S行列の対称性.

S. Coleman & J. Mandula PR ('67) 1251

R. Haag, J.T. Łopuszański, & M. Sohnius N.P. B88 ('75) 257.

i) particle finiteness ii) Lorentz inv. iii) Weak elastic analyticity
iv) scattering 有. v) momentum space で kernel 有.

1) $Q = \sum \int dp\,dq\; a^{+in}_{\alpha\lambda}(p)\, K_{\alpha\lambda\beta\nu}(p,q)\, a^{in}_{\beta\nu}(q)$

2) $U(\Lambda, x_0)$ poincaré 変換.

$$A \to f\cdot A = \int d^4x_0\, U^+(1,x_0)\, A\, U(1,x_0)\, \tilde{f}(x_0) \qquad [A,S]=0$$

$$\downarrow$$

$$f\cdot A(p',p) = f(p-p')\,A(p',p) \qquad [fA,S]=0.$$

$(U a^{in}(p) = e^{ipx_0}\, a^{in}(p)$ より$)$

p, p_0 と p', q' on shell. $q-a$, $p'-a$, $q'-a$ off shell にとる.

$p+q = p'+q'$ $\qquad$ a: space like.

$q_0 = p-a.$ $\qquad p_0 + q - a = (p'-a)+(q'-a)$

$(S(\varphi_1\otimes\varphi_2),\, AS[x_1\otimes x_2]) = (\varphi_1\otimes\varphi_2,\, A(x_1\otimes x_2))$, $A(x_1\otimes x_2)$
$= (Ax_1\otimes x_2)+(x_1\otimes Ax_2)$

g_i 任意... $0 \neq c$.

故に. $A(p',q)$ は $p'-p=0$ 以外に support を持ってはならない!!

$$A = \sum c\, \frac{\partial}{\partial p_{i_1}} \cdots \frac{\partial}{\partial p_{i_n}}\, \delta(p'-p)$$

covariant に書けば $\partial_i \to \frac{\partial}{\partial p_\mu} - \frac{p_\mu p_\nu}{m^2}\frac{\partial}{\partial p_\nu}$

$\uparrow$ (違った mass を結びつける...)

$$A = \sum_{n=0}^{\infty} A^n(p)_{\mu_1\cdots\mu_n} \frac{\partial}{\partial p_{\mu_1}} \cdots \frac{\partial}{\partial p_{\mu_n}} \qquad [A,\, p^\mu p_\mu] = 0.$$

ii). 重心系

$\langle pq|U(\theta)S|pq\rangle \neq 0 \quad 0<\theta<\delta$ $\qquad$ $|pq\rangle$: class 有の仮定. θ の連続性.

$\mathfrak{z} = \{A;\, [A\, p_\mu]=0,\, A\in\mathcal{O}\!\mathcal{L}\}$ $\qquad A(p)$: 多項式.

$\downarrow$ $K(p)$: trans less part $=0$. (中で可換?).

$B(p,q) = B(p)\otimes 1 + 1\otimes B(q)$

内元 B. trans less part B^x. $K(p)$ は $B^x = 0$ の場合.

$K(p,q) = K(p) \cap K(q)$.

$\langle pq \cdots | U(\theta) U^{-1}(\theta) B U(\theta) S | pq \cdots \rangle$

B は $B^{k}=0$ のとき, $U^{-1}BU$ は $B^{k}=0$. $B^{k} \neq 0$ なら ここ以上の固有値. なぜ B と UBU^{+} で違う固有値の $|pq\cdots\rangle$ 有.

の $|pq\cdots\rangle$ をとって. $\langle pq\cdots | U(\theta) S | pq\cdots \rangle = 0$.

つまり. $K(p,q) = K(p',q')$ (集合として) $K(p,q)$ は $p+q$ の函数.

<u>statement.</u> (p,q) で $B^{k}(pq)=0$ なら 全ての で $B^{k}=0$.

$K(p,q) = K(p',q')$ $p+q = p'+q'$.

$K(p) \supset K(p,q)$ $K(p') \supset K(p',q') = K(p,q)$

$K(p) \cap K(p') = K(p\,p') \supset K(p,q)$ $\Rightarrow$ $K(p,q) \supset K(p,p')$ (q.e.d.)

—— ∘ ——

$B^{k}=0$ は不変.

B^{k} は代数的に閉じている. 内部自由度 finite 代数 compact 群.

Lorentz 群との かかわり. homomorphism.

semi simple ×. (compact 群のみ)

$U(1)$ × $[T_3\ \phi] = \lambda\phi$ $[\phi\ \phi] = 0$ $f(\phi) = \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \begin{pmatrix} T_3 \\ \phi \end{pmatrix}$

← $U(1)$ でない.

故に B^{k} は内部自由度のみ.

—— ∘ ——

残り は. $\mathrm{Tr}\, B(p) + \mathrm{Tr}\, B(q) = \mathrm{Tr}\, B(p') + \mathrm{Tr}\, B(q')$ $(p+q = p'+q')$

← 違えば 散乱生じる.

故に $\mathrm{Tr}\, B(p)$ は p の一次 $a(p_\mu + q_\mu) + b$.

—— ∘ ——

$S^{(N)}$

$[P_{\mu_1} [P_{\mu_2} \cdots A] \cdots] = A^{(N)}_{\mu_1 \cdots \mu_N}(p) = a_{\sigma \mu_1 \cdots \mu_N} p^{\sigma} + b_{\mu_1 \cdots \mu_N}$

$0 = [p^{\mu} P_{\mu} [\cdots A] \cdots] = \sigma_{\sigma\mu_1 \cdots \mu_N} p^{\sigma} p^{\mu_1} + b_{\mu_1 \cdots \mu_N} p^{\mu_1} = 0$

$b = 0$

$\sigma_{\sigma \mu_1 \cdots \mu_N} = -\sigma_{\mu_1 \sigma \cdots \mu_N}$ より $N=1$ で $A[\sigma\mu]$ のみ 可.

$A - M_{\mu\nu}$ は P と可換 故に. $ap+b$ となる..

故に. $M_{\mu\nu}$ P_μ 内部自由度 のみ. (massless の時除く.)

A. Haag, Kopuszánski & Sohnius.

fermion operator まで含めて考える

$S^{(N)}$ Kernel N回交換子の集合.

$S^{(0)}$ Spin 1 以上の Boson op なし. (Coleman & Mandula.)

Q は $(\frac{1}{2}, 0)$ $(0\ \frac{1}{2})$ のみ. しかもこれらは

$QQ^{+} + Q^{+}Q = 0$ (← これは $aP_\mu + b$ しか許され…)

N = 1.

$[P_\mu\ G] \in S^{(0)}$ $\quad$ P_μ $(\frac{1}{2} \frac{1}{2})$, scalar (0.0) より

Boson. G: $(\frac{1}{2}\ \frac{1}{2})$ A_μ, (0.0) D, $\underbrace{(1\ 0)\ (0.1)}_{S_{\mu\nu}\ T_{\mu\nu}}$ ($M_{\mu\nu}$), (1.1) のみ.

$[P_\mu^2\ G] = 0$ より

$M_{\mu\nu}$

$[P_\mu\ D] = i P_\mu$ $\quad$ (massless $\quad$ } の二つのみ.

N = 2. $S^{(1)}$ は (0.0) (1.0) (0.1) のみ.

$(\frac{3}{2}\ \frac{1}{2})$ は $[P[PA]]$ 型の Jacobi より 無し.

$(\frac{1}{2}\ \frac{1}{2})$ K_μ.

$[P_\mu\ K_\nu] = a\, g_{\mu\nu} D + b\, M_{\mu\nu} + c\, \epsilon_{\mu\nu\kappa\lambda} M_{\kappa\lambda}$

P P K の Jacobi より, $c = 0$, $b = -a$.

$D \neq 0$ が K が存在するための必要条件.

N > 2 の Boson op なし.

Fermion type. $S^{(1)}$: $(\frac{1}{2}, 0)$ $(1, \frac{1}{2})$

$(1, \frac{1}{2})$ は $\{P^2\ Q\} = 0$, $\{[P\ Q^{(1)}]\ Q\}$ の Jacobi よりなし

$(\frac{1}{2}, 0)$: $Q^{(1)}_\alpha$

$[P_{\alpha\dot\beta}\ Q^{(1)}_\gamma] = i\, \epsilon_{\alpha\gamma} \bar{Q}_{\dot\beta}$ $\qquad$ $\bar{Q} \in S^{(0)}$

N > 1 なし.

O − 土

massive

$$\{ Q^L_\alpha\ Q^M_\beta \} = \varepsilon_{\alpha\beta} Z^{LM} \quad , \quad [Z^{LM}, {}^\forall\Lambda] = 0$$

$$\{ Q^L_\alpha\ \bar{Q}^M_{\dot\beta} \} = \delta^{LM} \sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$[Q^L_\alpha\ B_\ell] = \Sigma\, S_\ell^{LM} Q^M_\alpha$$

$$[B_\ell\ B_m] = i \Sigma\, C^k_{\ell m} B_k \qquad C^k_{\ell m} \text{ 内部自由度に対する structure const.}$$

$$[Q^L_\alpha\ P_\mu] = [B_\ell\ P_\mu] = [B_\ell\ M_{\mu\nu}] = 0$$

$$[Q^L_\alpha\ M_{\mu\nu}] = \tfrac{1}{2} (\sigma_\mu)_\alpha{}^\beta\ Q^L_\beta$$

massless $(Z^{LM} = 0)$

$$\{ Q^{(1)L}_\alpha ,\ Q^{(1)M}_\beta \} = 0 \qquad [P_\mu\ K_\nu] = 2i (g_{\mu\nu} D - M_{\mu\nu})$$

$$\{ Q^L_\alpha ,\ Q^{(1)M}_\beta \} = 0$$

$$\{ Q^{(1)L}_\alpha\ \bar{Q}^{(1)M}_{\dot\beta} \} = \delta^{LM} K_{\alpha\dot\beta}$$

$$\{ Q^L_\alpha\ Q^{(1)M}_\beta \} = \delta^{LM} \epsilon_{\alpha\beta} D - \delta^{LM} M_{\alpha\beta} + i\, \epsilon_{\alpha\beta} B^{LM}$$

← B_k の linear combination.

$$\{ Q^L_\alpha\ D \} = \tfrac{1}{2} i\, Q^L_\alpha$$

$$[P_{\alpha\dot\beta}\ \bar{Q}^{(1)L}_{\dot\gamma}] = 2i\, \epsilon_{\dot\beta\dot\gamma} Q^L_\alpha \ , \ [\bar{Q}^{(1)L}_{\dot\alpha}\ D] = -\tfrac{1}{2} i\, \bar{Q}^{(1)L}_{\alpha}$$

$$[Q^{(1)L}_\alpha\ D] = -\tfrac{1}{2} i\, Q^{(1)L}_\alpha \qquad [\bar{Q}^{(1)L}_{\dot\alpha}\ B_\ell] = S_\ell^{LM} \bar{Q}^{(1)M}_{\dot\alpha}$$

$$[\bar{Q}^{(1)}_{\dot\alpha}\ B_\ell] = \Sigma\, S_\ell^{LM} \bar{Q}^{(1)M}_{\dot\alpha} \qquad (S_\ell^{LM} \text{ は } B_\ell \text{ の表現行列})$$

$$[Q^{(1)}_\alpha\ B_\ell] = \Sigma\, t_\ell^{LM} Q^{(1)M}_\alpha$$

$$[K_\mu\ K_\nu] = 0 \qquad [K_\mu\ D] = -i K_\mu \qquad [K_\mu\ B_\ell] = 0$$

$$[D\ B_\ell] = 0 \qquad [K_{\alpha\dot\beta}\ Q^L_\gamma] = 2i\, \epsilon_{\alpha\gamma} \bar{Q}^{(1)L}_{\dot\beta} \qquad [K_{\beta\dot\alpha}\ \bar{Q}^L_{\dot\gamma}] = 2i\, \epsilon_{\dot\alpha\dot\gamma} Q^{(1)L}_\beta$$

—— ∘ ——

$$\begin{pmatrix} M_a{}^b & -iP_{a\dot b} \\ & M^{\dot a}{}_{\dot b} \end{pmatrix} \qquad \begin{pmatrix} M_a{}^b & -iP_{a\dot b} & iQ_{aB} \\ iK^{\dot a b} & M^{\dot a}{}_{\dot b} & i\bar{Q}'^{\dot a}{}_B \\ iQ'^{bA} & -i\bar{Q}_{\dot b}{}^A & \Lambda^A{}_B \end{pmatrix} + \begin{pmatrix} \tfrac{1}{2}\delta_a^b D & \\ & -\tfrac{1}{2}\delta_{\dot a}^{\dot b} D \end{pmatrix}$$

$$\gamma_4 V^\dagger \gamma_4 = V^{-1} \qquad U(2,2/1) \qquad OSp(1/4)$$

← SO(5) の二価表現

Grading Lie Algebra. (K.G. Kac : Commun. Math. Phys. <u>53</u> (1977) 31)

$$g = \textstyle\sum_\alpha g_\alpha \qquad \alpha \in Z_2 \qquad [\Lambda_\alpha\ \Lambda'_\beta]_{\epsilon(\alpha,\beta)} \in g_{\alpha+\beta}$$

