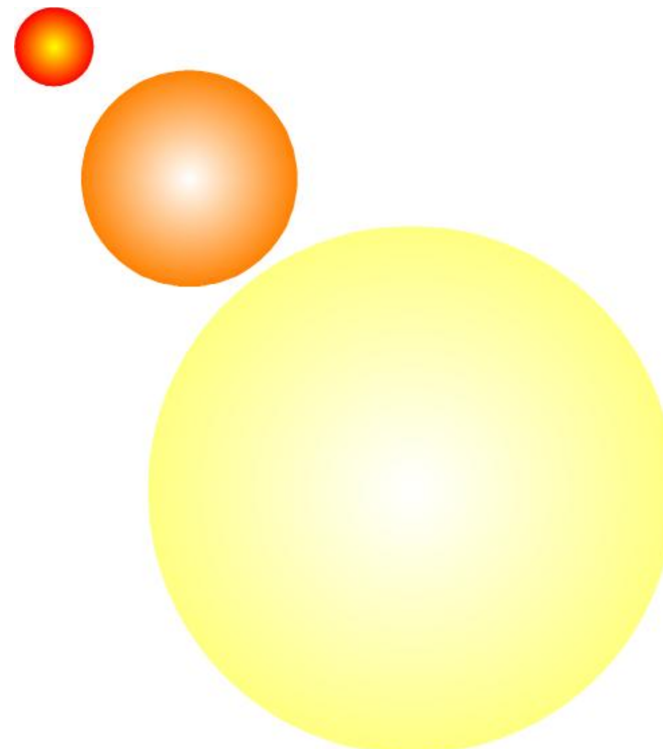
Inflationary cosmology after Planck

Takahiro Tanaka (YITP)

Big Bang Cosmology

The universe starts with a fireball.

- Friedmann Universe
  ～Hubble's law
- Nucleosynthesis
  Baryon/Photon $\sim 10^{-9\sim-10}$
- Cosmic Microwave
  Background

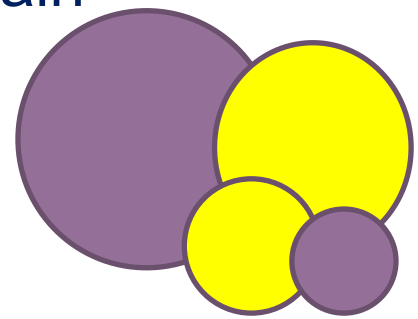


Beginning of Particle Cosmology

Various motivations for inflation @1980

- Large domain of baryon/anti-baryon domain
- Superhorizon scale correlation
- Monopole problem

K. Sato



- Horizon problem
- Flatness problem

A. Guth

Inflation solves all problems just by assuming an early phase of exponential expansion of the universe.

- Initial singularity avoidance

A. Starobinsky

$$L_{grav} = M_{pl}^2 \left(R + R^2 / 6M^2 \right)$$

+ Initial density perturbation

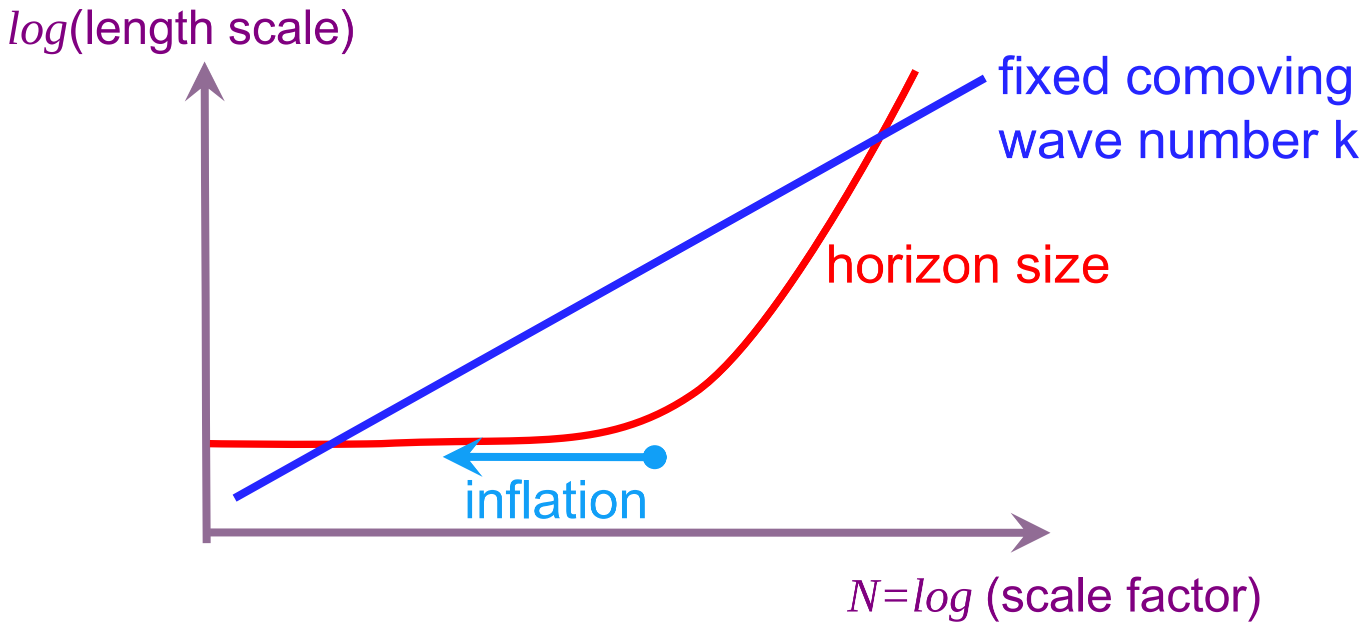
Evolution of scales

Horizon scale: H^{-1}

Distance that light can travel for the cosmic expansion timescale

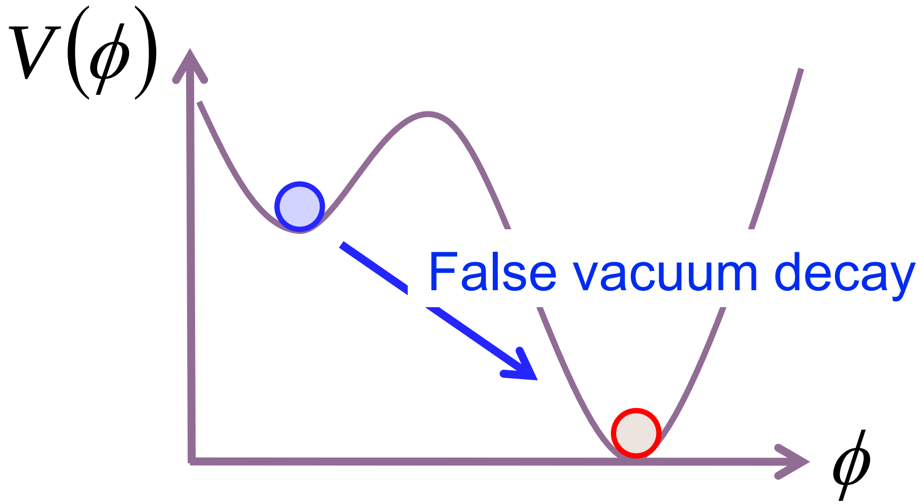
Comoving wavenumber: $\mathbf{k} = -i\dfrac{\partial}{\partial \mathbf{x}}$

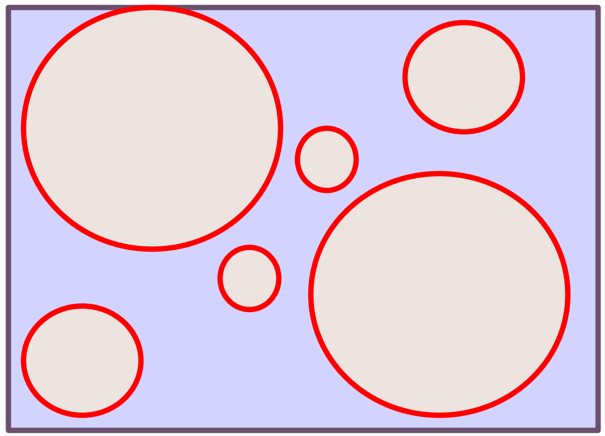
In linear perturbation, different $\mathbf{k}$ modes evolve independently.



Various inflation models

False vacuum inflation (Old inflation)





Small nucleation rate for inflation

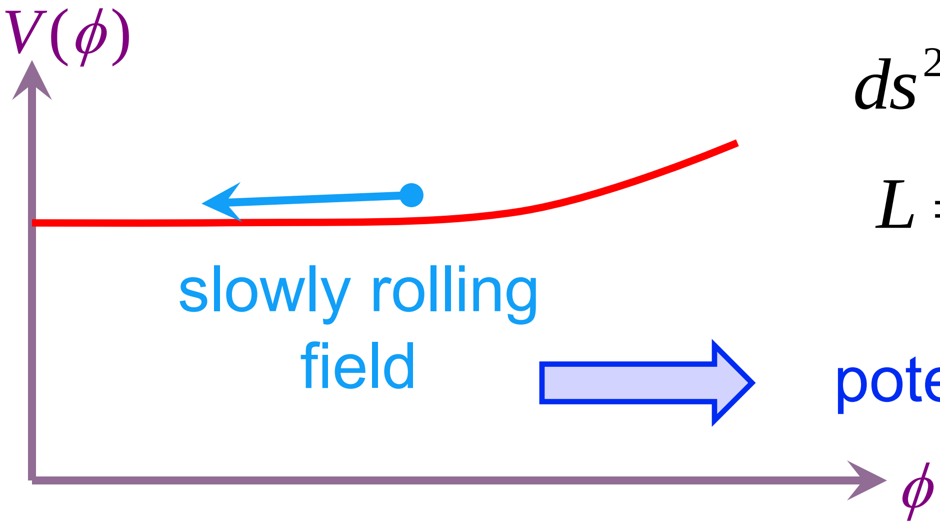
⇨ Some region continues to inflate

⇨ Difficult to terminate inflation / too large fluctuation

A synchronized clock to control the transition is necessary

Standard slow roll inflation

New inflation, Chaotic inflation A. Linde (1982,3)



$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

$$L = -\frac{1}{2}(\nabla\phi)^2 - V(\phi)$$

⇒ potential term dominant

Expansion rate: $H^2 = \frac{8\pi G}{3}\rho \approx \frac{8\pi G}{3}V$

$H \equiv \frac{\dot{a}}{a}$ ⇒ $a \propto e^{Ht}$

exponential expansion

ϕ plays the role of synchronized clock

Generation of density perturbation

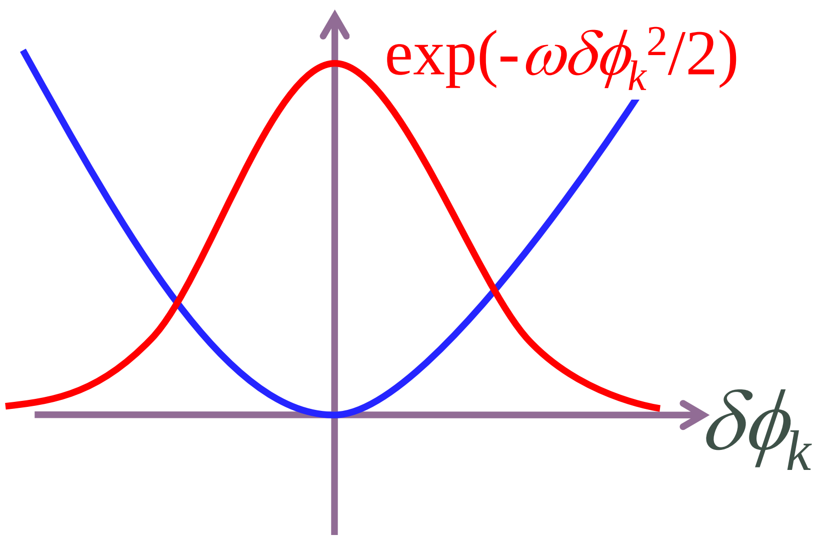
Quantum fluctuation of inflaton ϕ during inflation:

