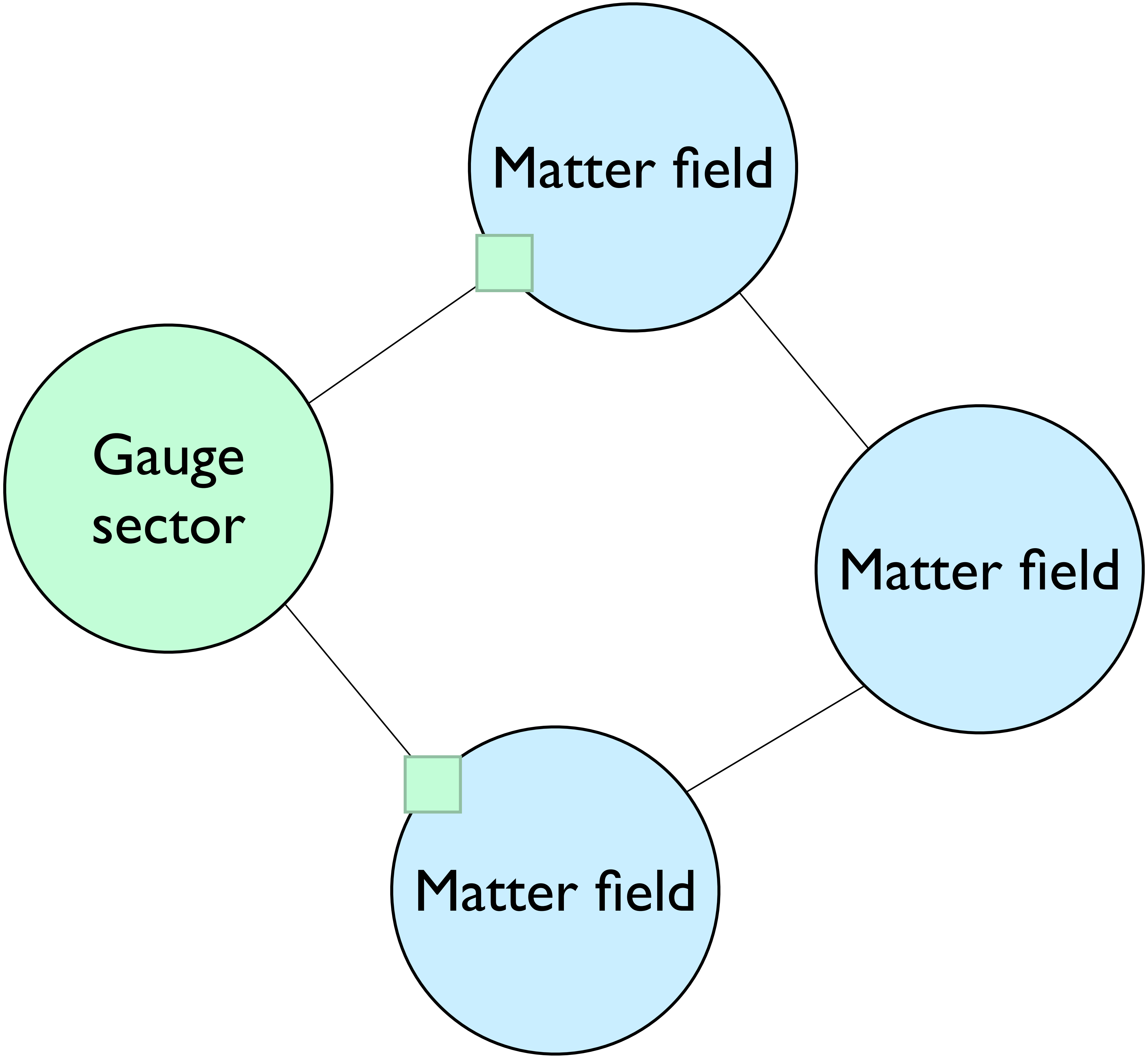
Matter field
Gauge
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Matter field
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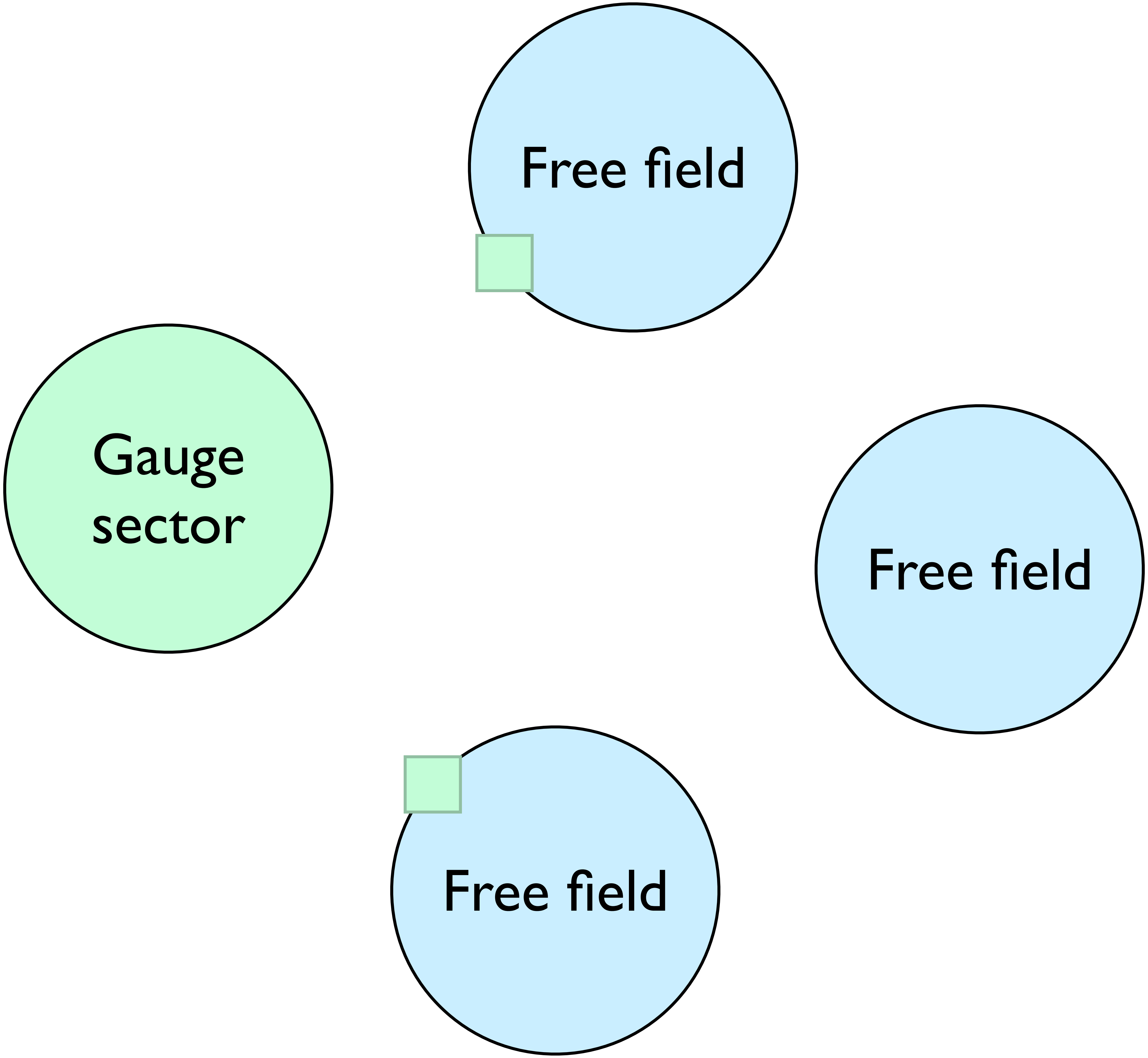
Free field
Gauge
sector
Free field
Free field

- A conventional QFT problem:
  - Pick elementary fermions and bosons with symmetry currents.
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  - Add interaction terms
- We then analyze the system.

A trivial Conformal Field Theory

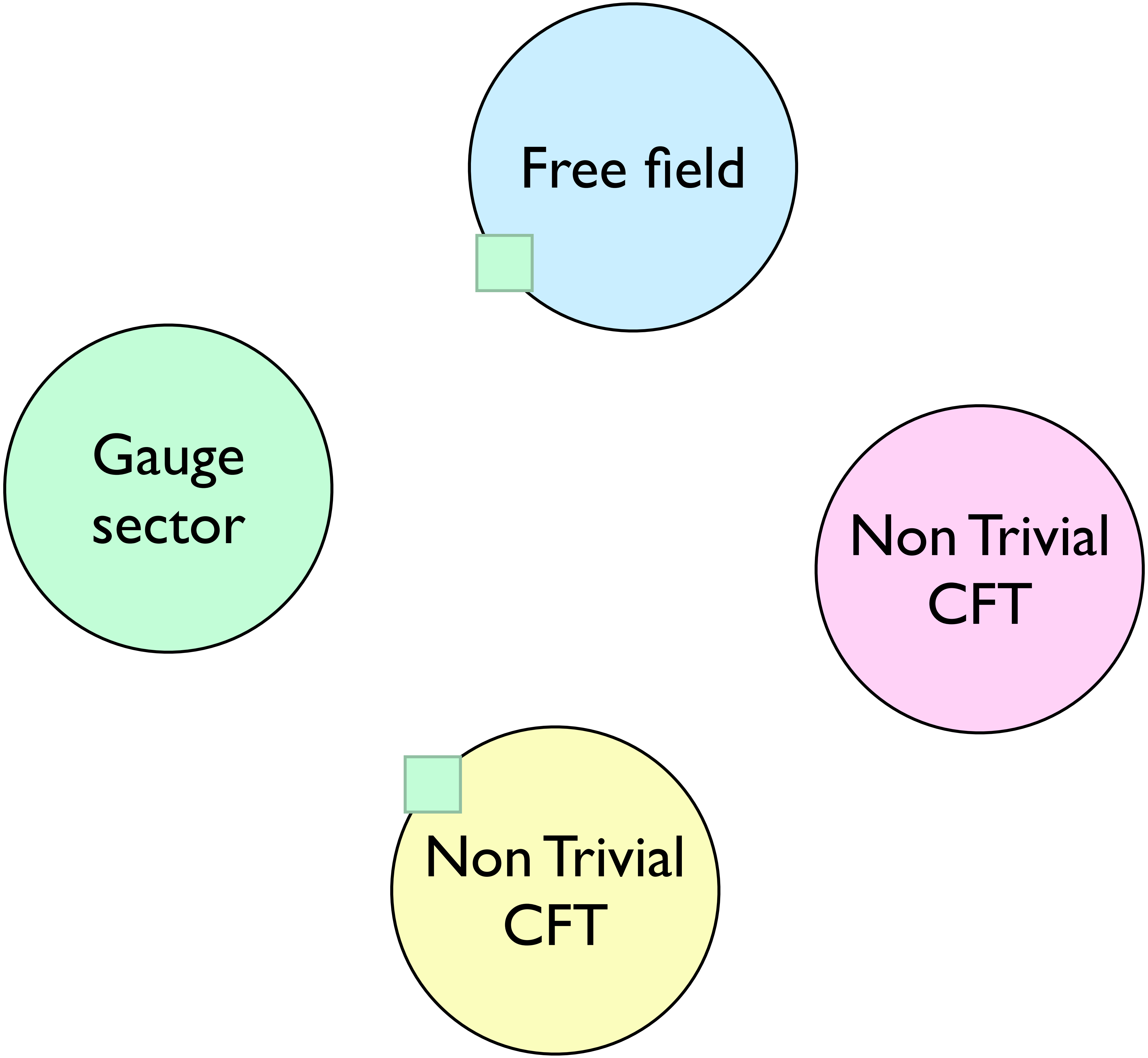
- A conventional QFT problem:
  - Pick elementary fermions and bosons with symmetry currents.
  - Couple gauge fields to currents.
  - Add interaction terms
- We then analyze the system.

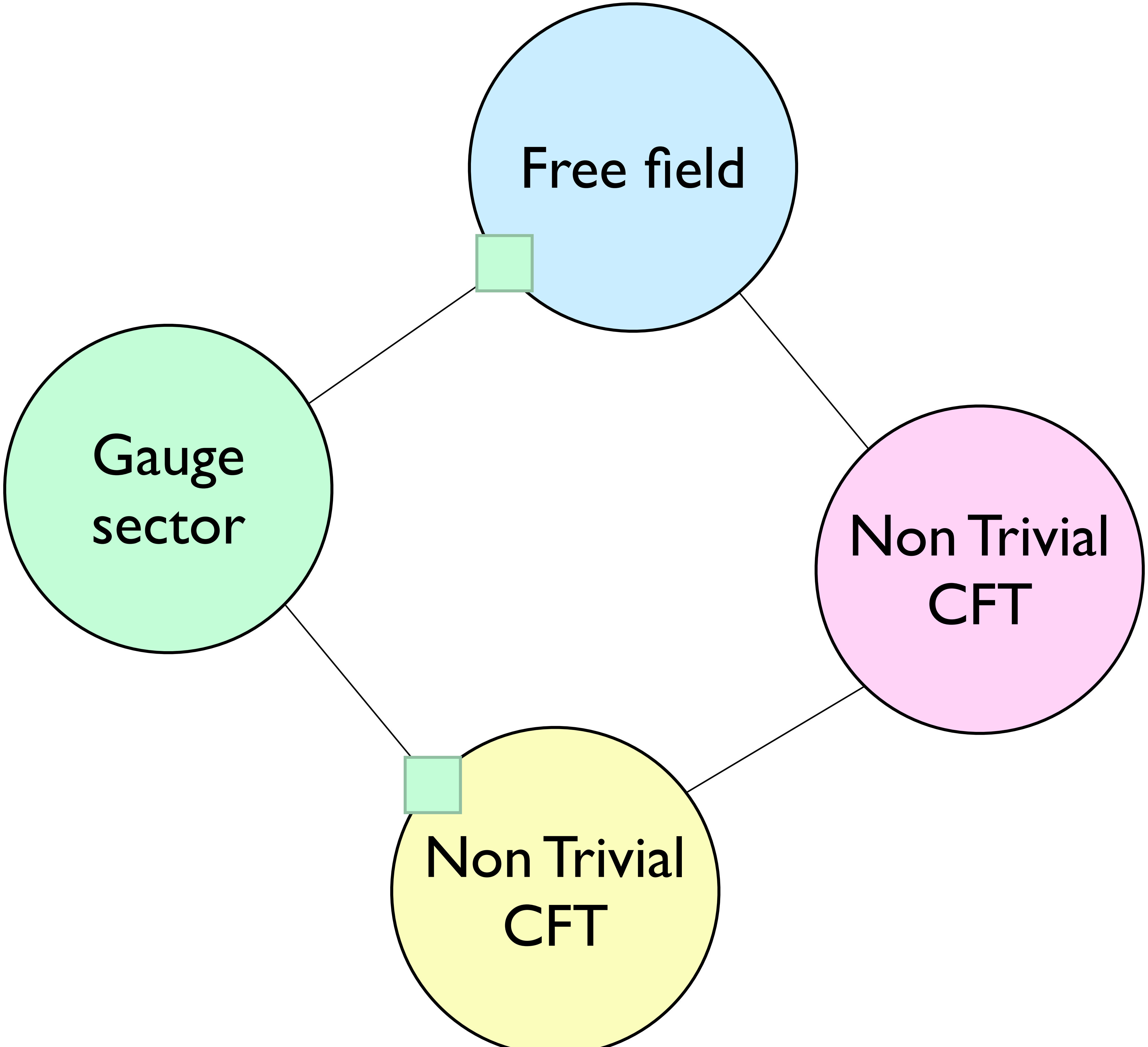
- A conventional QFT problem:
  - Pick a trivial CFT
    with symmetry currents.
  - Couple gauge fields to currents.
  - Add interaction terms
- We then analyze the system.

- A non-conventional QFT problem:
  - Pick a non-trivial CFT with symmetry currents.
  - Couple gauge fields to currents.
  - Add interaction terms
- We then analyze the system.

- A non-trivial CFT:
  - is still a quantum field theory
  - has "composite" operators, correlation functions...
  - in general, you don't know if it has a Lagrangian in terms of elementary fields.

- A non-conventional QFT problem:
  - Pick a non-trivial CFT with symmetry currents.
  - Couple gauge fields to currents.
  - Add interaction terms
- We then analyze the system.


Free field
Gauge sector
Non Trivial CFT
Non Trivial CFT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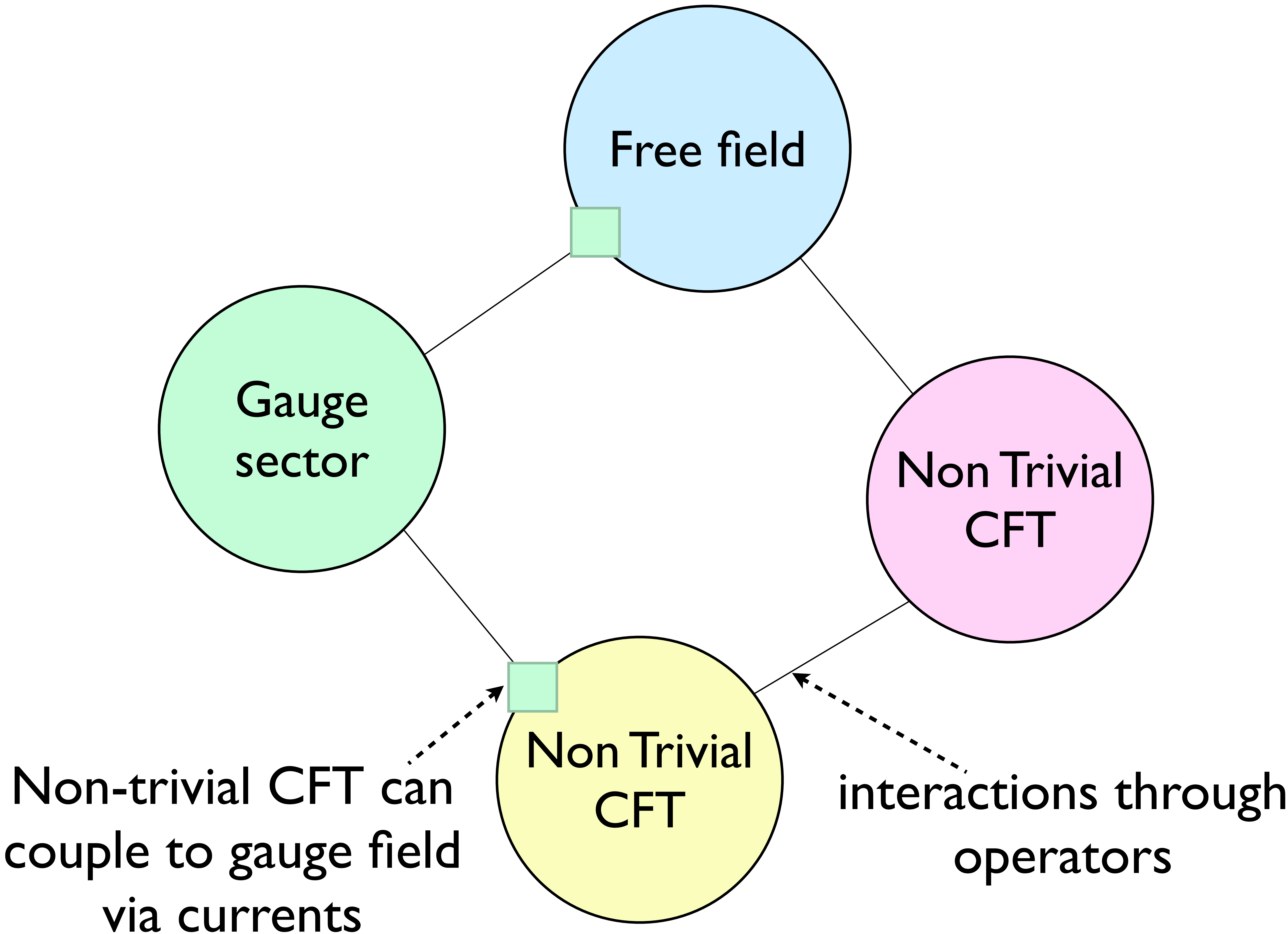
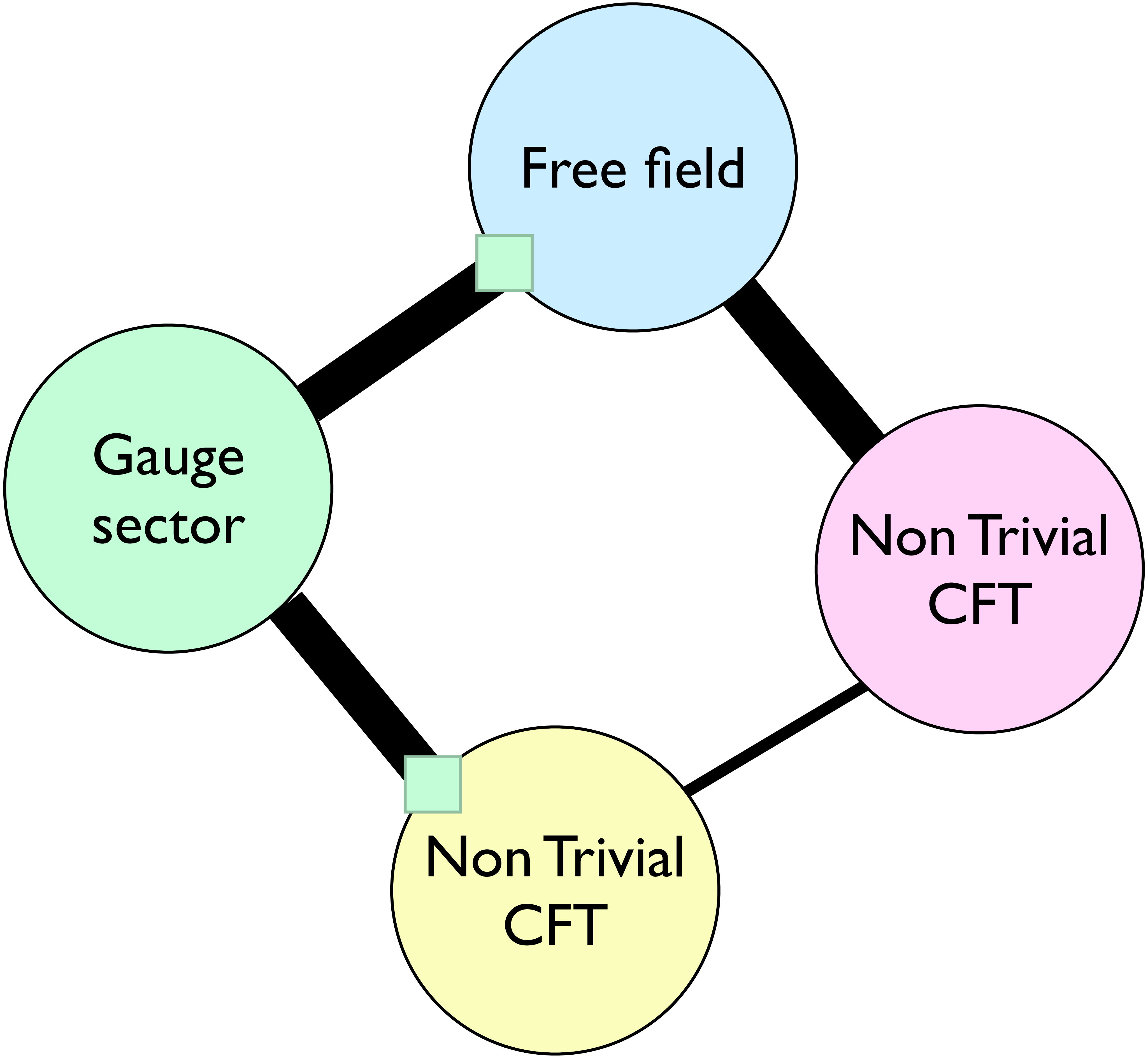
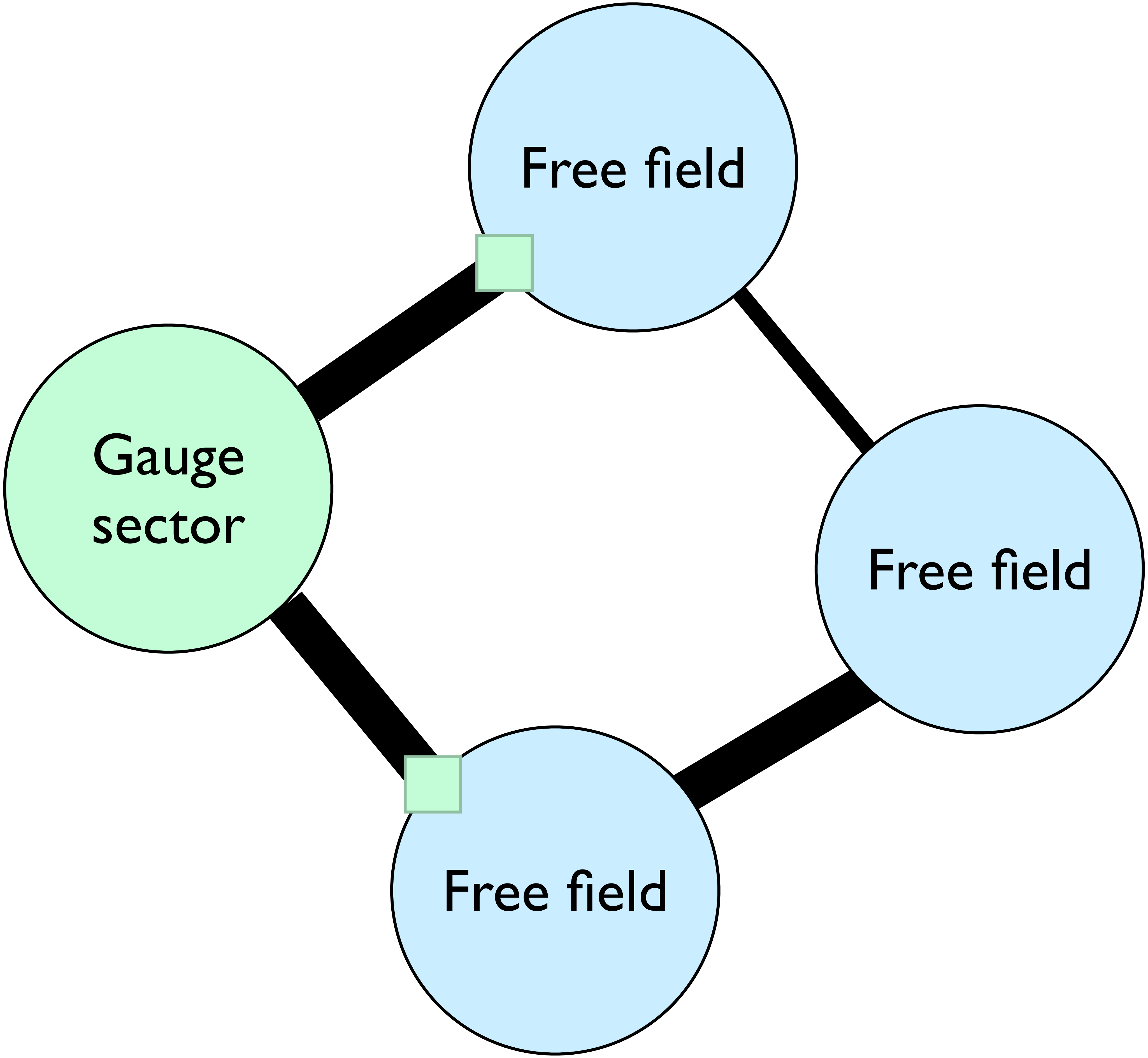
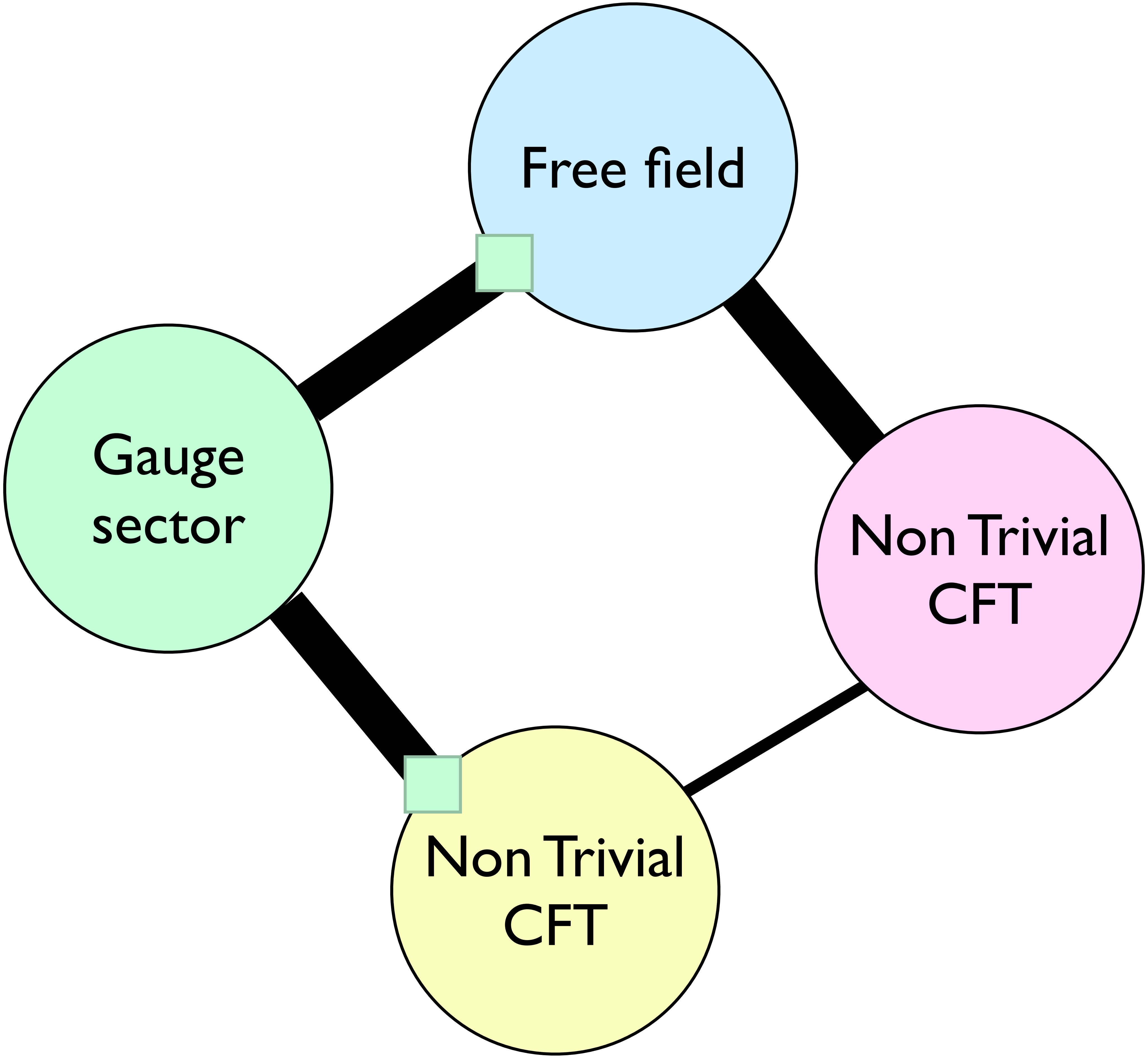