← [illegible]

$$\left(\begin{array}{c|c} \cdot & \\ \hline & \cdot \end{array} \right) \qquad \text{str}, \ \text{sdet}, \ldots \qquad \epsilon(1,1) = + \quad \text{他}\ \epsilon = -$$

§ Super Symmetric Theory A. Salam & J. Strathdee

Fortschritte der Physik 26 (78)

◎ η. θ majorana $\quad (\eta = C^T \gamma_4^T \eta^*)$

· $\bar\eta \gamma_\mu \theta = (\bar\eta_{\dot a}\ \eta^a)\begin{pmatrix} & i \\ -i & \end{pmatrix}\begin{pmatrix} & \sigma_\mu{}_{a\dot b} \\ \sigma_\mu^{\dot a b} & \end{pmatrix}\begin{pmatrix}\theta_b \\ \bar\theta^{\dot b}\end{pmatrix} = i(\bar\eta_{\dot a}\sigma_\mu^{\dot a b}\theta_b - \eta^a \sigma_{\mu a\dot b}\bar\theta^{\dot b})$

· $(\bar\eta\theta)^* = \bar\eta\theta$

· $Q(\eta) = \bar\eta Q = i(\bar\eta_{\dot a}\bar Q^{\dot b} - \eta^a Q_a)$

$Q^\dagger(\eta) = Q(\eta)$

$\varepsilon_{ab} = \begin{pmatrix} & -1 \\ 1 & \end{pmatrix} \qquad \varepsilon^{ab} = -\varepsilon_{ab}.$

◎ (x, θ) space での表現.

$$\begin{pmatrix} \sigma_{\mu\nu\, a}{}^b & -iP_{a\dot b} & iQ_a \\ 0 & \sigma_{\mu\nu}{}^{\dot a}{}_{\dot b} & 0 \\ 0 & i\bar Q_{\dot b} & 0 \end{pmatrix}$$

$\{Q_a\ \bar Q_{\dot b}\} = -P_{a\dot b}$ i) $Q_a\ \bar Q_{\dot b}$ は $\frac{\partial}{\partial x_\mu}$ を含む.

$\{Q_a\ Q_b\} = 0$ ii) $\{P\,Q\} = 0$ より. x_μ は含まない.

$\{P_{a\dot b}\ Q_c\} = 0$ iii) $\{Q, \bar Q\} \neq 0$ より. θ と $\frac{\partial}{\partial\theta}$ がともに必要.

$$Q_a = \frac{i}{2}\sigma_{\mu a\dot b}\bar\theta^{\dot b}\frac{\partial}{\partial x_\mu} + \varepsilon_{ab}\frac{\partial}{\partial\theta_b}$$

$$Q_{\dot a} = \frac{i}{2}\sigma_{\mu b\dot a}\theta^b\frac{\partial}{\partial x_\mu} + \varepsilon_{\dot a\dot b}\frac{\partial}{\partial\bar\theta_{\dot b}}$$

$\{Q(\eta)\ \theta_a\} = i\eta_a$

$\{Q(\eta)\ \bar\theta^{\dot a}\} = i\bar\eta^{\dot a}$

$\{Q(\eta), \underset{\text{Dirac}}{\eta}\} = i\eta \qquad \eta = \begin{pmatrix}\eta_a \\ \bar\eta^{\dot a}\end{pmatrix}$

$\{Q(\eta)\ x_\mu\} = \frac{i}{2}\bar\eta\sigma_\mu\theta$

⌜ $\{\underset{\text{Dirac}}{Q_\alpha}\ Q_\beta\} = -(\sigma_\mu C)_{\alpha\beta}\frac{\partial}{\partial x_\mu} \qquad \{Q_\alpha\ \bar Q_\beta\} = (\sigma_\mu\partial_\mu)_{\alpha\beta}$ ⌟

◎ covariant differentiation.

$$\phi(x, \theta) = A + \bar\theta\psi + \frac{1}{4}\bar\theta\theta F + \frac{1}{4}\bar\theta\gamma_5\theta G + \frac{1}{4}\bar\theta i\gamma_\nu\gamma_5\theta V_\nu + \frac{1}{4}\bar\theta\theta\bar\theta\chi + \frac{1}{32}(\bar\theta\theta)^2 D.$$

これは既約でない.

$$A_\pm = \frac{1}{4}\left(A - \frac{1}{\partial^2} D\right) \mp \frac{i}{2} \frac{1}{\partial^2} \partial_\mu V_\mu \qquad A_1 = \frac{1}{2}\left(A + \frac{1}{\partial^2} D\right)$$

$$\psi_\pm = \frac{1}{2} \frac{1 \pm i\gamma_5}{2} \left(\psi - \frac{1}{i\partial\!\!\!/}\chi\right) \qquad \psi_1 = \frac{1}{2}\left(\psi + \frac{1}{i\partial\!\!\!/}\chi\right)$$

$$F_\pm = \frac{1}{2}(F \mp iG) \qquad \psi_{1\mu} = \left(\eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right) V_\nu$$

$\delta :\ \theta(\eta)$

$$\delta A_\pm = \bar{\eta}_\mp \psi_\pm$$

$$\delta \psi_\pm = F_\pm \eta_\pm - i\partial\!\!\!/ A_\pm \eta_\mp \qquad \text{chiral scalar rep.}$$

$$\delta F_\pm = -\bar{\eta}_\pm i\partial\!\!\!/ \psi_\pm$$

$$\delta A_1 = \bar{\eta}\psi_1$$

$$\delta \psi_1 = \frac{1}{2}(i\gamma_\mu \gamma_5 V_{1\mu} - i\partial\!\!\!/ A_1)\eta \qquad \text{transverse vector rep.}$$

$$\delta V_{1\mu} = -\bar{\varepsilon}\sigma_{\mu\nu} i\gamma_5 i\partial_\nu \psi_1$$

既約成分を特徴づける.

$$z_\mu = x_\mu - \frac{1}{4}\bar{\theta}\gamma_\mu\gamma_5\theta \qquad \bar{\theta}\gamma_\mu\gamma_5\theta \qquad (\theta_+\theta_+)^2 = 0 \qquad \theta = \begin{pmatrix}\theta_+ \\ \theta_-\end{pmatrix}$$

$$\delta z_\mu = i\bar{\eta}_+\gamma_\mu\theta_+$$

$$\phi(z, \theta_+) = A_+(z) + \bar{\theta}_-\psi_+(z) + \frac{1}{2}\bar{\theta}_-\theta_+ F_+(z)$$

$$= e^{-\frac{1}{4}\bar{\theta}\partial\!\!\!/\gamma_5\theta}\left(A_+(x) + \bar{\theta}_-\psi_+ + \frac{1}{2}\bar{\theta}_-\theta_+ F_+(x)\right) = \phi_+(x, \theta)$$

これは $D = \left(\frac{\partial}{\partial\bar{\theta}} - \frac{i}{2}\partial\!\!\!/\theta\right)$ で特徴づけられる.

$$[D\ J_{\mu\nu}] = \frac{1}{2}\sigma_{\mu\nu}D \qquad [D, P_\mu] = 0 \quad , \{D\ Q\} = 0$$

$$D\phi_+ = \frac{1+\gamma_5}{2} D\phi_+ \qquad D_-\phi_+ = 0$$

$$\phi_+(x\,\theta)\phi_+'(x\,\theta) = \phi_+''(x, \theta)$$

$$A_+'' = A_+ A_+'$$

$$\psi_+'' = A_+\psi_+' + \psi_+ A_+'$$

$$F_+'' = A_+ F_+' + {}^T\psi_+ C^{-1}\psi_+' + F_+ A_+'$$

$$v(\phi_+) = e^{-\frac{1}{4}\bar{\theta}\gamma\gamma_5\partial\theta}\left\{ v(A_+) + \bar{\theta}_-\psi_+ v'(A_+) + \frac{1}{2}\bar{\theta}_-\theta_+\left(F_+ v'(A_+) + \frac{1}{2}\psi_+ C^{-1}\psi_+ v''(A_+)\right)\right.$$

$$v'(\rho) = \frac{\partial}{\partial\rho} v(\rho) \qquad v''(\rho) = \frac{\partial^2}{\partial\rho^2} v(\rho)$$

- ψ_{-+} : negative chirality with respect to its external spinor index and positive chirality with respect to its internal structure

$$(1-i\gamma_5)\psi_{-+}=0$$

$$D_-\psi_{-+}=0$$

$$\psi_{-+}(x,\theta)=e^{-\frac{1}{4}\bar\theta\gamma\gamma_5\partial\theta}\left(U_-(x)+M_\mu(x)\gamma_\mu\theta_+ +\frac{1}{2}\bar\theta_-\theta_+V_-(x)\right)$$

where U_- and V_- are negative chiral spinors and M_μ is a 4 Vector

- ψ_{++}

$$(1+i\gamma_5)\psi_{++}=0 \quad \text{and} \quad D_-\psi_{++}=0$$

$$\psi_{++}(x,\theta)=e^{-\frac{1}{4}\bar\theta\gamma_\mu\gamma_5\partial\theta}\left[U_+(x)+\left(D(x)+\frac{1}{2}\sigma_{\mu\nu}F_{\mu\nu}(x)\right)\theta_+ +\frac{1}{2}\bar\theta_-\theta_+V_+(x)\right]$$

where U_+, V_+ are positive chiral spinors, D is a scalar and $F_{\mu\nu}$ is a self dual antisymmetric tensor

- real transverse vector representative

ψ_{-+} で reality condition をおける.

$$V_-=i\gamma_5 C\bar V_-^T$$ これが超対称性と compatible であるためには.

$$M_\mu=\frac{1}{2}(V_{1\mu}-i\partial_\mu A_1)$$

$U_-=\psi_{1-}$ $\qquad$ A_1 real scalar

$V_-=i\gamma\partial\psi_{1+}$ $\qquad$ $\psi_{1\pm}$ Majorana.

- ψ_{++} に対しても. $\qquad \psi_{++}=i\gamma\partial D_-\Phi_1 \qquad \Phi_1=(A\ \ \psi\ \ V)$ (Sc. Sp. Vec.)

$$E(\phi,\phi^*)_D=\partial_\mu\phi^\alpha\frac{\partial^2E}{\partial\phi^\alpha\partial\phi^{*\beta}}\partial_\mu\bar\phi^\beta+F^\alpha\frac{\partial^2E}{\partial\phi^\alpha\partial\bar\phi^\beta}\bar F^\beta+\cdots$$

- $\phi'=\bar\phi \qquad \phi\phi'=\phi(z,\theta_+)\phi'(z^*,\theta_-)$

$$A''=A_+A_-'$$

$$\psi''=A_+\psi_-'+\psi_+A_-' \qquad F''=A_+F_-'+F_+A_-' \qquad G''=-iA_+F_-'+iF_+A_-'$$

$$V_\nu''=(i\partial_\nu A_+)A_-'+A_+(-i\partial_\nu A_-')+\psi_+^TC^{-1}\gamma_\nu\psi_-'$$

$$\chi''=(i\partial_\nu A_+)\gamma_\nu\psi_-'+\gamma_\nu\psi_+(i\partial_\nu A')-i(\gamma\partial\psi_+)A_-'-A_+(i\gamma\partial\psi_-')+2\psi_+F_-'+2F_+\psi_-'$$

$$D''=(-\partial^2A_+)A_-'+2(\partial_\nu A_+)(\partial_\nu A_-')+A_+(-\partial^2A_-')+4F_+F+2\psi_+^TC^{-1}(i\overleftarrow{\partial}-i\overrightarrow{\partial})\psi$$

特に $\frac{1}{4}(\phi\phi')_D=\partial A_+\partial A_-'+F_+F_-+\psi_+^TC^{-1}(i\overleftarrow{\partial}-i\overrightarrow{\partial})\psi_-'$

$$v(\phi)v(\phi')^*=\partial v(\phi)\partial v^*(\phi)+F\,v'(\phi)v^{*\prime}(\phi)F'+\cdots$$

$v(\phi)=e^{-\frac{1}{4}\bar\theta\gamma_5\partial\theta}[v(\phi)\cdots]$

Nonlinear Realization in Supersymmetric Theories

(With M. Bando, T. Kuramoto & S. Uehara)

PL 138 (1984) 94
PTP 72 (1984) 313
PTP 72 (1984) 1207

§0 はじめに

i) Low energy Theorem

F.E. Low PR 96 (1954) 1428

$$\sigma^{e\gamma} = \sigma_{Thomson} + o(k)$$

$$\frac{e^2}{M^2}$$

C.A. + NG Boson R. Dashen & M. Weinstein PR 183 (1969) 1261

↳ S. Coleman, J. Wess & B. Zumino PR 177 (1969) 2239

N.G. Boson ξ $S.B : G \to H$

$\xi \in G/H$ $\xi \xrightarrow{g} g\xi h^{-1}(\xi, g)$ $(h \in H \quad g \in G)$

Matter N : ρ_0 : Hの表現 ($\rho \supset \rho_0$, ρ : Gの表現)

$N \xrightarrow{g} \rho_0(h(\xi, g))N$

linear base

$$(\rho(\xi)N)' = \rho(g)\rho(\xi)N.$$

ii) SuperSymmetry

	Scale of S.B	
Super Sy.	Λ_S	$\Lambda_G \gg \Lambda_S$
Global Sy	Λ_G	

$G \to H \wedge$ S.B. SuperSy は Normal phase.