Mukhanov, Viatcheslav(1981)
Hawking(1982)
Starobinsky(1982)
Guth, Pi (1982)
Bardeen(1982)
Kodama-Sasaki
PTP supplement (1984)

$$\Box\delta\phi + V''\delta\phi = 0 \Rightarrow \left[\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right]\delta\phi_k = 0$$

$$\frac{k^2}{a^2} \equiv \omega^2$$

Time-dependent harmonic oscillator



1) $\omega \gg H$: sub-horizon

$\omega \searrow$ ⇨ wider wave function

$\frac{\dot{\omega}}{\omega^2} << 1$:adiabatic evolution

2) $\omega \ll H$: super-horizon

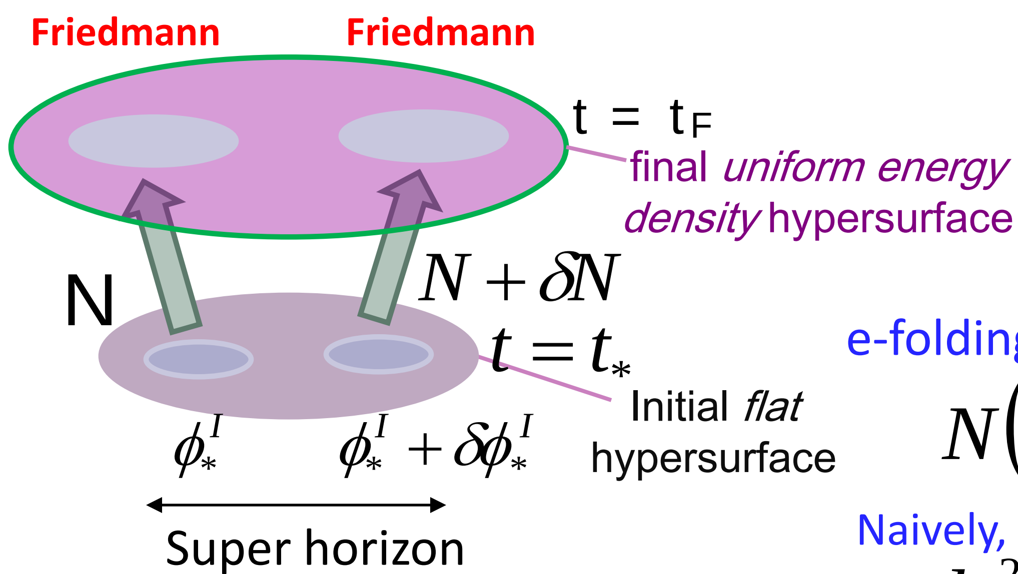
freeze out at $\omega \lesssim H$

$(k/a)^3 \delta\phi_k^2 \sim H^2$

Super-horizon dynamics – δN formalism-

- Super-horizon dynamics is locally described by the FRW universe.

Starobinsky (1985)
Salopek & Bond (1990)
Sasaki & Stewart(1996)
Sasaki & TT(1998),
Lyth et al.(2005)



e-folding number

$$N\left(t_f; t_*, \phi^I(\boldsymbol{x})\right) = \int_{t_*}^{t_f} H\,dt$$

Naively,

$$ds^2 = -dt^2 + \underline{a^2 e^{2\zeta}}\,\delta_{ij}dx^i dx^j$$

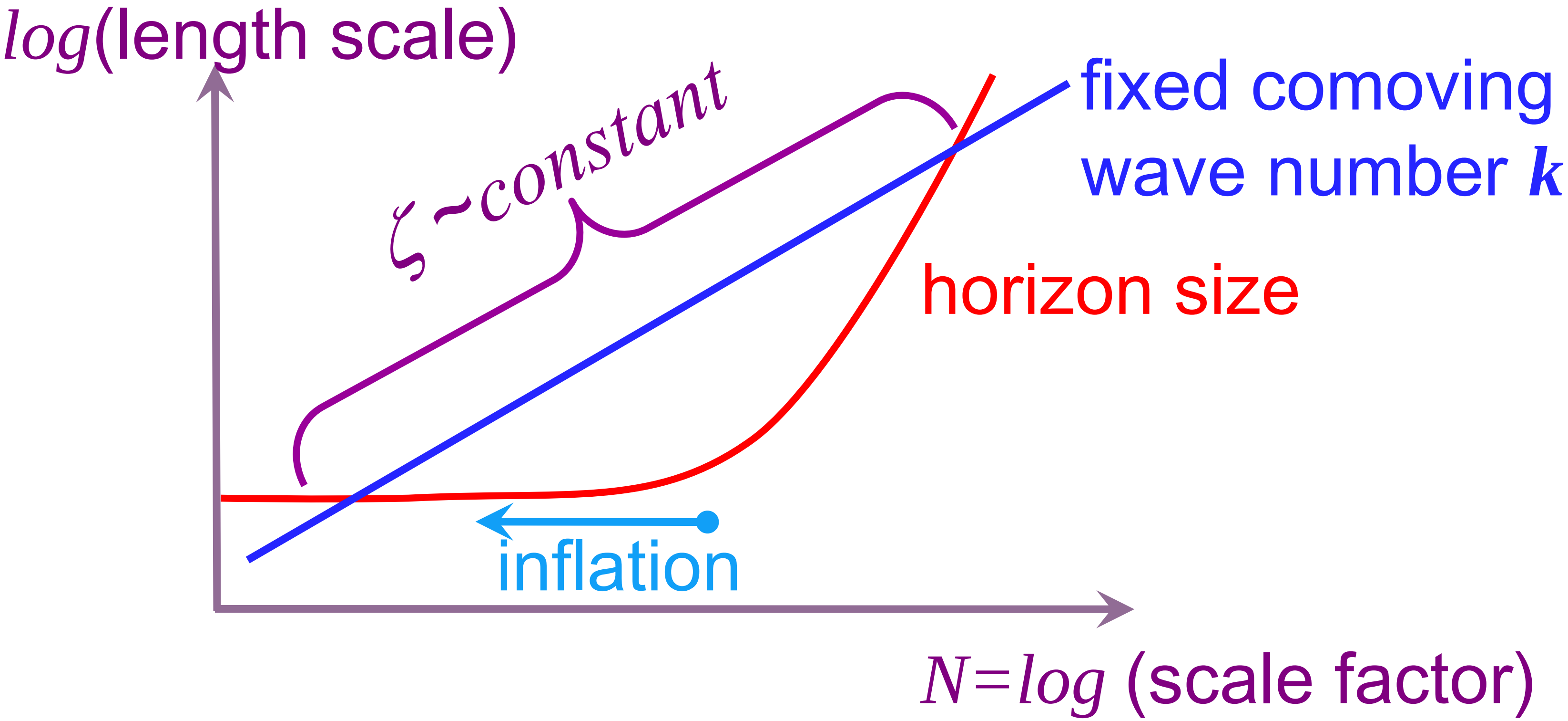
$$e^{2\left(N+\delta N\left(t_f; t_*, \phi^I\right)\right)}$$

$$\boxed{\zeta\left(t_f, \boldsymbol{x}\right) \approx \delta N\left(t_f; t_*, \phi^I(\boldsymbol{x})\right)}$$

ζ is conserved for single field inflation on super horizon scale.

$$\zeta \approx H\delta t = H\frac{\delta\phi}{\dot{\phi}} \approx \frac{H^2}{\dot{\phi}}$$

Single field inflation



Tensor perturbations

$$L_{grav} = \frac{M_{pl}^2}{2} R \approx -\frac{M_{pl}^2}{4} \nabla_\rho h_{\mu\nu} \nabla^\rho h^{\mu\nu} + \cdots$$

$$\left(\frac{\delta T}{T}\right)_{tensor} \approx \delta h_{\mu\nu} \approx \frac{\delta\psi_{\mu\nu}}{M_{pl}} \approx \frac{H}{M_{pl}}$$

canonically normalized gravitational wave perturbation

Direct probe of the energy scale of inflation.

Formulas for slow roll inflation

Slow roll parameters

$$\varepsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{V''}{V}$$

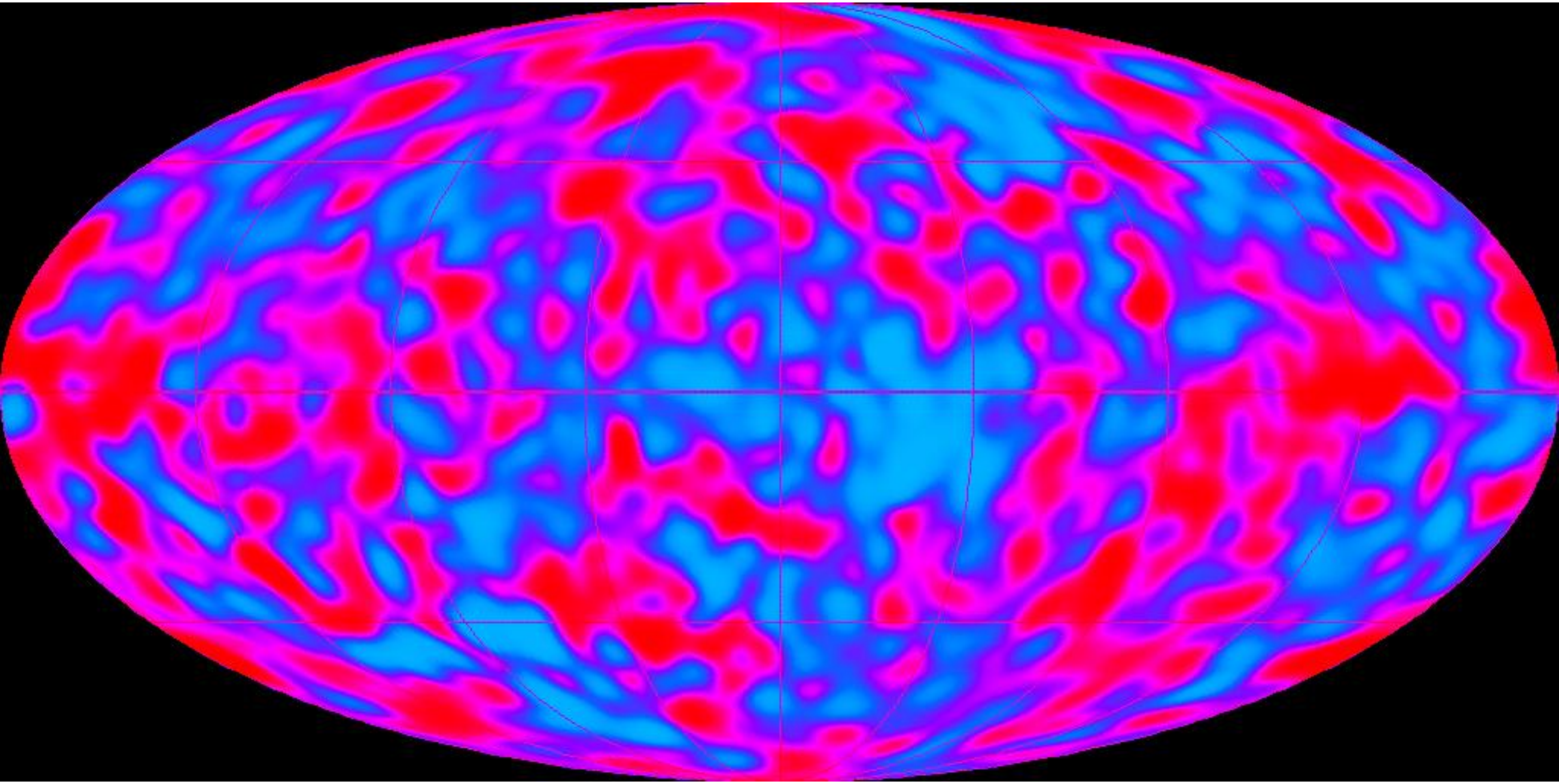
$$\Delta_\zeta^2 = \frac{V}{24\pi^2\varepsilon}$$:squared amplitude of curvature perturbation