Free field
Gauge sector
Non Trivial CFT
Non Trivial CFT
Non-trivial CFT can couple to gauge field via currents
interactions through operators



Free field
Gauge sector
Non Trivial CFT
Non Trivial CFT


Free field
Gauge sector
Free field
Free field


Free field
Gauge sector
Non Trivial CFT
Non Trivial CFT

- A non-conventional QFT problem:
  - Pick a non-trivial CFT with symmetry currents.
  - Couple gauge fields to currents.
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- We then analyze the system.

elementary fermions +gauge fields + interactions

↓ to low energy

strongly coupled physics

a non-trivial CFT +gauge fields + interactions

↓ to low energy

strongly coupled physics

If

Elementary scalars+fermions
+ gauge fields
+interactions

↓ to low energy

a non-trivial CFT

Then

a non-trivial CFT +gauge fields + interactions

↓ to low energy

strongly coupled physics

Then

Elementary scalars+fermions
+ gauge fields
+interactions

↓ to low energy

a non-trivial CFT +gauge fields + interactions

↓ to low energy

strongly coupled physics

Elementary scalars+fermions
+ gauge fields
+interactions

+gauge fields + interactions

to low energy

strongly coupled physics

Elementary scalars+fermions
+ gauge fields
+interactions

to low energy

Might be absent.

a non-trivial CFT

a non-trivial CFT

a non-trivial CFT

Can be called a non-Lagrangian theory.

Elementary scalars'+fermions'
+ gauge fields'
+interactions'

Elementary scalars+fermions
+ gauge fields
+interactions

Elementary
scalars''+fermions''
+ gauge fields''
+interactions''

to low energy

a non-trivial CFT

It can have multiple dual descriptions
and no one description captures all features of it.

6d "non-Lagrangian" theory
on N M5-branes

↓ KK reduction

a non-trivial CFT

The only known construction can be
a reduction from higher-dimensional theory

- To fully understand dualities involving M5-branes, non-Lagrangian theories can’t be avoided.
- The reason is that the 6d theory on N M5-branes on general space is in itself non-Lagrangian.

a non-trivial CFT +gauge fields + interactions

↓ to low energy

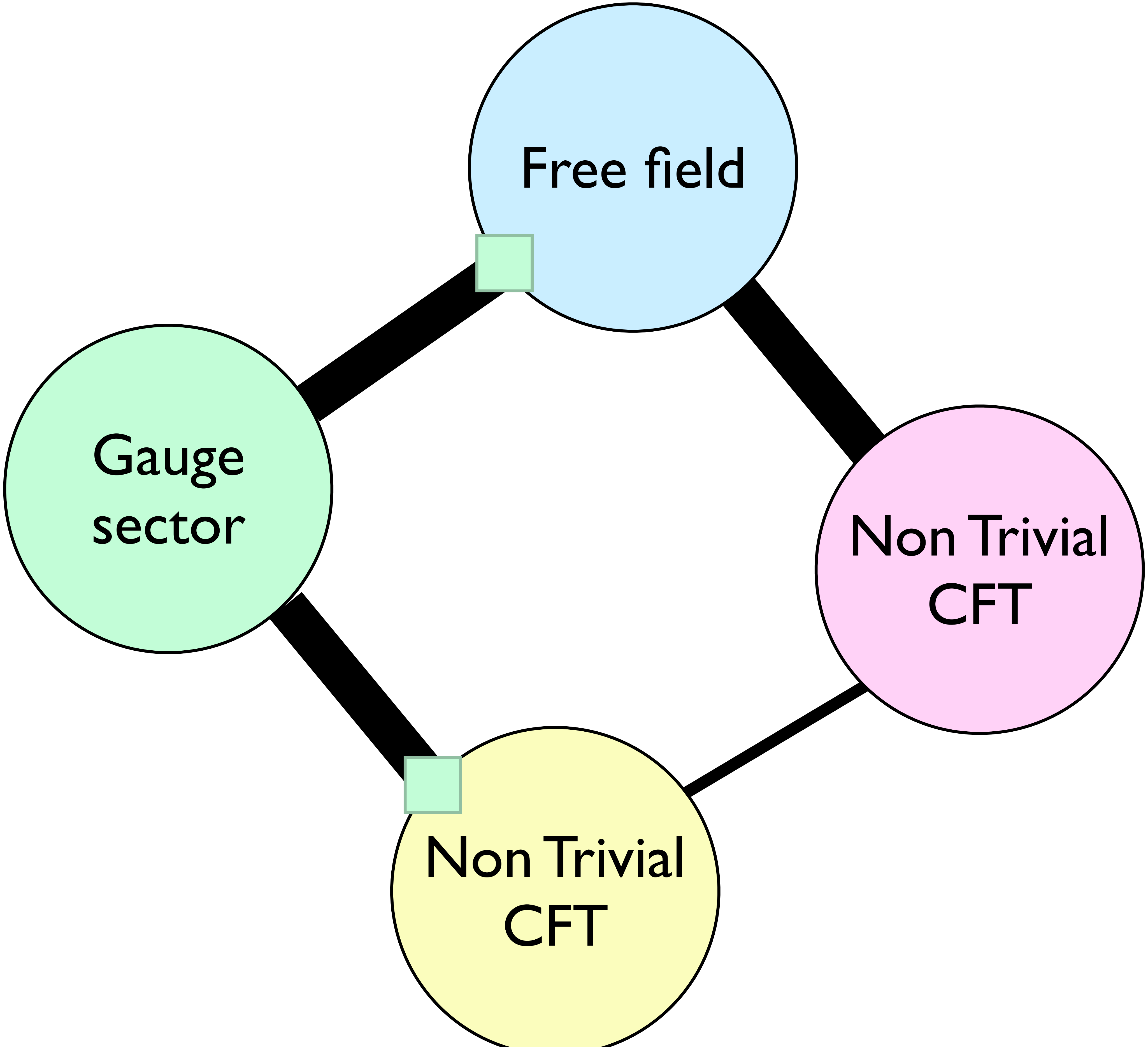
strongly coupled physics

So, let's consider this theory as the starting point.

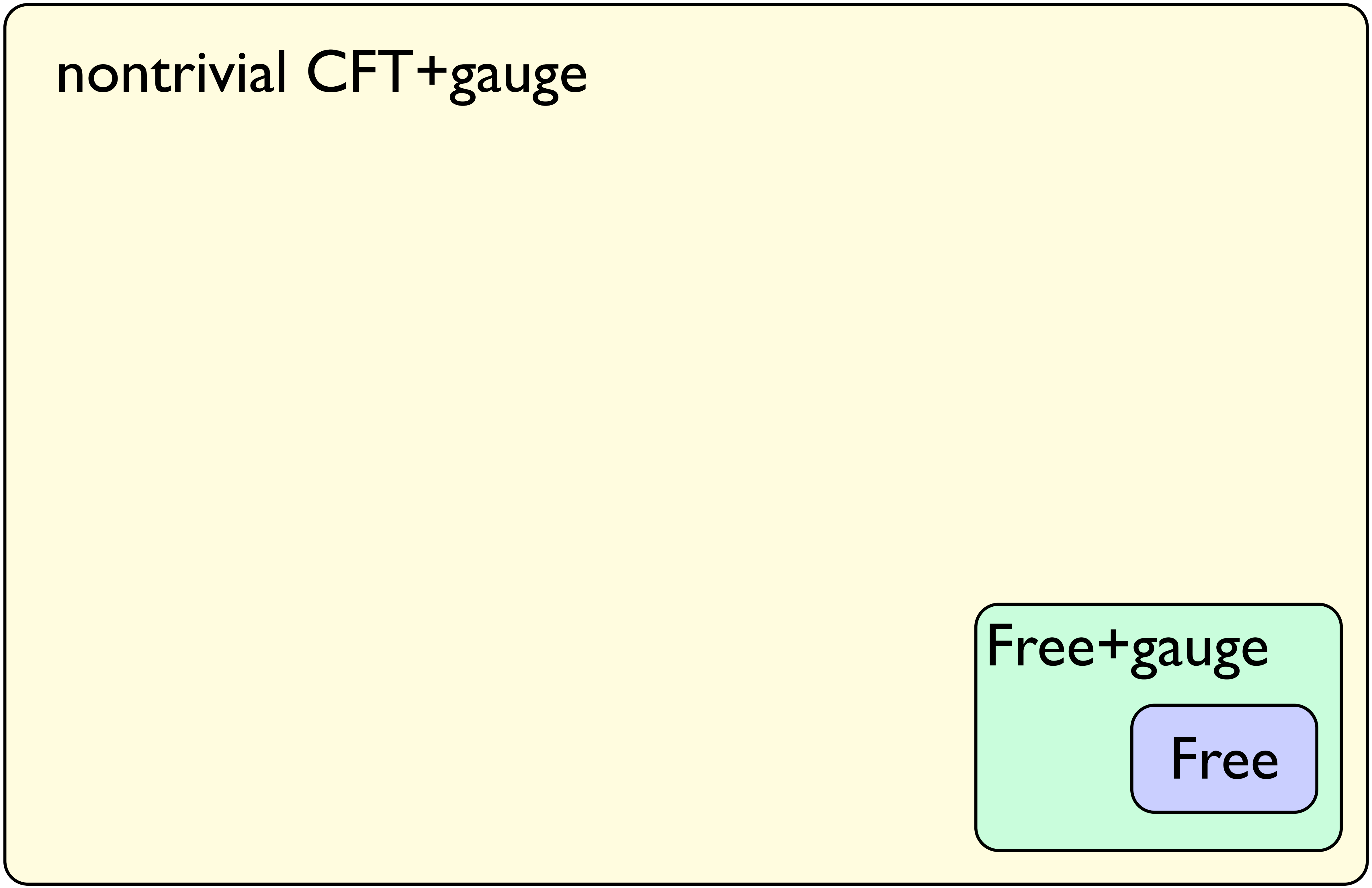
a non-trivial CFT +gauge fields + interactions

↓ to low energy

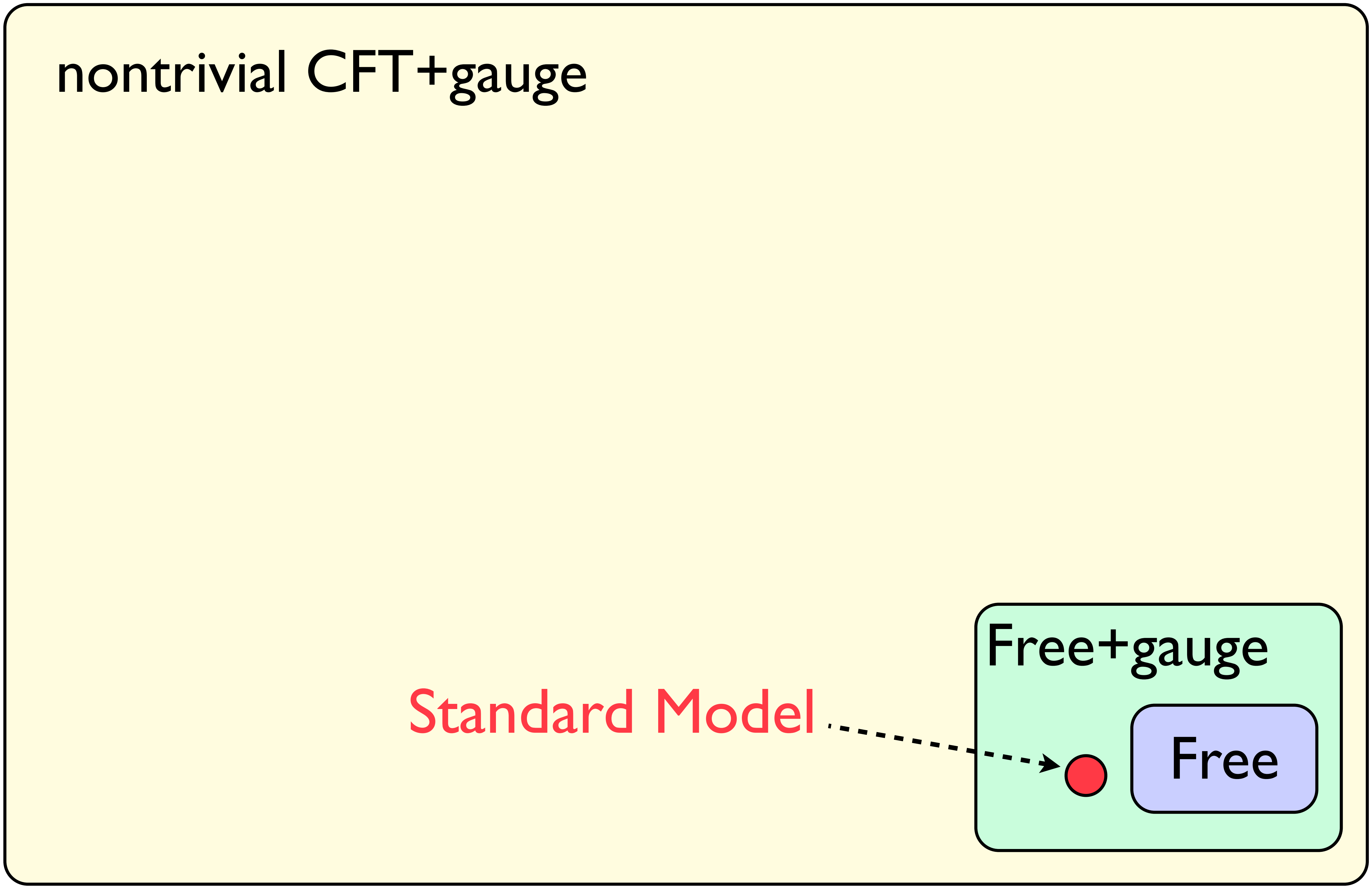
strongly coupled physics


Free field
Gauge sector
Non Trivial CFT
Non Trivial CFT

The space of QFTs (my subjective impression)



The space of QFTs (my subjective impression)



- If you’re interested in QFT in general, and if you start your work by writing down a Lagrangian of the form

  scalar + fermion + gauge + interactions,

- You might be missing a lot ! Be mindful !

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How to study them?

- No general framework yet available.
- N=2&1 supersymmetric ones are tractable, thanks to supersymmetry.
- Many N=2 examples are now known.

1996	Minahan-Nemeschansky's E_n theory
2007	Argyres-Seiberg duality
2007	Argyres-Wittig's examples
2009	Gaiotto's T_N theory
	My work on SO version
2010	Chacaltana-Distler's examples
2013	My work with friends ...

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2013	My work with friends ...

- As an example, let me discuss Minahan-Nemeschansky's theory.

- A bit of preparation is necessary.

- We'll start from $\mathcal{N}$=2 SQCD:

$\mathcal{N}$=2 supersymmetric SU(N_c) with N_f flavors

As $\mathcal{N}$=1 supersymmetric gauge theory,

	SU(N_c) vector multiplet	W_α
	SU(N_c) adjoint chiral	Φ
N_f	SU(N_c) fundamental chiral	q
N_f	SU(N_c) antifundamental chiral	$\tilde{q}$

$$W = q\Phi\tilde{q}$$

1-loop β function = $2N_c - N_f$

$\mathcal{N}$=2 supersymmetric SU(N_c) with N_f flavors

In components,

Two gauginos

SU(N_c) vector → A_μ $\quad \lambda_\alpha \quad \lambda_\alpha'$

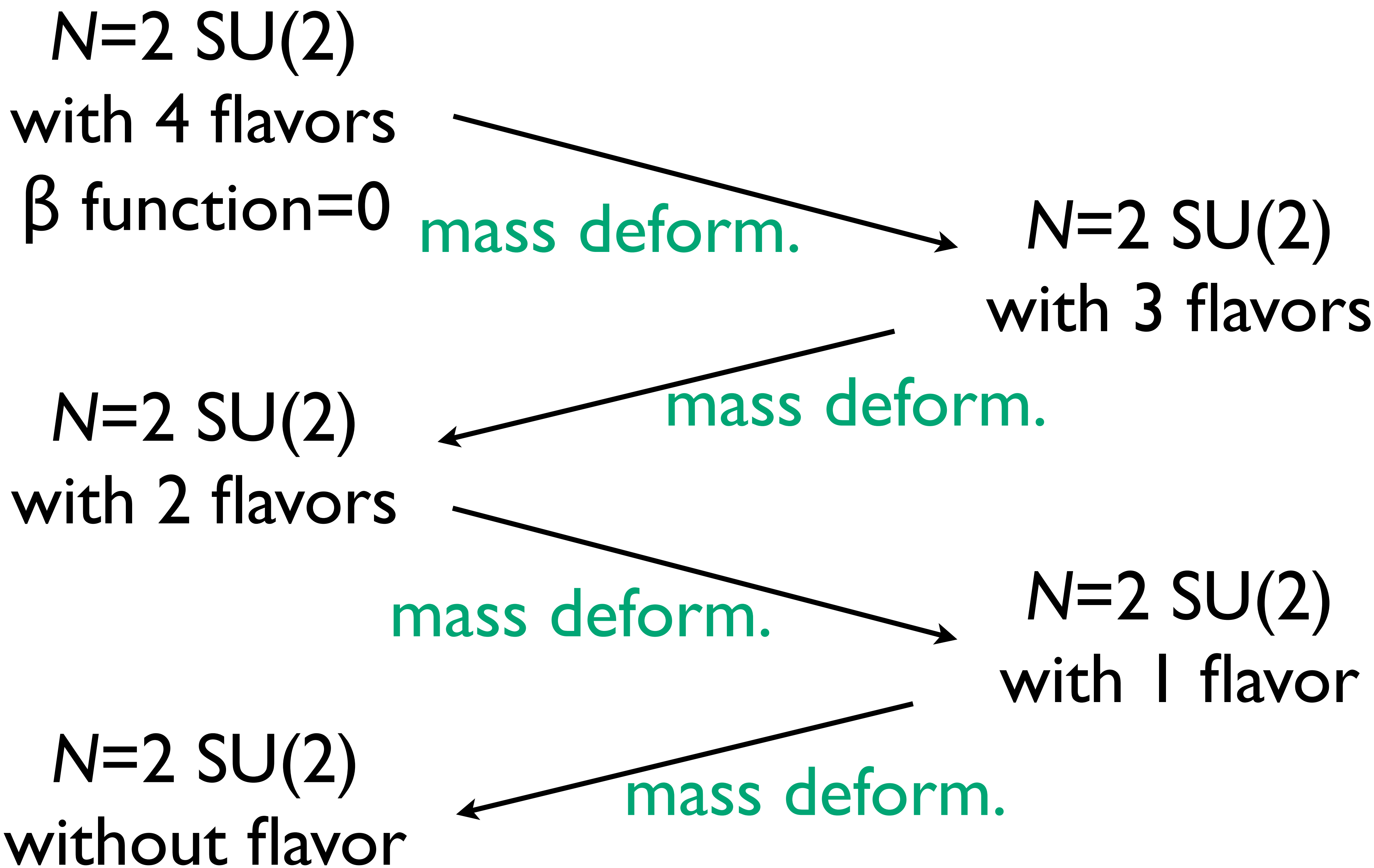
SU(N_c) adjoint scalar → Φ

N_f SU(N_c) fundamental fermion+scalar $\quad \psi \quad q$

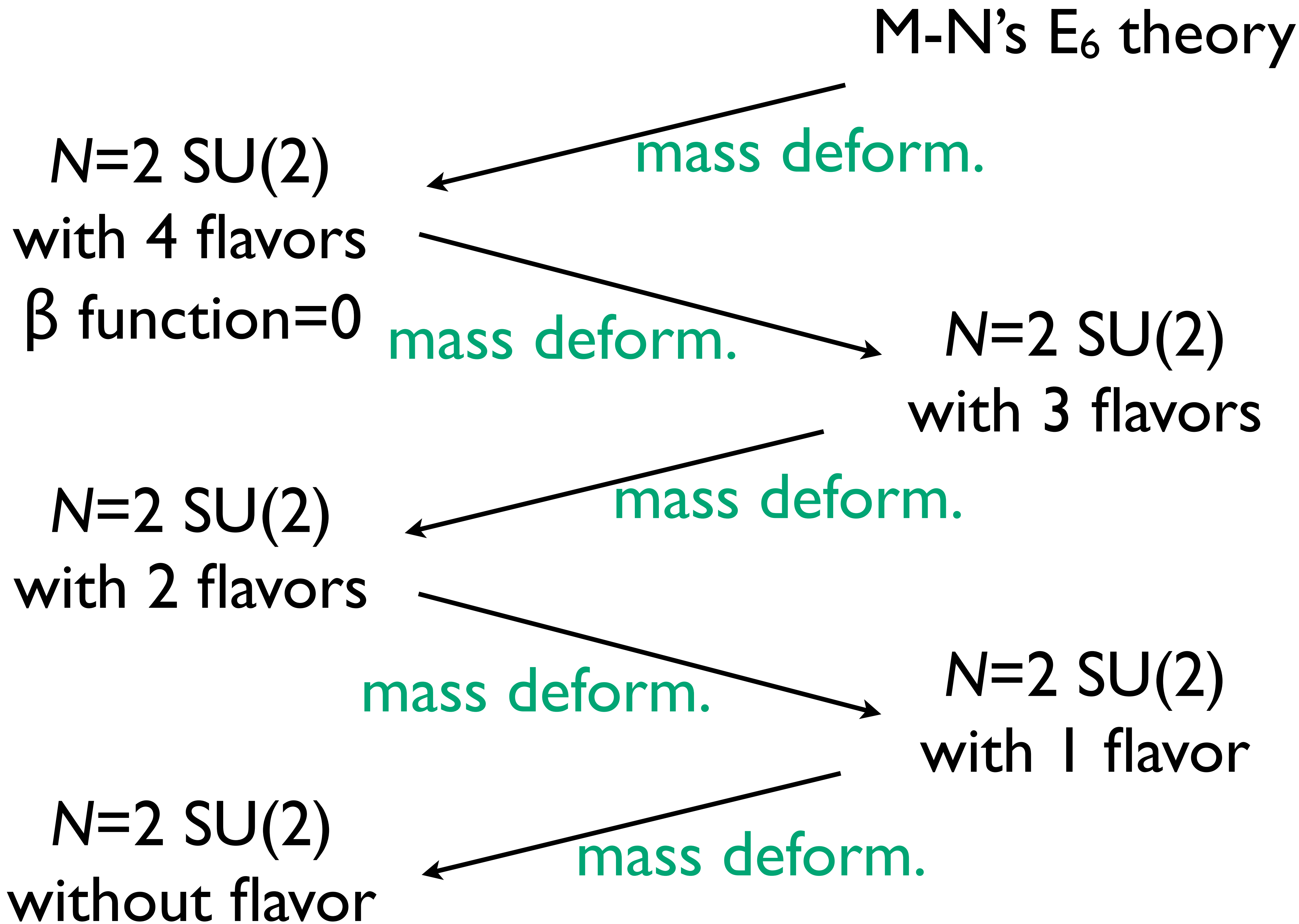
N_f SU(N_c) antifundamental fermion+scalar $\quad \tilde{q} \quad \bar{\psi}$

$$W = q\Phi\tilde{q}$$

1-loop β function = $2N_c - N_f$



$\mathcal{N}$=2 SU(2) with 4 flavors β function=0
mass deform.
$\mathcal{N}$=2 SU(2) with 3 flavors
mass deform.
$\mathcal{N}$=2 SU(2) with 2 flavors
mass deform.
$\mathcal{N}$=2 SU(2) with 1 flavor
mass deform.
$\mathcal{N}$=2 SU(2) without flavor


M-N's E_6 theory
mass deform.
$\mathcal{N}$=2 SU(2)
with 4 flavors
β function=0
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with 2 flavors
mass deform.
$\mathcal{N}$=2 SU(2)
with 1 flavor
mass deform.
$\mathcal{N}$=2 SU(2)
without flavor

M-N's E_6 theory

is $\mathcal{N}=2$ supersymmetric.

has E_6 flavor symmetry.

has no Lagrangian description
with E_6 symmetry.