の Paradigm 有.

iii) N.G. Boson ← chiral super multiplet.

Boson$_1$, Boson$_2$ } fermion.

B_1 B_2 共に NG Boson : P type

B_1 B_2 の一方のみ NG Boson : M type

NG boson, QNG boson } QNG Fermion

$$N_{QF} = N_p + M_M \quad \dim G/H = 2N_p + N_M.$$

iii) $N_M=0$ の場合 (Zumino's Discussion : PL 87B (79) 203)

$$F(\Phi^\alpha, \bar{\Phi}^\beta)_D = \frac{\partial^2 F}{\partial\phi^\alpha \partial\bar{\phi}^\beta} \partial\phi^\alpha \partial\bar{\phi}^\beta + \cdots$$

ϕ に対し Kählerian.

G/H が Kählerian の場合. Ptype N.G. Boson のみでよい.

既約 Kählerian $U(n+m)/U(n)\times U(m)$, $SO(2m)/U(m)$, $Sp(m)/U(m)$
$SO(m+2)/SO(m)\times U(1)$, $E_6/Spin(10)\times U(1)$ $E_7/E_6\times U(1)$

例 $U(n+m)/U(n)\times U(m)$

$$(C^{n+m})^n = \left(\begin{array}{c} // \\ /_{\!\!/} \end{array}\right)\, n+m \qquad C^{n+m}/GL(n,C) \sim \overset{\text{代表元}}{\begin{pmatrix} 1 \\ \Delta \end{pmatrix}}$$

$$g\begin{pmatrix} 1 \\ \Delta \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} 1 \\ \Delta \end{pmatrix} = \begin{pmatrix} A+B\Delta \\ C+D\Delta \end{pmatrix} = \begin{pmatrix} 1 \\ \Delta' \end{pmatrix}(A+B\Delta)$$

$$\Delta \xrightarrow{g} \Delta' = (C+D\Delta)/(A+B\Delta)$$

$$\xi = \begin{pmatrix} 1 \\ \Delta \end{pmatrix} \qquad (\xi^*\xi)' = (A+B\Delta)^{*-1}\xi^*\xi(A+B\Delta)^{-1}$$

$$\xi' = g\xi(A+B\Delta)^{-1}$$

$$(\log\det \xi^*\xi)' = \log\det(\xi^*\xi) + \log\det(A+B\Delta)^{*-1} + \log\det(A+B\Delta)^{-1}$$

$$\Gamma_{U(n+m)/U(n)\times U(m)} \ni \Xi = e^{\begin{pmatrix} & \phi \\ \phi^\dagger & \end{pmatrix}} \qquad \rho = \Xi\begin{pmatrix} 1 & \\ & 0 \end{pmatrix}\Xi^*$$

Boson Part: $T_r\, \partial\rho\, \partial\rho^*$ = 一致 (2 '')

- Reducible Kählerian の場合?
- $N_M \neq 0$ の場合?

$\Longrightarrow$ 構成的手法?

一般的に何か可能か?

§1. $\hat{H}$-Structure theorem and $\hat{H}$ representation Theorem.

i) chiral superfield Φ^A $\quad G$: 表現 ρ

domain of ρ $G \to G^c$ $\qquad \underset{G}{\bar{\rho}} \to \underset{G^c}{\bar{\rho}}$ or ${}^t\rho^{-1}$? $\quad$ こちら.

G^c に対し解析的.

$\Sigma(\phi) = \{\phi;\ \phi$ は V_{eff} の min. point $\}$ $\qquad \rho(G)\Sigma = \Sigma$

$\Sigma(\phi) \ni \phi_0.$ $\qquad \rho(H)\phi_0 = \phi_0$ (Isotropy group, stability group little group)

$\Downarrow$

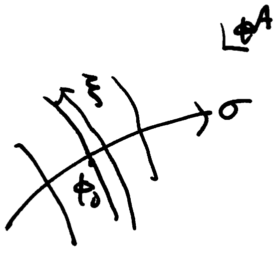
$\rho(H^c)\phi_0 = \phi_0 \qquad H^c$: H の complexification

$\hat{H} \overset{def}{=} \{g:\ g \in G^c,\ \rho(g)\cdot\phi_0 = \phi_0\} \qquad \hat{H} \supset H^c \qquad \hat{H} \cap G = H$

ii) $\xi \in G^c/\hat{H}$, $\qquad \phi^A = \phi^A(\xi, \sigma)$

($\rho(\xi)$ は ϕ_0 に effective に作用している.)

$\phi^A(\xi,\sigma) = \rho(\xi)\phi^A(0\,\sigma)$



$V_{eff} = F_\alpha g^{\alpha\beta}(\xi,\sigma)\bar{F}_\beta \qquad (\alpha, \beta = \xi, \sigma)$

$\bar{F}_\alpha = -(g^{-1})^{\alpha\beta}\dfrac{\delta W}{\delta\phi_\beta} \qquad W_F(\cancel{\xi}\ \sigma)$

$\delta\psi = F\theta$

[SuperSy は Normal plane より
$\bar{F}_\sigma \big|_{\xi=\sigma=0} = 0$

$F_\xi \big|_{\sigma=0} = 0$, $\qquad F_\sigma \big|_{\sigma=0} = 0$

Kugo, Ojima Yanagida
(MPI-PAE/PTh 74/83)
PL

ξ は massless. $\qquad (\xi \in G^c/\hat{H})$

Theorem ($\hat{H}$ - Structure Theorem)

$\hat{h} = h^c + r$

r : nilpotent ideal and $X(\in r)$, $ad(X)^N = 0$

—— o ——

Levi's Theorem $\qquad \hat{h} = h_0 \oplus r'$ ← radical

↑

semisymple

$\dfrac{\delta W}{\delta\phi_\beta} = 0 \quad W$ は G^c 不変.

①より. ① $\rho(G^c)\Sigma = \Sigma.$

$(h_0)_R$ は hermite

② $r' = \text{" } U(1)\text{'s factor}$
$+\ r$

③ $\left(\ \right)$ より.
$(\hat{h} \cap g_R = h.)$

$\left[\begin{array}{l} SU(3)^c \supset SU(2) \\ \qquad\qquad \uparrow \\ \qquad g\,SU(2)_H\,g^{-1} \\ \qquad g g^* \neq 1 \end{array} \right]$

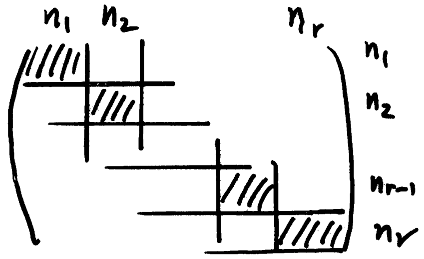
$$N_{QNGB} = \dim[G^c/\hat{H}] - \dim[G/H]$$
$$= \dim(G/H) - \dim R$$

iii) $\hat{H}$ representation theorem.

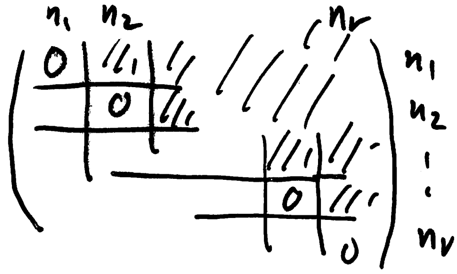
G^c の表現 ρ

<u>Theorem</u>

$\rho(h^c) =$



$\rho(r) =$

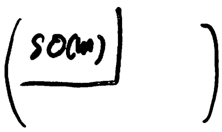


H の G への うめこみ.

i) $G \supset \rho(H)$

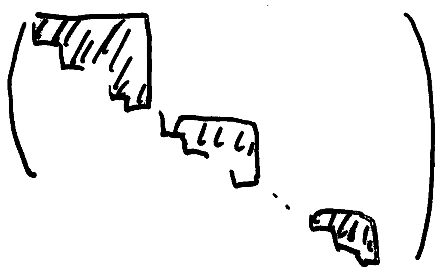
ρ なにでもよい. 可約でもよい.

ii) $G = U(n) \quad H = SO(m)$



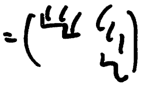
更に $G = U(N)$

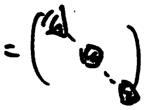
$\rho(\hat{h}) =$



$G = SO(N)$ or $Sp(N)$ $\qquad$ Z: completely reducible

$$\hat{h} = \begin{bmatrix} W & X & S \\ & Z & -P_0 X^T \\ & & -W^T \end{bmatrix} \qquad P = \begin{bmatrix} & & 1 \\ & P_0 & \\ \varepsilon 1 & & \end{bmatrix}$$

W=


Z=


$$\varepsilon = \begin{cases} 1 & : SO \\ -1 & : Sp \end{cases} \qquad S^T + \varepsilon S = 0, \quad Z^T P_0 + P_0 Z = 0 \quad P_0^T = \varepsilon P_0$$
$$P_0^2 = \varepsilon$$

具体的例

1. $U(n+m)/U(n)\otimes U(m)$

$$\hat{h}^c = \begin{pmatrix} /// & \\ & /// \end{pmatrix} \qquad r = 0 \text{ or, } \quad r = \begin{pmatrix} & /// \\ & \end{pmatrix}$$

I $N_{QNGB} = \dim G/H - \dim R$ ($\dim R = 0$)
全て M type.

II $N_{QNGB} = 0$
全て P type.

II で $G^c/\hat{H} = \begin{pmatrix} 1 & 0 \\ \Delta & 1 \end{pmatrix}$

2 $U(n_1+n_2+n_3)/U(n_1)\otimes U(n_2)\otimes U(n_3)$

$$r = \begin{pmatrix} 0 & * & * \\ & 0 & * \\ & & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & * \\ & 0 & * \\ & & 0 \end{pmatrix} \begin{pmatrix} 0 & * & 0 \\ & 0 & 0 \\ & & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ & 0 & * \\ & & 0 \end{pmatrix} \begin{pmatrix} 0 & * & 0 \\ & 0 & 0 \\ & & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & * \\ & 0 & 0 \\ & & 0 \end{pmatrix}$$
$$\begin{pmatrix} 0 & 0 & 0 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$$

(一般的 rule RIFP 548.)
PTP 72 (1984), 313 ; 1207 BKMU

§2. Effective Lagrangian の作り方.

$$\xi' \xrightarrow{g} g\xi \cdot \hat{h}^{-1}(\xi, g) \qquad \xi \in G^c/\hat{H}$$

A. ρ : G^c の表現. e_a : $\rho(\hat{H})e_a = e_a$.

$$[\rho(\xi)e_a]' = \rho(g)\rho(\xi)\rho(\hat{h}^{-1}(\xi, g))e_a$$
$$= \rho(g)\rho(\xi)e_a$$

「Lerche MPI-PAE/Pth 59/83」

ⓐ $SU(n+m)/SU(n)$

$$H : \begin{pmatrix} //// & \\ & 1 \end{pmatrix}^{n}_{m} \quad : \quad e_a = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}^{n}_{n+i}$$

ⓑ $\rho(\hat{H})\phi_0^A = \phi_0^A$

ⓒ $U(n)/O(n)$ $O(n)$ の metric J

$$(\xi J \xi^T)' = g\xi \underbrace{\hat{h}^{-1} J\, {}^t\hat{h}^{-1}}_{J}\, {}^t\xi\, {}^tg = g\xi J\, {}^t\xi\, {}^tg$$

$\hat{h}^c$ の変換の kinetic part.

一般に $\hat{h}$ の maximal Torus < G の maximal Torus の時 e_a 存!!

B. (Zumino type)

ρ: G^c の表現. $\rho(\hat{H})\eta = \eta\rho(\hat{H})\eta$, $\eta^2 = \eta$.

例 $U(n_1+\cdots+n_r)/U(n_1)\otimes\cdots\otimes U(n_r)$

$R = \begin{pmatrix} \ddots & 0 \\ & \ddots \end{pmatrix}$ $\eta = \begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & & 1 & & \\ & & & 0 & \\ & & & & \ddots \; 0 \end{pmatrix}$ $\hat{h} = \begin{pmatrix} \ddots & \\ & \ddots \end{pmatrix}$

$(\ \ )\eta = (\in|\ )$

$\eta(\ \ ) = (\uparrow)$

$[\log\det_\eta \rho(\xi^*\xi)]'$

$= \log\det_\eta \rho(h^{-1})^* + \log\det_\eta \rho(\xi^*\xi) + \log\det_\eta \rho(h^{-1})$

補正するもの.