$$n_s - 1 = -6\varepsilon + 2\eta$$:tilt of the spectrum $\Delta_\zeta \propto k^{n_s - 1}$

$$r \equiv \frac{\Delta_h^2}{\Delta_\zeta^2} = 16\varepsilon$$:tensor-to-scalar ratio

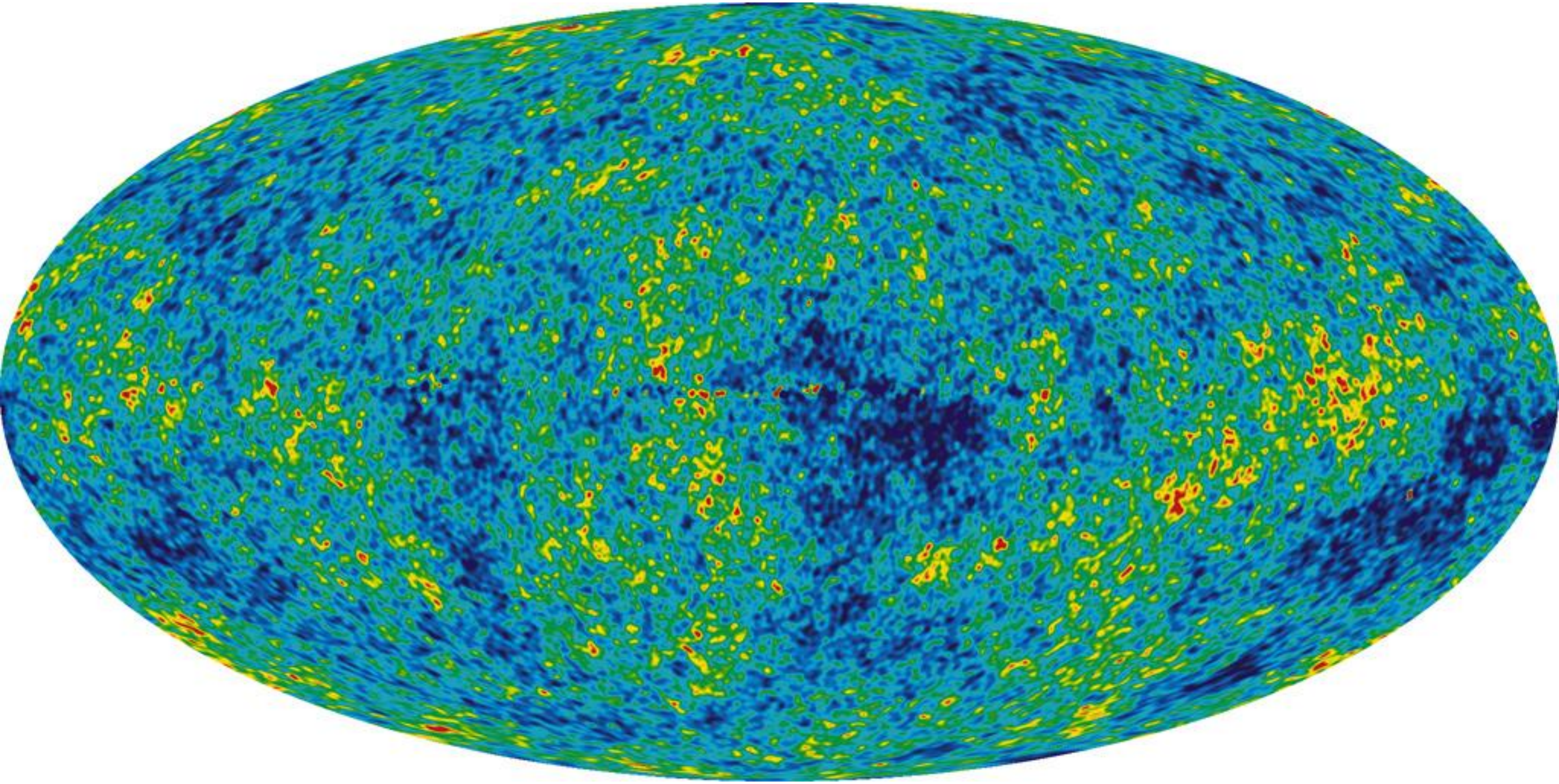
$$n_t = -2\varepsilon$$:tilt of tensor perturbation

CMB map by COBE satellite



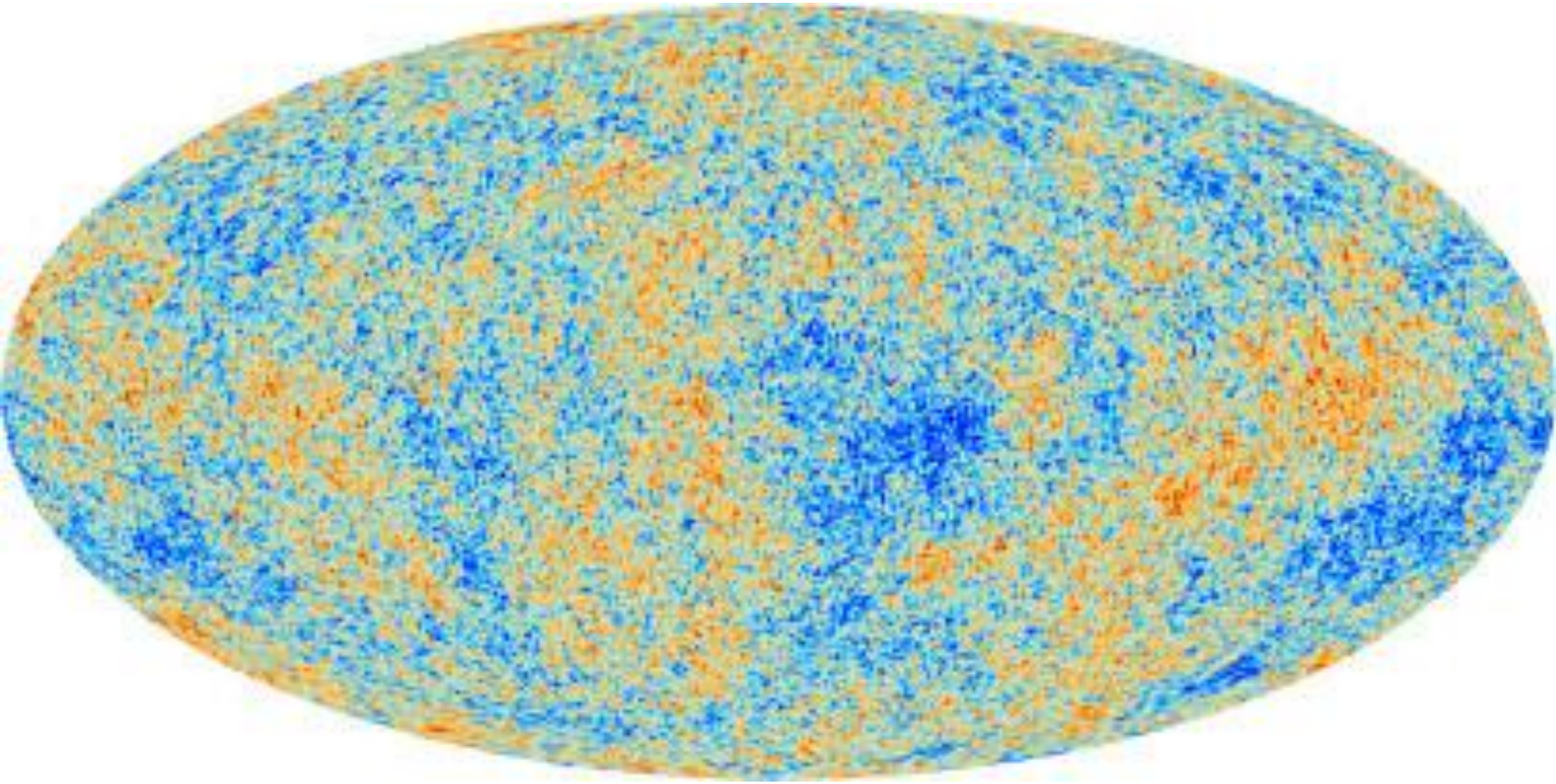
Amplitude of fluctuation is about 10^{-5}.

CMB map by WMAP satellite



Amplitude of fluctuation is about 10^{-5}.

