

益川塾セミナー

14 July 2012, 京都産業大学

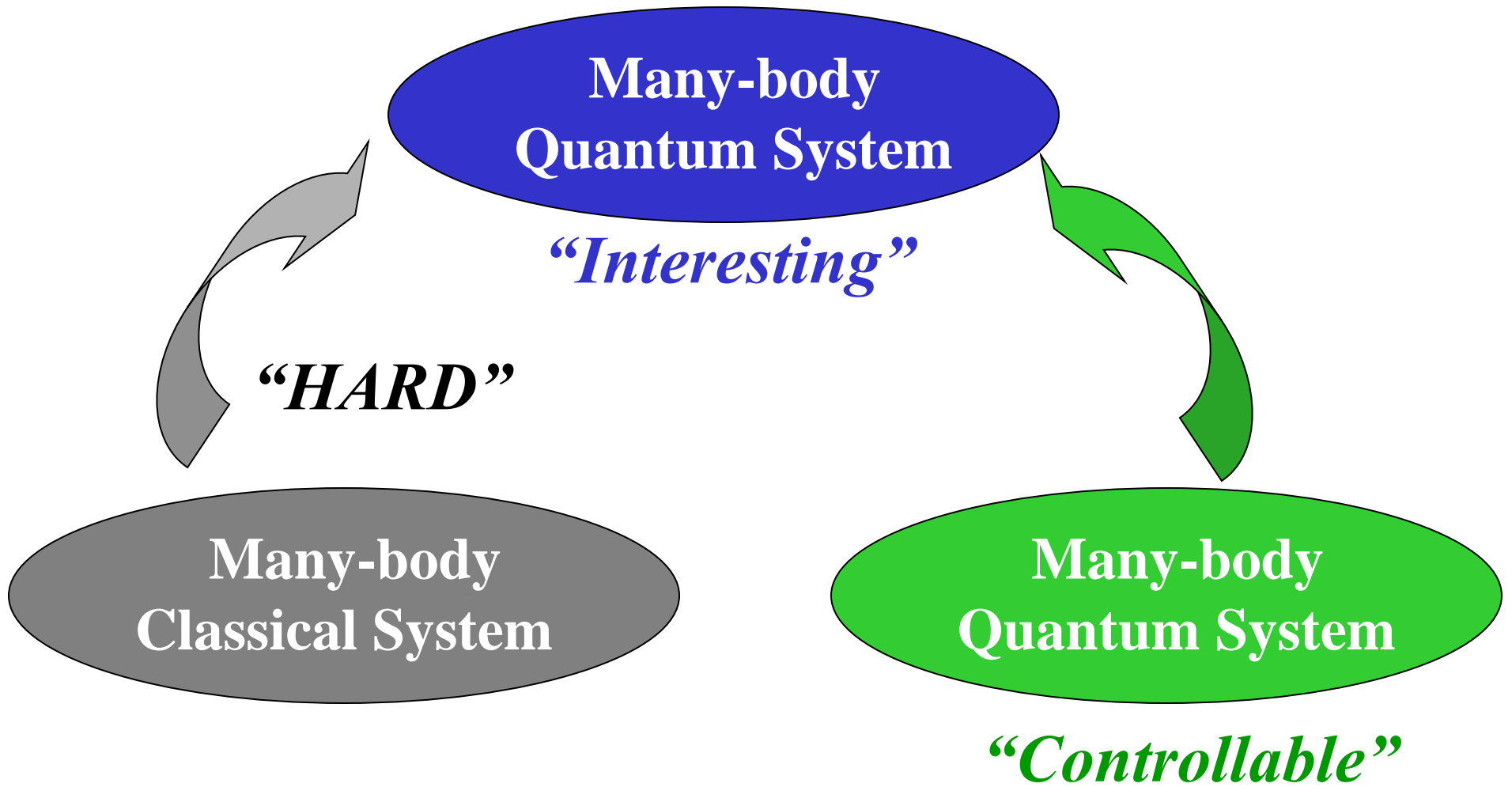
超低温原子気体の精密量子制御：
強相関量子多体系の量子シミュレーションと
基礎物理学への応用