注 pure realization の場合.

$g^c = r + h^c + r^*$.

$\xi^*\xi = \underset{R^c\in r^c,}{e^{R^*}}\ e^{H_{s.s}}\ \underset{U(1)}{e^{\xi\Lambda^\alpha}} \cdot \underset{R'\in r}{e^{R'}}$ s.s : semi simple

補正するもの h^c の $U(1)$ factor の部分. G: 単純群 $\rho \to \rho_\alpha$ と 取る η.

次に ρ 一般の場合.

$\log\det_\eta\rho(h^{-1})$ は同様に h の $U(1)$ factor の部分が補正するもの故

$[\log\det_\eta\rho(\xi^*\xi)]' = \log\det_\eta\rho(h^{-1})^* + \underbrace{\log\det_\eta\rho(\xi^*\xi)}_{\nu_0} + \log\det_\eta\rho(h^{-1})^*$

$\log\det_\eta\rho(\xi^*\xi) - b_i\log\det_{\eta_i}\rho_i(\xi^*\xi)$ は G 不変 (D による補正に)

C. type.

$P_\eta = \rho(\xi)\eta\cdot\frac{1}{[\rho(\xi^*\xi)]_\eta}\eta\rho(\xi^*)$ $P_\eta^2 = P_\eta$.

$P' = \rho(g)P_\eta\rho(g)^*$.

$\mathrm{Tr}\,P_{\eta_1}P_{\eta_2}$, $\mathrm{Tr}\,P_{\eta_2}P_{\eta_2}P_{\eta_3}$, ...

ただし. $\eta_1\eta_2 = \eta_1$ なら $P_{\eta_1}P_{\eta_2} = P_{\eta_1}$.

(pure realization の場合 全て $\eta_1\eta_2 = \eta_1$ となる)

古典群の場合.

$\mathcal{L}_{eff}$ は. $\xi^\dagger\xi$ の函数.

$\eta_i \xi^\dagger \xi \eta_j$ 前の index は $\eta_i \hat{h}^{-1\dagger} \eta_i$ で 後の index は $\eta_j \hat{h}^{-1} \eta_j$ で動く.

それ等は. $[\eta_a \hat{h}^{-1} \eta_a]^{-1}$ で つぶされているはず.

$\eta_a \hat{h}^{-1} \eta_a$ が const であれば (A type であれば)

$\xi = (\cdots)\cdot(\cdots)$ $\qquad \rho(\xi^\dagger\xi)^{-1}_\eta = [\rho(\xi^{-1})\,\rho(\xi^{\dagger})^{-1}]_\eta$

↳ は η を通りぬけて消える.

故に. C type は max R から R が小さくなった分だけ sq'd.

§3. Matter Field.

G の表現. ρ. $\hat{H}$ の表現. ρ_0. ρ_0 の表現ベクトル N ($\rho \supset \rho_0$)

$$N' = \rho_0(h(\xi, g))\,N$$

$$[\rho(\xi)N_0]' = \rho(g)\,\rho(\xi)\,\rho(h^{-1})\,\rho_0(h)\,N = \rho(g)\,\rho(\xi)\,N.$$

$$h(\xi, g_1 g_2) = h(g_2 \xi h^{-1}(\xi, g_2), g_1)\cdot h(\xi, g_2)$$

$G = U(N) \qquad H = U(n_1) \oplus \cdots \oplus U(n_r) \qquad n_1 + \cdots + n_r = N$

$$\hat{h} = \begin{pmatrix} \ddots & & \\ & \boxed{\ \ddots\ } & \\ & & \ddots \end{pmatrix}_{ij} \qquad \rho_{ij} = \boxed{\ \ddots\ }_{ij}$$

$$N(ij)' = \rho_{ij}(h)\,N_{ij}$$

§1. Low Energy Theorem.

F.E. Low PR 96 (1954) 1428
110 (1958) 974

$\sigma^{e\gamma} = \sigma_{\text{Thomson}} + O(k)$
$\frac{e^4}{M^2}$

Nambu Jona-Lasinio PR 122 ('61) 345 ← Chiral Sy.

⋮

R. Dashen & Weinstein PR 183 (1969) 1261

SU(2)⊗SU(2) $\delta q = i\epsilon^\alpha \lambda^\alpha q$ current

$\delta_5 q = i\epsilon_5^\alpha \lambda^\alpha \gamma_5 q \rightarrow A_\mu^\beta$

$\langle \pi^\alpha(q) | A_\mu^\beta(0) | 0 \rangle = -i(q^\mu / 2f_\pi)\,\delta_{\alpha\beta}$ $\pi \to \mu\nu$

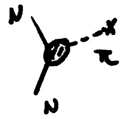
$\langle \pi^\alpha(q) | \partial A^\beta(0) | 0 \rangle = (m_\pi^2 / 2f_\pi)\,\delta_{\alpha\beta}$ $(2\sqrt{2} f_\pi)^{-1} = .96\, m_\pi$

i) Goldberger Treiman

$\langle N(p') | A^\mu(0) | N(p) \rangle = \bar{u}(p') \{ \gamma^\mu \gamma_5\, g_A(q^2) + q^\mu \gamma_5\, h_A(q^2) \} \frac{1}{2}\tau \cdot u(p)$

$q \overset{\text{def}}{=} p' - p.$ ($p' + p$ は charge conj' より 0)

$\partial A = 0$ $2m_N g_A(q^2) + q^2 h_A(q^2) = 0$ $h_A = -\frac{1}{q^2} 2m_N g_A(q^2)$



$\frac{1}{2} f_\pi^{-1} f_{\pi N} = 2m_N g_A(0)$

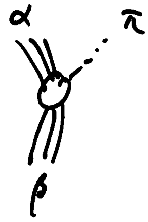
$m_\pi \neq 0$ より

$\partial A = \epsilon$ $\rightarrow$ $m_\pi^2 \propto \epsilon$

$Q^\alpha(t) = \int d^3x\, V_0^\alpha(x,t)$, $Q_5^\alpha = \int d^3x\, A_0^\alpha(x,t)$

$[Q^\alpha Q^\beta] = i f^{\alpha\beta\gamma} Q^\gamma$, $[Q^\alpha Q_5^\beta] = i f^{\alpha\beta\gamma} Q_5$, $[Q_5^\alpha Q_5^\beta] = i f^{\alpha\beta\gamma} Q^\gamma$

$\partial_\mu \langle \alpha^{out} | A_\mu^\gamma(q) | \beta^{in} \rangle = \langle \alpha^{out} | \partial A^\gamma | \beta^{in} \rangle$



$\langle \alpha\pi\ out | \beta\ in \rangle \cdot \frac{i}{q^2 - m_\pi^2}\, q^2\, \frac{1}{2f_\pi} + \partial_\mu \langle \alpha\ out | \tilde{A}_\mu^\gamma(q) | \beta\ in \rangle$

$= \langle \alpha\pi\ out | \beta\ in \rangle \cdot \frac{i}{q^2 - m_\pi^2}\, m_\pi^2 \cdot \frac{1}{2f_\pi} + \langle \alpha\ out | \partial\tilde{A} | \beta\ in \rangle$

$\frac{i}{2f_\pi} \langle \alpha\pi\ out | \beta\ in \rangle + \partial_\mu \langle \alpha\ out | \tilde{A}_\mu^\gamma(q) | \beta\ in \rangle = \underset{O(\epsilon)}{\langle \alpha\ out | \partial\tilde{A} | \beta\ in \rangle}$

$$\langle\alpha|T\,\partial A^{\delta}\partial A^{\gamma}|\beta\rangle = \partial_\mu\langle\alpha|T A^{\delta}_\mu \partial A^{\gamma}|\beta\rangle - \langle\alpha|[A^{\delta}_0 \partial A^{\gamma}]\delta(t-t')|\beta\rangle$$

$$= \partial_\mu\partial_\nu\langle\alpha|T A^{\delta}_\mu A^{\gamma}_\nu|\beta\rangle - \partial_\mu\langle\alpha|[A^{\delta}_\mu A^{\gamma}_0]|\beta\rangle - \langle\alpha|[A^{\delta}_0 \partial A^{\gamma}]|\beta\rangle$$

$$\qquad\qquad \overset{\parallel}{\epsilon^{\delta\gamma\eta}\,\partial_\mu\langle\alpha|V^{\eta}_\mu|\beta\rangle}$$

$$\frac{1}{(2f_\pi)^2}\langle\alpha+\pi^{\delta}(q)+\pi^{\gamma}(k)\ \mathrm{out}|\beta\ \mathrm{in}\rangle$$

$$= q_\mu k_\nu\langle\alpha|T\tilde{A}_\delta{}^{\mu}\tilde{A}_\gamma{}^{\nu}(k)|\beta\rangle + q_\mu\epsilon_{\gamma\delta\eta}\langle\alpha|V^{\mu}_{\rho}(q+k)|\beta\rangle$$

$$- \langle\alpha|[A^{\delta}_0 \partial A^{\gamma}]|\beta\rangle + \frac{1}{2f_\pi}\langle\alpha+\pi_\gamma(k)|\partial\tilde{A}_\delta|\beta\rangle$$

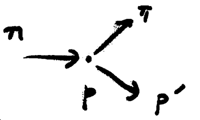
$$+ \frac{1}{2f_\pi}\langle\alpha+\pi_\delta(q)|\partial\tilde{A}_\gamma(k)|\beta\rangle - \langle\alpha|T\,\partial\tilde{A}_\delta\,\partial\tilde{A}_\gamma|\beta\rangle$$

$$\Rightarrow \frac{1}{(2f_\pi)^2}\langle\alpha\,\pi_\delta\pi_\gamma|S|\beta\rangle = q_\mu k_\nu\langle\alpha|T\tilde{A}_\delta^{\mu}\tilde{A}_\gamma^{\nu}(k)|\beta\rangle$$

$$+ q_\mu\,\varepsilon_{\gamma\delta\eta}\langle\alpha|V^{\mu}_{\rho}(k+q)|\beta\rangle$$

$N\pi\to N\pi$ $\qquad q_\mu\,\varepsilon_{\gamma\delta\eta}\,\tau^{\eta}\,\gamma_\mu(2f_\pi)^2.$

W. I. Weinberger PR 143 ('66) 1302
S. L. Adler PR 140 ('65) 736
Y. Tomozawa. N.C. A46 707 ('66)



$$\frac{1}{g_A^2} = 1 + \frac{2M_N^2}{\pi g_{\pi N}^2}\int_\mu^\infty \frac{k\,d\nu}{\nu^2}\left(\sigma_{\pi^- p}(\nu) - \sigma_{\pi^+ p}(\nu)\right) \qquad \nu = pq$$

$q = p' - p$

$$\frac{1}{4f_\pi^2} = \frac{f_{\pi N}^2}{4m_N^2 g_A^2} = \frac{f_{\pi N}^2}{4m_N^2} + \frac{1}{2\pi}\int\frac{k\,d\nu}{\nu^2}\left[\sigma_{\pi^- p}(\nu) - \sigma_{\pi^+ p}(\nu)\right]$$

——— o ———

Basic Identity $\qquad SU(2)\otimes SU(2)$

$\partial_\mu A^{\mu}_\alpha = 0 \qquad m_\pi^2 = 0 \qquad \langle\pi_\alpha(q)|A_\beta(0)|0\rangle = -\frac{i q^\mu}{2f_\pi}\delta_{\alpha\beta}$

$$[V^{(0)}_\alpha(x)\ V^{\mu}_\beta(y)]_{x_0=y_0} = i\,\delta^3(x-y)\,\varepsilon_{\alpha\beta\gamma}V^{\mu}_\gamma(x) + \mathrm{S.T.}$$

$$[V^{(0)}\quad A\quad] = \quad A \quad + \mathrm{S.T}$$

$$[A^0\quad A\quad] = \quad V$$

covariant な取りあつかいならS. T. 無視してよい

Theorem 1.