CMB map by Planck satellite



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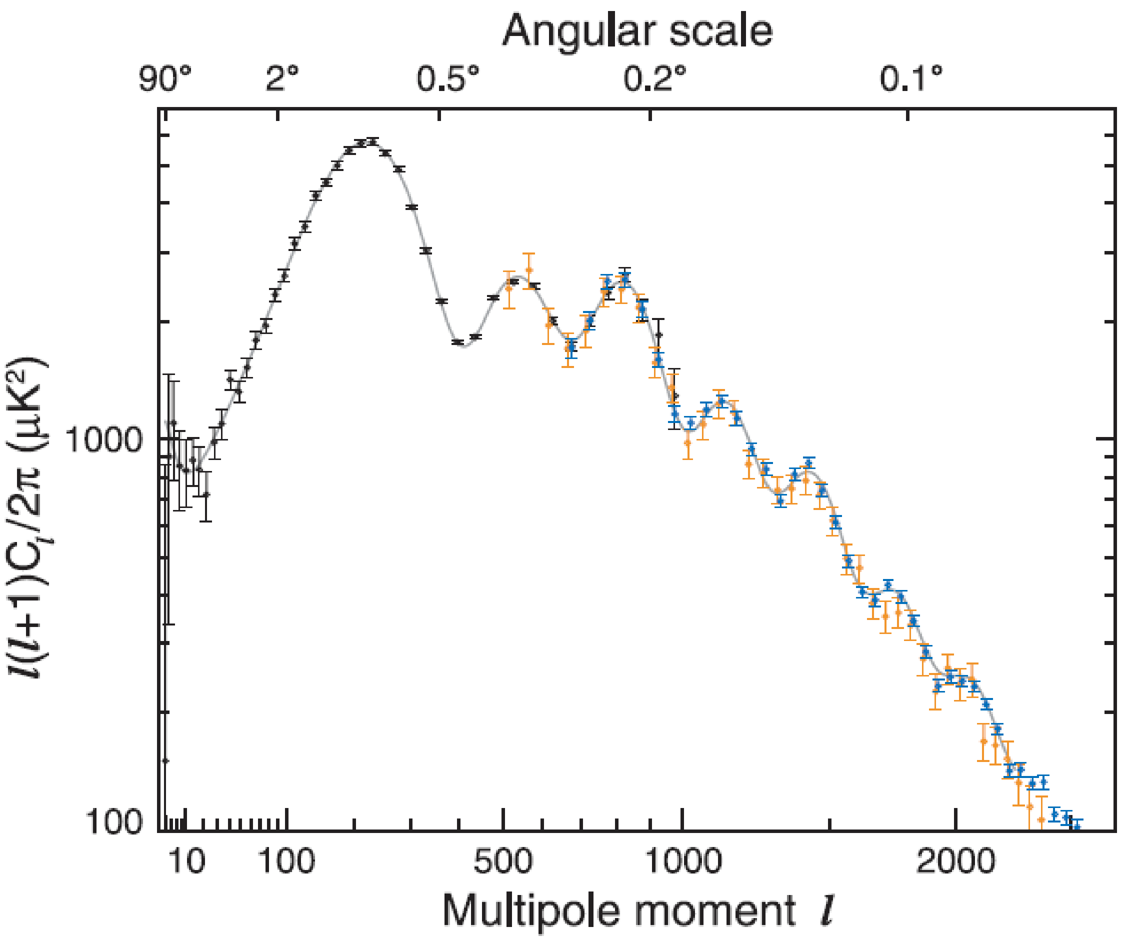
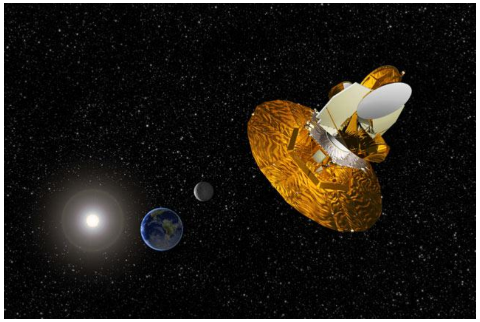
Success of inflationary model

WMAP9yr+SPT+ACT





WMAP

$$-0.0011 < \Omega_K < 0.0066$$

$$\Omega_\Lambda = 0.712 \pm 0.010$$

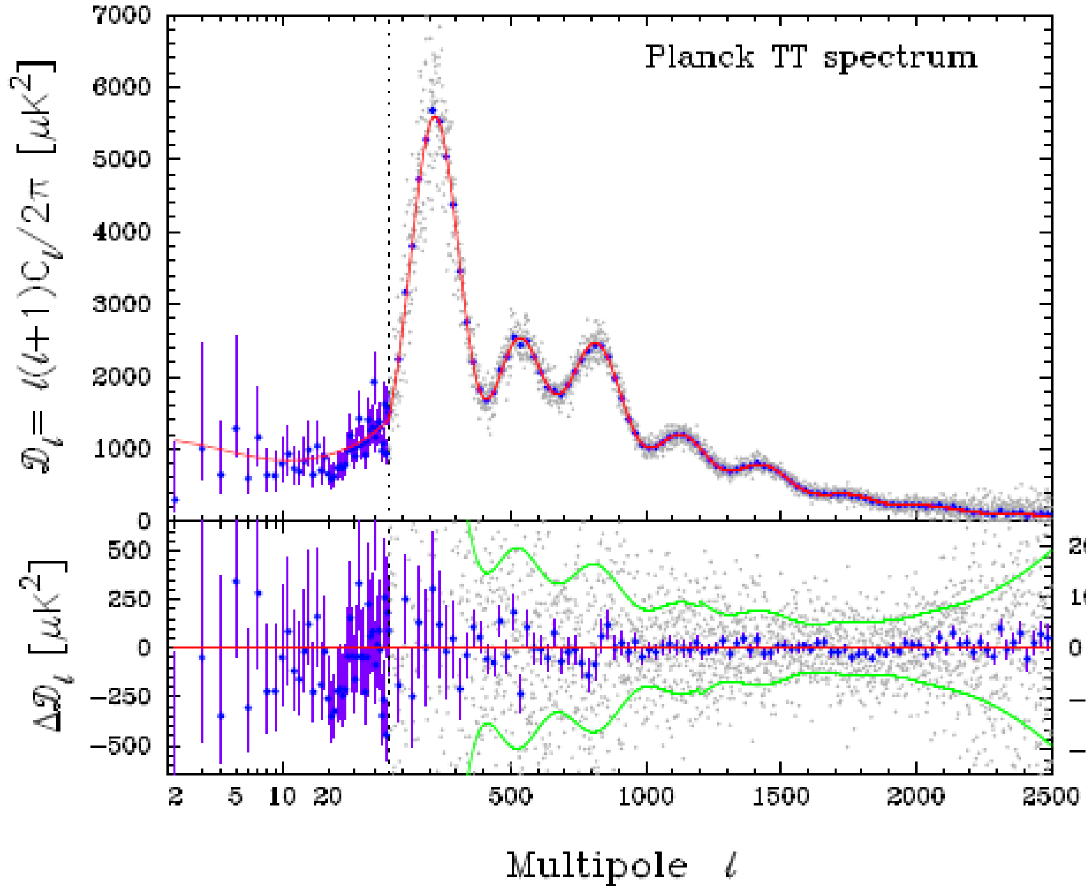
$$\Delta_\zeta^2 = 2.427 \times 10^{-9}$$

$$n_s = 0.971 \pm 0.010$$

$$r < 0.13$$

Planck(2013)

http://www.sciops.esa.int/index.php?project=PLANCK&page=Planck_Published_Papers



$$-0.0075 < \Omega_K < 0.0052\,(2\sigma)$$

$$\Omega_\Lambda = 0.685^{+0.018}_{-0.016}$$

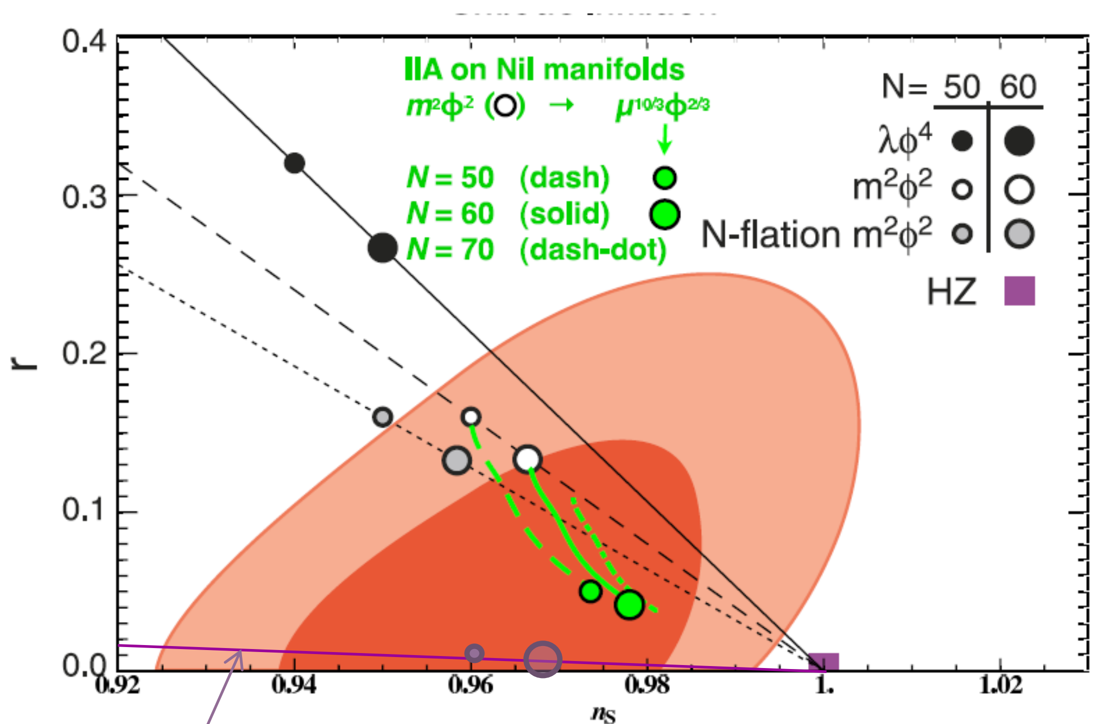
$$\Delta^2_\zeta = 2.195\times 10^{-9}$$

$$n_s = 0.9603 \pm 0.0073$$

$$r_{0.002} < 0.11(2\sigma)$$

Constraints on inflation models (WMAP)

Scalar-tensor ratio from power spectrum fit



Spectral tilt

WMAP+BAO+H_0 (not latest)

- Starobinsky inflation

$$L_{grav} = M_{pl}^2\left(R + R^2/6M^2\right)$$

⟺

Einstein gravity + single scalar

$$V = \frac{3M^2}{4}\left(1 - e^{-\sqrt{2/3}\phi}\right)^2$$

Constraints on inflation models (after Planck)

+ $dn_s/d(\log k)$ (slightly misleading)
power low potentials are not saved

+ N_{eff} (effective number of n species)