M-N's E_6 theory

is N=2 supersymmetric.

has E_6 flavor symmetry.

has no Lagrangian description
with E_6 symmetry.

proof:

any N=2 susy gauge theory is
an N=1 susy gauge theory with superpotential

$$W = q\Phi\tilde{q} .$$

Its flavor symmetry can be explicitly found;
it can only be SU, SO or Sp.

- How do you know such a thing exists?

- If you accept that string theory exists as a consistent quantum theory,
you can use type-E_6 7-brane of F-theory probed by a D3-brane.

- If you prefer purely field theoretical approach, you can proceed as follows.

- It takes some efforts, so please be patient.

Construction of M-N E_6 theory

The 1-loop beta function of

$\mathcal{N}$=2 SU(N_c) with $2N_c$ flavors $q, \tilde{q}$

vanishes, and the coupling constant

$$\tau = 4\pi i/g^2 + \theta/2\pi$$

is exactly marginal.

Q. What happens if you send $g \to \infty$?

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A. Depends on θ.

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Q. What happens if you send $g \to \infty$?

A. Depends on θ.

General θ is complicated, so let's just consider CP invariant cases $\theta=0, \pi$

You can study what happens in the extremely strong coupling limit

$$g \to \infty$$

because the Seiberg-Witten curve contains the info of all the masses and all the multiplicities of all SUSY particles in the system.

When $\theta=0$:

Study what happens to the Seiberg-Witten curve when $g \to \infty$.

It happens that

the Seiberg-Witten curve at the coupling g

is equal to

the Seiberg-Witten curve at the coupling $g' = 1/g$

When $\theta=0$:

So, it is quite likely that

$\mathcal{N}=2$ SU(N_c) with $2N_c$ flavors $q, \tilde{q}$
at coupling g

is equal to

$\mathcal{N}=2$ SU(N_c) with $2N_c$ flavors $q', \tilde{q}'$
at coupling $g'=1/g$

When $\theta=\pi$:

Study what happens to the Seiberg-Witten curve when $g \to \infty$.

It happens that

the Seiberg-Witten curve at the coupling g

is not equal to

the Seiberg-Witten curve at the coupling $g'=1/g$

when $N_c > 2$.

When $\theta=\pi$:

Rather, it happens that

the Seiberg-Witten curve of
$\mathcal{N}=2$ SU(N_c) with $2N_c$ flavors $q, \tilde{q}$
at coupling g

is equal to

the Seiberg-Witten curve of SU(2) gauge theory
at coupling g'=1/g ,
coupled to one flavor plus something.

When $\theta=\pi$:

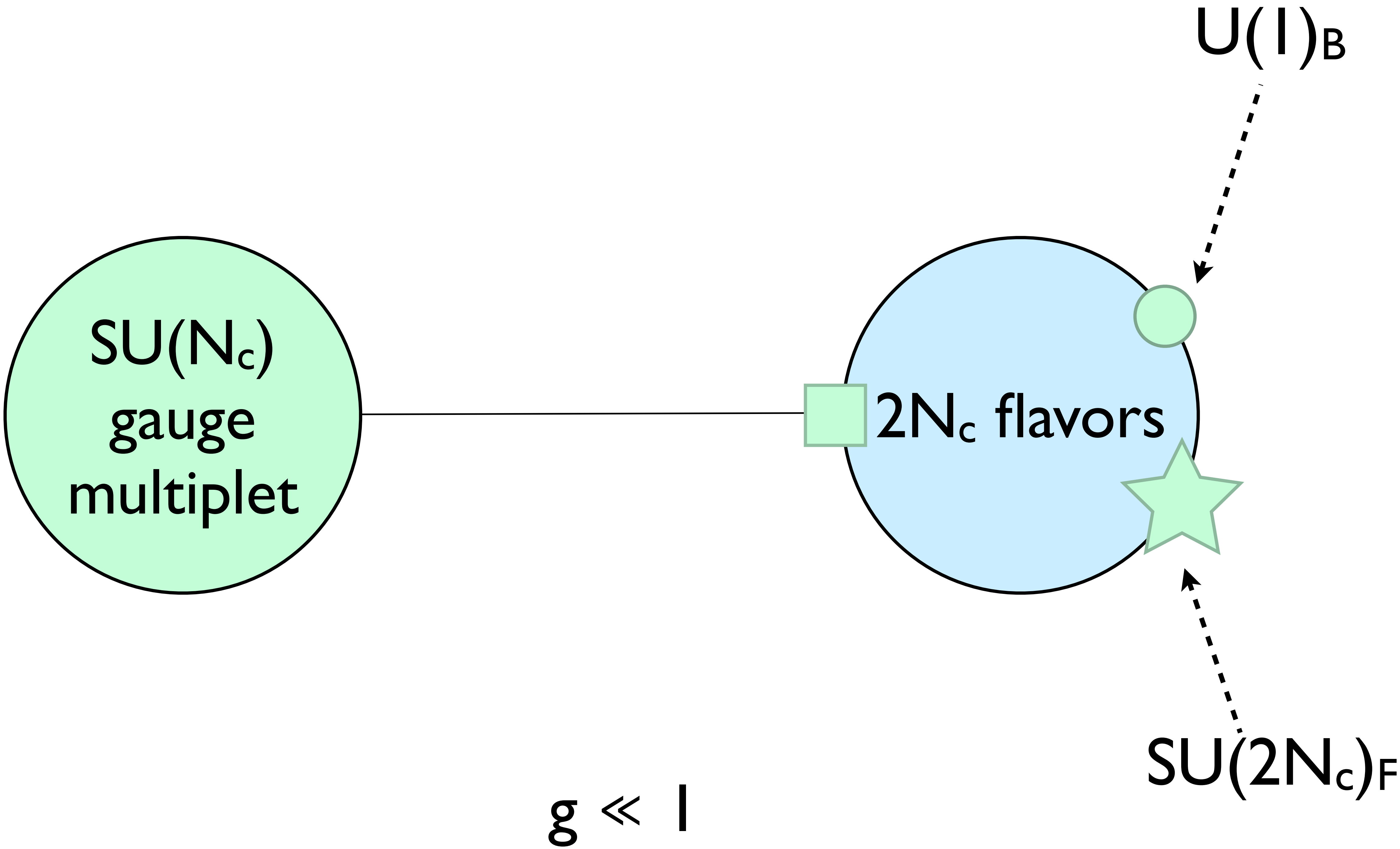
It is then likely that

$\mathcal{N}=2$ SU(N_c) with $2N_c$ flavors $q, \tilde{q}$
at coupling g

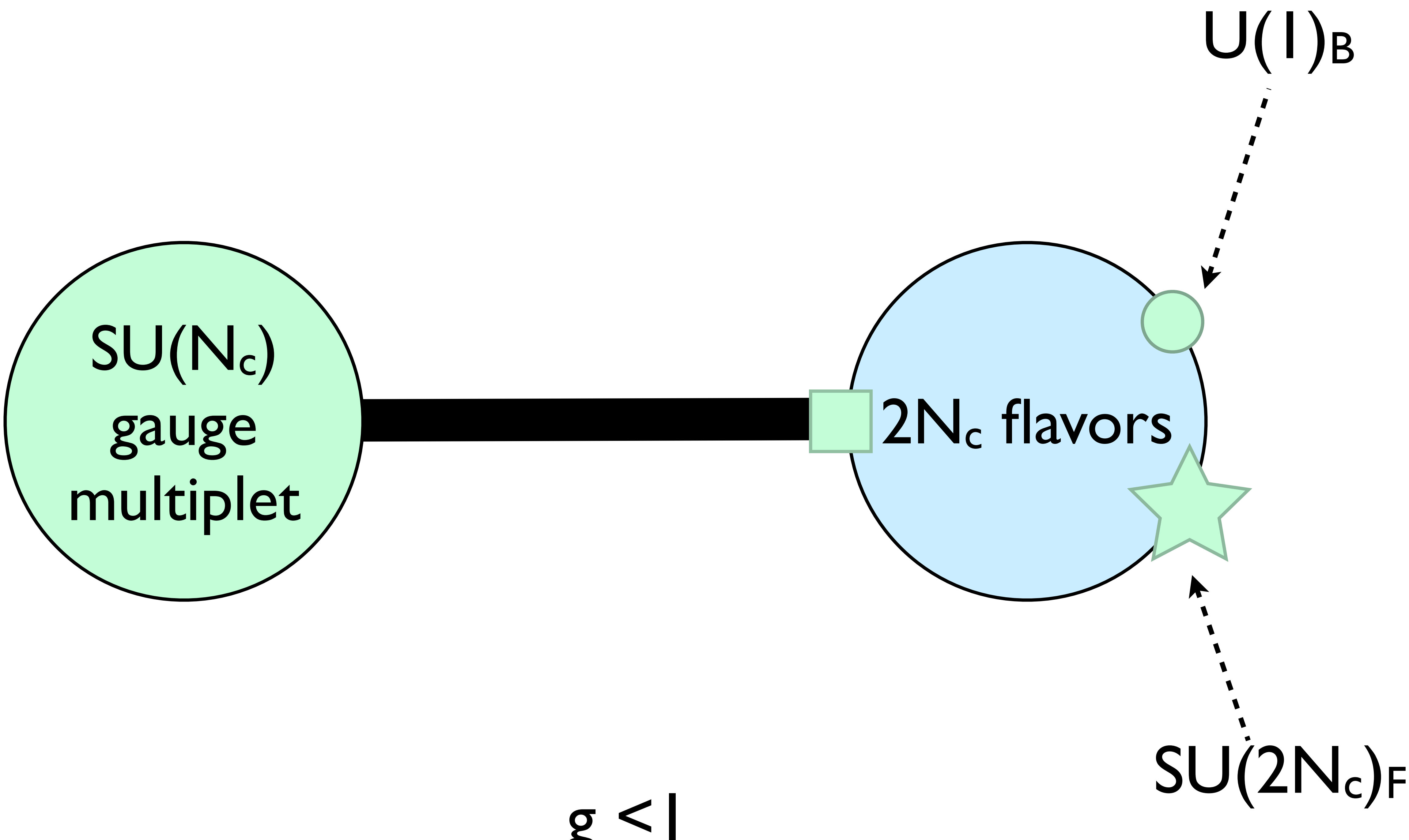
is equal to

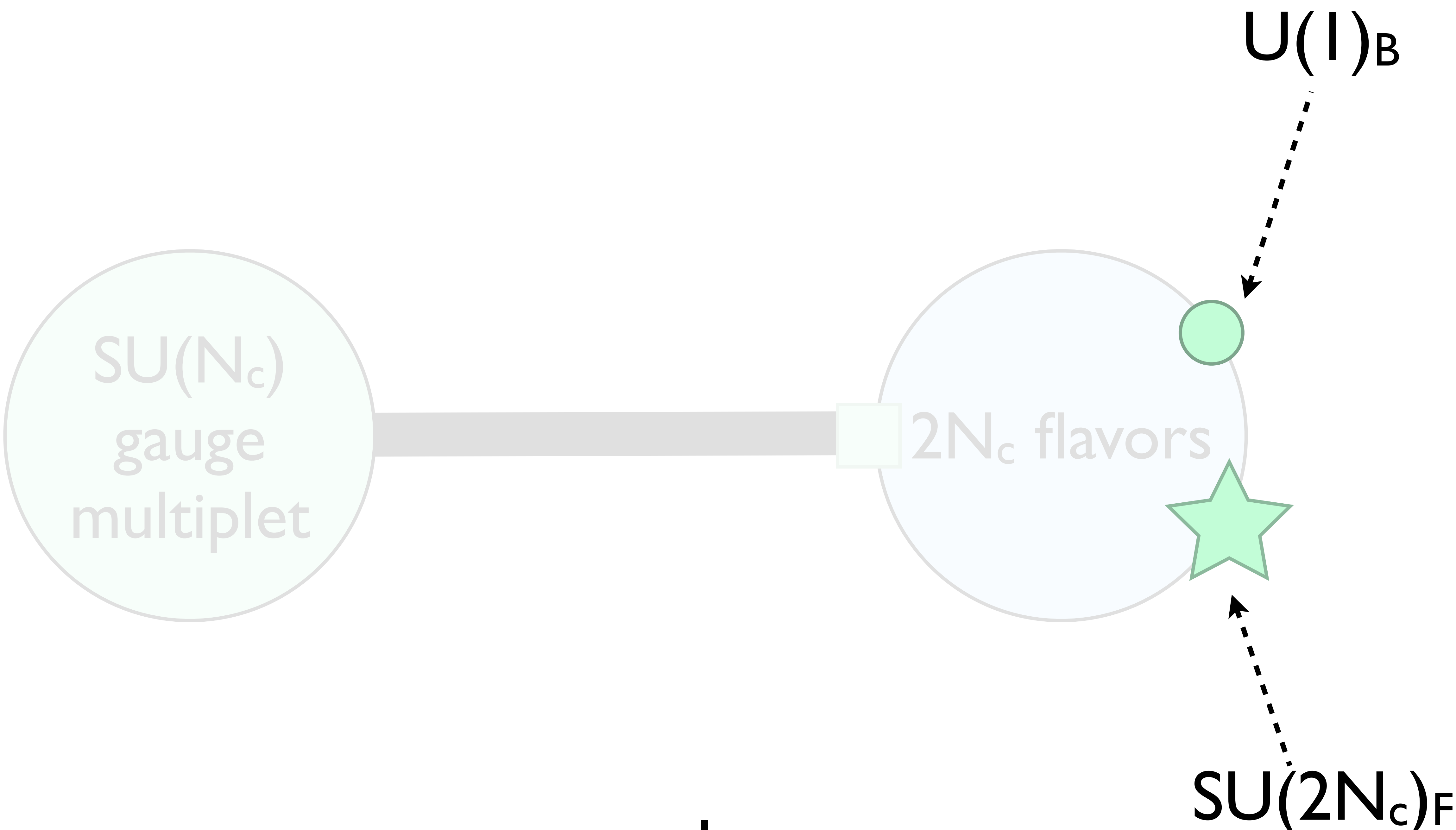
$\mathcal{N}=2$ SU(2) gauge theory
at coupling g'=1/g ,
coupled to one flavor plus something.

When $\theta=\pi$:

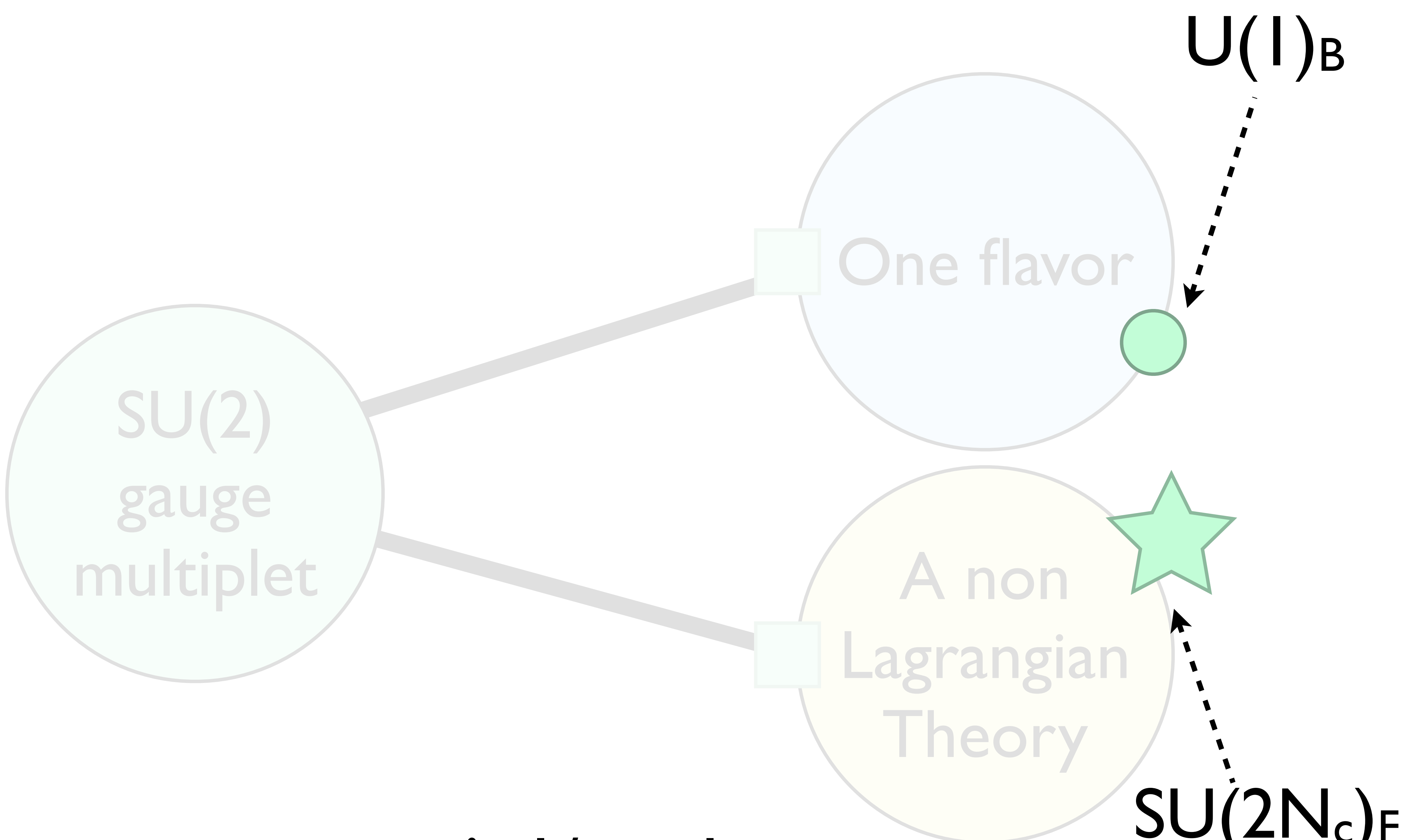


When $\theta=\pi$:



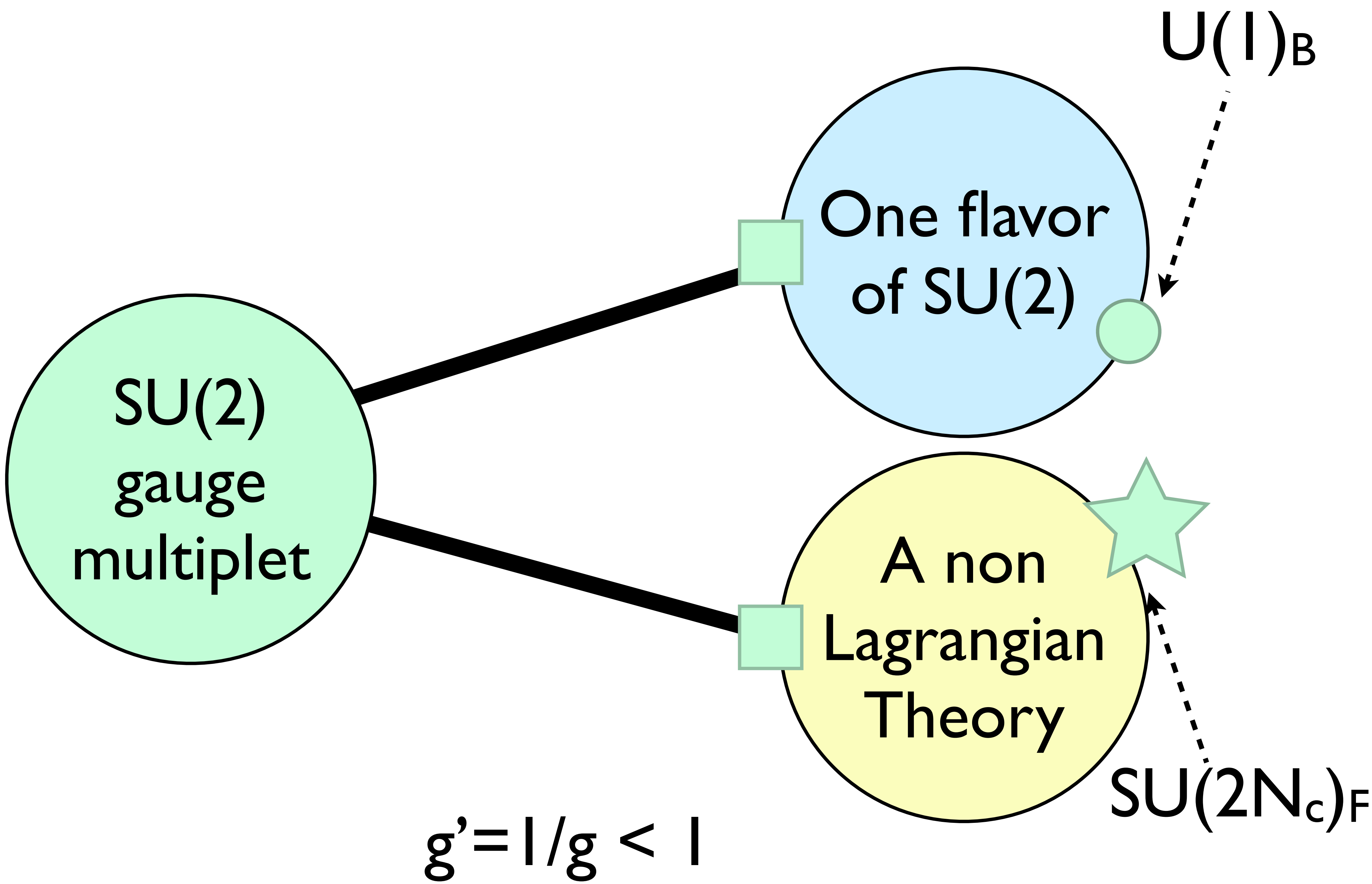


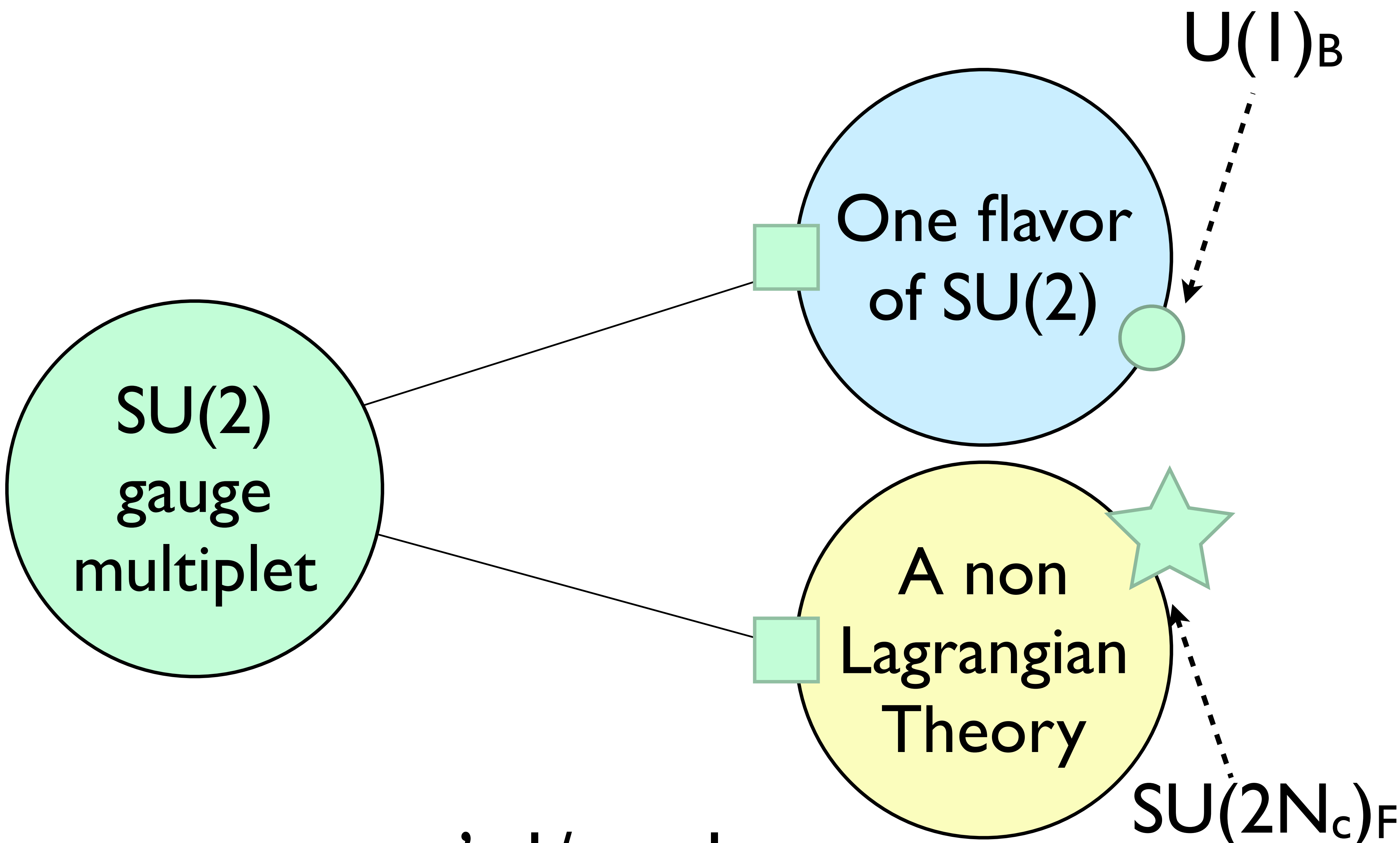
$g \sim 1$



U(1)B
One flavor
SU(2)
gauge
multiplet
A non
Lagrangian
Theory
SU(2Nc)F

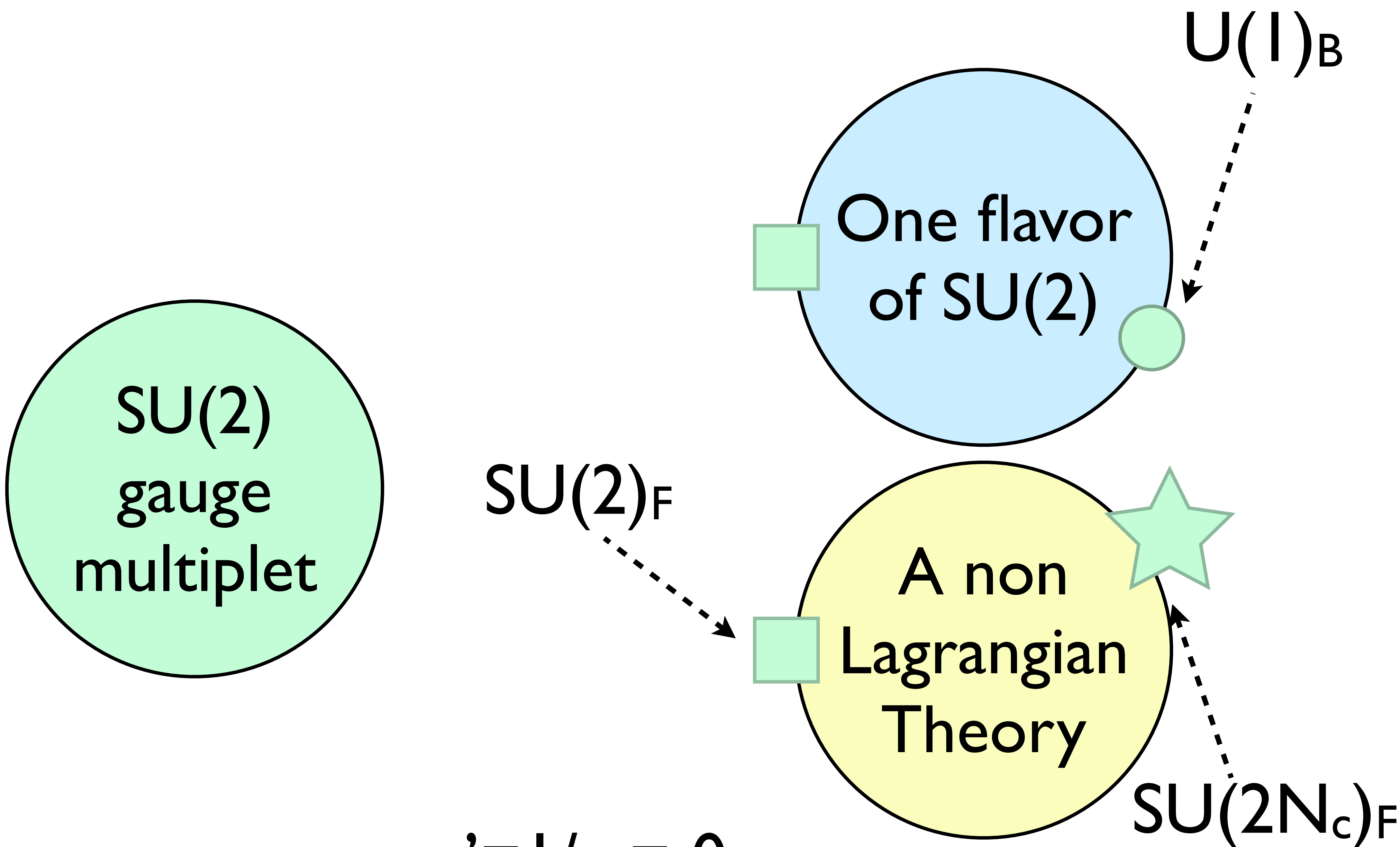
$g' = 1/g \sim 1$


U(1)B
One flavor of SU(2)
SU(2) gauge multiplet
A non Lagrangian Theory
SU(2Nc)F
g'=1/g < 1