× . · isospin の外,内積

φ : 任意のテスト函数.

$$\langle\alpha|T\left(\exp\left[+2if_\pi\int d^4x\,\varphi\cdot\partial_\mu A^{\mu}(x)\right]\right)|\beta\rangle$$

$$= \langle\alpha|T\left(\exp\left[+i\int d^4x\,L(x)\right]\right)|\beta\rangle$$

$$L(x) \overset{\text{def}}{=} \frac{-2f_\pi}{1+f_\pi^2\varphi^2}\left[\partial_\mu\varphi\cdot A^\mu + f_\pi(\varphi\times\partial_\mu\varphi)\cdot V^\mu\right]$$

$$-\int_0^1 du\, S^{-1}(u,x_0)\left[f_\pi G(\varphi^2)\,\varphi\cdot\partial A\right]S(u,x_0)$$

$$+\,S^{-1}(1,x_0)\left[2f_\pi\varphi\cdot\partial A\right]S(1,x_0)$$

$$S(u,x_0) \overset{\text{def}}{=} \exp\left[+iuf_\pi\int d^3x\, G(\varphi^2)(\varphi\cdot A^0)(x)\right]$$

$$G(\varphi^2) = 2\,\mathrm{Tan}^{-1}(f_\pi\sqrt{\varphi^2})/f_\pi\sqrt{\varphi^2}$$

(proof) $h(x_0) \overset{\text{def}}{=} 2f_\pi\int d^3x\, \varphi(x)\,\partial_\mu A^\mu(x)$, $U(x_0) \overset{\text{def}}{=} T\exp\left[i\int_{-\infty}^{x_0}dx_0'\, h(x_0')\right]$

$g(x_0) = f_\pi\int d^3x\, G(\varphi^2)\,\varphi(x)\cdot A^0(x)$ $\qquad S(x_0) = \exp[+ig(x_0)]$

$\Rightarrow \quad \underset{(x_0\to\infty)}{V(x_0) = S^{-1}(x_0)\,U(x_0)}$ $\qquad$ (φは適当な函数と思えば $S(x_0) \xrightarrow[x_0\to\infty]{} 1$

$$\langle\alpha|T(\exp[i2f_\pi\int d^4x\,\varphi\cdot\partial A])|\beta\rangle = \lim_{x_0\to\infty}\langle\alpha|U(x_0)|\beta\rangle = \lim_{x_0\to\infty}\langle\alpha|V(x_0)|\beta\rangle$$

$$\frac{\partial}{\partial x_0}V(x_0) = i\{-\underbrace{[-iS^{-1}(x_0)\tfrac{d}{dx_0}S(x_0)]}_{(イ)} + \underbrace{S^{-1}(x_0)\,h(x_0)\cdot S(x_0)}_{(ロ)}\}\cdot V(x_0) \equiv iL(x_0)$$

$$(ロ) = S^{-1}(1,x_0)\left[2f_\pi\int d^3x\,\varphi\cdot\partial_\mu A\right]S(1,x_0) \qquad S(u,x_0) \overset{\text{def}}{=} e^{iug(x_0)}$$

$$(イ) = f_\pi\int_0^1 du\,\{\exp[-if_\pi u\int d^3x\, G(\varphi^2)\varphi\cdot A^0]\int d^3y\,\partial_\mu(G(\varphi^2)\varphi)\,A^\mu\cdot\exp[if_\pi u\int d^3x\, G(\varphi^2)A^0]$$

$$+f_\pi\int_0^1 du\,\{\exp[-if_\pi u\int d^3x\, G(\varphi^2)\varphi\cdot A^0]\int d^3y\, G(\varphi^2)\varphi\cdot\partial_\mu A^\mu\cdot\exp[if_\pi u\int d^3x\, G(\varphi^2)A^0]$$

(但 ∂_i は上と下を合わせて zero)

$$(イ)の第一項 = \int d^3x\,\frac{2f_\pi}{1+f_\pi^2\varphi^2}\{\partial_\mu\varphi\cdot A^\mu(x) + f_\pi(\varphi\times\partial\varphi)\cdot V^\mu\}$$

Corollary 1 $\quad \partial A = 0$ のとき.

$$1 = T\left[\exp\left\{+i\int d^4x\,\frac{-2f_\pi}{1+f_\pi^2\varphi^2}\left[\partial_\mu\varphi\cdot A^\mu + f_\pi(\varphi\times\partial_\mu\varphi)\cdot V^\mu\right]\right\}\right]$$

Theorem 2.

$$\langle\alpha\,\pi(\varepsilon_1 q_1)\cdots\pi(\varepsilon_n q_n)|S|\beta\rangle = f_\pi^n\langle\alpha|U^n[q_1\cdots q_n]|\beta\rangle$$

$$U^n = T\exp\left\{-i\int d^4x\, 2f_\pi\left(\partial_\mu\varphi\cdot\tilde{A}^\mu + \frac{f_\pi}{1+f_\pi^2\varphi^2}\left(f_\pi(\varphi\times\partial_\mu\varphi)\cdot V^\mu(x) - f_\pi^2\varphi^2\partial_\mu\varphi A^\mu\right)\right)\right\}$$

の f_π^n の係数.

$$\varphi(x) = \sum\varepsilon_j e^{iq_j\cdot x}$$

(proof) $A^{\mu} = \pi^{a\alpha} + \tilde{A}^{\mu}$, $\varphi = \xi_i e^{i g_i \cdot \lambda} s_i$

$\Pi \frac{\partial}{\partial s_i} [\quad]_{s_i} = 0$ より $\pi^{a\alpha}$ の より

$\langle \alpha\, \pi \cdots \pi | \beta \rangle = f_\pi^n \langle \alpha | V^{(n)} | \beta \rangle$

Theorem 3 $\mathcal{L}_{eff} = \mathcal{L}_{free} + \mathcal{L}_{int}$

$$\mathcal{L}_{int} = -\frac{2f_\pi}{1+f_\pi^2\varphi^2}(\partial_\mu\varphi\, a^\mu + f_\pi\varphi\times\partial\varphi\cdot v^\mu) - \frac{1}{2}\left[\frac{\partial_\mu\varphi\partial_\mu\varphi}{(1+f_\pi^2\varphi^2)^2} - \partial_\mu\varphi\partial_\mu\varphi\right]$$

(proof) $\langle\alpha|A|\beta\rangle$, $\langle\alpha|V|\beta\rangle$ は a , v の …

$\langle|\hat{A}A|\rangle + \langle|AA|\rangle$

$\langle AA\rangle = \langle\tilde{A}\tilde{A}\rangle + \frac{k_\mu k_\nu}{q^2} 2f_\pi$

$\mathcal{L}_{int}$ $\mathcal{L}_{int}$

$$\frac{1}{f_\pi^2}\left[-2f_\pi\frac{\varphi^2}{1+f_\pi^2\varphi^2} + \frac{f_\pi^4(\varphi^2)^2}{(1+f_\pi^2\varphi^2)^2}\right]\partial_\mu\varphi\partial\varphi$$

に対応

$$= \left[\frac{1}{f_\pi^2}\left(\frac{f_\pi\varphi^2}{1+f_\pi^2\varphi^2} - f_\pi\right)^2 - 1\right]\partial_\mu\varphi\partial\varphi = \frac{1}{1+f_\pi^2\varphi^2}\partial_\mu\varphi\partial_\mu\varphi - \partial_\mu\varphi\partial_\mu\varphi$$

AAV φ の 4個 の …. φ に対応.

$$M = \frac{1-if_\pi\varphi}{1+if_\pi\varphi} \qquad \varphi = \tau^\alpha\varphi^\alpha$$

M(φ) の 任意性.
point 変換. K.O.S. Th.
$G(\varphi^2)$ の 自由度.

$$\mathcal{L} = \frac{1}{8f_\pi^2}\mathrm{Tr}\,\partial_\mu M\partial_\mu M + \frac{1}{2}(\bar{\psi}\sqrt{M^+}(\gamma\partial + m)\sqrt{M}\psi + h.c) +$$

J. Schwinger PL 24B ('67) 47.

Cronin. PR 161 ('67) 1483.

$$\begin{cases} [\Lambda_5\ i\pi] = \lambda + \pi\lambda\pi. \\ [\Lambda, \pi] = -[\lambda\,\pi] \end{cases} \qquad \begin{matrix}[\Lambda_5\ M] = -\{\lambda M\} \\ [\Lambda\ M] = -[\lambda M]\end{matrix}$$

$$M = (N_L, \bar{N}_R)$$

Coleman Wess Zumino PR 177 ('69) 2239

S. Weinberg PRL 18('67), 188
J. Schwinger PL 24B('67) 473
Cronin PR 161 ('67) ~~47~~ 148
J.S. Chrischolm NP 26('61) 469
S. Kametuchi, L. O'raifeartaigh & A. Salam N.P. 28('61) 529
Colleman Wess Zumino PR 177 (1969) 2239

群作用の非線型表現

1. 一般論

0. homogeneous space, symmetric space

$$G\ ;\ M \xrightarrow[\text{transitive}]{} M$$

$H = \{g\ ;\ g\in G\ \ gP = P\ \ P\in M\}$ Pは固定しておく.

$$G/H \simeq M$$

$$V_Q = \{g\ ;\ gP = Q,\ g\in G\ \ P, Q\in M\}$$

$$V_Q \ni g_1, g_2 \qquad g_1^{-1} g_2 \in H$$

$$V_Q = g_Q H$$

1 Nonlinear Realization.

① $H \subset G$ H : subgroup. $G = \sum K_i H = K\cdot H$

coset G/H に代表元の選び方を決めておく. $K = \sum K_i$

$g = K_g h_g$ で K_g, h_g を定ぎする. $h_g \in H$ $K_g \in K \updownarrow M$

$$\underset{K}{\xi\ \in} \xrightarrow{g} g\xi \cdot h^{-1}_{g\xi}\ (\in K) = K_{g\xi} \qquad (\text{注}\ K_{gh} = K_g\ )$$

$$\xi \xrightarrow{g_1} g_1\xi h^{-1}_{g_1\xi} \xrightarrow{g_2} g_2 g_1 \xi h^{-1}_{g_1\xi} \cdot h^{-1}_{g_2 g_1 \xi h^{-1}_{g_1\xi}}$$

「 $h_{g_2 g_1 \xi h^{-1}_{g_1\xi}} \cdot h_{g_1\xi} = K^{-1}_{g_2 g_1 \xi h^{-1}_{g_1\xi}}\, g_2 g_1 \xi \underbrace{h^{-1}_{g_1\xi} K^{-1}_{g_1\xi}\, g_1\xi}_{=1}$

$= K^{-1}_{g_2 g_1 \xi} \cdot g_2 g_1 \xi$, ここで $K_{gh} = K_g$ (定ぎ) を使った 」

$$= K_{g_2 g_1 \xi}.$$

故に ξ へ … の作用は
[illegible] を満す.

(ロ) $U(g)\,\xi\,U^*(g) = g_1^{-1}\xi h^{-1}_{g_1^{-1}\xi} = \kappa_{g_1^{-1}\xi}$

$U(g)\,N\,U^*(g) = \rho(h_{g^{-1}\xi})N$

(N は H の ρ-線型表現に従う)

$U(g_1)U(g_2)\,\xi\,U^*(g_2)U^*(g_1)$ ($\kappa_{gh} = \kappa_g$ より)

$= \kappa_{g_2^{-1}\kappa_{g_1^{-1}\xi}} = \kappa_{g_2^{-1}g_1^{-1}\xi h^{-1}_{g_1^{-1}\xi}} = \kappa_{g_2^{-1}g_1^{-1}\xi} = \kappa_{(g_1g_2)^{-1}\xi}$

$= U(g_1g_2)\,\xi\,U^*(g_1g_2)$

$U(g_1)U(g_2)\,N\,U^*(g_2)U^*(g_1)$ $h_{gh_0} = h_g h_0$

$= \rho(h_{g_2^{-1}g_1^{-1}\xi h^{-1}_{g_1^{-1}\xi}})\rho(h_{g_1^{-1}\xi})N$

$= \rho(h_{g_2^{-1}g_1^{-1}\xi})N = U(g_1g_2)\,N\,U^*(g_1g_2)$

Remark

H の元で $\rho(h_0)N = U(h_0)\,N\,U^*(h_0)$ となる.

"$U(h_0)\,\xi\,U^*(h_0) = h_0\xi h_0^{-1}$ となる事有"

(ハ) 線型表現の base. は

ξN (正確には ρ を G まで拡張して $\rho(\xi)N$)

$U(g)\,\xi N\,U(g)^* = \kappa_{g^{-1}\xi}\cdot h_{g^{-1}\xi}N = g^{-1}\xi N$

㊁ 例. $G = G_0 \otimes G_0$ $\quad H = \{ h \otimes h ,\ h \in G_0 \}$

$G/H \cong K = \{ \xi \otimes \xi^{-1} ,\ \xi \in G_0 \}$ $\qquad (g_1 \otimes g_2)\cdot(h_1 \otimes h_2) = (g_1 h_1 \otimes g_2 h_2)$

$g_1 h_1 g_2 h_2 = 1 \qquad h_1 = g_1^{-1} h_0$

$$g^{-1}(\xi \otimes \xi^{-1}) = (g_L^{-1}\xi \otimes g_R^{-1}\xi^{-1}) = (K_{g^{-1}\xi}\, h_{g^{-1}\xi} \otimes K^{-1}_{g^{-1}\xi}\, h_{g^{-1}\xi})$$

より

$$K^2_{g^{-1}\xi} = g_L^{-1}\xi^2 g_R \qquad \text{故に} \quad \underline{U(g)\,\xi^2\, U^*(g) = g_L^{-1}\xi^2 g_R}$$

$$h_{g^{-1}\xi} = \sqrt{g_L^{-1}\xi^{-2} g_R}\cdot g_L^{-1}\xi$$

普通 $\xi^2 = M(\phi) = \dfrac{1+i\phi/f_\pi}{1-i\phi/f_\pi}$ と書く。 $\qquad N_L = \sqrt{M}\,N$

$N_R = \sqrt{M^{-1}}\,N$

$$\rho_{extend} = \underset{\substack{\downarrow \\ \xi N}}{\rho_H \otimes 1} + \underset{\substack{\downarrow \\ \xi^{-1} N}}{1 \otimes \rho_H}$$

G_0 : fundamental rep. ρ_f

$\rho_f \otimes 1 + 1 \otimes \rho_f$.