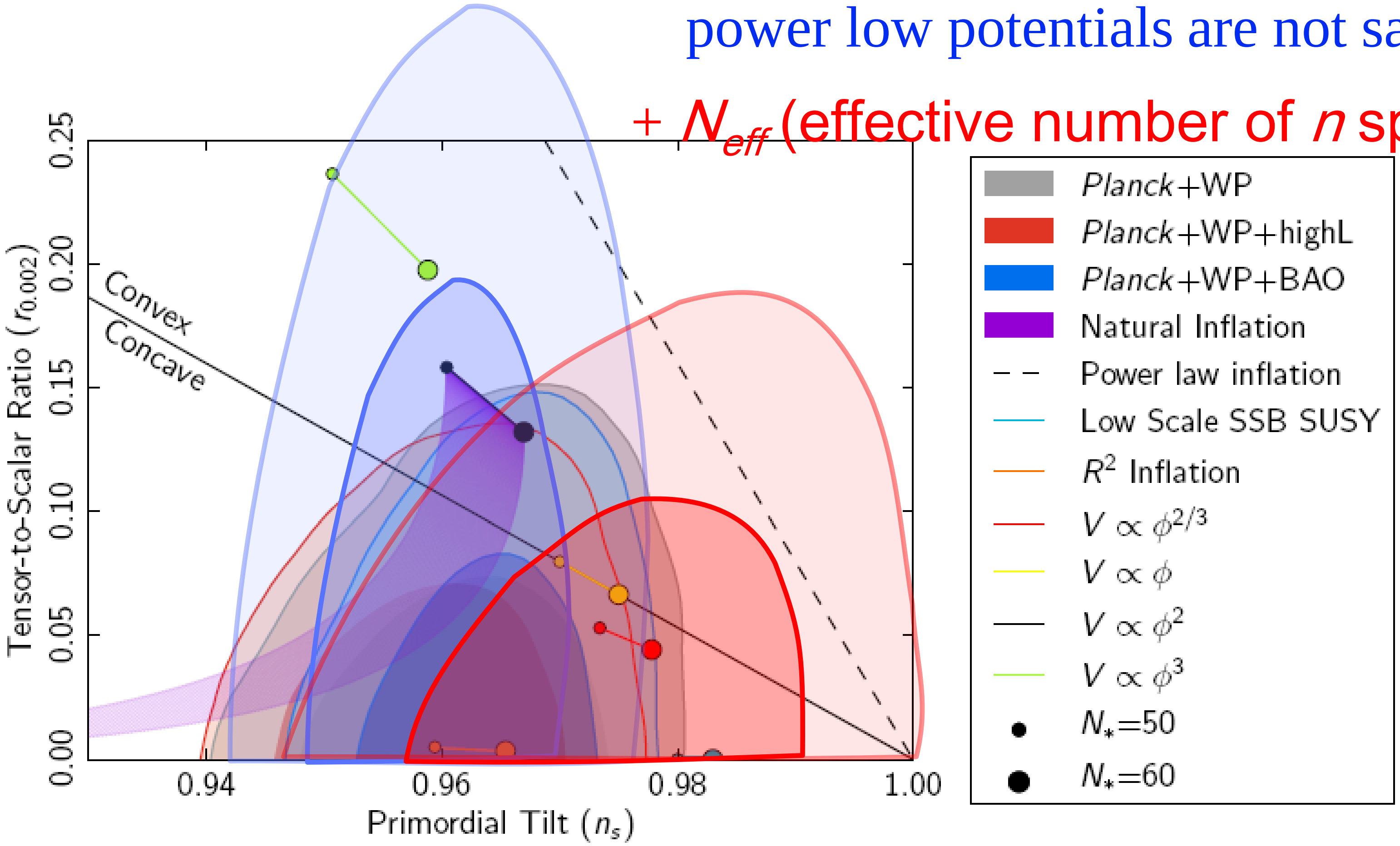


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Large tensor from inflation

Large tensor perturbation requires large field inflation

$$r = 16\varepsilon = 8\left(\frac{V'}{V}\right)^2 \approx \frac{8\Delta\phi^2}{N^2}$$: Lyth bound $$\frac{d\phi}{dN} = \frac{\dot{\phi}}{H} \approx -\frac{V'}{3H^2} \approx -\frac{V'}{V}$$

SUGRA:

Scalar field potential

$$V = e^K \left[K_{\Phi\bar{\Phi}}^{-1} \left| D_\Phi W \right|^2 - 3\left| W \right|^2 \right]$$

$$D_\Phi W = \frac{\partial W}{\partial \Phi} + \frac{\partial K}{\partial \Phi} W$$

$K_{\Phi\bar{\Phi}} \partial_\mu \Phi \partial^\mu \bar{\Phi}$:kinetic term

Canonical choice of Kähler potential is $K = \Phi\bar{\Phi}$, for which $K_{\Phi\bar{\Phi}} = 1$.

- Exponential growth of potential for ϕ > 1.
- η -problem: $m^2 = O(H^2)$

A solution is to choose $$K = -\frac{1}{2}\left(\Phi - \bar{\Phi}\right)^2 = \underline{\Phi\bar{\Phi}} - \frac{1}{2}\left(\Phi^2 + \bar{\Phi}^2\right)$$

kinetic term is canonical

Kawasaki, Yamaguchi and Yanagida (2000)

Realizing Large field inflation

String:

Moduli/brane in internal space:
$\Delta\phi >> M_{pl}$ ⇨ long internal space.

If the volume of internal space is large, it is also disfavored:

$$M_{pl}^2 \approx M_{10D}^8 Vol_6$$

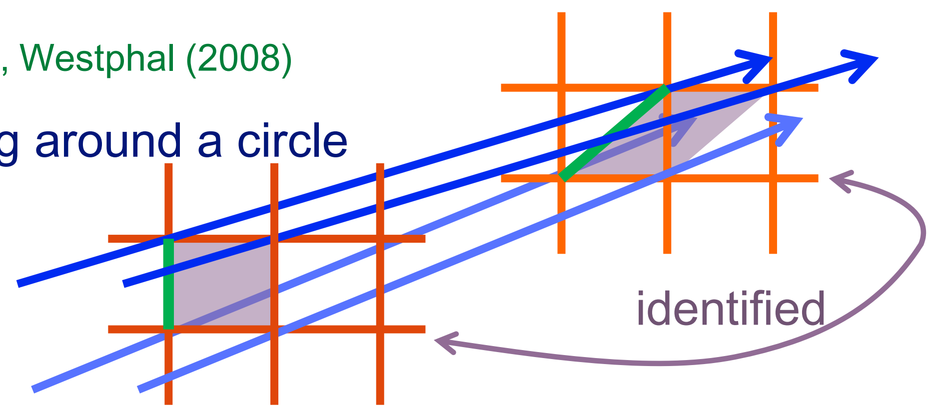
⇕ difficult to be compatible

Small ε and η ⇨ small backreaction to the whole internal space
⇨ strong stabilization

- Monodromy

Silverstein, Westphal (2008)

Roughly speaking, wrapping around a circle



- N-flation

Dimopoulos, Kachru, McGreevy, Wacker (2005)

N scalar fields ⇨ lager H

⇨ slower rolling ⇨ smaller ε and η

Constraint on non-Gaussianity

Non-Gaussianity ⟺ effects of non-linear dynamics during and after inflation

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{-\mathbf{k}-\mathbf{k}'} \rangle \neq 0$$

WMAP 9yr

$$-3 < f_{NL}^{local} < 77 \; (95\% CL)$$

$$-221 < f_{NL}^{equil} < 323 \; (95\% CL)$$

$$-445 < f_{NL}^{orthog} < -45 \; (95\% CL)$$

Planck

$$f_{NL}^{local} = 2.7 \pm 5.8 \; (68\% CL)$$

$$f_{NL}^{equil} = -42 \pm 75 \; (68\% CL)$$

$$f_{NL}^{orthog} = -25 \pm 39 \; (68\% CL)$$

$$\tau_{NL} < 2800 \; (95\% CL)$$

$$c_s \geq 0.02 \; (95\% CL)$$

Non-Gaussianity

- In the standard slow roll inflation, non-Gaussianity is extremely suppressed.

  ⇨ Non-Gaussianity requires non-standard inflation models.

- Non-linear dynamics gives non-linear mapping

$$\zeta_G \to \zeta = \zeta(\zeta_G)$$

Komatsu and Spergel (2001)

$$\zeta(\boldsymbol{x}) = \underline{\zeta_G(\boldsymbol{x})} + \frac{3}{5}\underline{f_{NL}}\,\zeta_G^2(\boldsymbol{x})$$

Gaussian variable　　Non-linear parameter

$$\left\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3} \right\rangle \equiv \delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\underline{B_\zeta(k_1,k_2,k_3)}$$

bispectrum

$$B_\zeta(k_1,k_2,k_3) = \frac{6}{5}\frac{f_{NL}}{(2\pi)^{3/2}}\left[P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_3)P_\zeta(k_1)\right]$$

In general, mapping is non-local.

Local interaction only ~ Super horizon dynamics

$$\zeta(t_c) \approx \delta N = N^*_a \varphi^a_* + \frac{1}{2} N^*_{ab} \varphi^a_* \varphi^b_* + \cdots$$

$$N_a(t) \equiv \left. \frac{\partial N(t_c, \phi)}{\partial \phi^a} \right|_{\phi^a = \phi^a(t)}$$

"*" indicates a time just after initial horizon crossing

$$N_{ab}(t) \equiv \left. \frac{\partial^2 N(t_c, \phi)}{\partial \phi^a \partial \phi^b} \right|_{\phi^a = \phi^a(t)}$$

$$\langle \zeta(\mathbf{x}_1) \zeta(\mathbf{x}_2) \zeta(\mathbf{x}_3) \rangle = N^*_a N^*_b N^*_c \underline{\langle \varphi^a_*(\mathbf{x}_1) \varphi^b_*(\mathbf{x}_2) \varphi^c_*(\mathbf{x}_3) \rangle}$$

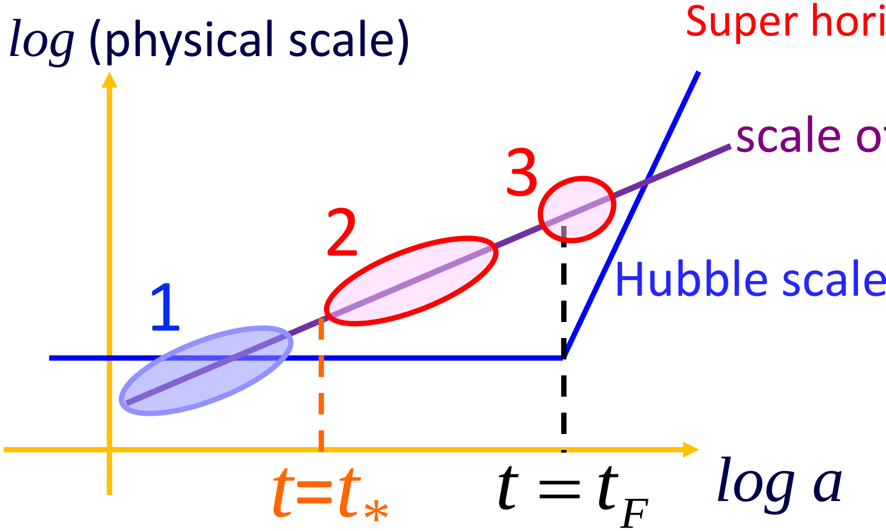
Early generation of non-Gaussianity

➔ suppressed by slow-roll parameters. (Seery & Lidsey (2005))

Exception is fast roll inflation.

$$+ \frac{1}{2} N^*_a N^*_b N^*_{cd} \left[\langle \varphi^a_*(\mathbf{x}_1) \varphi^b_*(\mathbf{x}_2) \varphi^c_*(\mathbf{x}_3) \varphi^d_*(\mathbf{x}_3) \rangle + \mathrm{perm} \right]$$

Super horizon part of non-Gaussianity



Non Gaussianity is produced

1) before horizon crossing

2) during super horizon evolution

3) at the end of or after inflation

2) or 3)are local ⇨

$$\frac{6}{5} f_{NL} \approx \frac{N^a_* N^b_* N^*_{ab}}{\left(N^c_* N^*_c\right)^2}$$