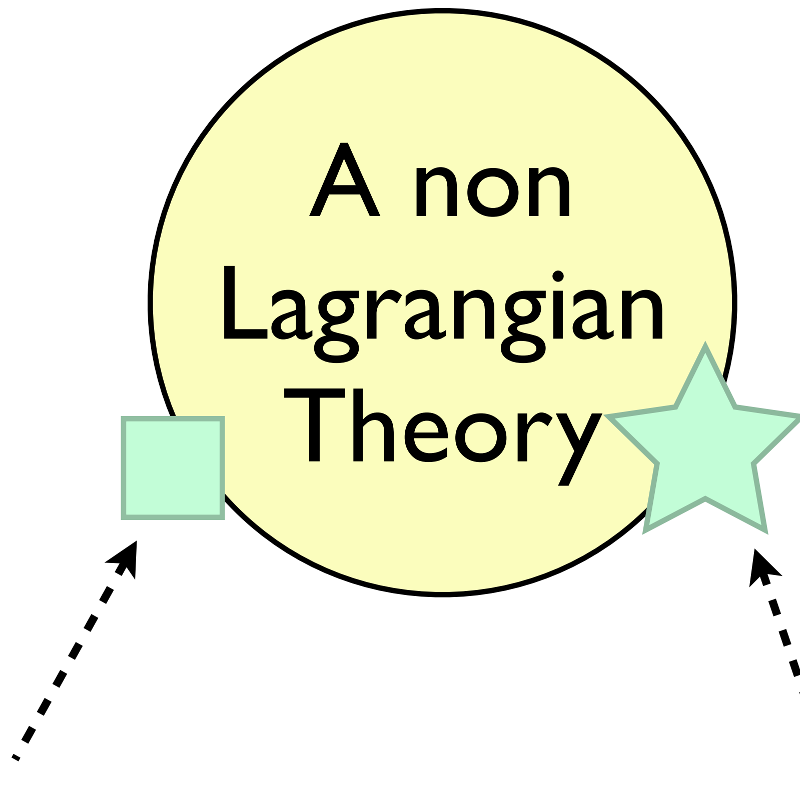
U(1)B
One flavor of SU(2)
SU(2) gauge multiplet
A non Lagrangian Theory
SU(2Nc)F

$g' = 1/g \ll 1$


U(1)B
One flavor of SU(2)
SU(2) gauge multiplet
SU(2)F
A non Lagrangian Theory
SU(2Nc)F

$g' = 1/g = 0$

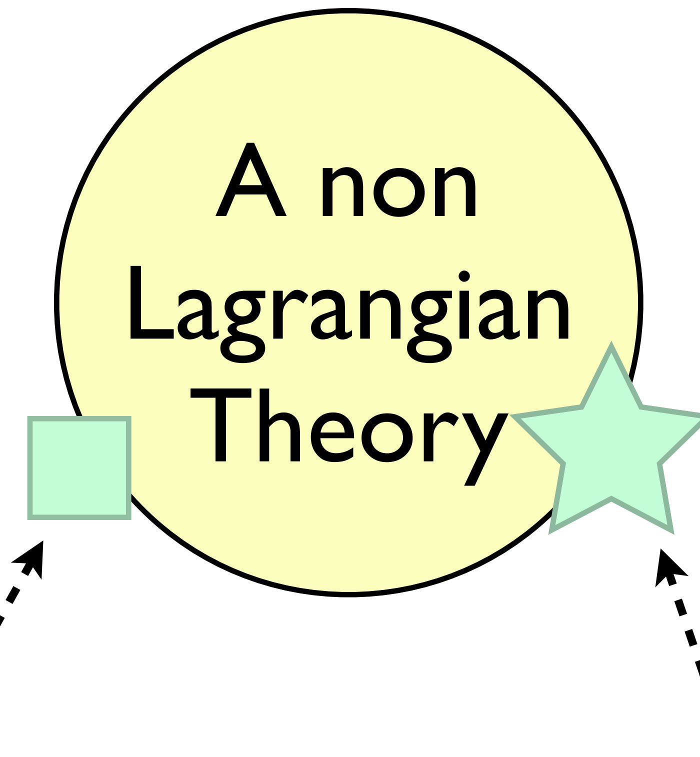
We isolated



with $SU(2)_F$ and $SU(2N_c)_F$

flavor symmetry.

When $N_c = 3$



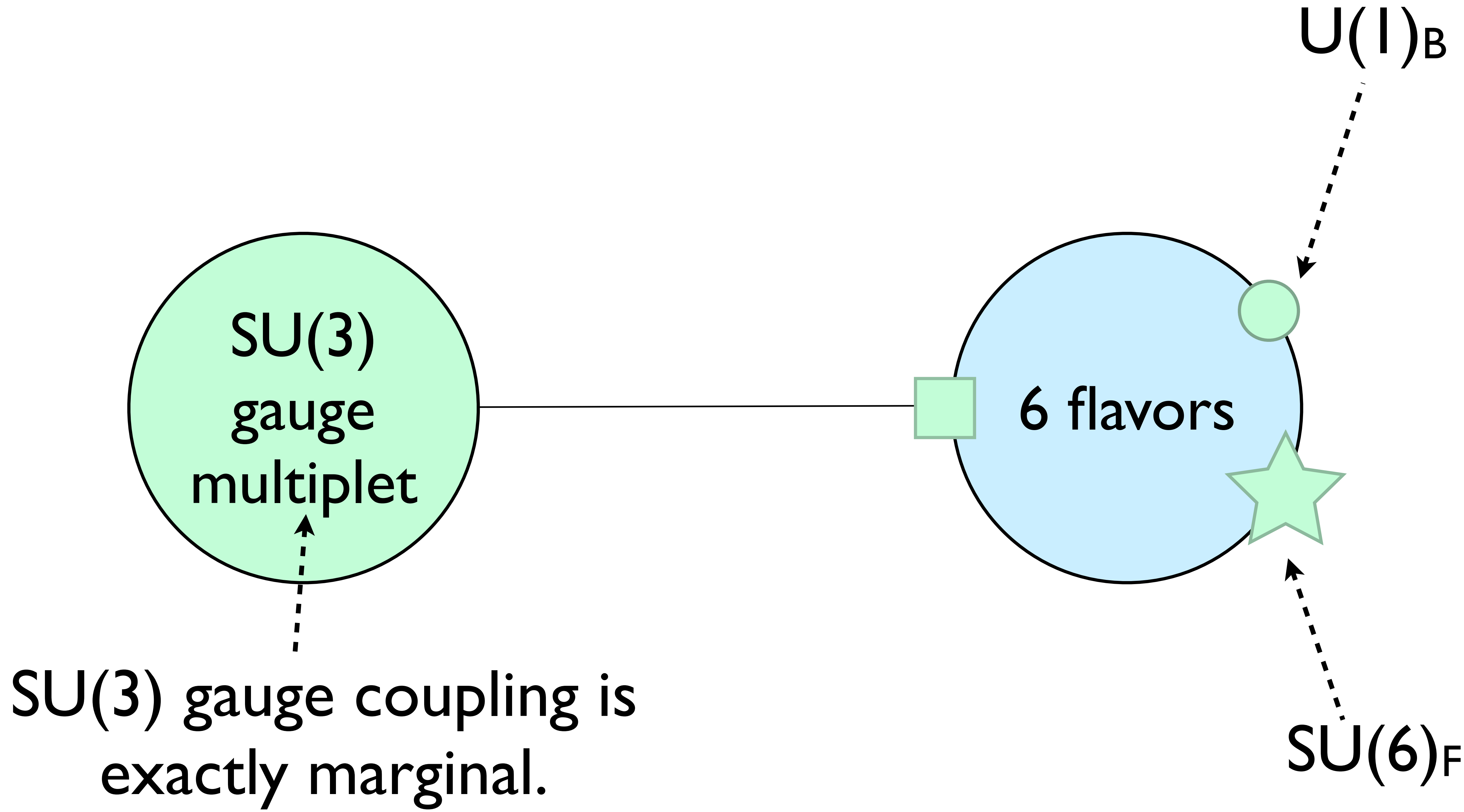
$SU(2)_F$ and $SU(2N_c)_F = SU(6)$

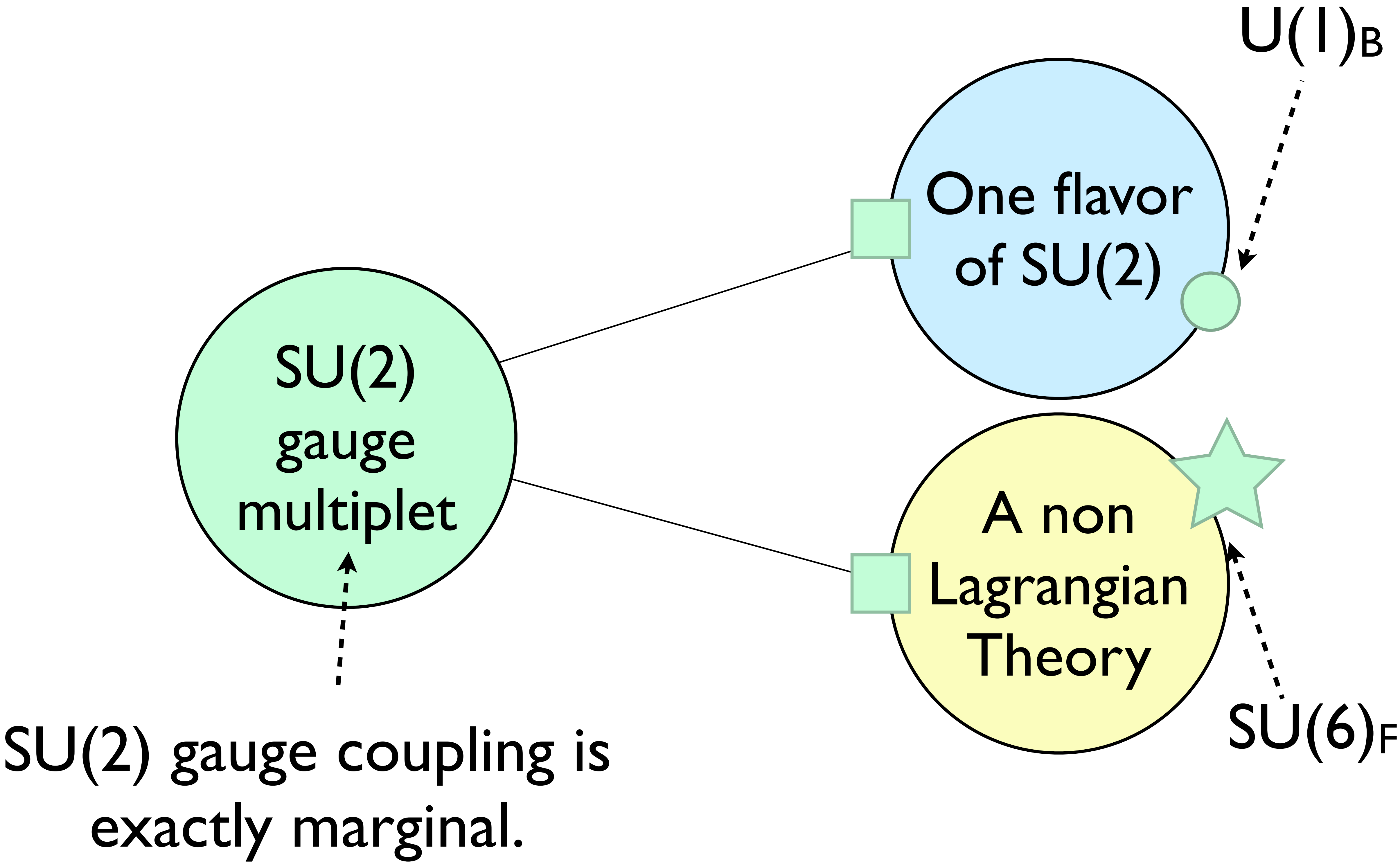
combines to E_6.

This is the Minahan-Nemeschansky theory.

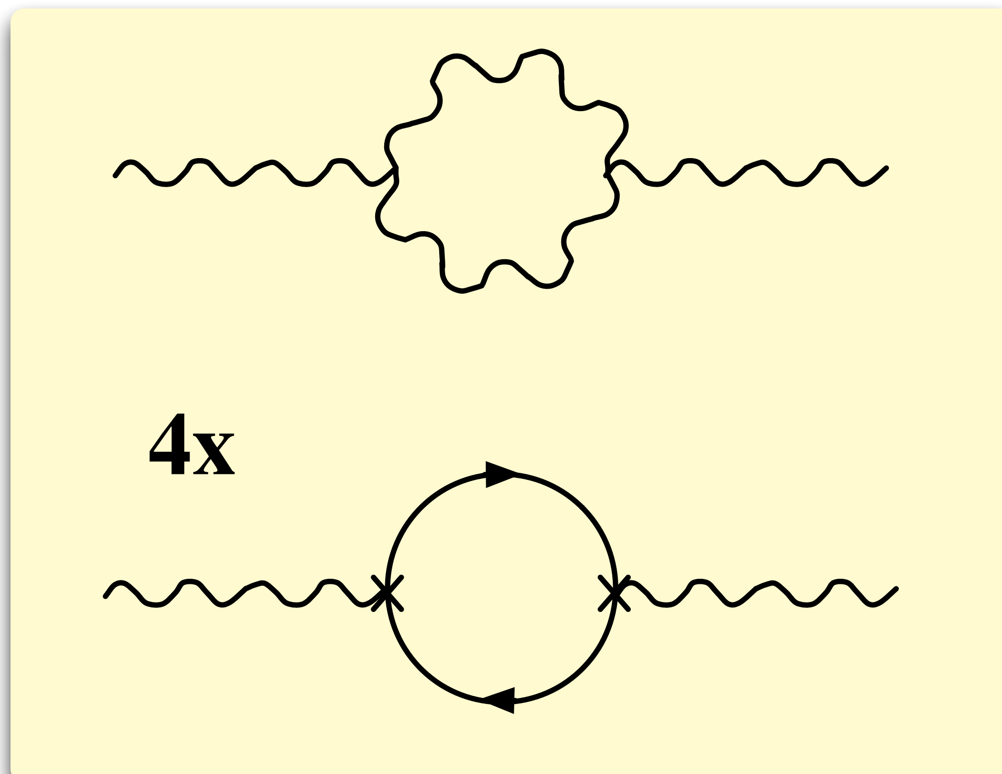
On current 2-pt functions

When $\theta=\pi$:

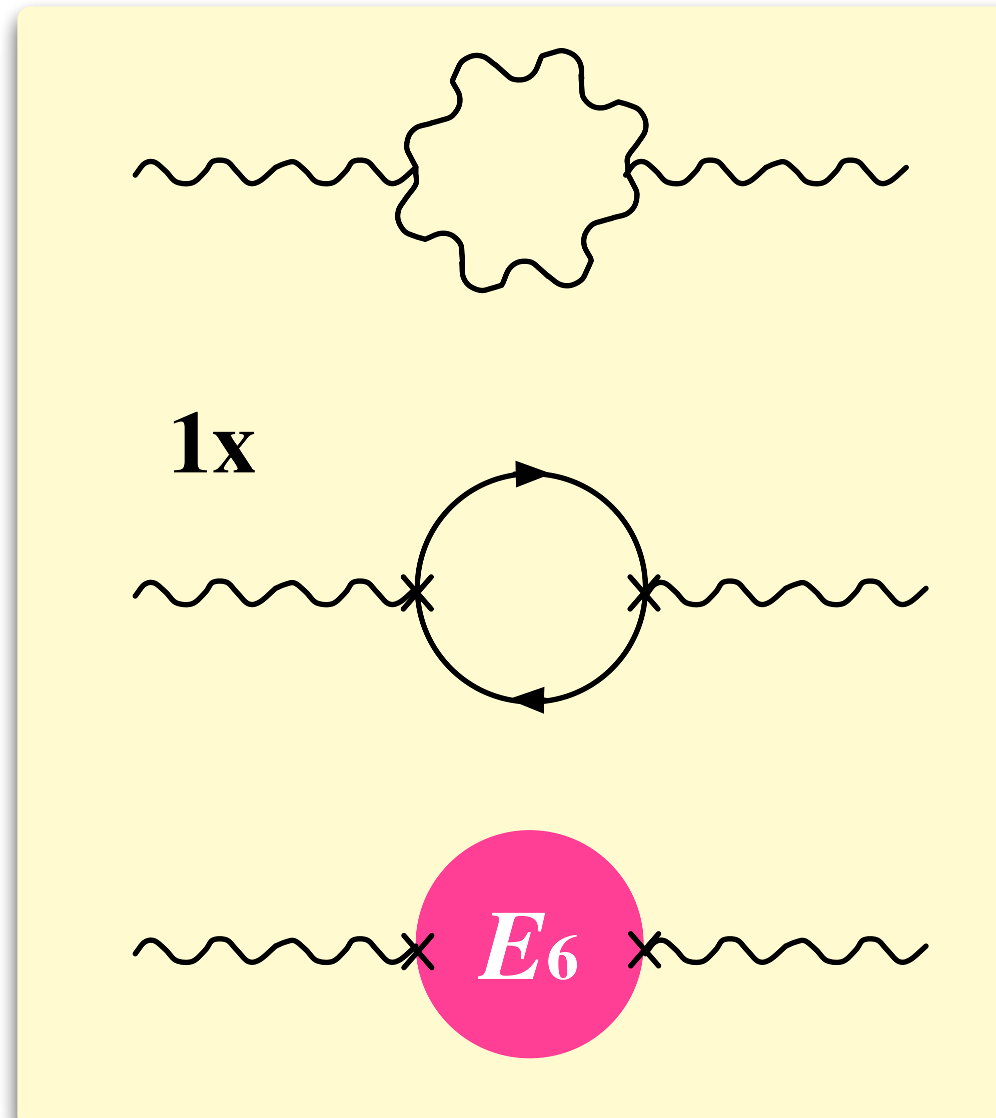


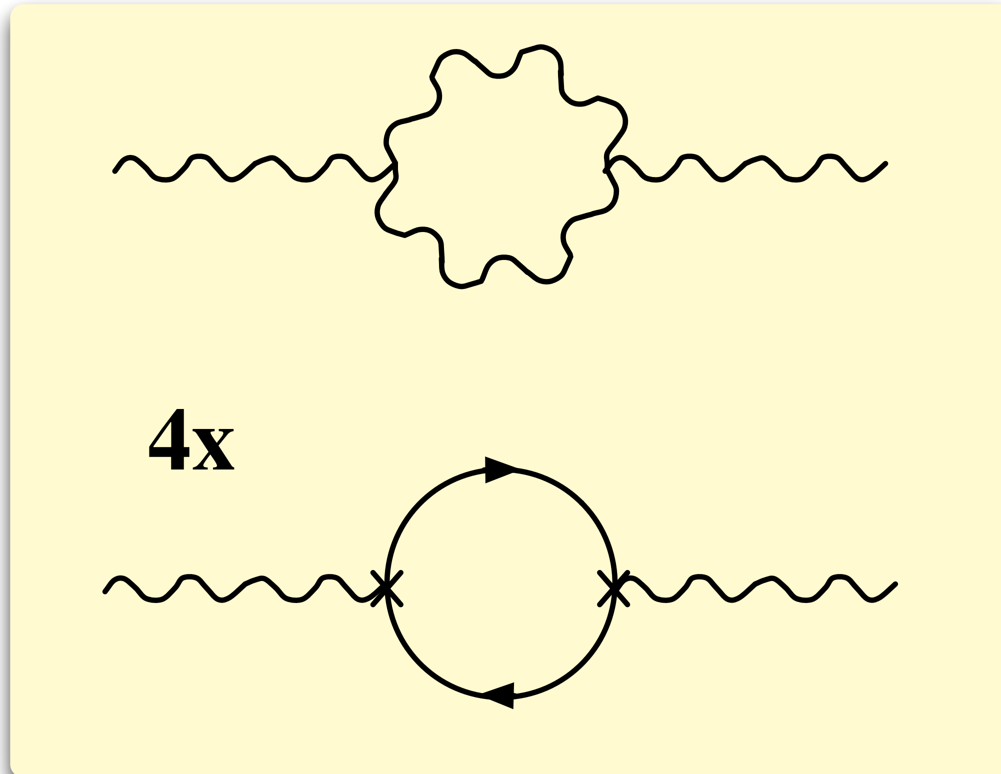

U(1)B
One flavor
of SU(2)
SU(2)
gauge
multiplet
A non
Lagrangian
Theory
SU(6)F
SU(2) gauge coupling is
exactly marginal.

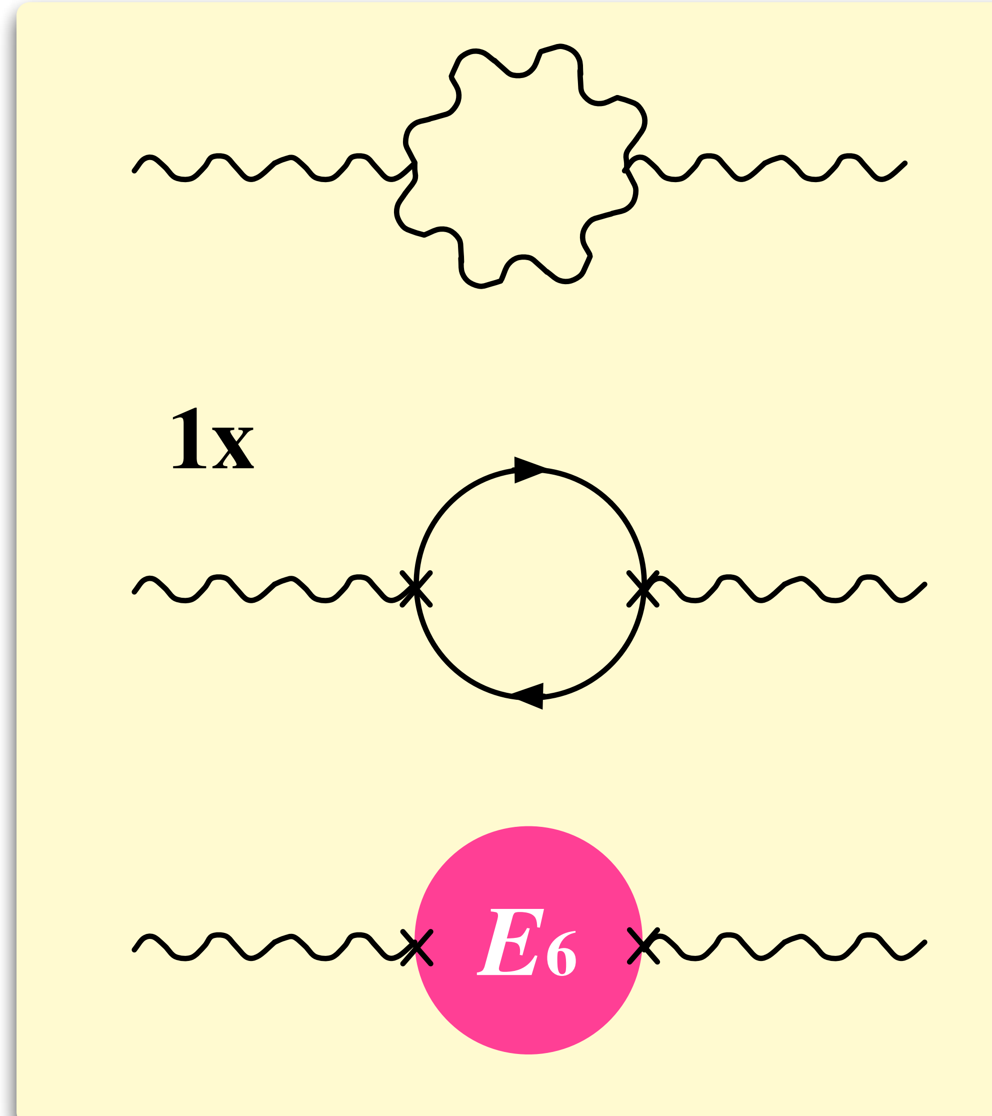
1-loop β func. of SU(2) with 4 flavors is zero.

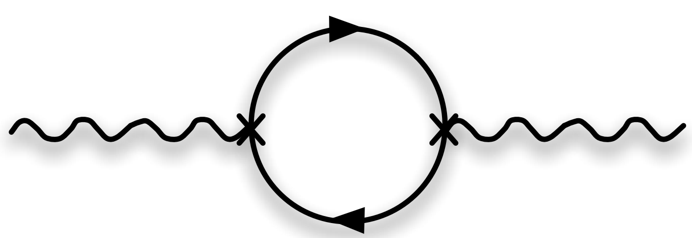


1-loop β func. of SU(2) with 1 flavor plus MN theory is zero.