㋭ Lattice gauge における。

ρ_0 : $\rho_L = \rho_f \otimes 1$

$\rho_R = 1 \otimes \rho_f$

$$(e^{igA_x})_a{}^j \qquad a \longrightarrow j$$

Non linear realization である。

2 ξの選び方

PTP 64 (1980) 1294

T. Morita Maruomori et al

① Gの generator space $\Lambda = H \oplus K$

$$e^{i\Lambda} = e^{iK}\cdot e^{iH} \qquad \text{proof. PTP}$$

② $$\frac{1+i\Lambda}{1-i\Lambda} = \frac{1+iK}{1-iK}\cdot\frac{1+iH}{1-iH}$$

$K = \xi^a \Lambda^a_K$

$H = h^a \Lambda^a_H$

ξの変換 $[\Lambda^a \xi] = c + \xi\cdot\xi$

§2. 非本質的非線型表現と一つの定理.

1 Linearization Theorem

PTP 43 (1970) 1334
T.M with C. Hattori et al

・非本質的非線型表現.

$$[\Lambda^\alpha, \phi_i] = -f_i^\alpha(\phi) \qquad \Lambda^\alpha ; G \text{ の generator}$$

線型化可能ならば. $f_i^\alpha(0)=0$ としてよい (origin 固)

$$f_i^\alpha(\phi) = A_{ij}^\alpha \phi_j + O(\phi^2)$$

$$[A^\alpha\ A^\beta] = i\, C^{\alpha\beta}_\gamma A^\gamma \qquad C : \text{str. con. of Lie algebra of } G$$

A の作る表現を D とせよ.

表現 f に対する.
Ω : G の covering space.

$$\chi_i(\phi) \overset{\text{def.}}{=} \sum_j \int_\Omega d\mu(g)\, D^{-1}(g)_{ij}\, U^{-1}(g)\, \phi_j\, U(g)$$

$$U^{-1}(g_0)\chi_i(\phi)U(g_0) = \sum \int_\Omega d\mu(g)\, D^{-1}(g)_{ij}\, U^{-1}(g_0)U^{-1}(g)\phi_i U(g)U(g_0)$$

$$= \sum \int_\Omega d\mu(g_0^{-1}g)\, D^{-1}(g g_0^{-1})_{ij}\, U^{-1}(g)\phi_i U(g)$$

$$= D_{ij}(g_0)\,\chi_j(\phi)$$

$$U^{-1}(g)\,\phi\, U(g) = D_{ij}(g)\phi_j + O(\phi^2)$$

Ω : finite ならば. $\quad \chi(\phi) = \phi + O(\phi^2)$

<u>**Theorem**</u> 表現 f の Ω が finite で $f(0)=0$ ならば
線型化出来る.

例えば. Compact 群で有限多価表現であり. $f_i^\alpha(\phi_0)=0$ の
解があれば 線型化出来る

有限多価 $\Longleftrightarrow$ G の基本群が高々 torsion のみ.

2 U(1) で

$$[\Lambda\ \varphi_i] = (a + b\varphi^\dagger\varphi)\varphi_i$$

$$[\Lambda\ \varphi_i^*] = -(a + b\varphi^\dagger\varphi)\varphi_i^*$$

$$e^{-i\Lambda\theta}\varphi_i e^{i\Lambda\theta} = e^{i\theta(a+b\varphi^\dagger\varphi)}\varphi_i$$

§3 graded Lie group の場合.

1 線型表現 と 非線型表現

① 変換.

Linear

$$\phi' = \phi(x + \frac{1}{2i}\bar\epsilon\gamma\theta,\ \theta-\epsilon)$$

Non Linear

$$\delta\lambda = \epsilon + \frac{1}{2i}\bar\epsilon\gamma_\mu\lambda\partial^\mu\lambda$$

$$\delta\sigma = \frac{1}{2i}\bar\epsilon\gamma_\mu\lambda\partial^\mu\sigma$$

$$\delta_2\delta_1\lambda = \frac{1}{2i}\bar\epsilon_1\gamma_\mu\epsilon_2\partial_\mu\lambda + \frac{1}{(2i)^2}[\bar\epsilon_2\gamma_\nu\lambda\bar\epsilon_1\gamma_\mu\partial_\nu\lambda + \bar\epsilon_1\gamma_\nu\lambda\bar\epsilon_2\gamma_\mu\partial_\nu\lambda]\partial_\mu\lambda + \frac{1}{(2i)^2}\bar\epsilon_1\gamma_\mu\lambda\bar\epsilon_2\gamma_\nu\lambda\partial_\mu\partial_\nu\lambda$$

故に

$$(\delta_1\delta_2 - \delta_2\delta_1)\lambda = \frac{-1}{i}\bar\epsilon_1\gamma_\mu\epsilon_2\partial_\mu\lambda$$

$$\delta_2\delta_1\sigma = \frac{1}{2i}\bar\epsilon_1\gamma_\mu\epsilon_2\partial_\mu\sigma + \frac{1}{(2i)^2}[\bar\epsilon_2\gamma_\nu\lambda\bar\epsilon_1\gamma_\mu\partial_\nu\lambda + \bar\epsilon_1\gamma_\nu\lambda\bar\epsilon_2\gamma_\mu\partial_\nu\lambda]\partial_\mu\sigma + \frac{1}{(2i)^2}\bar\epsilon_1\gamma_\mu\lambda\bar\epsilon_2\gamma_\nu\lambda\partial_\mu\partial_\nu\sigma$$

故に

$$(\delta_1\delta_2 - \delta_2\delta_1)\sigma = \frac{-1}{i}\bar\epsilon_1\gamma_\mu\epsilon_2\partial_\mu\sigma$$

$$\Delta x_\mu \overset{\text{def}}{=} \frac{1}{2i}(\bar\epsilon\gamma_\mu\lambda - \bar\lambda\gamma_\mu\epsilon),\quad \Delta dx \overset{\text{def}}{=} d\Delta x$$

$$(1+\delta)\omega_\mu = d[x+\Delta x] + \frac{1}{2i}[\bar\psi(x+\Delta x)\gamma_\mu d\psi(x+\Delta x) - d\bar\psi(x+\Delta x)\gamma_\mu\psi(x+\Delta x)]$$

② Action. (N.L の場合)

$x_\mu \to x'_\mu = x_\mu - \frac{1}{2i}a(\bar\xi\sigma_\mu\psi - \psi^\dagger\sigma_\mu\xi)$ $\psi \to \psi'_{(x')} = \psi + \xi$ のとき

$\omega_\mu = dx_\mu + \frac{a}{2i}(\bar\psi\sigma_\mu d\psi - d\bar\psi\sigma_\mu\psi)$ が不変 である

x' の変換によって…

$$S = \kappa\int\omega_1\omega_2\omega_3\omega_4$$

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(11) N から L を作る.

$$X_\mu(x,\theta) - \frac{1}{2i}\bar{\lambda}(X)\gamma_\mu\theta = x_\mu \quad \text{で} \quad X_\mu(x\ \theta) \text{ を定義}.$$

$$\begin{cases} \delta_N\lambda = \epsilon + \frac{1}{2i}\bar{\epsilon}\gamma_\mu\lambda\partial^\mu\lambda & \delta_L x = \frac{1}{2i}\bar{\epsilon}\gamma_\mu\theta \\ \delta_N\bar{\lambda} = \bar{\epsilon} + \frac{1}{2i}\bar{\epsilon}\gamma_\mu\lambda\partial^\mu\bar{\lambda} & \delta_L\theta = -\epsilon \end{cases}$$

$$[\delta_{\mu\nu} - \frac{1}{2i}(\partial_\nu\bar{\lambda})(X)\gamma_\mu\theta]\,\delta_N X_\nu = \frac{1}{2i}(\delta_N\bar{\lambda})(X)\gamma_\mu\theta$$

$$[\delta_{\mu\nu} - \frac{1}{2i}(\partial_\nu\bar{\lambda})(X)\gamma_\mu\theta]\,\delta_L X_\nu = \frac{1}{2i}(\bar{\epsilon}\gamma_\mu\theta - \bar{\lambda}\gamma_\mu\epsilon) = \frac{1}{2i}\bar{\epsilon}\gamma_\mu(\theta+\lambda)$$

$$[\qquad \text{"} \qquad](\delta_N X_\nu - \delta_L X_\nu) = \frac{-1}{2i}[\delta_{\mu\nu} - \frac{1}{2i}\partial^\nu\bar{\lambda}\gamma_\mu\theta]\,\bar{\epsilon}\gamma_\mu\lambda$$

zero eigen value なし 故に

$$\delta_N X_\mu + \bar{\epsilon}\gamma_\mu\lambda/2i = \delta_L X_\mu$$

$$\tilde{\lambda}(x,\theta) \overset{\text{def}}{=} \lambda(X) - \theta \qquad \tilde{\sigma} \overset{\text{def}}{=} \sigma(X)$$

$$\begin{aligned} \delta_N\tilde{\lambda} &= \epsilon + \frac{1}{2i}(\bar{\epsilon}\gamma_\mu\lambda\partial_\mu\lambda)(X) + \delta_N X_\mu(\partial_\mu\lambda)(X) \\ &= \epsilon + (\frac{1}{2i}\bar{\epsilon}\gamma_\mu\lambda + \delta_N X_\mu)(\partial_\mu\lambda)(X) = \epsilon + \delta_L X_\mu\partial_\mu\lambda(X) \\ &= \delta_L\tilde{\lambda} \end{aligned}$$

$$\begin{aligned} \delta_N\tilde{\sigma} &= \delta_N X_\mu(\partial_\mu\sigma)(X) + \frac{1}{2i}\bar{\epsilon}\gamma_\mu\lambda\,(\partial_\mu\sigma)(X) = \delta_L X_\mu(\partial_\mu\sigma)(X) \\ &= \delta_L\tilde{\sigma} \end{aligned}$$

2 graded Lie group.

K.G. Kac Comm. Math. Phys. 53 ('77) 31.

(1) Z_2 graded Lie group.

<u>A Type</u>

$$\left\{ M = \begin{array}{c} \begin{matrix} m & n \end{matrix} \\ \begin{pmatrix} A & B \\ C & D \end{pmatrix} \end{array} \begin{matrix} m \\ n \end{matrix} \quad \begin{matrix} A, D \text{ even Grassman} \\ B\ \ C \text{ odd Grassman} \end{matrix} \right\} = PL(m,n)$$

$$M^{ST} = \begin{pmatrix} A^T & -C^T \\ B^T & D^T \end{pmatrix} \qquad M^P = \begin{pmatrix} D^T & -B^T \\ C^T & A^T \end{pmatrix}$$

$$(MN)^{ST} = N^{ST}M^{ST} \qquad (MN)^P = N^P M^P$$

$s\cdot\mathrm{tr}\, M = \mathrm{tr} A - \mathrm{tr} D \qquad \mathrm{str}\, MN = \mathrm{str}\, NM$

$\mathrm{sdet}\, M = e^{\mathrm{str} \log M} \qquad \mathrm{sdet}(MN) = \mathrm{sdet}\, M \cdot \mathrm{sdet}\, N.$

$\left(M^{-1} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \qquad \mathrm{sdet}\, M = \det A \det D' \right)$

□ operation $\quad (\theta : \text{odd})$

$(a\theta)^{\square} = a^* \theta^{\square} \qquad \theta^{\square\square} = -\theta \qquad (\theta_1 \theta_2)^{\square} = \theta_1^{\square} \theta_2^{\square}$

$OSp(m.n) = \{ M \in PL(m, n) \quad M^{ST} H M = H \}$

$H = \begin{pmatrix} I_m & & \\ & & I_{n/2} \\ & -I_{n/2} & \end{pmatrix}$

$U(m.n) = \{ M \in PL(m.n) \quad M M^{st} = 1 \}$

$M^{st} = (M^{st})^{\square}$

$\boxed{\begin{array}{l} \underline{\text{C-type}}\ W(n) = \{ D ; \sum P_i \frac{\partial}{\partial \xi_i},\ P_i \in \Lambda(n) \} \\ S(n),\ H(n) \quad \hat{S} \quad \tilde{H} \\ \text{体積一定. Hamilton form 不変.} \sim n = \text{even} \\ \Lambda(n) \quad n \text{ 変数 Grassmann 数.} \end{array}}$

Cartan

primitive infinite Lie Algebra

<u>B Type</u>

Kac Moody Type

$[\Lambda^\alpha \Lambda^\beta] = f^{\alpha\beta\gamma} \Lambda^\gamma$

$\Lambda^\alpha_n = \Lambda^\alpha t^n$

$[\Lambda^\alpha_n \Lambda^\beta_m] = f^{\alpha\beta\gamma} \Lambda^\gamma_{n+m}$

t の入った grassmann 数を使う.