Non-Gaussianity produced at the end of or after inflation

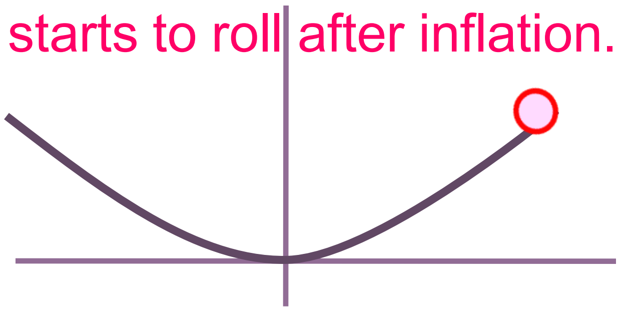
Curvaton (Lyth & Wands (2002))

Modulated reheating (Dvali, Gruzinov & Zaldarriaga (2004))

Modulated waterfall (Bernardeau, Kofman and Uzan (2004), Lyth (2004))

Ex.) Curvaton

σ starts to roll after inflation.



$$\zeta_\sigma \approx \frac{\delta\rho_\sigma}{\rho_{tot}} = r\left(2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma}\right)^2\right) \qquad r \equiv \frac{\rho_\sigma}{\rho_{tot}}$$

Suppose ζ_σ is the dominant component of fluctuation.

$$\rho_\sigma \approx \frac{m^2}{2}(\sigma + \delta\sigma)^2$$

Amplitude is observationally fixed.

$$P_\zeta \approx \left(r\frac{\delta\sigma}{\sigma}\right)^2 = 10^{-9}$$

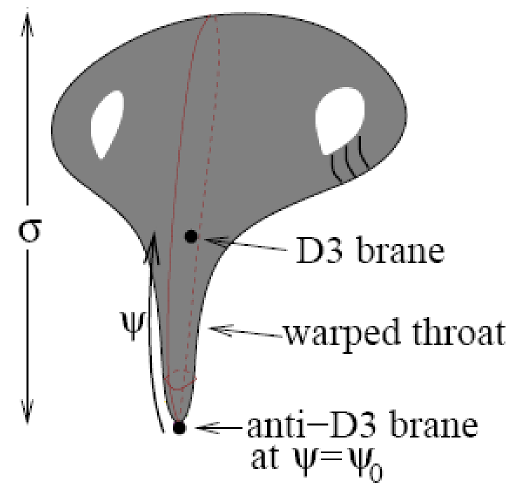
$$f_{NL} = \frac{1}{r}$$ can be as large as 10^5.

Non-local Non-Gaussianity from non-canonical kinetic term

- Typical example is DBI inflation — Alishhiha, Silverstein, Tong (2008)

Moving D3-brane in a higher-dimensional background ⇔ (AdS/CFT) Strong coupling large N CFT

$$ds^2 = h^{-1/2}(y^K) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^K) G_{IJ}(y^K) dy^I dy^J$$



$$\Rightarrow \quad L_{eff} = -\frac{1}{f}\sqrt{-\det\left(g_{\mu\nu} + f\, G_{IJ}\partial_\mu \phi^I \partial_\nu \phi^J\right)} - V(\phi)$$

$$f = \frac{h}{T_3} \qquad \phi^I = \sqrt{T_3}\,\delta y^I$$

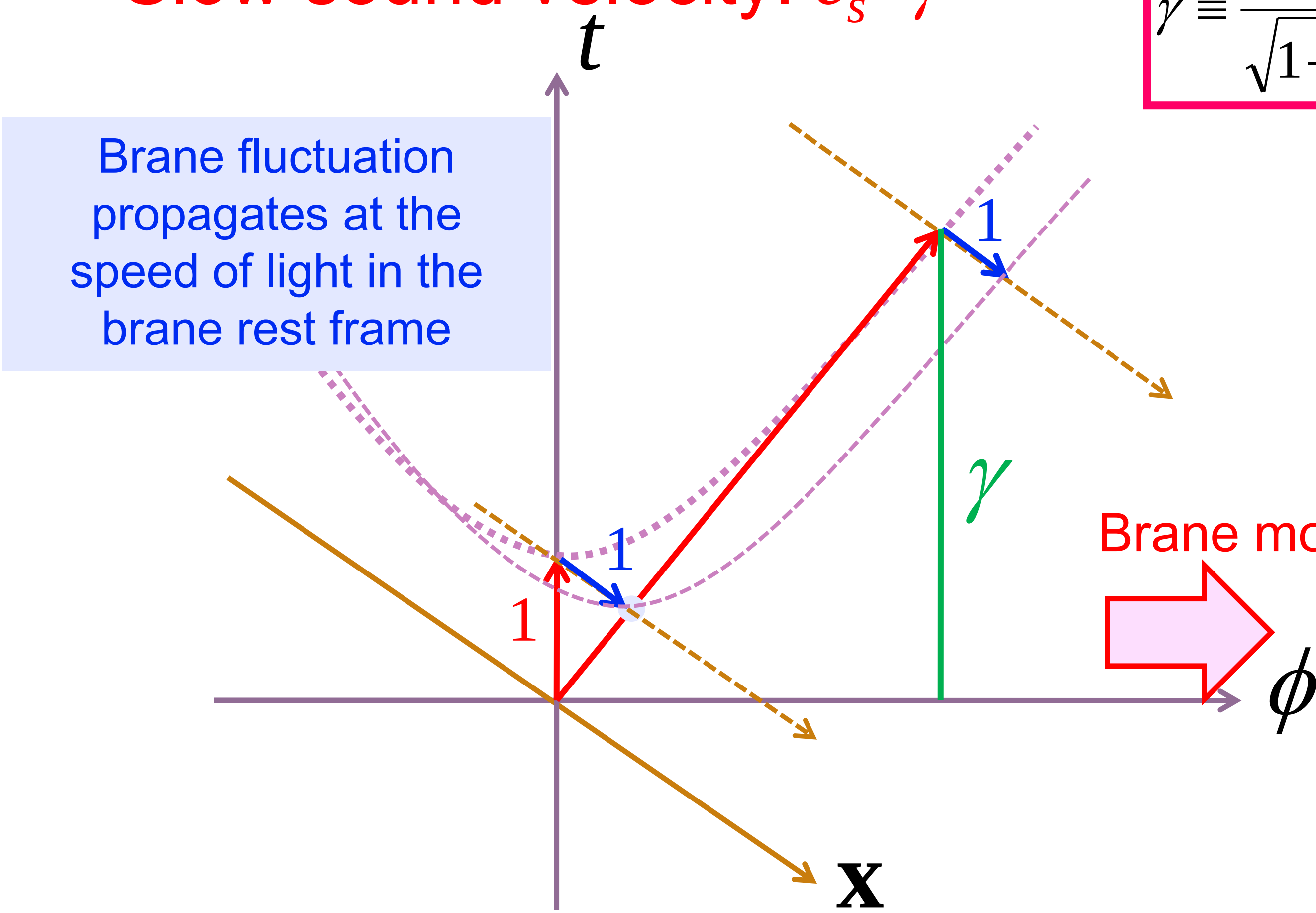
Speed limit:

$$\sqrt{-\det(\cdots)} \xrightarrow{\text{spatially homogeneous}} \sqrt{-g}\sqrt{\left(1 - f\dot{\phi}^2\right)}$$

Even if V' is large, $|\dot{\phi}| < f^{-1/2}$ ⇒ smaller ε and η

Slow sound velocity: $c_s = \gamma^{-1}$

$$\gamma \equiv \frac{1}{\sqrt{1 - f\dot{\phi}^2}}$$



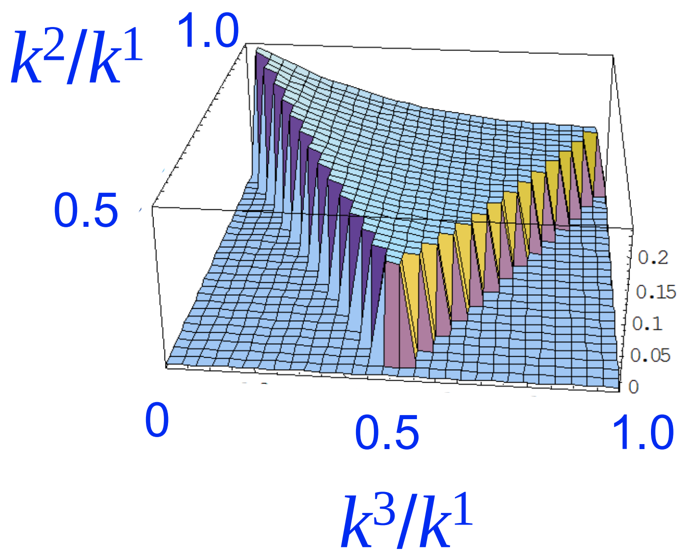
$$\Delta_\zeta^2 = \frac{H^2}{8\pi^2 \varepsilon c_s}$$

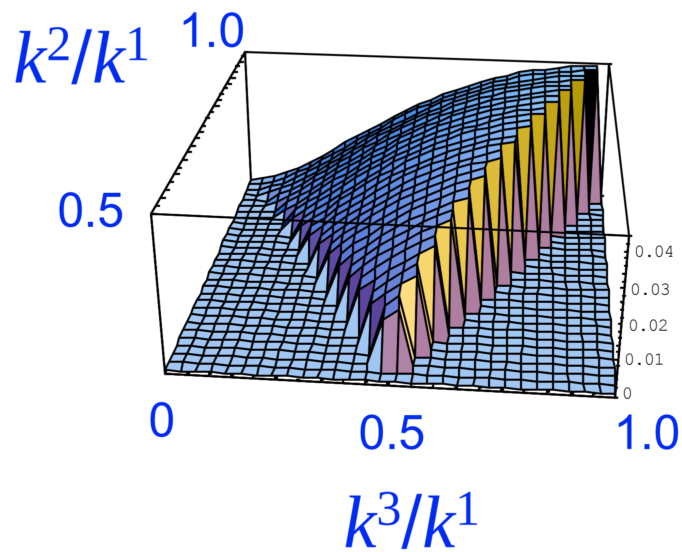
$$f_{NL}^{equil} = -\frac{35}{108}\frac{1}{c_s^2}$$ enhanced $$c_s \geq 0.02$$