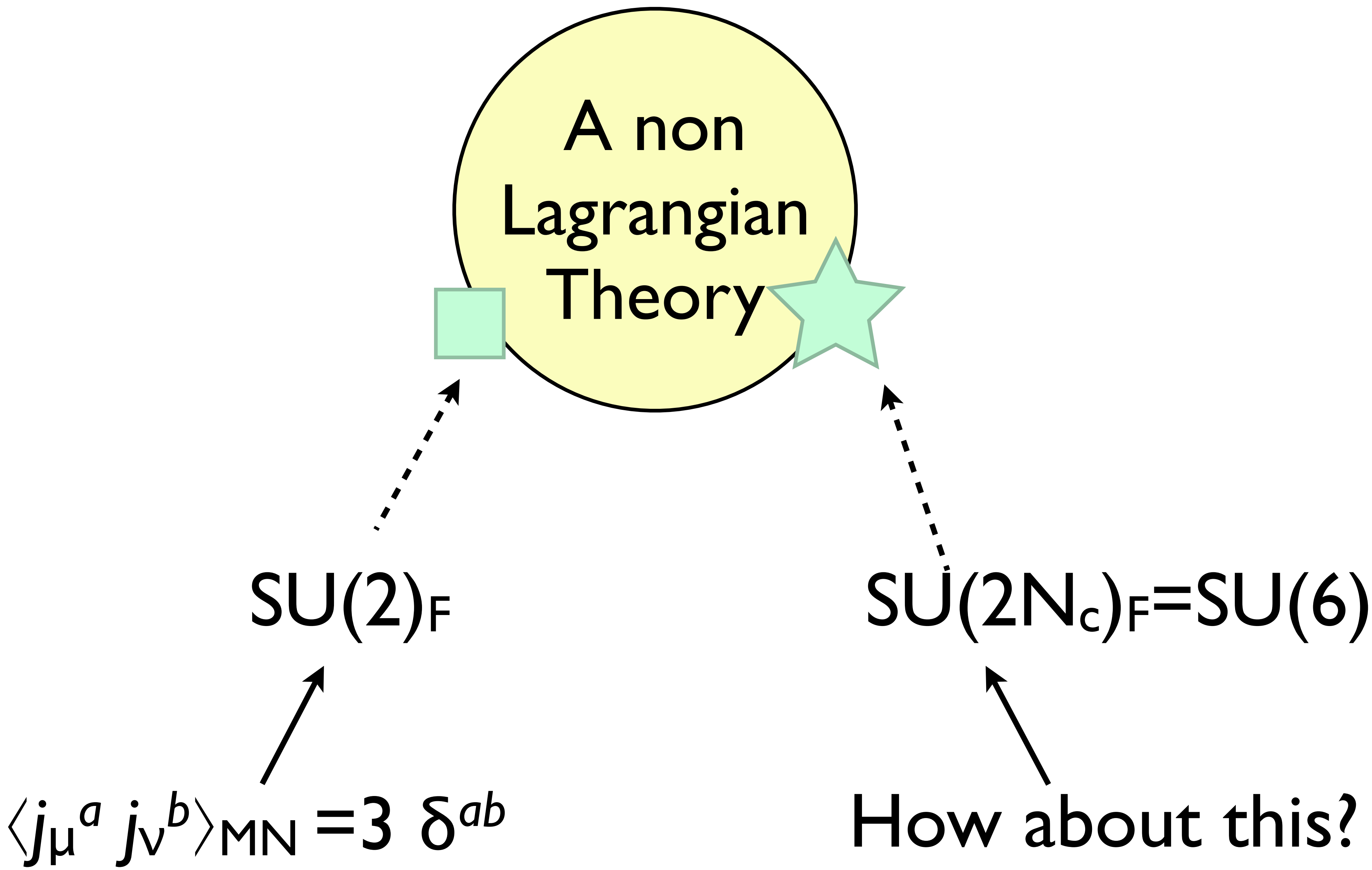


$\langle j_\mu{}^a \, j_\nu{}^b \rangle_{\text{doublet}} =$ 

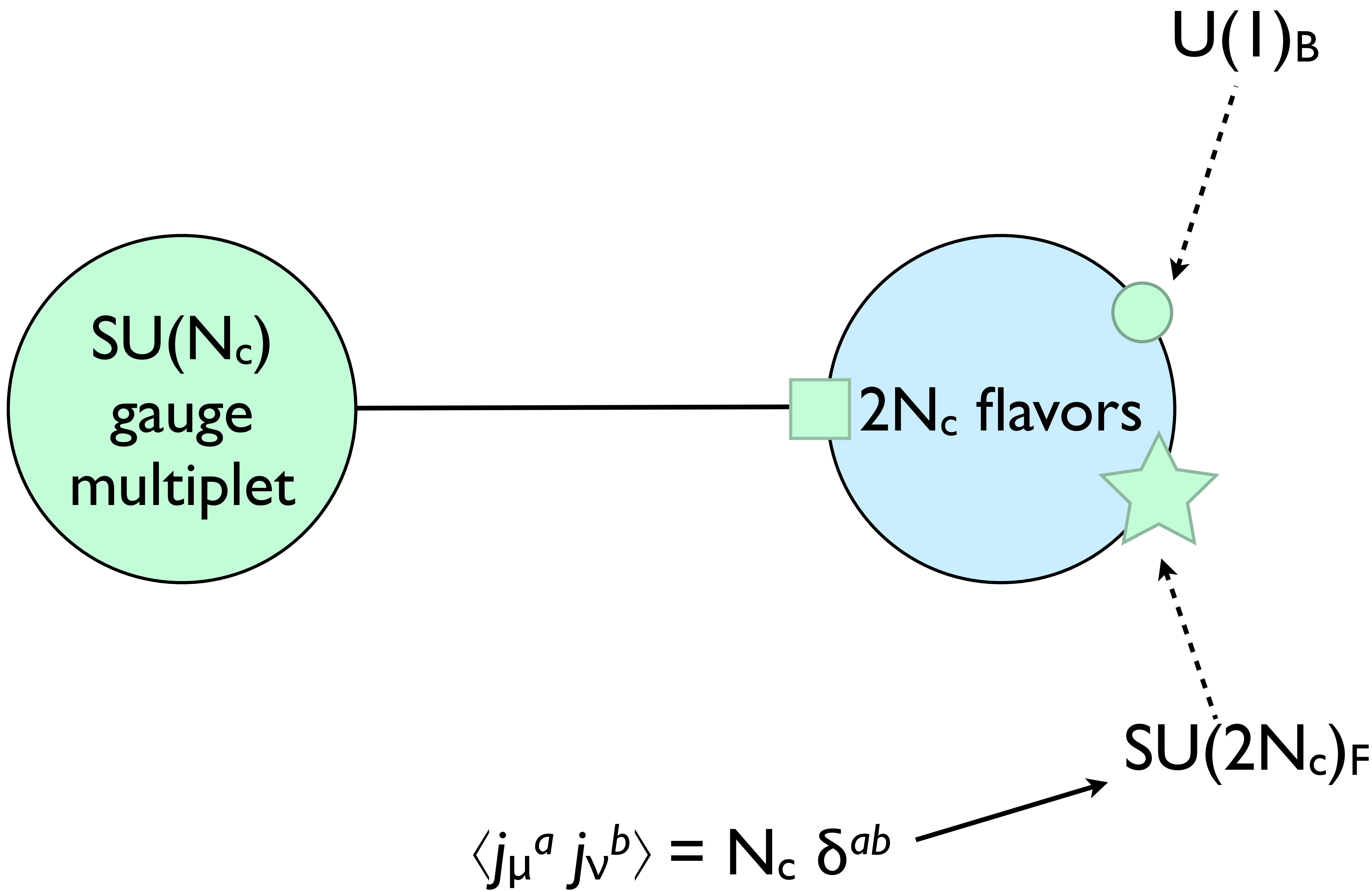
$\langle j_\mu{}^a \, j_\nu{}^b \rangle_{\text{MN}} =$ 

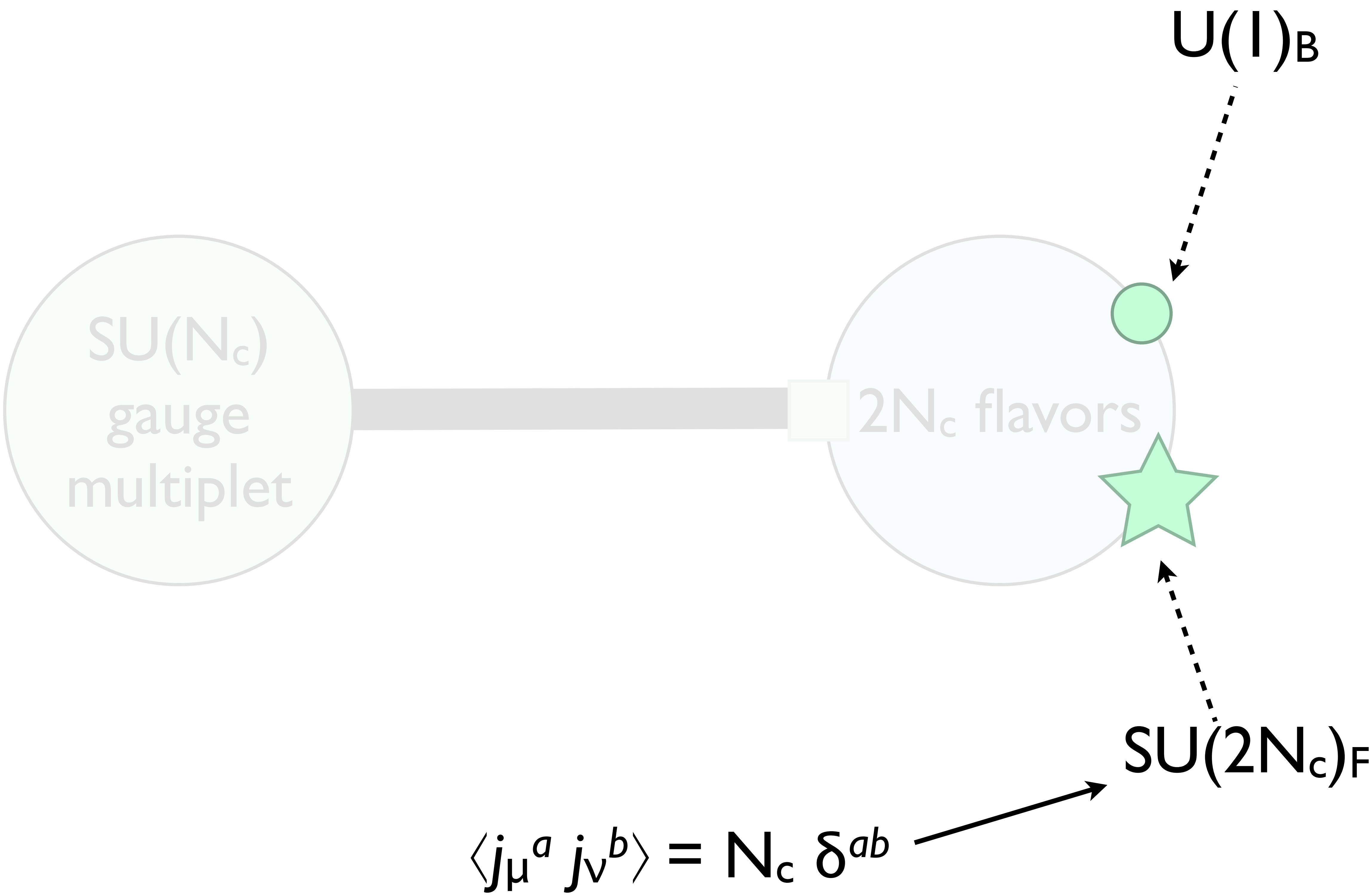
$$\langle j_\mu{}^a \, j_\nu{}^b \rangle_{\text{MN}} = 3 \langle j_\mu{}^a \, j_\nu{}^b \rangle_{\text{doublet}}$$
$$= 3 \ \delta^{ab}$$

The Minahan-Nemeschansky theory

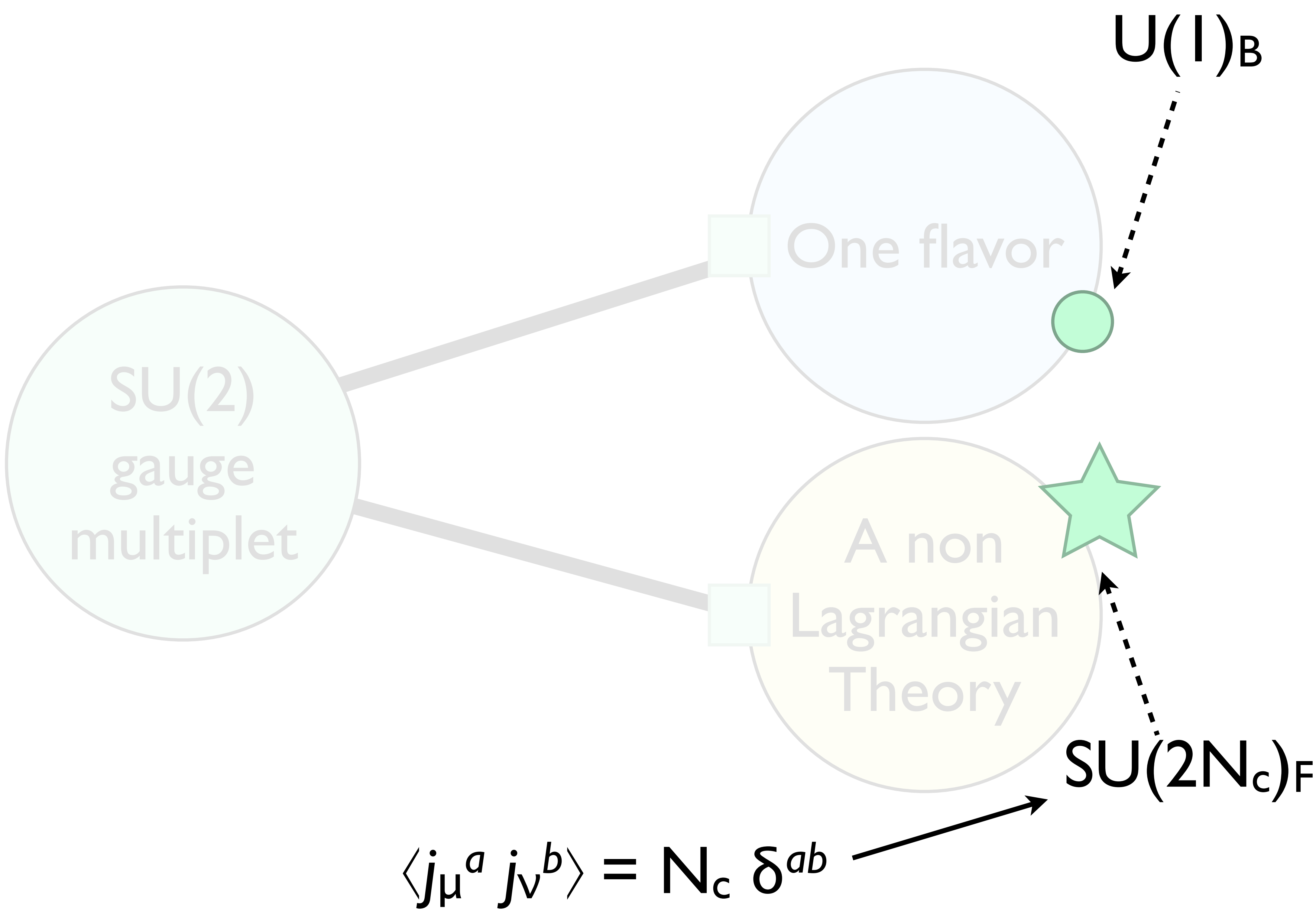


When θ=π :

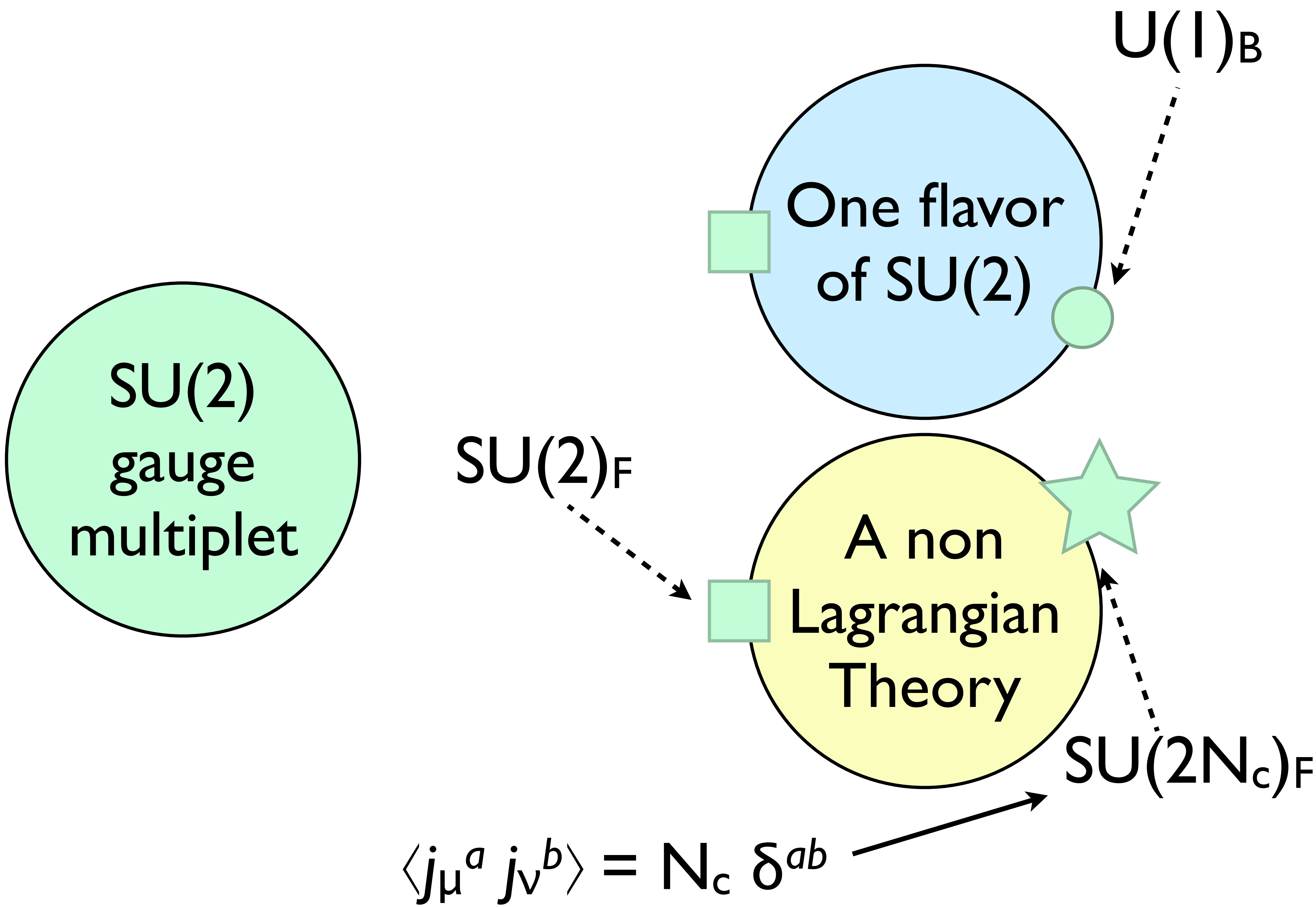




U(1)$_B$
SU(N$_c$) gauge multiplet
2N$_c$ flavors
SU(2N$_c$)$_F$
$\langle j_\mu^a \, j_\nu^b \rangle = N_c \, \delta^{ab}$

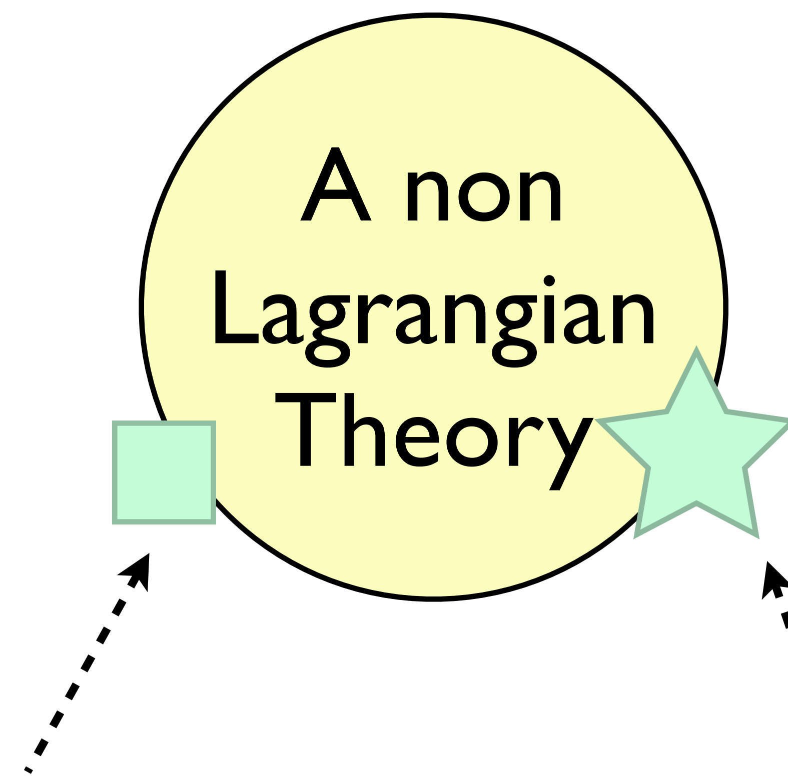


U(1)B
One flavor
SU(2)
gauge
multiplet
A non
Lagrangian
Theory
SU(2Nc)F
⟨jμa jνb⟩ = Nc δab



SU(2) gauge multiplet
One flavor of SU(2)
$U(1)_B$
$SU(2)_F$
A non Lagrangian Theory
$SU(2N_c)_F$
$\langle j_\mu^a \, j_\nu^b \rangle = N_c \, \delta^{ab}$

The Minahan-Nemeschansky theory

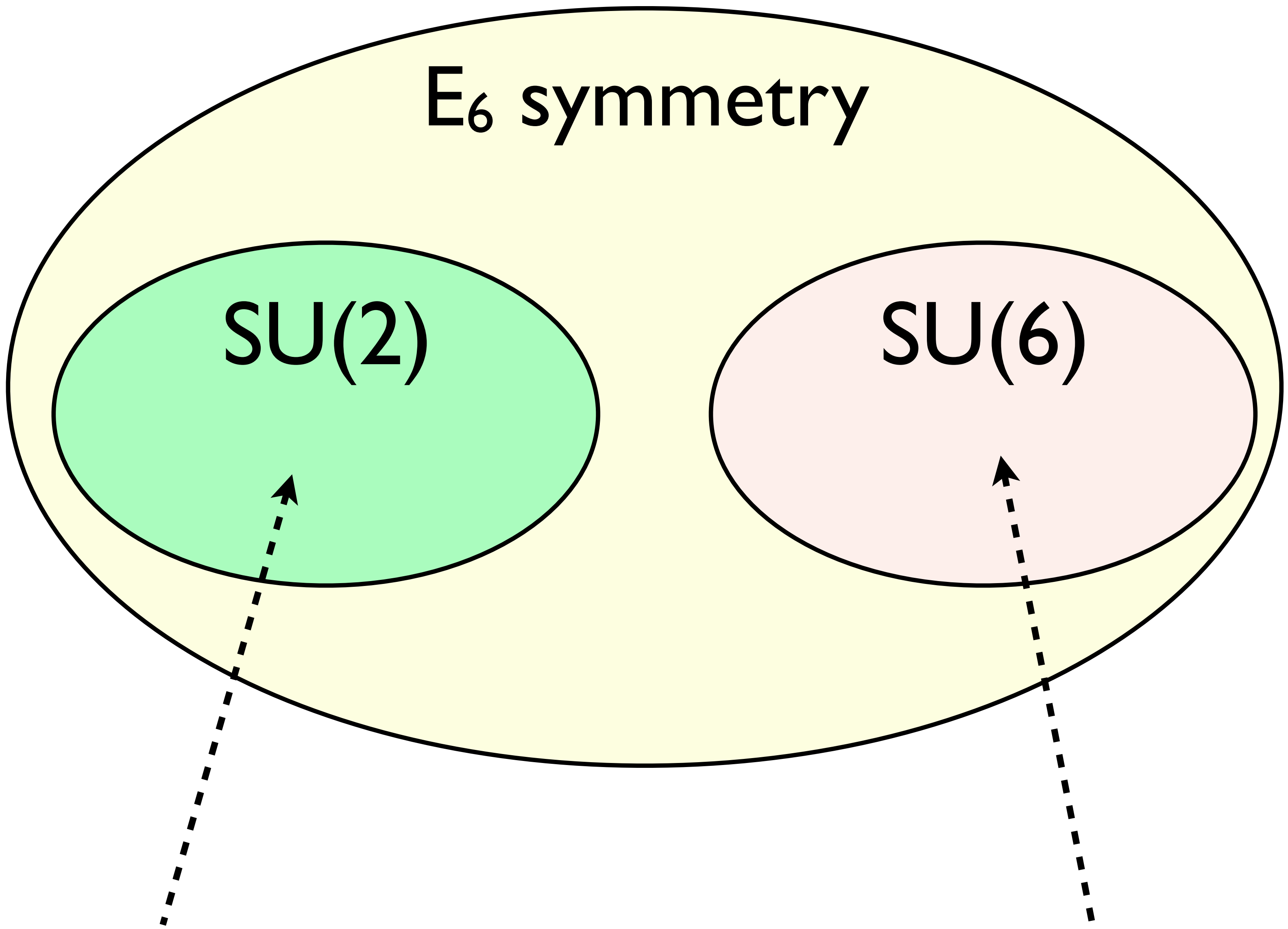


$SU(2)_F$ $\qquad\qquad$ $SU(2N_c)_F = SU(6)$

$\langle j_\mu^a \, j_\nu^b \rangle = 3\, \delta^{ab}$ $\qquad\qquad$ $\langle j_\mu^a \, j_\nu^b \rangle = N_c\, \delta^{ab}$

$= 3\, \delta^{ab}$

The Minahan-Nemeschansky theory



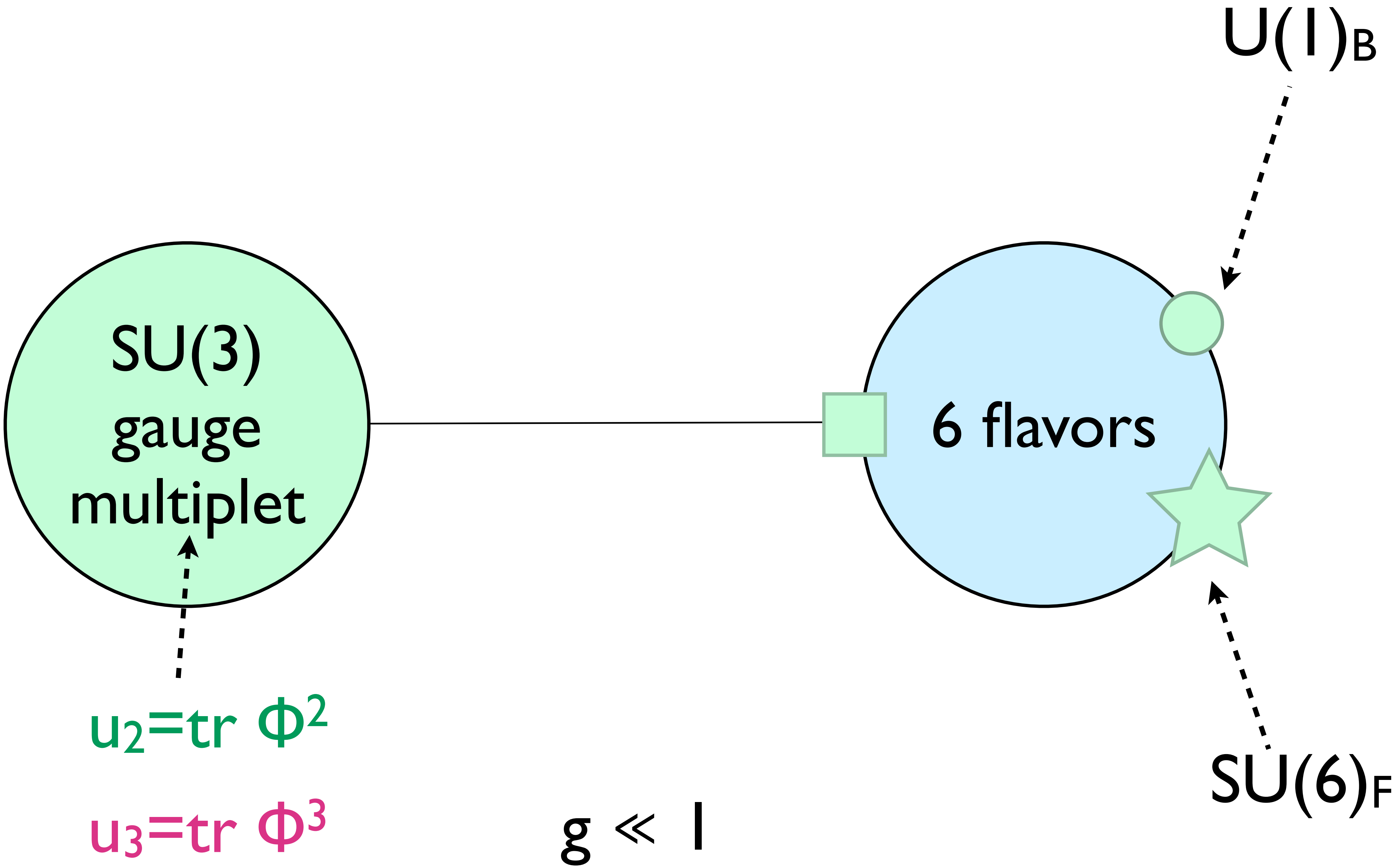
$\langle j_\mu^a \, j_\nu^b \rangle = 3\ \delta^{ab}$