↑ finite Lie super Algebra.

⓪ Haag Kopuszański Sohnius

$\begin{pmatrix} L & P & | & Q \\ & \tilde{L} & | & \\ \hline \bar{Q} & & | & \end{pmatrix} \updownarrow \text{spin 表現} \begin{pmatrix} M_a{}^b & -iP_{a\dot{b}} & Q_{aB} \\ iK^{\dot{a}b} & M^{\dot{a}}{}_{\dot{b}} & \bar{Q}'^{\dot{a}}{}_B \\ iQ'^{bA} & -i\bar{Q}_{\dot{b}}{}^A & \Lambda^A{}_B \end{pmatrix} + \begin{pmatrix} \frac{i}{2}\delta_a^b D & & \\ & -\frac{i}{2}\delta_{\dot{a}}^{\dot{b}} D & \\ & & \end{pmatrix}$

$U(2,2/n)$

(ハ) Linear 表現を base とすると
spin と似ており U(2,2(N)) wess では

U(2,2) でなく Sp(2) から出発する。
OSP(1/2)/O(1,3) ← Zumino $Sp(2) \cong SO(5)$ ↑ 2価.

(ニ) 物性での応用
spin glass quenching

3 Super Lie group の非線型表現.

(イ) 何が違うか。
x が動く事。 前は $\xi(x)$ で x を忘れていてよかった。

(ロ) G/H 上での表現. H は translation を含まない
K として Λ^α P_μ とする。

$$g = e^{i\varphi^\alpha \Lambda^\alpha} e^{ixP} \cdot h$$

$(1+\Delta g_0)g$ より $\delta^\circ \psi = \delta^\circ \psi(\varphi, x)$, $\delta^\circ x = \delta^\circ x(\varphi, x)$ を定義する。

これは $(\delta_1^\circ \delta_2^\circ - \delta_2^\circ \delta_1^\circ)\psi = \delta^\circ_{[1,2]}\psi$
$(\delta_1^\circ \delta_2^\circ - \delta_2^\circ \delta_1^\circ)x = \delta^\circ_{[1,2]} x$ が成立つ。
(前と同じ証明より)

(ハ) field $\psi(x)$ 上での表現.

$$\delta\psi \stackrel{\text{def}}{=} \delta^\circ\psi - \delta^\circ x_\mu \partial_\mu \psi$$

$$\partial A(\varphi, x)\big|_{\varphi=\psi(x)} \stackrel{\text{def.}}{=} \tilde{\partial}_\mu A(\varphi, x)$$

$$\delta_1\delta_2\psi = \delta_1^\circ(\delta_2^\circ\psi) - \delta_1^\circ(\delta_2^\circ x_\nu)\cdot\partial_\nu\psi$$
$$- \delta_1^\circ x_\mu [\partial_\mu(\delta_2^\circ\psi) - \partial_\mu(\delta_2^\circ x_\nu)\partial_\nu\psi]$$
$$- \delta_2^\circ x_\mu [\partial_\mu(\delta_1^\circ\psi) - \partial_\mu(\delta_1^\circ x_\nu)\partial_\nu\psi] + \delta_2^\circ x_\mu \delta_1^\circ x_\nu \partial_\mu\partial_\nu\psi$$

よって

$$[\delta_1\delta_2 - \delta_2\delta_1]\psi = [\delta_1^\circ, \delta_2^\circ]\psi - [\delta_1^\circ\delta_2^\circ](x_\nu)\partial_\nu\psi$$

(ニ) Matter field の変換.

$$\delta^0\sigma = \delta^0\sigma(x,\psi,\sigma)$$

Lie group の場合と同じ.

$$\delta^0 x_\mu = \delta^0 x_\mu(x,\psi)$$

$$\delta\sigma = \delta^0\sigma - \delta^0 x_\mu \partial_\mu \sigma$$

$$\begin{aligned}
\delta_1(\delta_2\sigma) = {} & \delta_1^0(\delta_2^0\sigma) - \delta_1^0(\delta_2^0 x_\mu)\partial_\mu\sigma \\
& - \delta_1^0 x_\mu \partial_\mu(\delta_2^0\sigma) - \delta_2^0 x_\mu \partial_\mu(\delta_1^0\sigma) \\
& + \delta_1^0 x_\nu \partial_\nu(\delta_2^0 x_\mu)\partial_\mu\sigma + \delta_2^0 x_\mu \cdot \partial_\mu(\delta_1^0 x_\nu)\partial_\nu\sigma \\
& + \delta_2^0 x_\mu \delta_1^0 x_\nu \partial_\mu \partial_\nu \sigma
\end{aligned}$$

$$\begin{aligned}
(\delta_1\delta_2 - \delta_2\delta_1)\sigma &= [\delta_1^0 \delta_2^0]\sigma - (\delta_1^0 \delta_2^0 X(x_\mu))\cdot\partial_\mu\sigma \\
&= \delta_{[12]}\sigma - \delta_{[12]}x_\mu \cdot \partial_\mu\sigma.
\end{aligned}$$

(ホ) 線型表現を作る.

$$X_\mu(\psi, \theta x)$$

$$\delta X_\mu = \delta_L X_\mu + \delta^0 x_\mu$$

$$(\delta_L : \theta x)$$

$$\sigma(X)$$

Comments on the Super space

§0 θとは何んであろうか.

- $\phi(x,\theta) = \phi_0(x) + \psi(x)\theta + (\cdots)\theta^2 + \cdots$
- $\theta \to \theta - \epsilon$
  $x \to x + \frac{1}{2i}\bar{\epsilon}\gamma\theta$ $\qquad \{\epsilon\ \theta\} = 0.$
- $\int d\theta$ $\qquad \theta \to \theta + \eta.$

θ 変数　変域は?　$\theta_i = \alpha_i\,\epsilon_i$　$\{\epsilon_i\ \epsilon_j\} = 0$?
x 変数　$x + \frac{1}{2i}\bar{\epsilon}\gamma\theta$?

$$\phi(\underset{?}{\uparrow},\ \underset{?}{\uparrow})$$

Axiom of Simplicity:

Storyを簡単明解にする目的のためには,
結果を導びくにあたっての, 前提条件についての言及することを
省略出来る。

§1. Supermaifolds, mapping, 微分, 積分 …

(イ) $\beta_\phi = 1 \qquad \beta_i\beta_j + \beta_j\beta_i = 0 \qquad 1 \le i, j \le L$

$\mu = (\mu_1, \mu_2, \cdots, \mu_k) \quad 1 \le \mu_1 < \mu_2 < \mu_3 < \cdots < \mu_k \le L \quad \mu_i$: integer.

$B_L \stackrel{\mathrm{def}}{=} \{\sum x_\mu \beta_\mu\} \qquad \dim B_L = 2^L$

$\|x\| = \sum_\mu |x_\mu| \qquad \beta_\mu = \beta_{\mu_1}\cdots\beta_{\mu_k}$

Proposition B_∞ is a Banach algebra

(i.e. $\|a+b\| \le \|a\| + \|b\|$, $\|ab\| \le \|a\|\|b\|$)

Definition $B_{L\underline{0}} = \{ \beta_\mu ;\ \mu = (\mu_1 \cdots \mu_k)$ ak even $\}$

$B_{L\underline{1}} = \{ \beta_\mu ;\ \mu = (\mu_1 \cdots \mu_k)$ ak odd $\}$

$(B_L = B_{\underline{0}} + B_{\underline{1}} \qquad B_{\underline{1}} \cdot B_{\underline{1}} \subset B_{\underline{0}})$ $\mathbb{Z}_2$-graded space

Definition $B_L^{m,n} = B_{L\underline{0}}^m \otimes B_{L\underline{1}}^n$

$B_{L\underline{0}}^m$ の point を $x = (x_1, \cdots, x_m)$

$B_{L\underline{1}}^n$ の point を $\theta = (\theta_1, \cdots, \theta_n)$ で表わす.

$B_{L\underline{0}}$ の元 x_i の中 1 に比例する成分を x_i^0

$x_i - x_i^0 = \tilde{x}_i \quad (\therefore \quad x = x^0 + \tilde{x} = (x_1^0 \cdots x_m^0) + (\tilde{x}_1 \cdots \tilde{x}_m))$

$\tilde{x}_i$; nilpotent (L : finit なら)

x^0 を body と呼ぶ. (または soul)

(ロ) 微分. ($L = \infty$ または十分大)

$\Phi : B_{m,n} \to B$

Definition

$$\| \Phi(a+h, b+k) - \Phi(a,b) - \sum_{i=1}^m h_i G_i \Phi - \sum_{j=1}^n k_j G_{j+m} \Phi \| / \|h+k\| \to 0$$

($\|h\| \to 0$ $\|k\| \to 0$ のとき)

$$\frac{\partial}{\partial x_i} \Phi \Big|_{x=a} = G_i \Phi \ , \quad \frac{\partial}{\partial \theta_j} \Phi \Big|_{\theta=b} = G_{j+m} \Phi \quad \text{と定義する.}$$

Remark $\frac{\partial}{\partial x}\Phi$, $\frac{\partial}{\partial \theta}\Phi$ 一意的に決る. (L=finit $G\Phi + \lambda\beta_{1 \cdots L}$ の不定性)

Remark.

$$f(\theta) = f_0 + f_1 \theta + \frac{1}{2!} f \theta^2 + \cdots$$

$\epsilon(h) = 0$ (body が 0)

$$f(a+h, b+k) = f(a,b) + \sum_{q=1}^{L} \frac{1}{q!} D^q f(a,b)[(h,k)]^q$$

(ハ) 積分(I)

(1) $f(x,\theta)$; $B_L^{m;n} \to B_L$ の積分とは

f の linear functional $F(f)$ を決めることである。

$F(f) \in B_L$, $F(cf) = cF(f)$ $F(f_1+f_2) = F(f_1)+F(f_2)$

Remark F の種類 $\Leftrightarrow$ 積分測度又は領域

(2) f は $C^{(\omega)}$ を要求

$\int d\theta$ に関しては、 $f(\theta) = \sum f_p \theta_{p_1}\theta_{p_2}\cdots\theta_{p_k}$

より $F(f) = \sum f_p c_p$ $(F \Leftrightarrow c_p)$

特に $\int d\theta = \frac{\partial}{\partial\theta_1}\cdots\frac{\partial}{\partial\theta_n}$ として、 $\mu(\theta) = \sum \mu_\sigma \theta_\sigma$ を適当に

選べば $F(f) = \int d\theta\, \mu(\theta) f(\theta)$ と表せる。

(3) $f(x)$ に関しては $x = x^0 + \tilde{x}$ と分ければ

$f(x) = \underset{\sim}{f}(x^0) + \sum (\tilde{x})^p \underset{\sim}{f}^{(p)}(x^0)$ より、

$F(f)$ とは $F(\underset{\sim}{f}, \underset{\sim}{f}^{(p)})$ のことである 特に $\tilde{F}(\underset{\sim}{f})$ と同じ…

特に $F(f) = \int dx^0\, \mu(x^0) f(x^0)$ と表せる。

特にまとめて $\overset{\forall}{F}(f(x,\theta)) = \int dx^0 d\theta \overset{\exists}{\mu}(x^0,\theta) f(x^0,\theta)$

↑任意　　　　　　↑ある

(ニ) 積分(II)

$(x,\theta) \to (X(x,\theta), \Theta(x,\theta))$ をとる。

(*) $\int dx\, d\theta\, f = \int dX\, d\Theta \left\| \frac{\partial(x\,\theta)}{\partial(X,\Theta)} \right\|_S f(x(X\Theta), \theta(X\Theta))$

$\frac{\partial(x,\theta)}{\partial(X,\Theta)} = s\cdot\det\left(\frac{\partial(x\,\theta)}{\partial(X\,\Theta)}\right)$

<u>Definition</u>

$$M = \begin{pmatrix} \text{even} & \text{odd} \\ \text{odd} & \text{even} \end{pmatrix} = \begin{pmatrix} A & \cdots \\ \cdots & \cdots \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} \cdots & \cdots \\ \cdots & \tilde{B} \end{pmatrix}$$

$$\mathrm{Sdet}\, M \overset{\mathrm{def}}{=} \det A \cdot \det \tilde{B}$$

$\mathrm{str}\,\Phi = \mathrm{Tr}\,\Phi_1 - \mathrm{Tr}\,\Phi_2$

(Remark $\quad M = e^{\Phi} \Longleftrightarrow \mathrm{Sdet}\, M = e^{\mathrm{str}\,\Phi}$) $\quad \Phi = \begin{pmatrix} \Phi_1 & * \\ * & \Phi_2 \end{pmatrix}$

(*) の証明.