$$r = 16\varepsilon c_s$$ suppressed

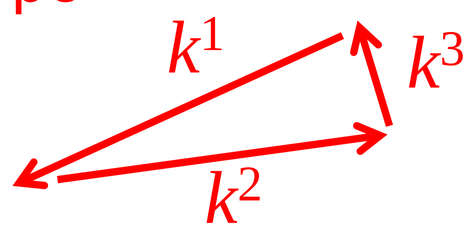
Curvaton bi-spectrum

DBI bi-spectrum

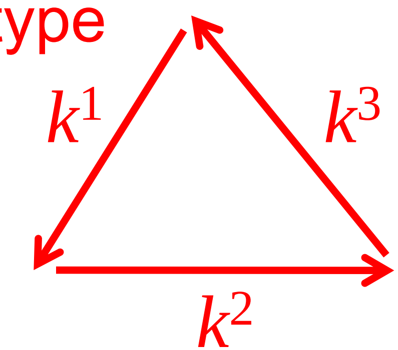




local-type



equilateral-type



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Some anomalies

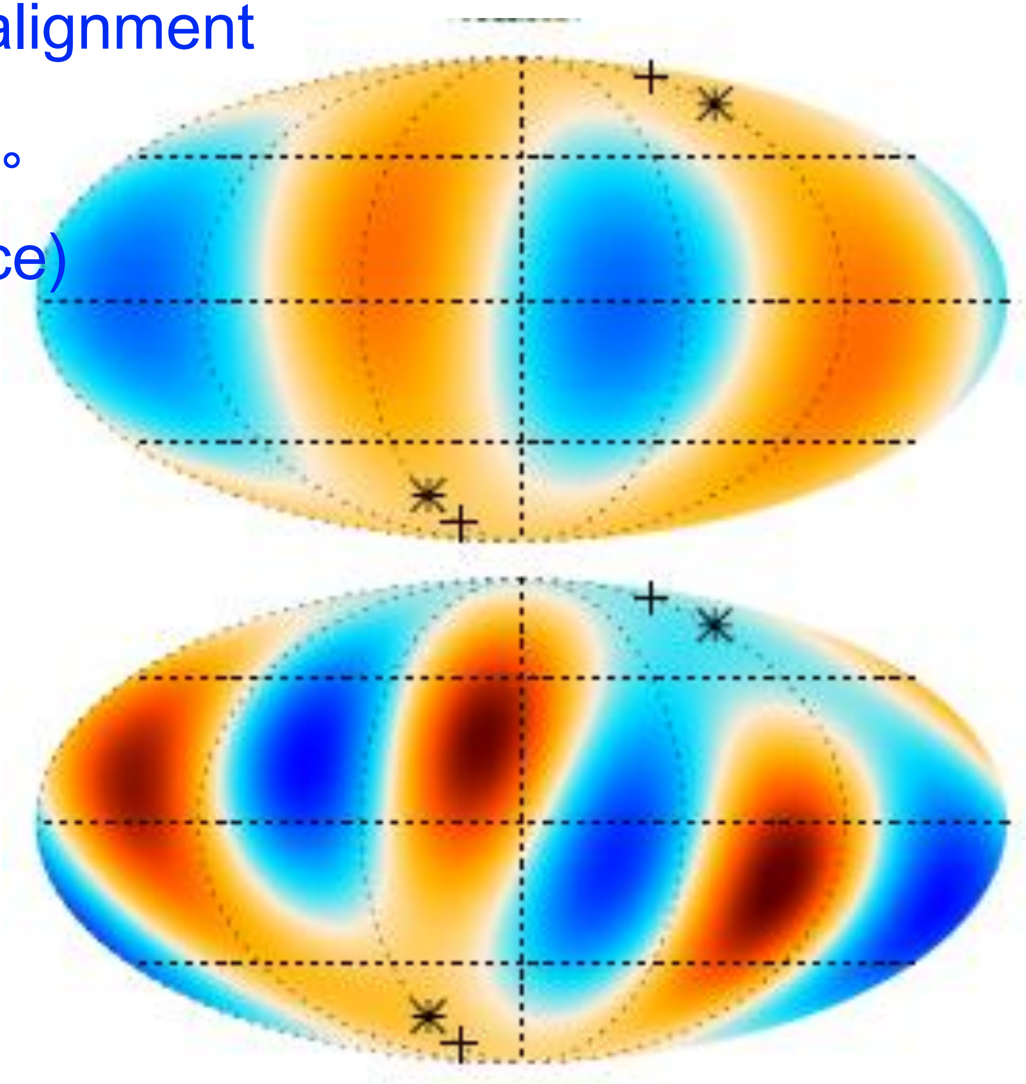
1303.5083で議論されているが、何を書いているのかわからん、かなりひどい論文

Quadrupole-octopole alignment

WMAP: 3°

→ Planck: 9° ～13°

(2σ level significance)

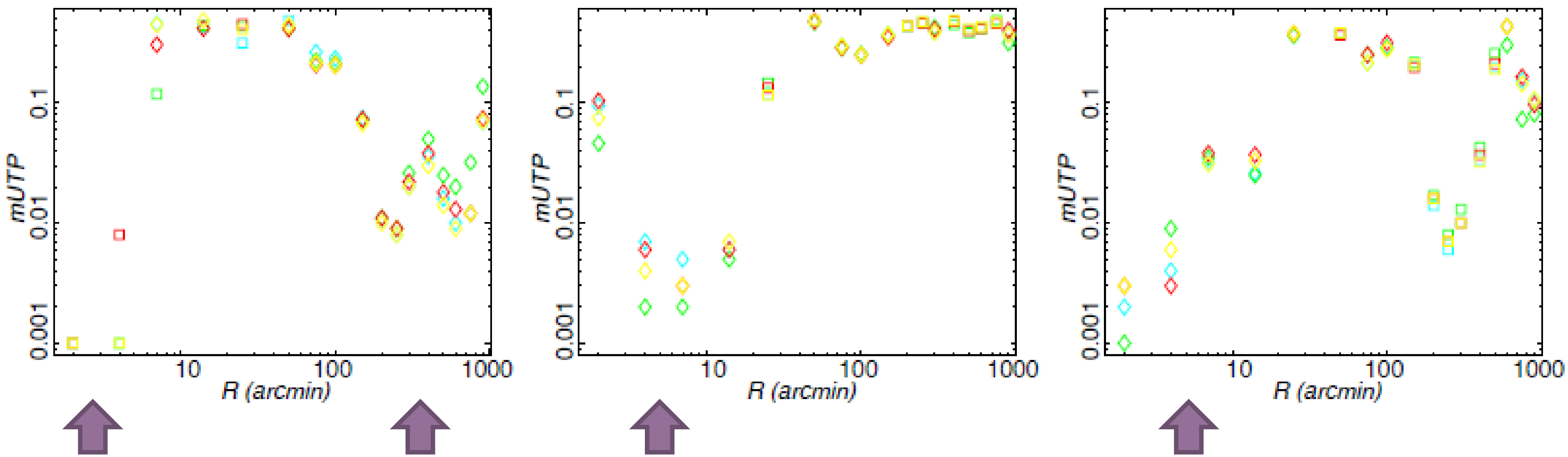


Some anomalies

Wavelet statistics:

Upper tail probabilities = the fraction of the simulations that present a value of a given statistic equal to or greater than the one obtained for the data

variance | skewness | curtosis



Too large | Too small | Too small | Too small

Some anomalies

Hemispherical asymmetry

number of pixels over the full sky is $12 \times N_{side}^2$

Mask	Variance	Skewness	Kurtosis
U73, f_{sky} =73%	0.017	0.189	0.419
CL58, f_{sky} =58%	0.003	0.170	0.363
CL37, f_{sky} =37%	0.030	0.314	0.266
Ecliptic North, f_{sky} =36%	0.001	0.553	0.413
Ecliptic South, f_{sky} =37%	0.483	0.077	0.556
Galactic North, f_{sky} =37%	0.001	0.788	0.177
Galactic South, f_{sky} =36%	0.592	0.145	0.428

Lower tail probabilities for N_{side} =2048
Small number means that the data is statistically very unlikely.

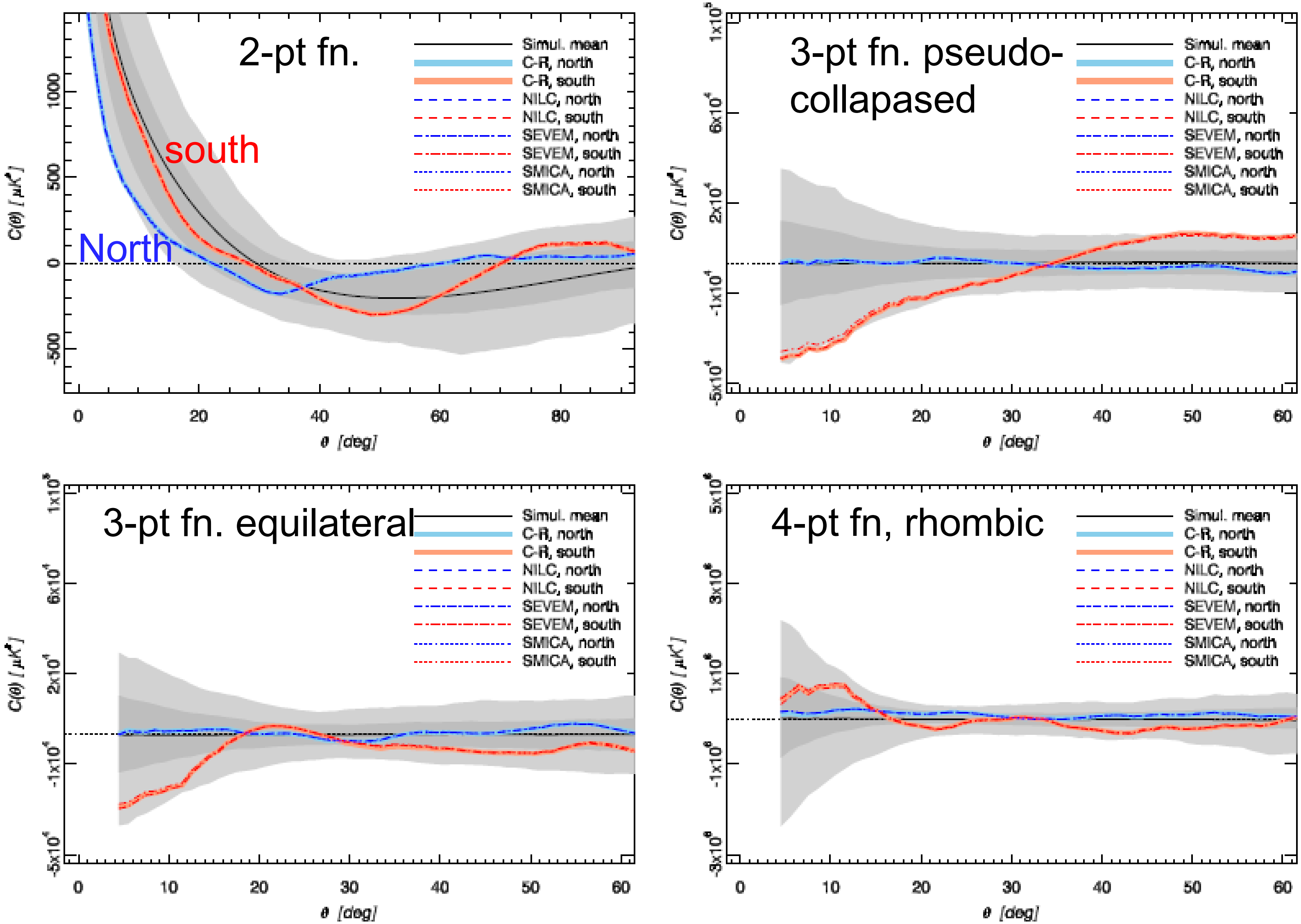
Mask	C-R	NILC	SEVEM	SMICA
	Variance			
U73, f_{sky} = 78%	0.019	0.017	0.014	0.019
CL58, f_{sky} = 58%	0.004	0.003	0.003	0.003
CL37, f_{sky} = 37%	0.028	0.017	0.018	0.016
Ecliptic North, f_{sky} = 39%	0.001	0.001	0.001	0.002
Ecliptic South, f_{sky} = 39%	0.464	0.479	0.454	0.490
	Skewness			
U73, f_{sky} = 78%	0.016	0.015	0.023	0.012
CL58, f_{sky} = 58%	0.208	0.139	0.162	0.147
CL37, f_{sky} = 37%	0.517	0.467	0.503	0.469
Ecliptic North, f_{sky} = 39%	0.502	0.526	0.526	0.521
Ecliptic South, f_{sky} = 39%	0.004	0.006	0.008	0.004