$\langle j_\mu^a \, j_\nu^b \rangle = 3\ \delta^{ab}$

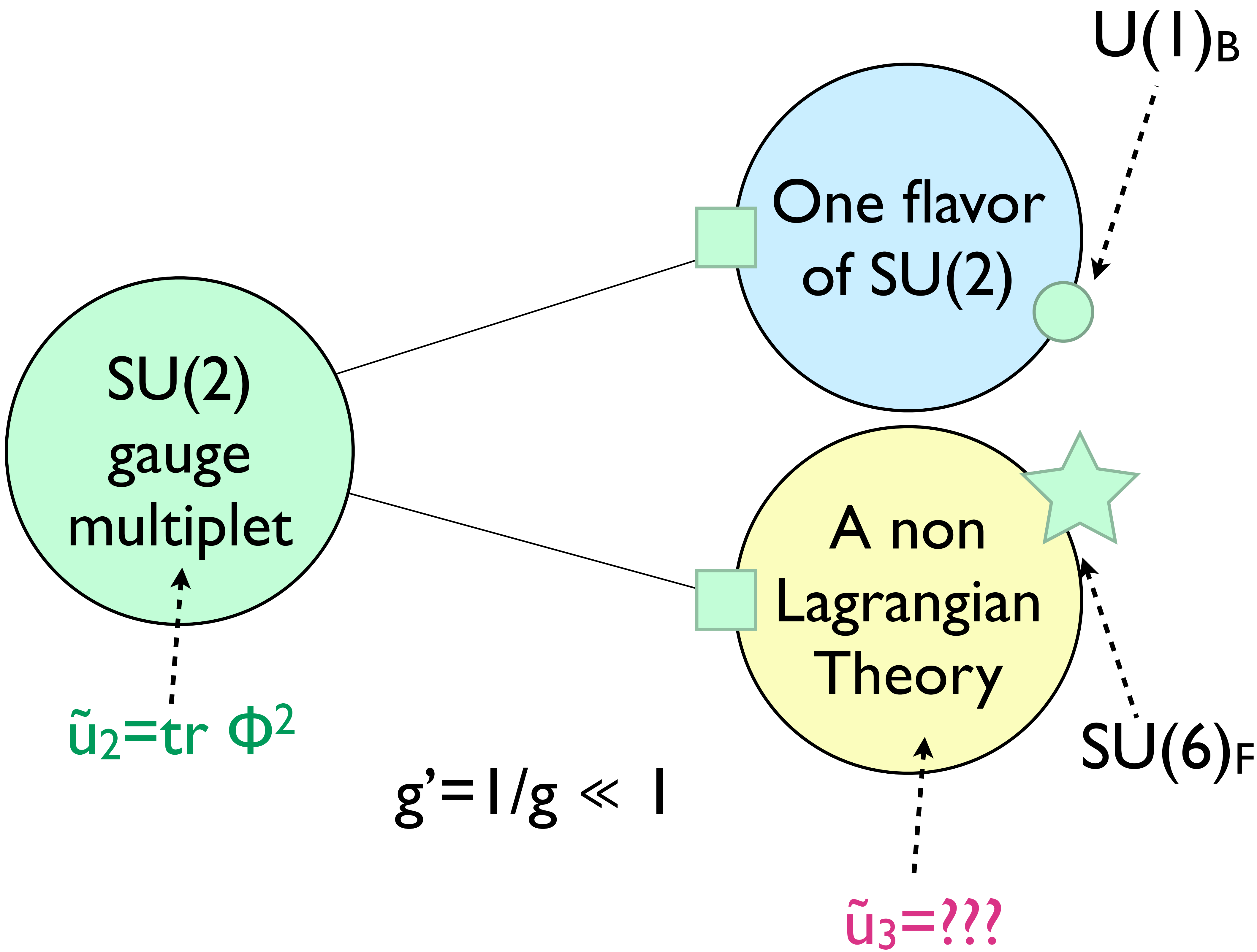
Compatible!

On chiral operators

When $\theta=\pi$:



$g \ll 1$


U(1)B
One flavor of SU(2)
SU(2) gauge multiplet
A non Lagrangian Theory
SU(6)F
ũ2=tr Φ2
g'=1/g ≪ 1
ũ3=???

The Minahan-Nemeschansky theory

has a tr $\Phi^?$ -like operator *u* of dimension 3.

All BPS operators can be determined this way.

The Minahan-Nemeschansky theory

For example, it has 78 operators of dimension 2 transforming as the adjoint of E_6

M-N's E_6 theory

↓ mass deform.

$\mathcal{N}$=2 SU(2) with 4 flavors

SU(2) adjoint Φ

q

$\tilde{q}$

Higgsing

$\langle u \rangle \neq 0$

→

$u = \mathrm{tr}\, \Phi^2$

$\mathcal{N}$=2 U(1) with massive charged fields

M-N's E_6 theory

Higgsing

$\langle u \rangle \neq 0$

u=???

$\mathcal{N}$=2 U(1)
with massive
charged fields

mass deform.

mass deform.

$\mathcal{N}$=2 SU(2) with 4 flavors

SU(2) adjoint Φ

q

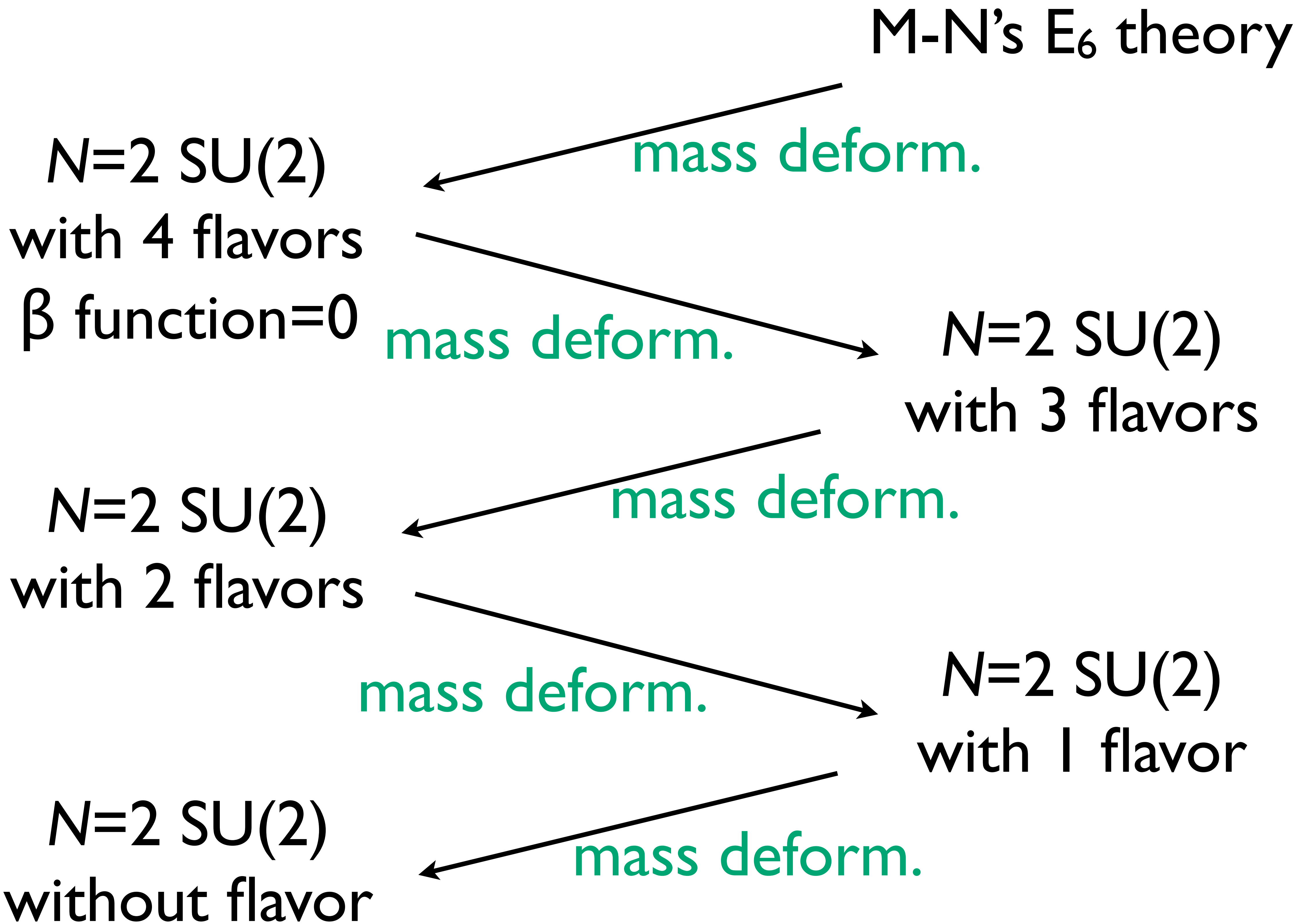
$\tilde{q}$

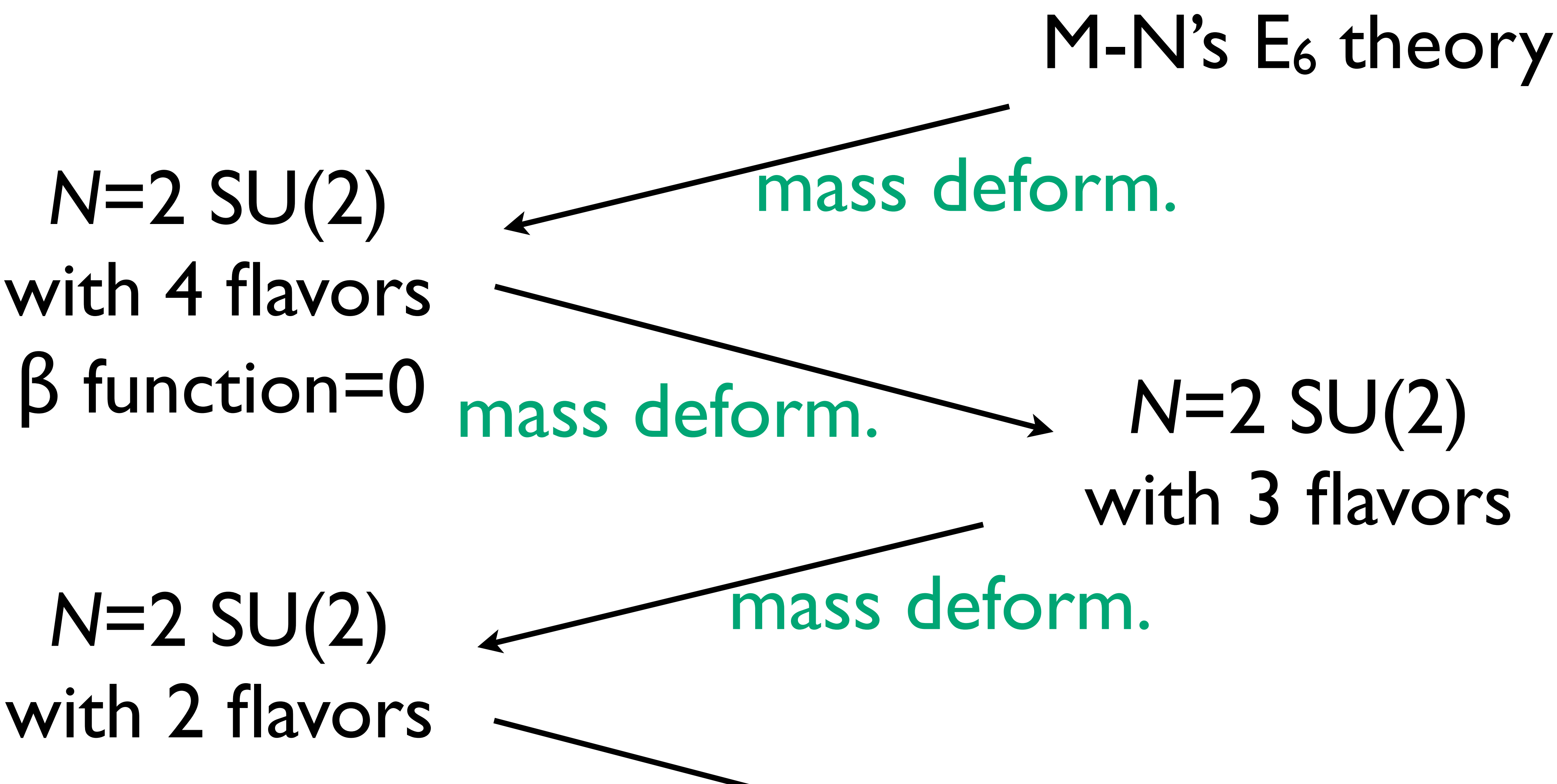
Higgsing

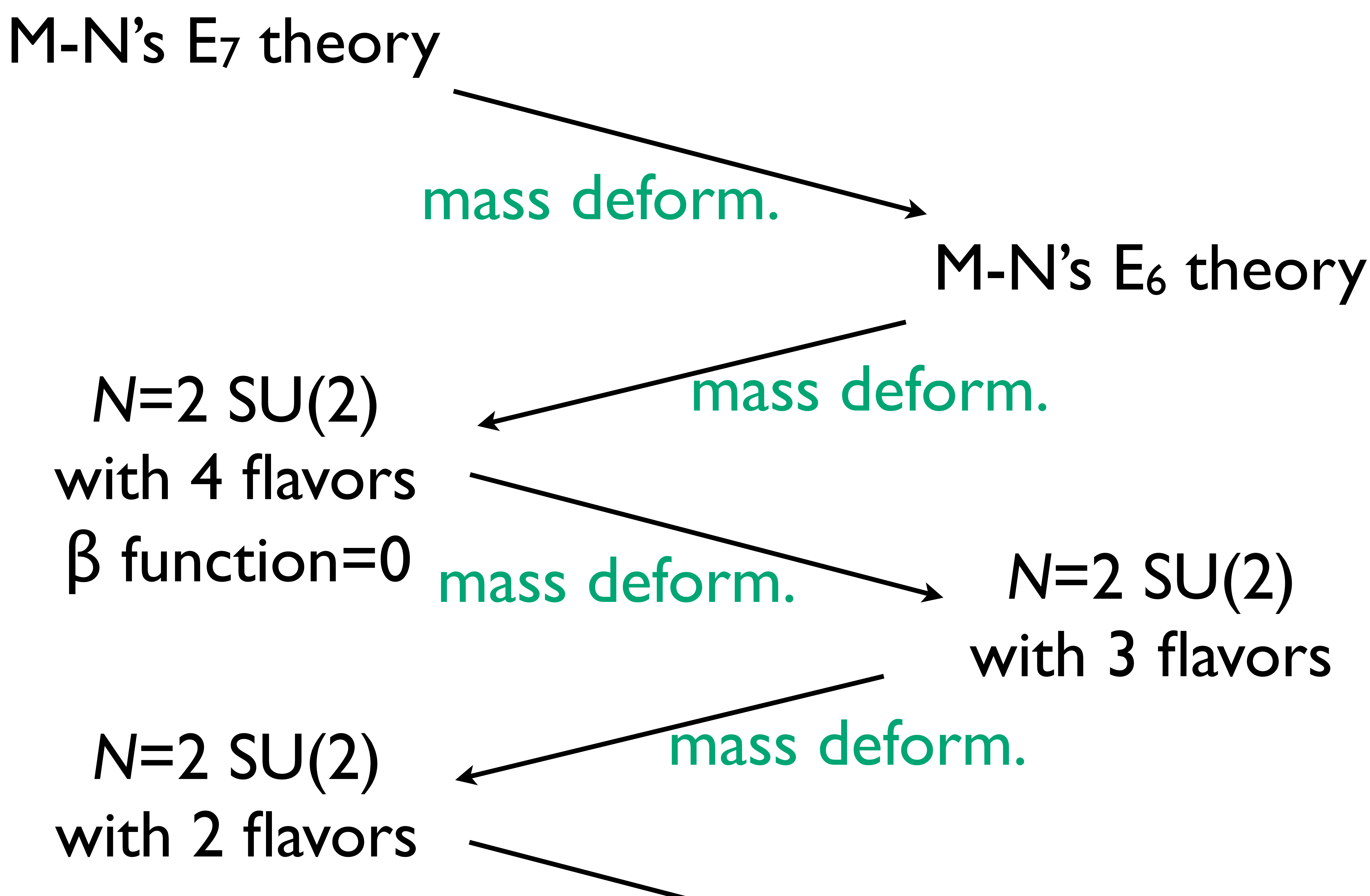
$\langle u \rangle \neq 0$

$u = \mathrm{tr}\,\Phi^2$

$\mathcal{N}$=2 U(1)
with massive
charged fields


M-N's E_6 theory
mass deform.
$\mathcal{N}$=2 SU(2)
with 4 flavors
β function=0
mass deform.
$\mathcal{N}$=2 SU(2)
with 3 flavors
mass deform.
$\mathcal{N}$=2 SU(2)
with 2 flavors
mass deform.
$\mathcal{N}$=2 SU(2)
with 1 flavor
mass deform.
$\mathcal{N}$=2 SU(2)
without flavor


M-N's E_6 theory
mass deform.
$\mathcal{N}$=2 SU(2)
with 4 flavors
β function=0
mass deform.
$\mathcal{N}$=2 SU(2)
with 3 flavors
mass deform.
$\mathcal{N}$=2 SU(2)
with 2 flavors


M-N's E7 theory
mass deform.
M-N's E6 theory
mass deform.
N=2 SU(2)
with 4 flavors
β function=0
mass deform.
N=2 SU(2)
with 3 flavors
mass deform.
N=2 SU(2)
with 2 flavors

M-N's E_8 theory

mass deform.

M-N's E_7 theory

mass deform.

M-N's E_6 theory

mass deform.

$\mathcal{N}$=2 SU(2)
with 4 flavors
β function=0

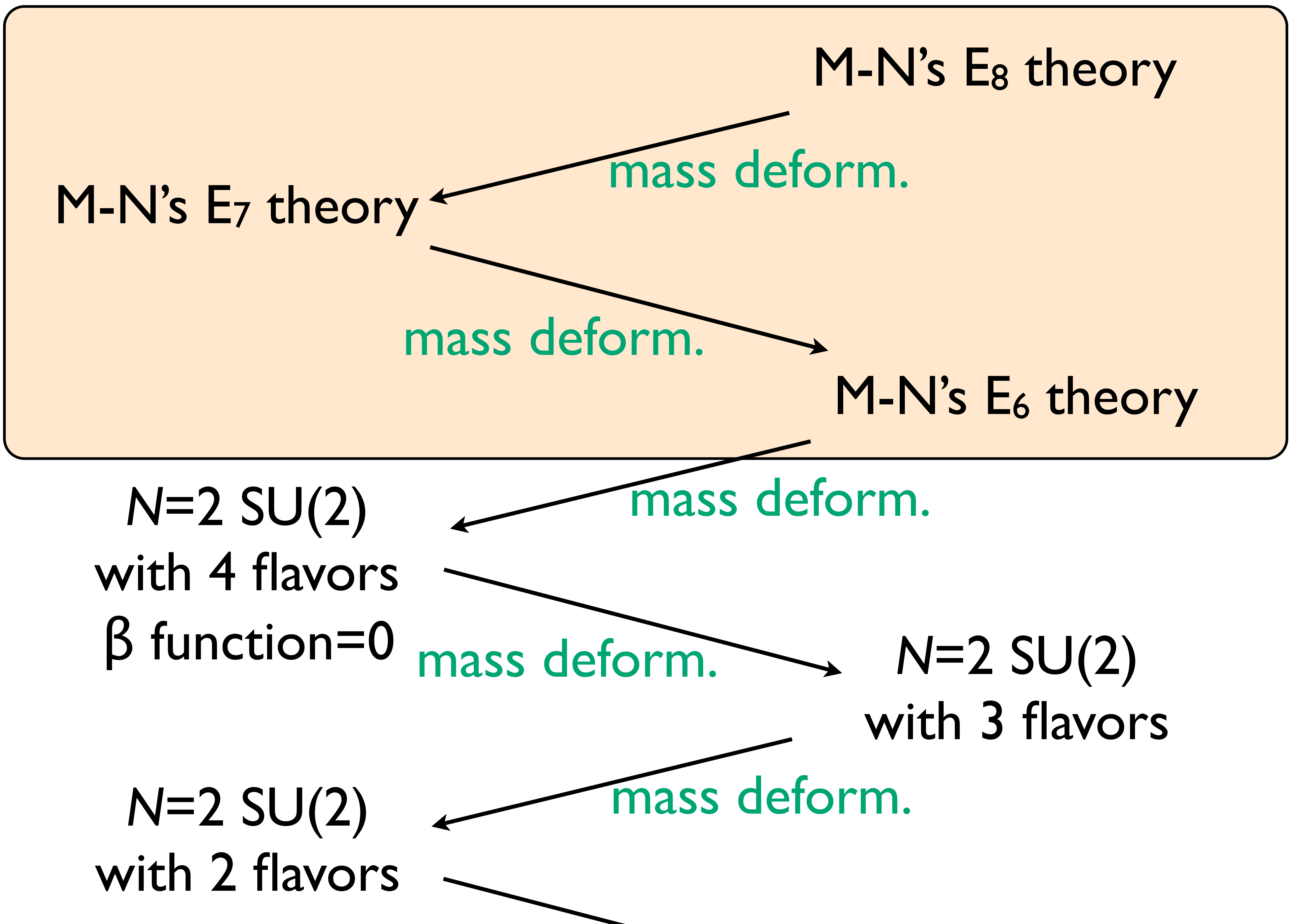
mass deform.

$\mathcal{N}$=2 SU(2)
with 3 flavors

mass deform.

$\mathcal{N}$=2 SU(2)
with 2 flavors

Non-Lagrangian!



M-N's E_6 theory

$\langle u \rangle \neq 0$

u=???

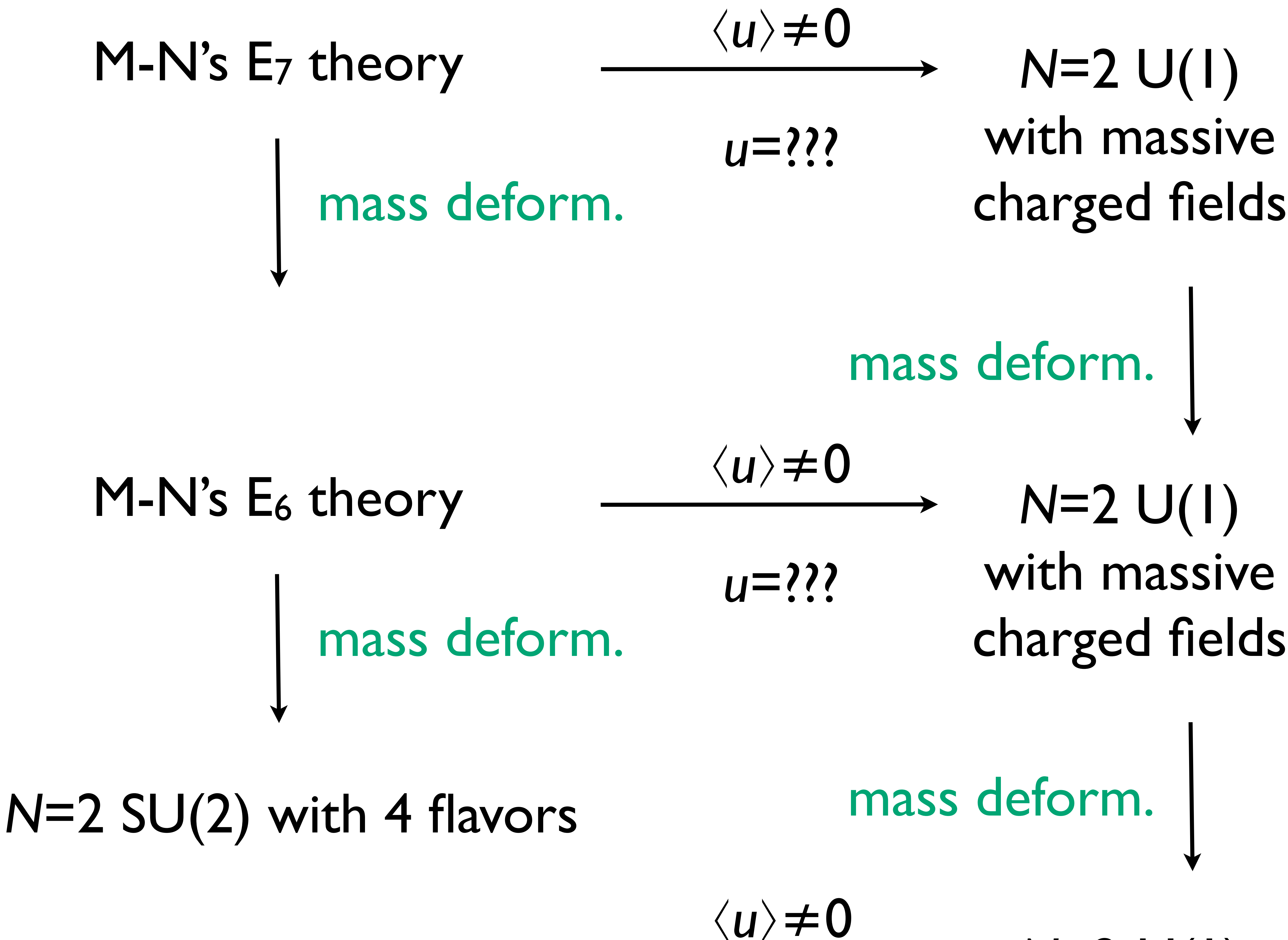
$\mathcal{N}$=2 U(1)
with massive
charged fields

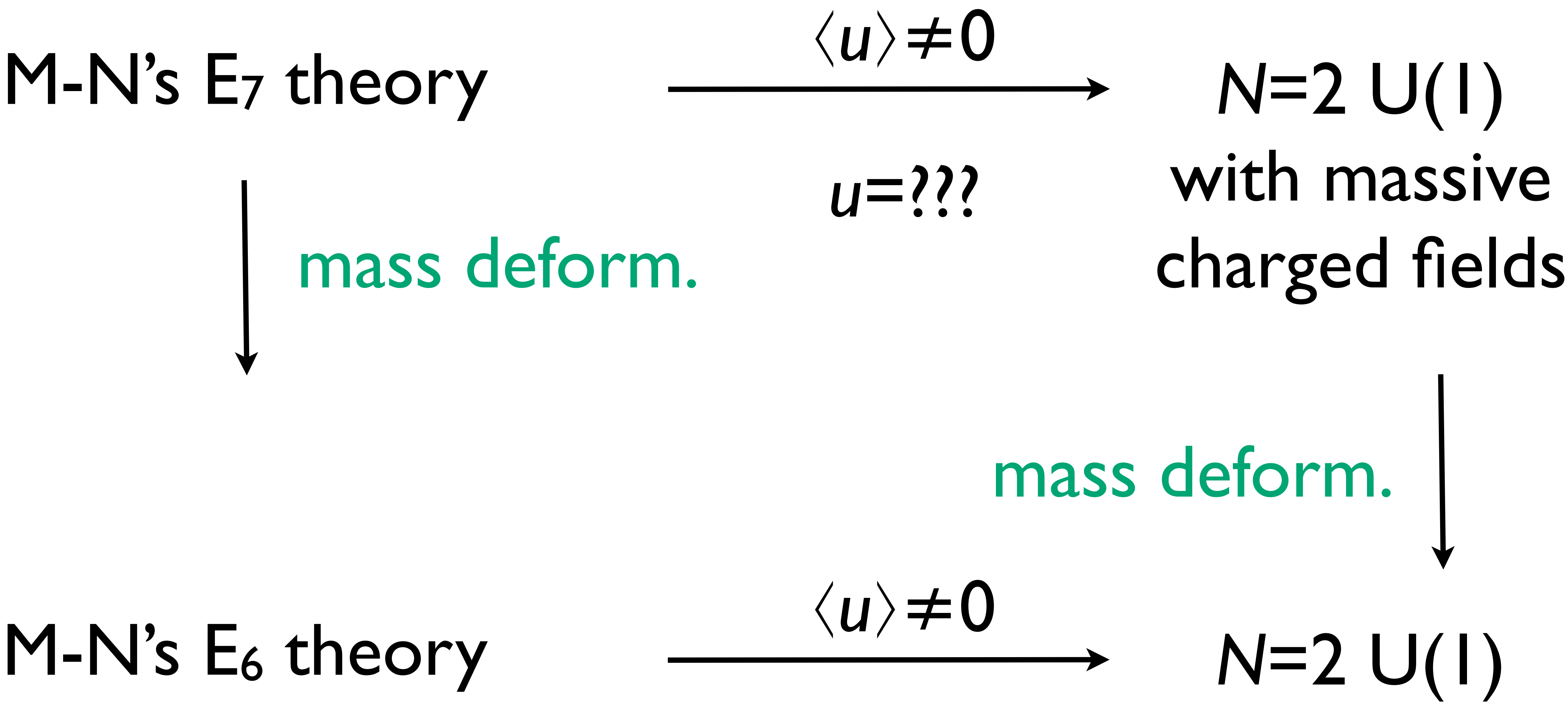
mass deform.