$g.\ (x, \theta) \Rightarrow (X, \Theta)$

① $g_A : \begin{array}{l} x = x(X, \Theta) \\ \theta = \Theta \end{array} \qquad g_B : \begin{array}{l} x = x(X) = X \\ \theta = \theta(X, \Theta) \end{array}$

$g = g_A \cdot g_B$ と出来る.

② $g_{Ai} \quad \begin{array}{l} x = X + \theta_i\, \Delta x(X, \theta_{i+1} \cdots \theta_n) \\ \theta = \Theta \end{array}$

$g_A = g_{A1} \cdots g_{An}$ と出来る. $\quad$ ($x = f(X) + \cdots$ は trivial)

$$\int dx\,d\Theta\ \mathrm{Sdet}(\quad)\ F(x\ \theta)$$

$$= \int dx\,d\theta\, F + \int dX\,d\Theta \left[\frac{\partial}{\partial X_a} \theta_i\, \Delta x_a \cdot F + \overbrace{\theta_i\, \Delta x_a \frac{\partial}{\partial X_a} F}^{=0} \right]$$

③ $g_B ; \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix} = \begin{pmatrix} f_1(\Theta_1 \cdots \Theta_n) \\ \vdots \\ f_n(\Theta_1 \cdots \Theta_n) \end{pmatrix} \qquad \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_\ell \end{pmatrix} = \begin{pmatrix} f_1 \\ \vdots \\ f_\ell \end{pmatrix}$ を $\theta_1\ \Theta_\ell$ について解く.

$$\Theta_1 = F_1^{(\ell)}(\theta_1 \cdots \theta_\ell, \Theta_{\ell+1} \cdots \Theta_n)$$

$$\vdots$$

$$\Theta_\ell = F_\ell^{(\ell)}(\theta_1 \cdots \theta_\ell\ \Theta_{\ell+1} \cdots \Theta_n)$$

(定義より). $\quad F_a^{(\ell)}(f_1(\Theta_1 \cdots \Theta_n) \cdots f_\ell, \Theta_{\ell+1} \cdots \Theta_n) = \Theta_a$

$$f_\ell(F_a^{(\ell)}\ \Theta_{\ell+1} \cdots \Theta_n) = \theta_\ell$$

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$$\tilde{g}_\ell : \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix} = \begin{pmatrix} f_1(\Theta_1 \cdots \Theta_n) \\ \vdots \\ f_\ell \\ \Theta_{\ell+1} \\ \vdots \\ \Theta_n \end{pmatrix}$$

$$g_{B\ell} : \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix} = \begin{pmatrix} \Theta_1 \\ \vdots \\ \Theta_\ell \\ f_{\ell+1}(F_\ell(\Theta)\ \Theta_{\ell+1} \cdots \Theta_n) \\ \Theta_{\ell+2} \\ \vdots \\ \Theta_n \end{pmatrix}$$

$$g_{B\ell}\tilde{g}_\ell = \tilde{g}_{\ell+1}$$

故に g_B は $g_{B\ell}$ の積で表わされる.

$$\theta_1 = \Theta_1$$
$$\vdots$$
$$\theta_{\ell-1} = \Theta_{\ell-1}$$
$$\theta_\ell = f(\Theta_1 \cdots \Theta_n) = \Theta_\ell(a + r_0(\Theta_1 \cdots \overset{\vee}{\ell} \cdots \Theta_n)) + r_1(\Theta_1 \cdots \overset{\vee}{\ell} \cdots \Theta_n)$$

$$\left(\frac{\partial \theta}{\partial \Theta}\right) = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & a+r_0 & \\ & & & & \ddots \end{pmatrix} \qquad \mathrm{sdet}\left(\frac{\partial \theta}{\partial \Theta}\right) = \frac{1}{a+r_0}$$

$$F(\cdots \Theta_\ell(1 + a - 1 + r_0) + r_1, \cdots)$$
$$= F(\Theta) + [(a-1+r_0)\Theta_\ell + r_1]\frac{\partial}{\partial \Theta_\ell}F$$
$$= (a+r_0)F - \frac{\partial}{\partial \Theta_\ell}\{[(a-1+r_0)\Theta_\ell + r_1]F\}$$

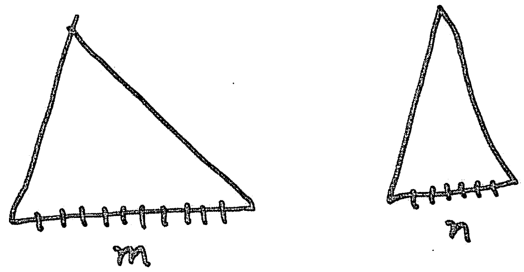
$$\therefore\ \mathrm{sdet}(\ )F(\theta' \cdots) = F - \frac{\partial}{\partial \Theta_\ell}\left[\frac{1}{a+r_0}\{\cdots\}\right]$$

(r_0 は Θ_ℓ を含まない

§2. 愚にも付かぬこと。"再び θ とは何んであろうか"

空間座標 x は何を表わすか.

ギリシヤ　x : 有理数　$\sqrt{2}$



例えば 万有引力の法則.

x : 実数.

R を超えよ ??

イ)　${}^*R$　超実数　ultrafilter

↑ Axiom of choice

ロ)　Superspace

θ まで論.

§ Fermion Path Integral

0) Grassmann 変数

$$\xi_i \xi_j + \xi_j \xi_i = 0 \qquad (1 \le i, j < \infty)$$

Complex Num.

$$V^{(o)} = \left\{ \sum_{\substack{n=0 \\ i_1 < i_2 \cdots < i_{2n+1}}}^{\infty} c_{i_1 \cdots i_{2n+1}} \xi_{i_1} \xi_{i_2} \cdots \xi_{i_{2n+1}} \right\} \qquad c_i \in \mathbb{C}$$

$$V^{(e)} = \left\{ \sum_{\substack{n=0 \\ i_1 < i_2 \cdots < i_{2n}}}^{\infty} c_{i_1 \cdots i_{2n}} \xi_{i_1} \xi_{i_2} \cdots \xi_{i_{2n}} \right\}$$

$\eta \in V^{(o)}$ (Fermionic 変数

$\phi \in V^{(e)}$ (Bosonic 変数)

i)

$\{\psi_i \, \psi_j^*\} = \delta_{ij}$, the is zero $\quad i, j = 1, 2, \cdots, n$

$$\{ | \rangle \} = \{ \sum c_i \psi_{i_1}^* \cdots \psi_{i_s}^* |0\rangle \} \Longleftrightarrow V(\eta^*)$$

$$\eta^* \in V^{(o)}$$

$$\{ \langle | \} \Longleftrightarrow V(\eta)$$

$$| \rangle_f = | f(\psi^*) \rangle \sim f(\eta^*)$$

$$_f\langle | \to (| \rangle_f)^* \sim f^*(\eta^*)$$

ii) $f(\eta_i \, \eta_j^*)$ の 積分

$$\int \Pi d\eta d\eta^* \, c f(\eta, \eta^*) = c \int \Pi d\eta d\eta^* f(\eta, \eta^*)$$

$$\int \Pi d\eta d\eta^* (f_1 + f_2) = \int \Pi d\eta d\eta^* f_1 + \int \Pi d\eta d\eta^* f_2$$

$$f(\eta \, \eta^*) \xrightarrow{F} c \text{ への線型写像.}$$

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$$f(\eta,\eta^*) = \sum_{\substack{i_1<i_2\cdots<i_s \\ j_1<j_2\cdots<j_r}} c_{i_1\cdots i_s j_1\cdots j_r}\, \eta_{j_r}\cdots\eta_{j_1}\,\eta^*_{i_1}\cdots\eta^*_{i_s}$$

$$F(f) = \sum c_{ji} \underset{\sim}{F}^{ji}$$

$\eta_i \to \eta_i + \lambda$, $\eta^*_j \to \eta^*_j + \lambda^*$ で不変なもの

$c\,\varepsilon^{j_1\cdots j_n}\,\varepsilon^{i_1\cdots i_n} = F$, 他は zero

特に $C = 1$ を

$\int d\eta\, d\eta^* f(\eta,\eta^*)$ で表わす.

他の測度は　$d\eta\, d\eta^*\, \mu(\eta,\eta^*)$ と表わせる.
　　　　　　　　　　　　　← があって

$$\langle f(n) | g(n) \rangle = \int \pi\, d\eta\, d\eta^* \prod_{i=1}^{n} (1+\eta_i\eta_i^*) f^*(\eta^*)\, g(\eta^*)$$

?: $\langle 0|(c^*+d^*\psi)(a+b\psi^*)|0\rangle = c^*a + d^*b$.

$$\int d\eta\, d\eta^* (1+\eta\eta^*)(c^*+d^*\eta)(a+b\eta^*) = c^*a + d^*b$$

iii) $\mathbb{1} = \sum |n\rangle\langle n| \longrightarrow 1+\eta_2^*\eta_1$

$\mathbb{1} f(\psi^*)|\rangle \Rightarrow \int d\eta_1 d\eta_1^* (1+\eta_1\eta_1^*)(1+\eta_2^*\eta_1) f(\eta_1^*)$

$= \int d\eta_1 d\eta_1^* (1+\eta_1(\eta_1^*-\eta_2^*)) f(\eta_1^*)$

$= \int d\eta_1 d\eta_1^*\, e^{\eta_1(\eta_1^*-\eta_2^*)} f(\eta_1^*)$

?: 実際に $f(\eta_1^*) = a + \eta_1^* b$ を代入.

(39) $|0\rangle\langle 0| \rightarrow 1$

$|\psi\rangle\langle\psi| \rightarrow \eta_2^* \eta_1$

$$\underbrace{\int d\eta_1 d\eta_1^* (1+\eta_2\eta_1^*)}_{\text{内積}} (1+\eta_2^*\eta_1) \otimes = \int d\eta_1 d\eta_1^* e^{\eta_1(\eta_1^*-\eta_2^*)} \otimes$$

完全系に

$$\sum_f |f(\eta_2)\rangle\langle f(\eta_1)| = \int d\eta_1 d\eta_1^* e^{\eta(\eta_1^*-\eta_2^*)} \otimes$$

iv) $$\langle g(\eta)| H(\psi\psi^*) |f(\eta)\rangle = \int d\eta d\eta^* \underbrace{e^{\eta_i\eta_i^*}}_{\Pi(1+\eta_i\eta_i^*)} g(\eta^*) H(\eta,\eta^*) f(\eta)$$

v) $$\langle g_F| \overset{|\eta_F^*\rangle\langle\eta_{N-1}|}{U_N U_{N-1}} \cdots \overset{|\eta_I^*\rangle\langle\eta_I|}{U_1} |f_I\rangle$$

$$= \int d\eta_F d\eta_F^* \int d\eta_{N-1} d\eta_{N-1}^* \cdots d\eta_I d\eta_I^*$$

$$\times e^{\eta_{N-1}(\eta_{N-1}^*-\eta_F^*) + \cdots + \eta_I(\eta_I^*-\eta_1^*)}$$

$$\times e^{\eta_F\eta_F^*} f(\eta_F) U_N(\eta_{N-1}\eta_{N-1}^*) \cdots U_1(\eta_I\eta_I^*) f(\eta_I^*)$$

$$U_i = e^{-i\Delta t H(\psi\psi^*)}$$

[---- 落す.
行列にする所]

$\mathcal{L} = \psi^* i\frac{\partial}{\partial t}\psi - H$ とすると

$\Longrightarrow \int d\psi d\psi^* e^{i\int dt \mathcal{L}}$

$\left[\begin{array}{l}\eta^* \to \psi \\ \eta \to \psi^*\end{array}\right]$

vi) $\eta_i^{*\prime} = A_{ij}\eta_j^*$ とする.

$\eta_1^* \cdots \eta_n^* = \det A^{-1}\, \eta_1^{*\prime} \cdots \eta_n^{*\prime}$

$1 = \int d\eta^* \eta_1^* \cdots \eta_n^* = \int d\eta^{*\prime} \cdot \mu (\det A)^{-1} \eta_1^{*\prime} \cdots \eta_n^{*\prime}$

$\mu = (\det A)$

Bosonic 変数と混合しているときは、 <u>Sdet</u>

v) 内 Trace

$A = a + b\eta + c\eta^* + d\eta\eta^*$

$\int d\eta d\eta^* (1+\eta\eta^*)\underline{(1+\eta\eta^*)}\, A = 2a + d$

$\uparrow$ $\quad$ $\langle 1$ の内積

$\langle n | A | n \rangle$ の内積 (Boson

$e^{\eta_{N-1}(\eta^*_{N-1} - \eta^*_N)}$

$\eta_N = -\eta_{N-1}$ とするとよい.

[例] $\langle 0 | A(\eta,\eta^*) | 0 \rangle$ $\quad$ $\langle 1 | A(\eta\eta^*) | 1 \rangle$

$\int d\eta d\eta^* (1+\eta\eta^*) A(\eta,\eta^*) + \int d\eta d\eta^* (1+\eta\eta^*)\eta A(\eta\eta^*)\eta^*$]