Lower tail probabilities for N_{side} =16

Some anomalies

Hemispherical asymmetry

$N_{side} = 64$



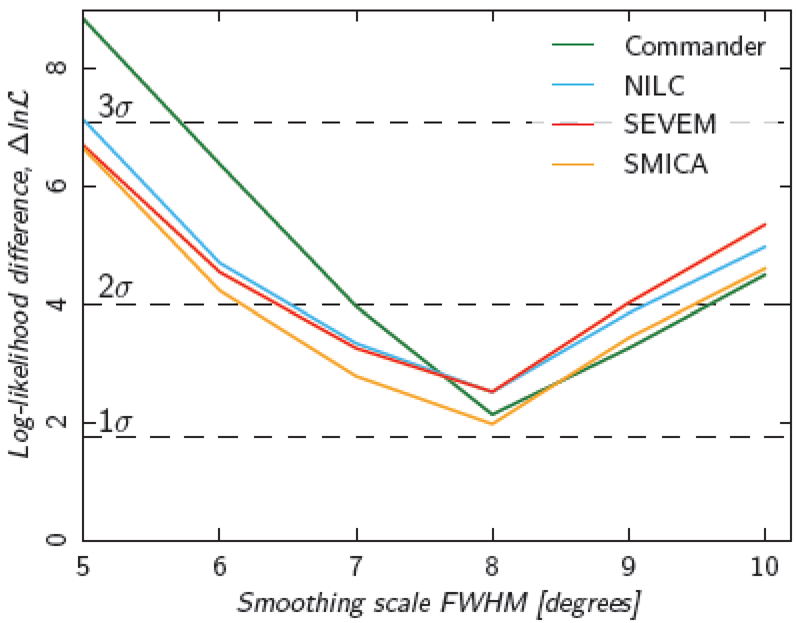
North sky is too featureless?

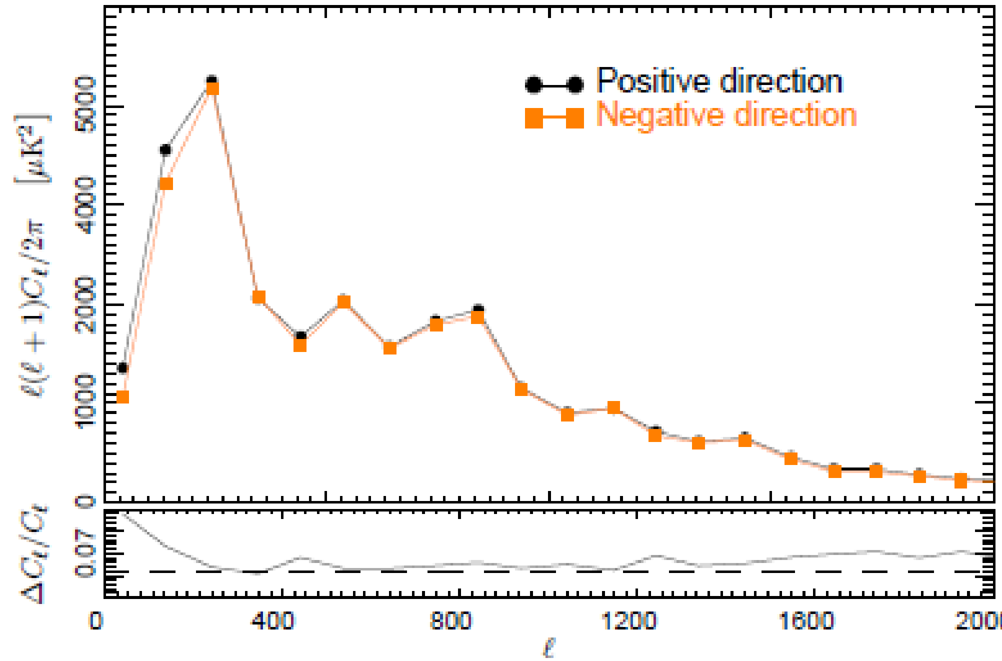
Some anomalies

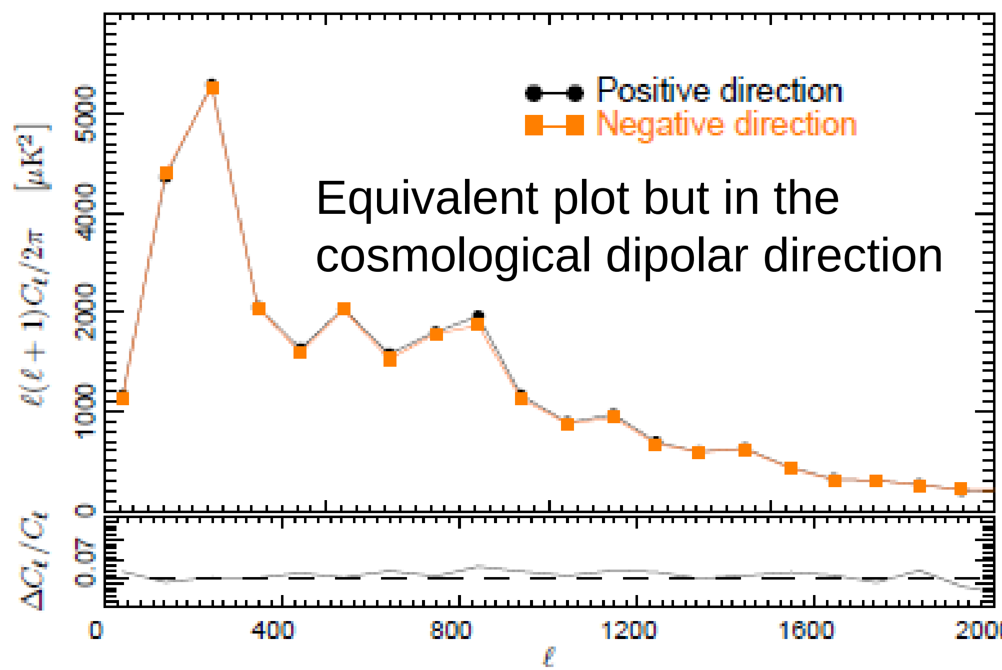
Dipolar power modulation

Model fit:

$$T(\boldsymbol{n}) = (1 + A\boldsymbol{p}\cdot\boldsymbol{n})T^{Iso}(\boldsymbol{n})$$

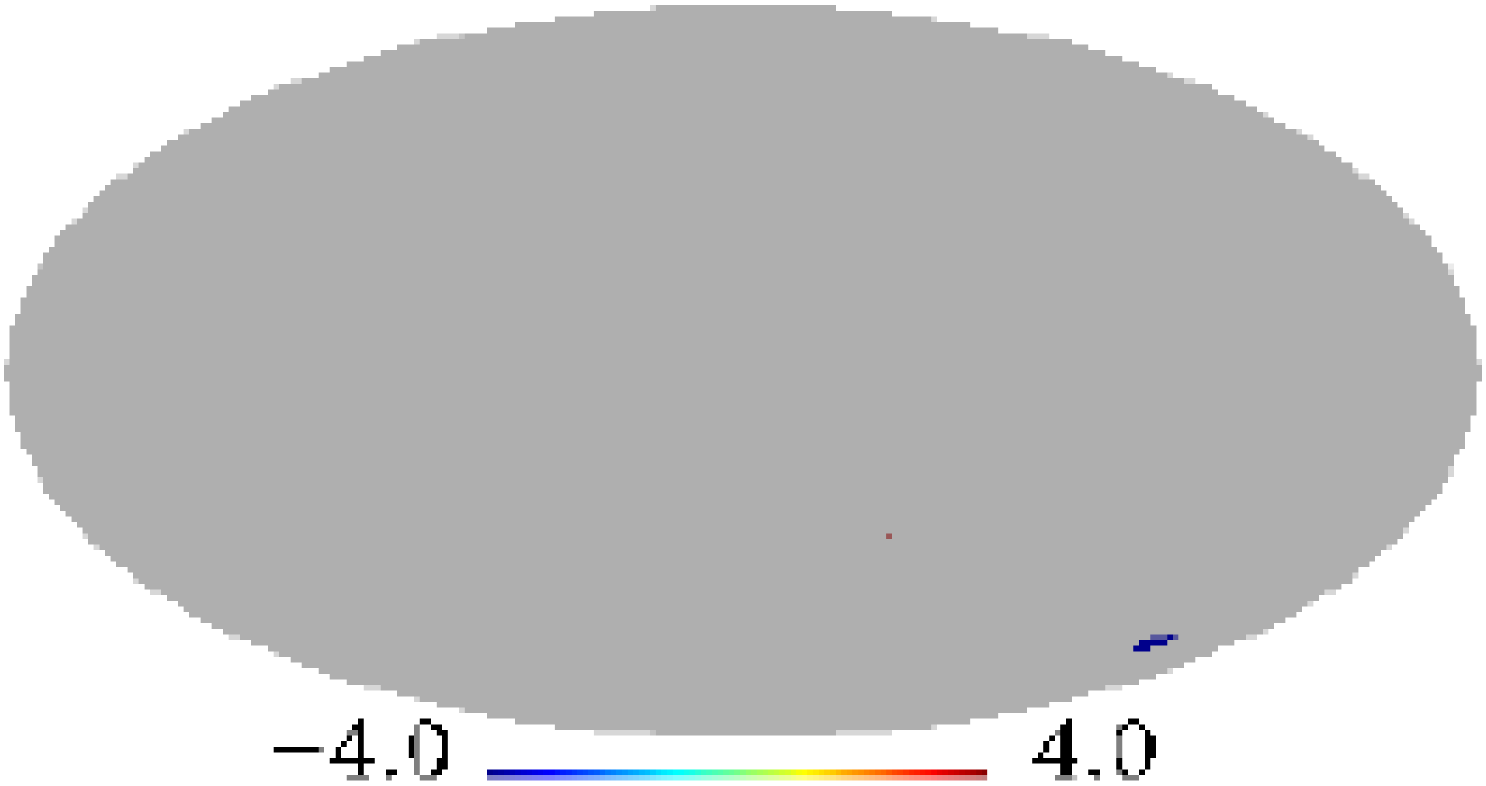






Some anomalies

Cold/hot Spots



Further steps from observations

Constraints on tensor perturbations from future observations:

r < 0.13 : WMAP

r < 0.05 : Planck (polarization data is not released yet. Coming in 2014)

r < 0.01 : QUIET, PolarBeaR, BICEP2, SPTpol, EBEX, Spider…

r < 0.001 : LiteBIRD, EPIC, PIXIE, COrE, B-Pol

Summary

- Tensor perturbations and non-Gaussianities in CMB are still key issues for understanding inflationary cosmology.
- Observations of the next generation will reduce the precision of tensor perturbation by factor 1/10 or more.
- Various inflation models make different prediction about tensor amplitude and amplitude/shapes of non-Gaussianities.
- Once they are detected, they become powerful tools to distinguish different models of inflation.