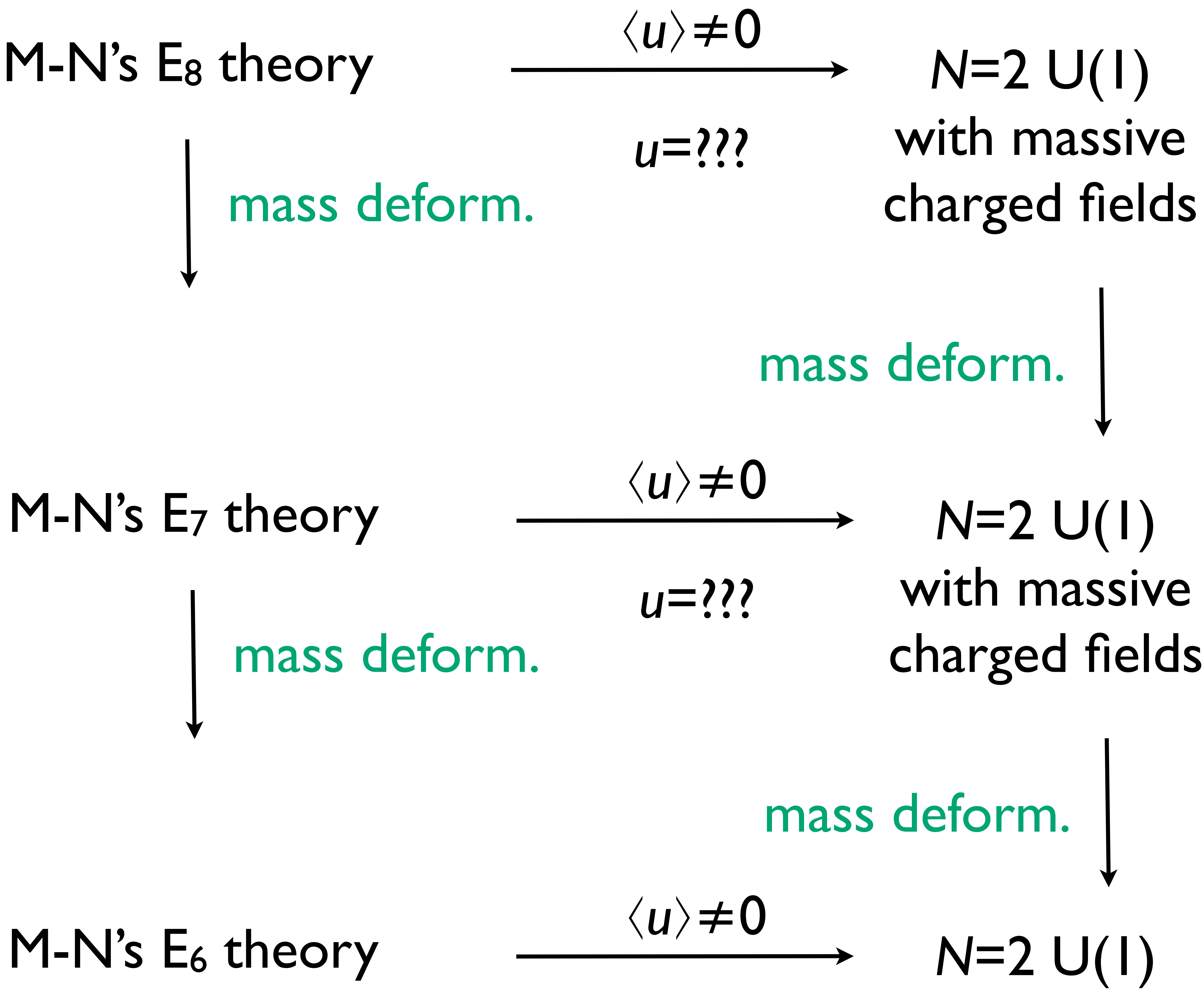
$\mathcal{N}$=2 SU(2) with 4 flavors

mass deform.

$\langle u \rangle \neq 0$


M-N's E7 theory
⟨u⟩≠0
u=???
N=2 U(1)
with massive
charged fields
mass deform.
mass deform.
M-N's E6 theory
⟨u⟩≠0
u=???
N=2 U(1)
with massive
charged fields
mass deform.
N=2 SU(2) with 4 flavors
mass deform.
⟨u⟩≠0


M-N's E_7 theory
$\langle u \rangle \neq 0$
$\mathcal{N}$=2 U(1) with massive charged fields
u=???
mass deform.
mass deform.
M-N's E_6 theory
$\langle u \rangle \neq 0$
$\mathcal{N}$=2 U(1)


M-N's E_8 theory
$\langle u \rangle \neq 0$
u=???
N=2 U(1)
with massive
charged fields
mass deform.
mass deform.
M-N's E_7 theory
$\langle u \rangle \neq 0$
u=???
N=2 U(1)
with massive
charged fields
mass deform.
mass deform.
M-N's E_6 theory
$\langle u \rangle \neq 0$
N=2 U(1)

M-N's E_n theory

is $\mathcal{N}$=2 supersymmetric.

has E_n flavor symmetry.

has an operator u which can be given a vev so that the theory becomes U(1) + massive

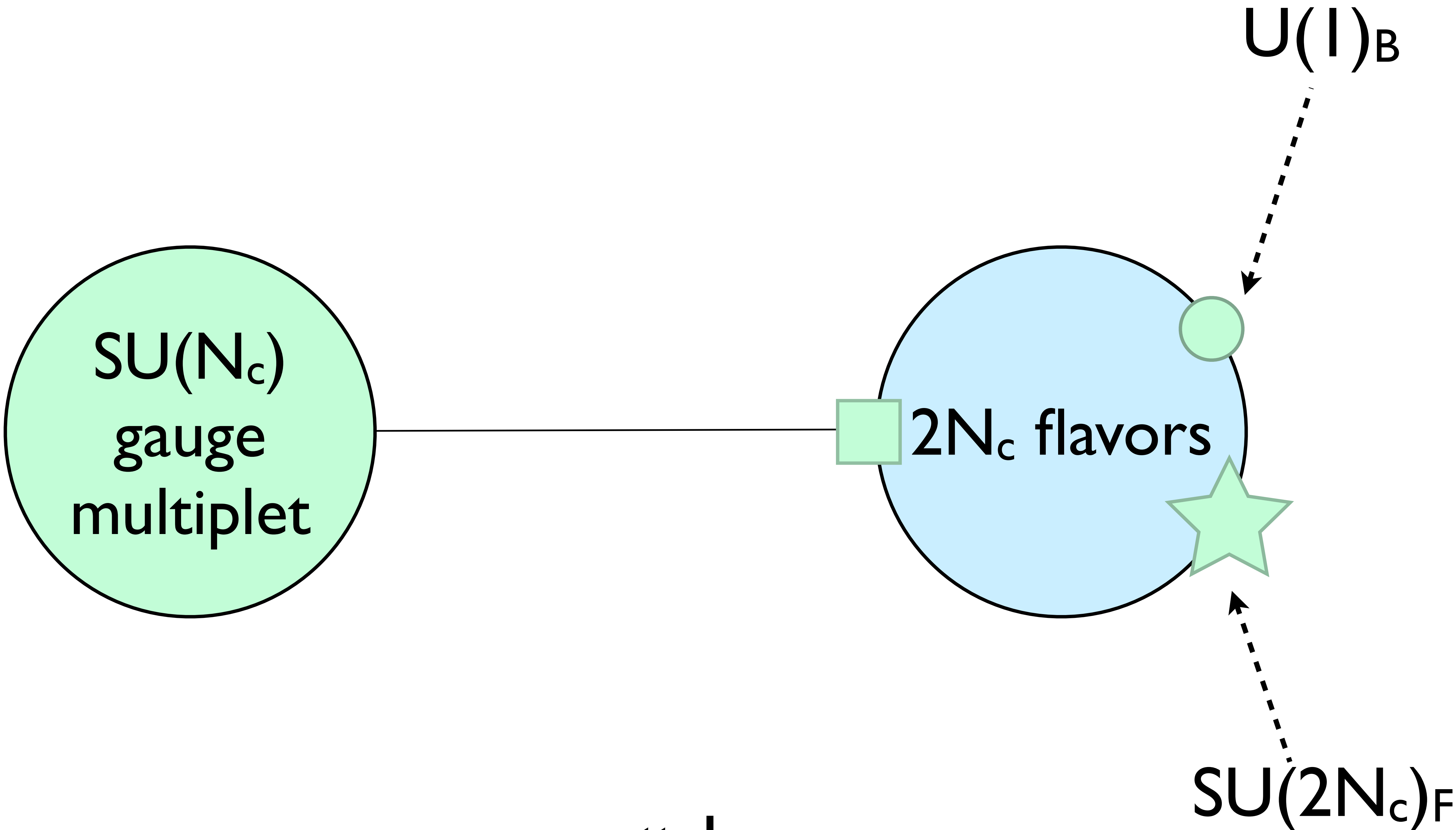
is just a natural cousin of SU(2) with flavors

	6	M-N's E_8 theory
	4	M-N's E_7 theory
	3	M-N's E_6 theory
scaling dimension of u =	2	SU(2) with 4 flavors
	4/3	SU(2) with 2 flavors
	3/2	SU(2) with 3 flavors
	6/5	SU(2) with 1 flavors

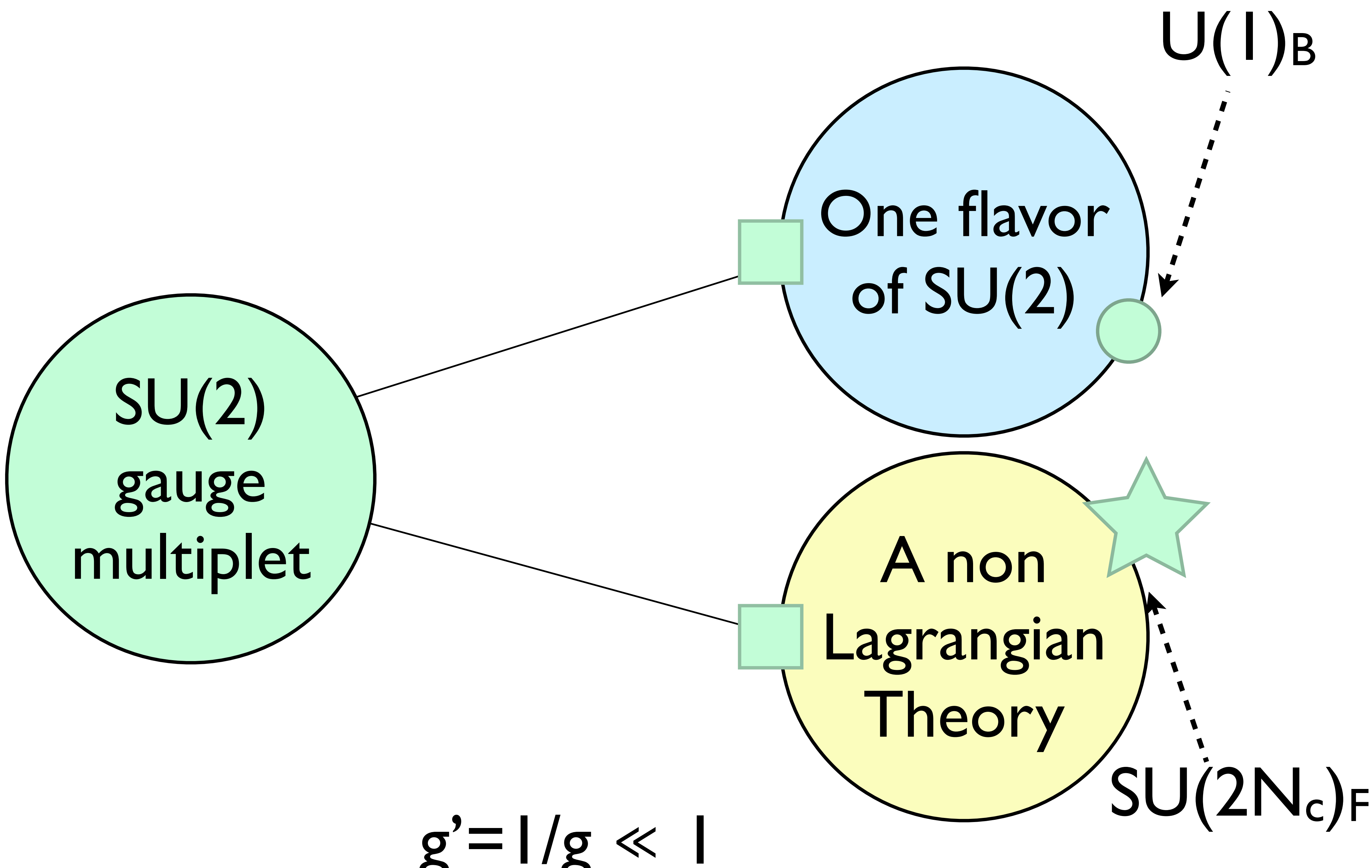
tr Φ^2 → u

Non-Lagrangians are “everywhere”

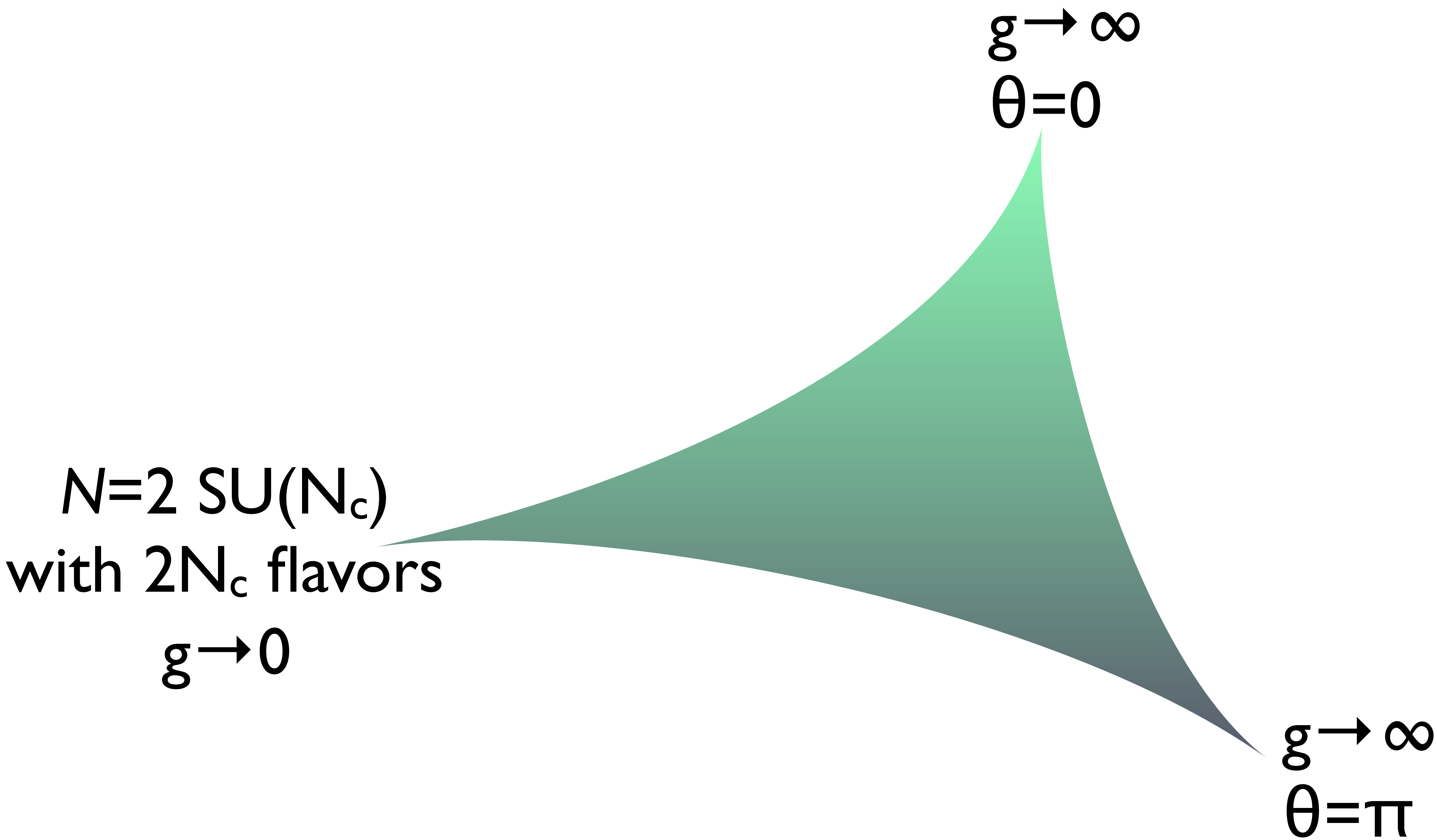
When θ=π :

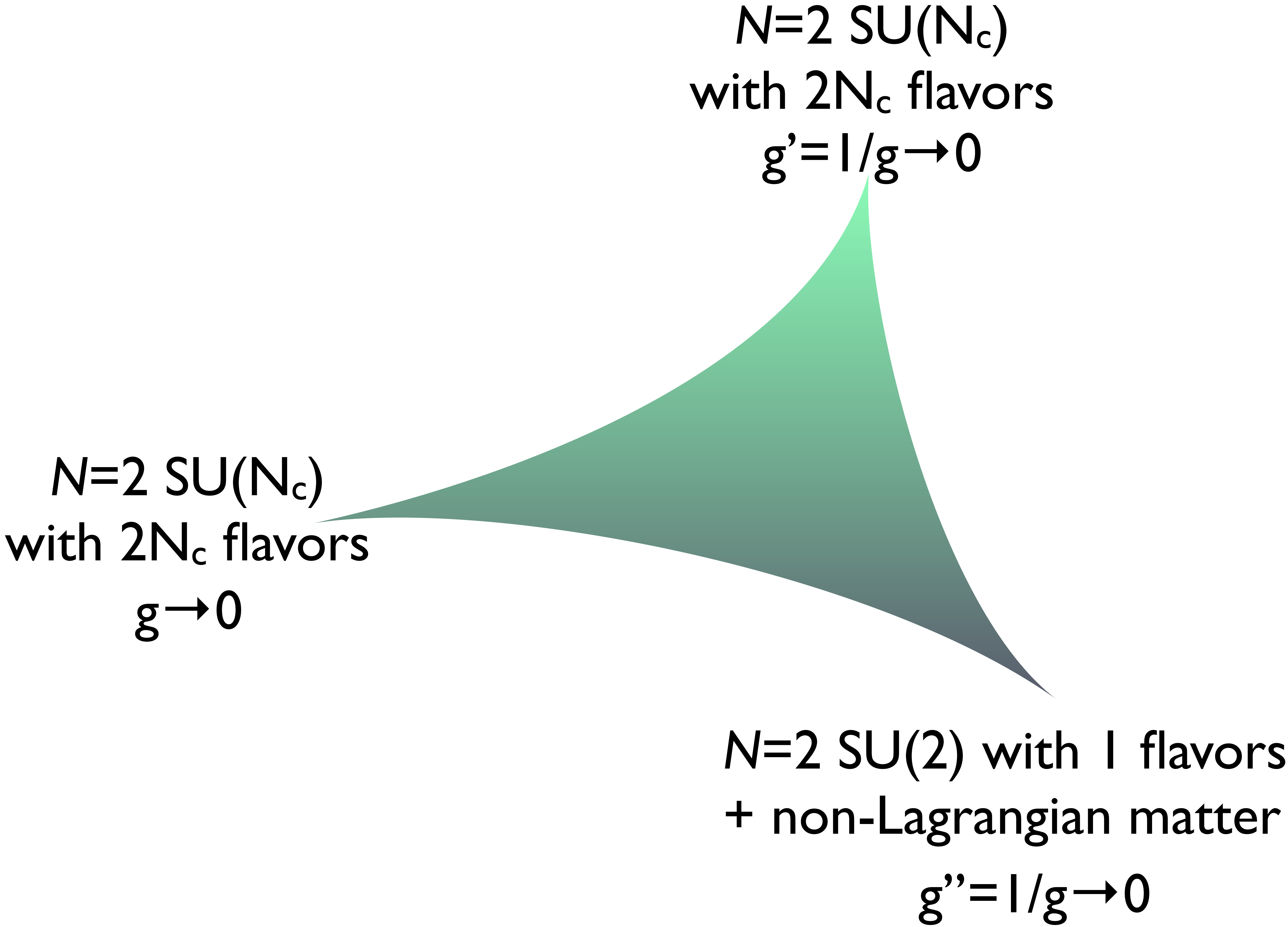


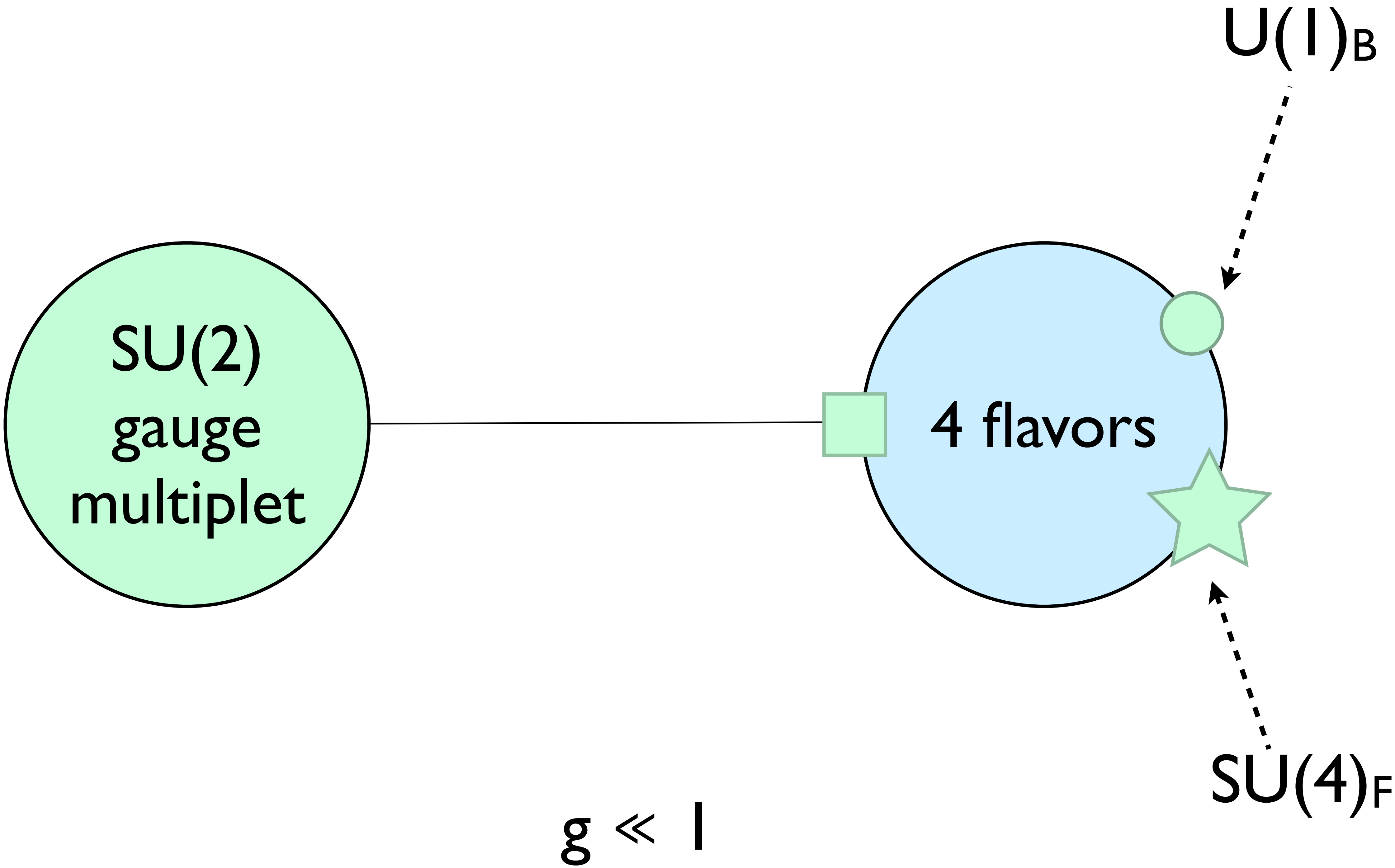
$g \ll 1$

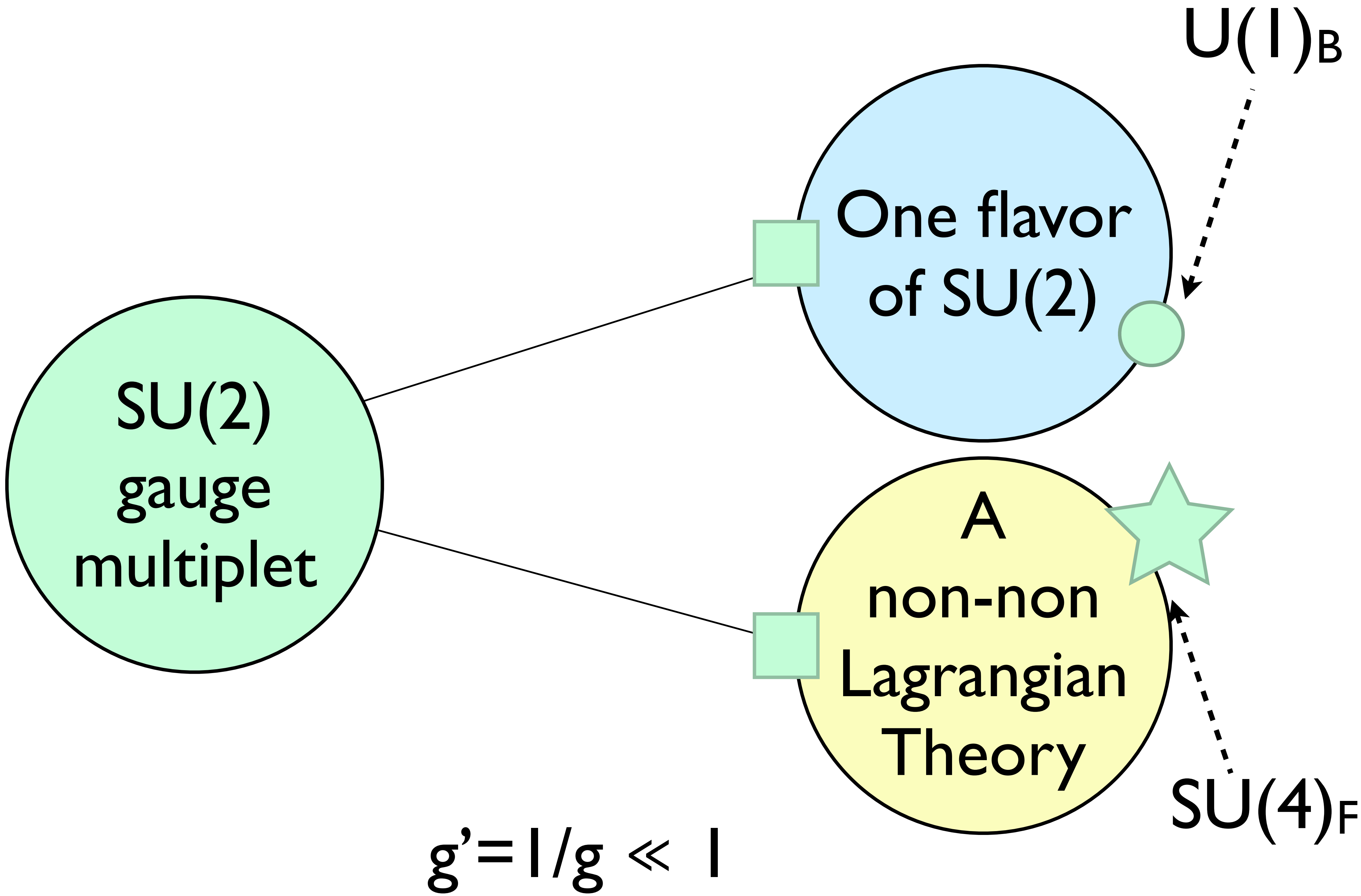


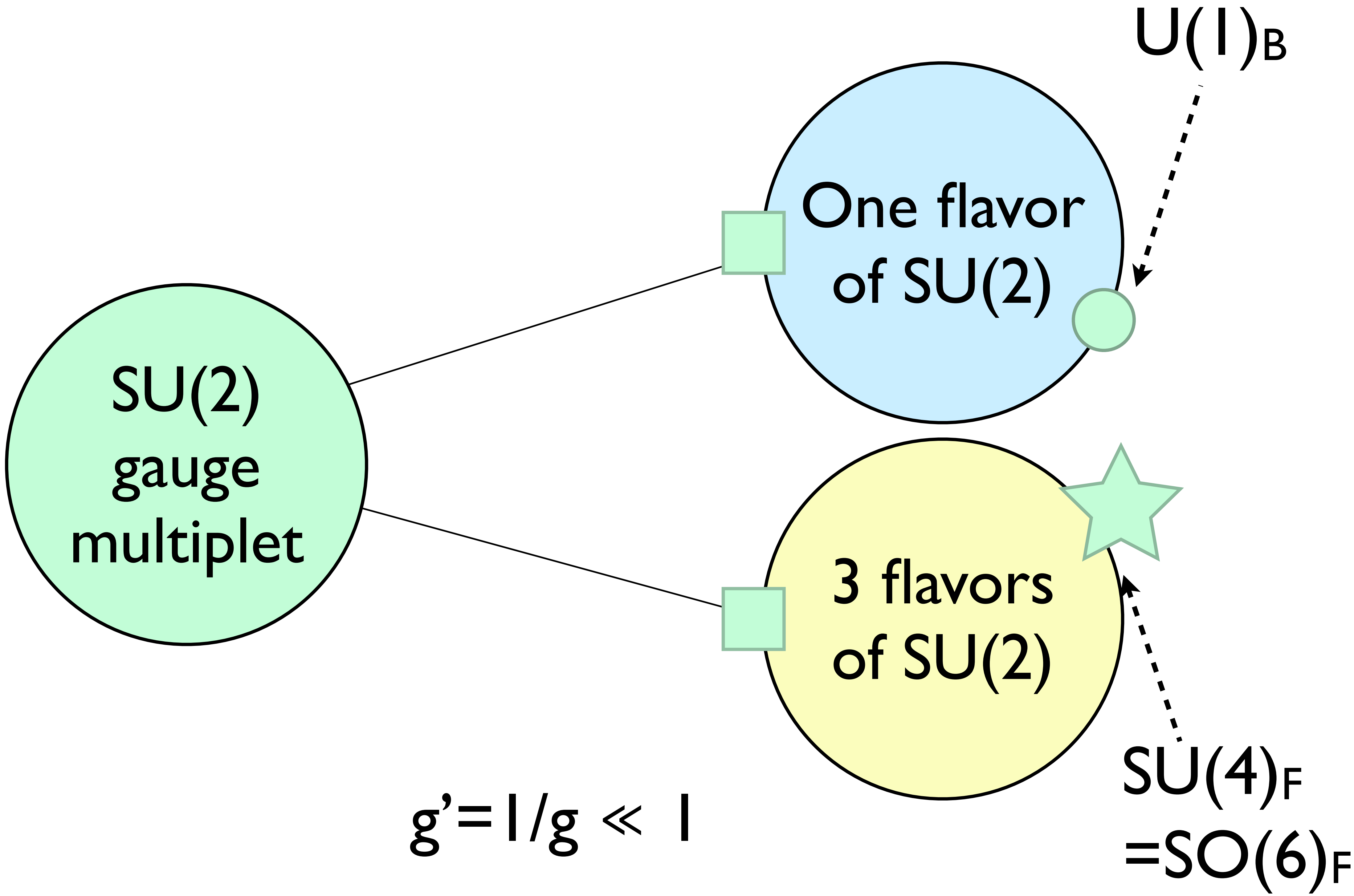
U(1)B
One flavor of SU(2)
SU(2) gauge multiplet
A non Lagrangian Theory
SU(2Nc)F
g'=1/g ≪ 1


g→∞
θ=0
N=2 SU(N_c)
with 2N_c flavors
g→0
g→∞
θ=π

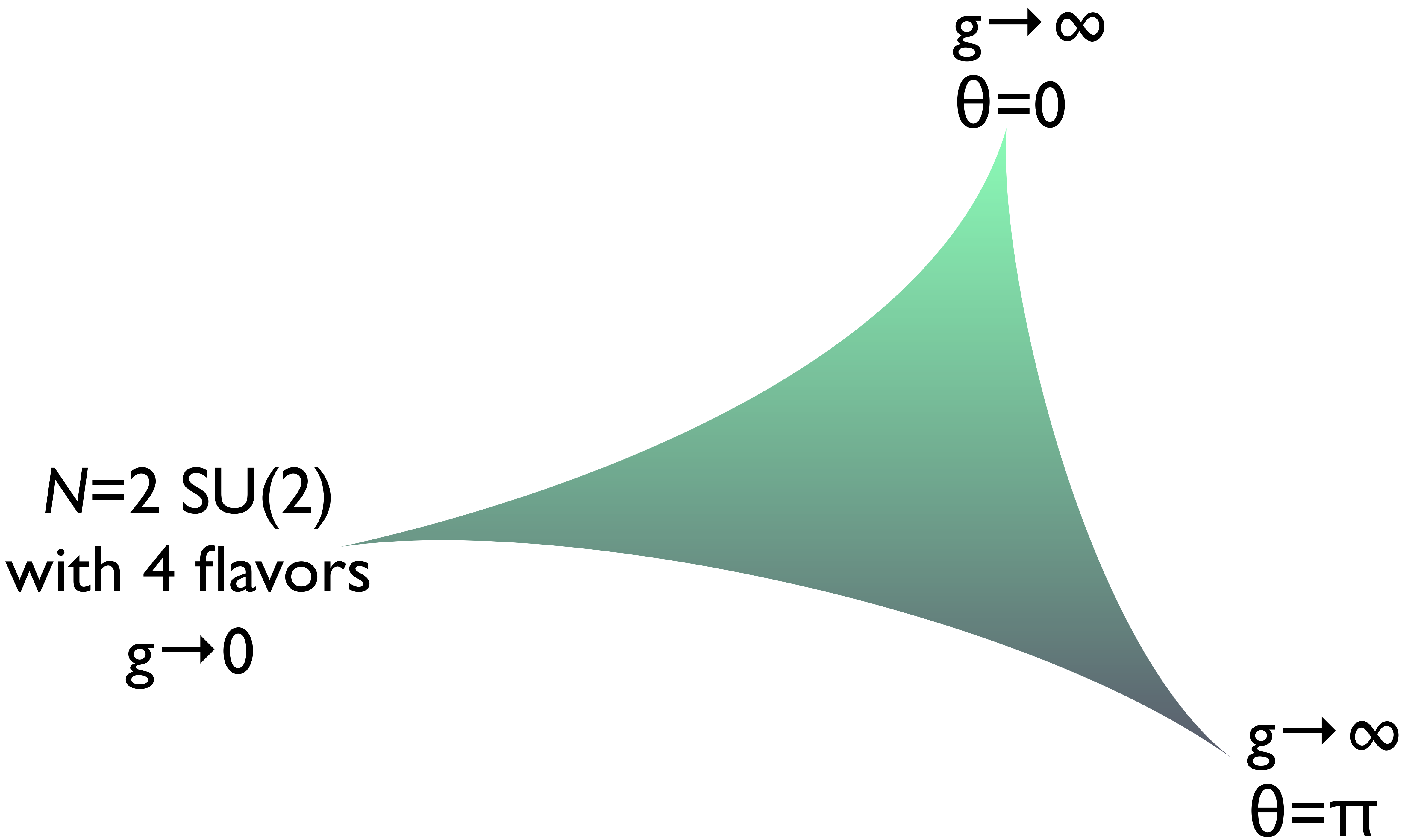

$\mathcal{N}$=2 SU(N_c)
with 2N_c flavors
g'=1/g→0
$\mathcal{N}$=2 SU(N_c)
with 2N_c flavors
g→0
$\mathcal{N}$=2 SU(2) with 1 flavors
+ non-Lagrangian matter
g''=1/g→0


U(1)$_B$
SU(2)
gauge
multiplet
4 flavors
SU(4)$_F$
g ≪ 1

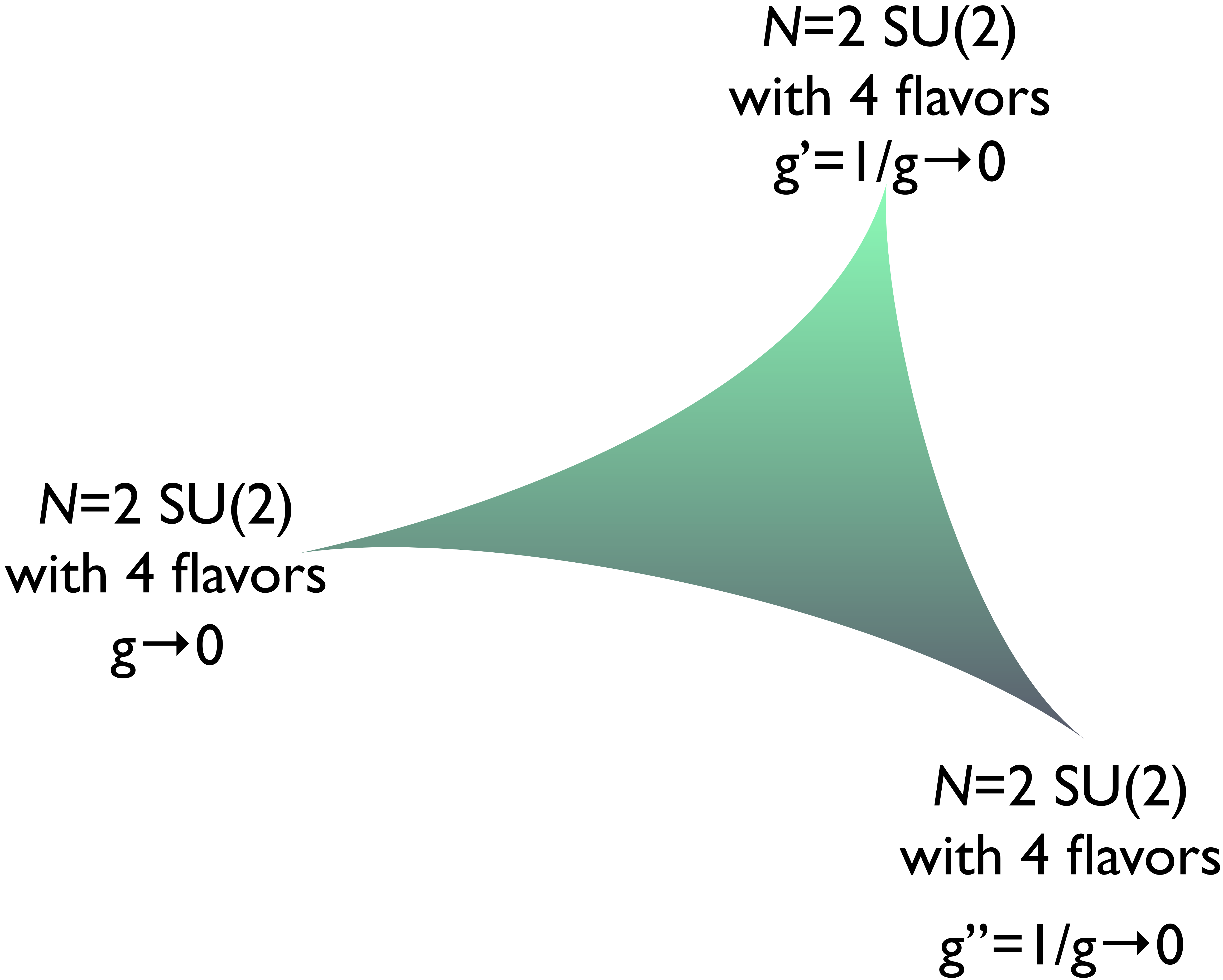

SU(2)
gauge
multiplet
One flavor
of SU(2)
$U(1)_B$
A
non-non
Lagrangian
Theory
$SU(4)_F$
$g' = 1/g \ll 1$



U(1)B
One flavor of SU(2)
SU(2) gauge multiplet
3 flavors of SU(2)
SU(4)F =SO(6)F
g'=1/g ≪ 1

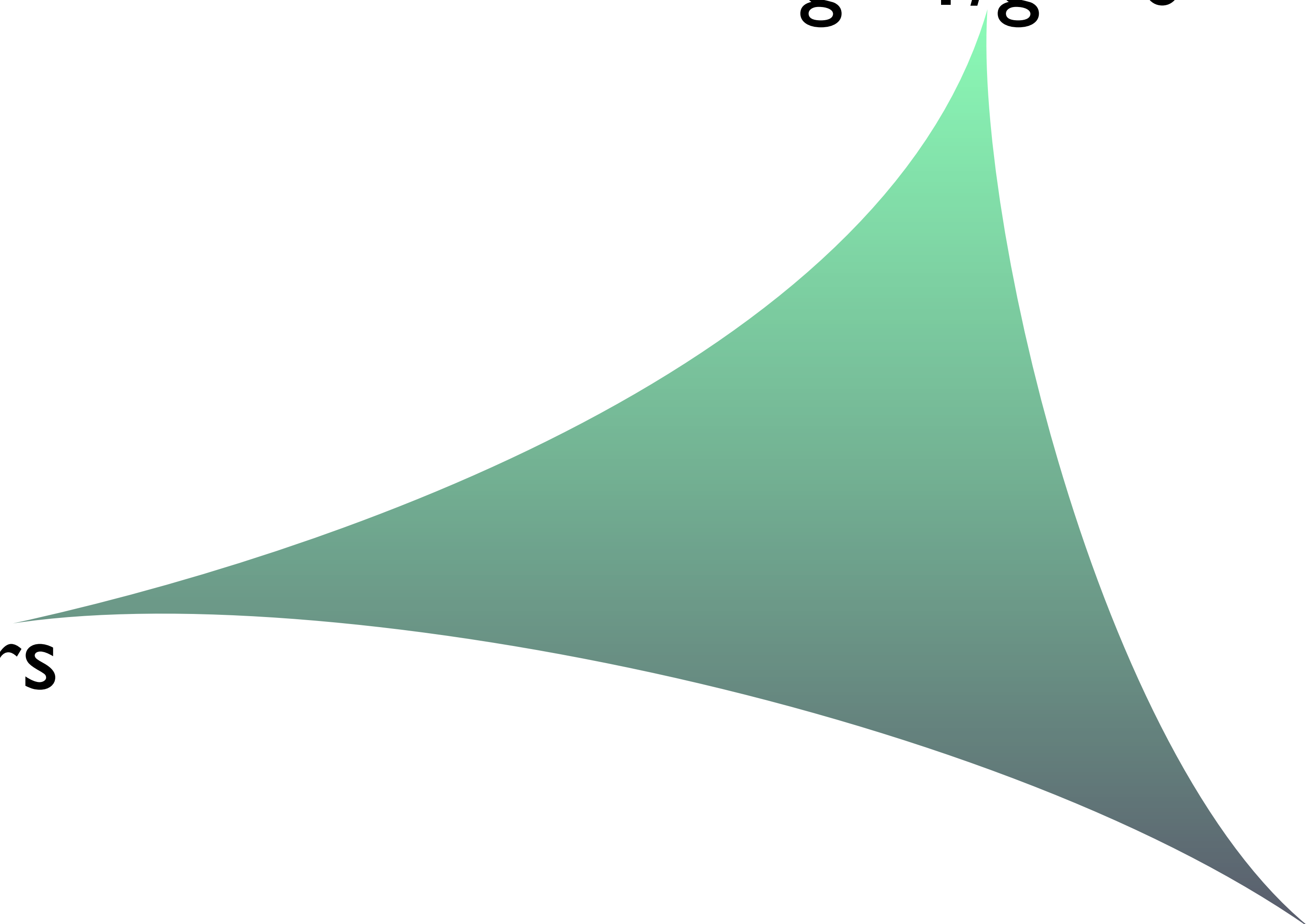


g→ ∞
θ=0
N=2 SU(2)
with 4 flavors
g→0
g→ ∞
θ=π



N=2 SU(2)
with 4 flavors
g'=1/g→0
N=2 SU(2)
with 4 flavors
g→0
N=2 SU(2)
with 4 flavors
g''=1/g→0

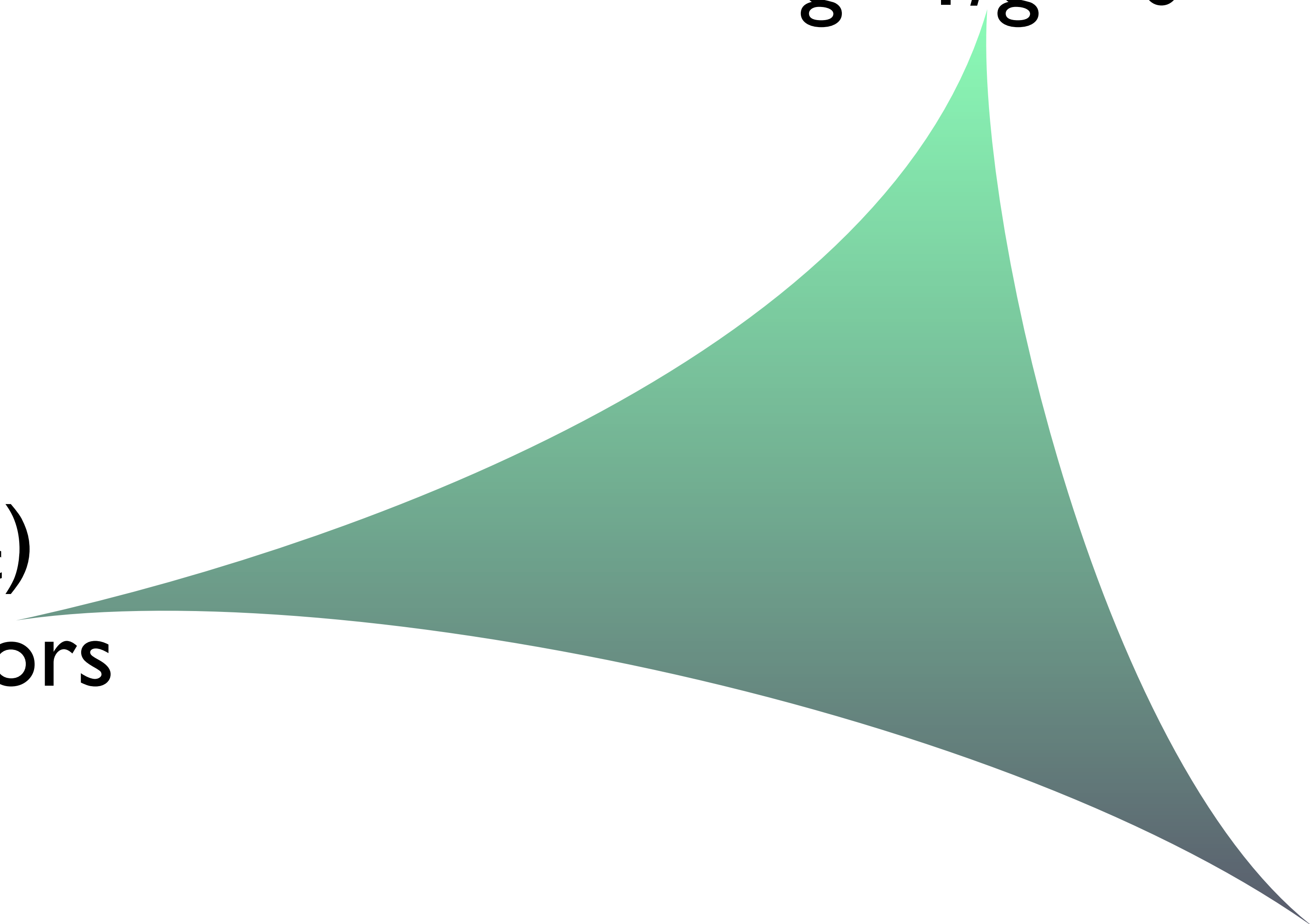
N=2 SU(2)
with 4 flavors
g'=1/g→0



N=2 SU(2)
with 4 flavors
g→0

N=2 SU(2)
with 4 flavors
g''=1/g→0

Known as the triality since 1994 !



Natural generalization needs non-Lagrangian theories!

- Non-Lagrangian sector を何故考えるか？
- Non-Lagrangian sector の一つの例
- Non-Lagrangian sector を使った系の解析

SUSY breaking with non-Lagrangian sector

丸吉一暢、立川裕二、顔文斌、米倉和也

[1308.0064]

- SUSY breaking model by Izawa-Yanagida-Intriligator-Thomas used a SUSY SU(2)=Sp(1) gauge theory.
- Sp(n) generalization was soon found.
- No SU(n) generalization was found ...
- You need a non-Lagrangian sector !

IYIT model の復習

- N=1 SU(2) ゲージ理論、matter 超場は

$$Q_{aiu}$$

a=1,2 ; i=1,2 ; u=1,2

- a をゲージの脚と思う。

- 強結合になる。

- N=1 SU(2) ゲージ理論、matter 超場は

$$Q_{aiu}$$

 a=1,2 ; i=1,2 ; u=1,2

- $M_{(ij)} = Q_{aiu} Q_{bjv} \varepsilon^{ab} \varepsilon^{uv}$

- $N_{(uv)} = Q_{aiu} Q_{bjv} \varepsilon^{ab} \varepsilon^{ij}$ がゲージ不変

- 添え字を上げて M_i^j , N_u^v という行列と思うと便利。

- 古典的には $\mathrm{tr}\, M^2 = \mathrm{tr}\, N^2$

- $N=1$ SU(2) ゲージ理論、matter 超場は

$$Q_{aiu}$$

$a=1,2$; $i=1,2$; $u=1,2$

- $M_{(ij)} = Q_{aiu} Q_{bjv} \varepsilon^{ab} \varepsilon^{uv}$

- $N_{(uv)} = Q_{aiu} Q_{bjv} \varepsilon^{ab} \varepsilon^{ij}$ がゲージ不変

- 添え字を上げて M_i^j , N_u^v という行列と思うと便利。

- 量子的には $\mathrm{tr}\, M^2 = \mathrm{tr}\, N^2 + \Lambda^4$

- 添え字を上げて M_i^j , N_u^v という行列と思うと便利。
- 量子的には $\mathrm{tr}\, M^2 = \mathrm{tr}\, N^2 + \Lambda^4$
- IYIT のアイデア:
  - ゲージ singlet S_i^j , T_u^v を足す
  - Superpotential $W = \mathrm{tr}\, SM + \mathrm{tr}\, TN$
  - S,T で変分: M=N=0。
  - 上の等式と矛盾→ SUSY が破れる。

- 我々のやったこと:
- Q_{iau} に SU(2) を結合させるのが IYIT。
- Q_{iau} を M.-N. の E_6 理論に置き換え、SU(2) を SU(3) に置き換える。何故？

- IYIT で重要だったこと:
  - Q_{aiu} で a=1,2; i=1,2; u=1,2
  - M_i^j , N_u^v で $\mathrm{tr}\, M^2 = \mathrm{tr}\, N^2$

- M.N. の E_6 理論には 「μ」というオペレーターで E_6 の adjoint で変換。
- E_6 を部分群 SU(3)xSU(3)xSU(3) で分解
- μ : 78 個 → 27+27+8+8+8
  - Q_{aiu}、Q'^{aiu}、 L_a^b 、 M_i^j 、N_u^v
  - $\mathrm{tr}\, L^2 = \mathrm{tr}\, M^2 = \mathrm{tr}\, N^2$
  - a=1,2,3 ; i=1,2,3 ; u=1,2,3

- M.N. の E_6 理論には
  - Q_{aiu}、 Q'^{aiu} 、 $L_a{}^b$ 、 $M_i{}^j$ 、$N_u{}^v$
  - $\mathrm{tr}\, L^2 = \mathrm{tr}\, M^2 = \mathrm{tr}\, N^2$
  - $a=1,2,3\ ;\ i=1,2,3\ ;\ u=1,2,3$
- 添え字 a に N=1 SU(3) ベクトル超場を結合させる。
  - $\mathrm{tr}\, M^2 = \mathrm{tr}\, N^2 + \Lambda^6$ に変形。

- M.N. の E_6 理論には M_i^j 、N_u^v
  - $\mathrm{tr}\, M^2 = \mathrm{tr}\, N^2$
- 添え字 a に $\mathcal{N}=1$ SU(3) ベクトル超場。
  - $\mathrm{tr}\, M^2 = \mathrm{tr}\, N^2 + \Lambda^6$ に変形。
- ゲージ singlet S_i^j ,T_u^v を足す
- Superpotential $W = \mathrm{tr}\, SM + \mathrm{tr}\, TN$
  - S,T で変分: M=N=0。
  - 上の等式と矛盾→ SUSY が破れる。

Summary

- 超対称場の理論の双対性を考えていると、ゲージ場、フェルミオン場、スカラー場の組み合わせでは足りない。
- ヘンテコな強結合 (non-Lagrangian) CFT も必要。
- それを使って解析も出来なくはない。