Kyoto University, JST

Y. Takahashi



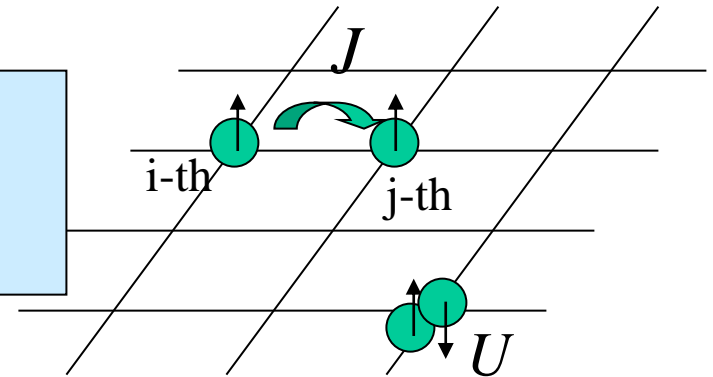
Quantum Simulation



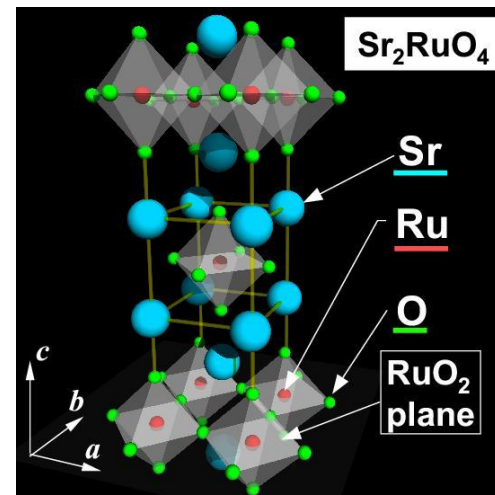
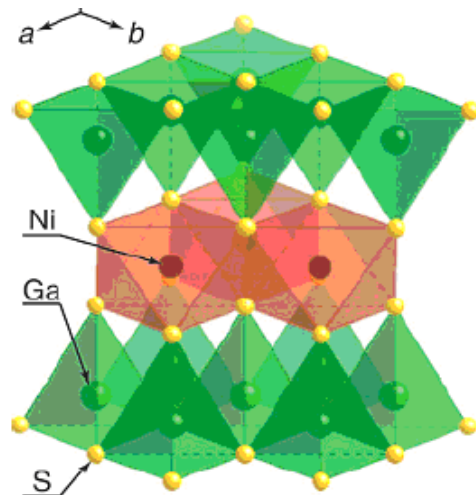
Quantum Simulation

Hubbard Model:

$$H = -J \sum_{\langle i,j \rangle} c_i^\dagger c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



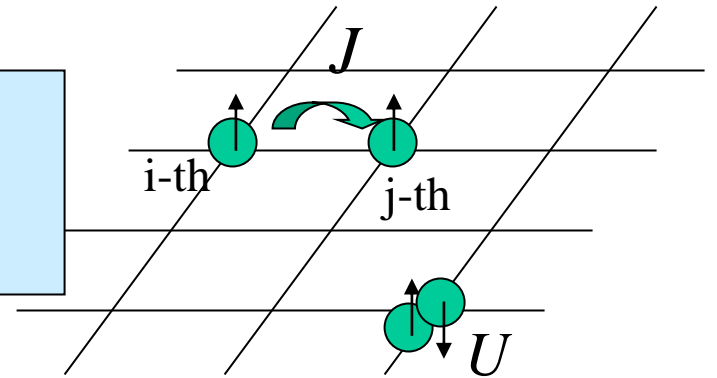
→ Magnetism, Superconductivity



Quantum Simulation

Hubbard Model:

$$H = -J \sum_{\langle i,j \rangle} c_i^+ c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



→ Numerical Calculation

DMFT(動的平均場)

Gutzwiller

QMC(量子モンテカルロ)

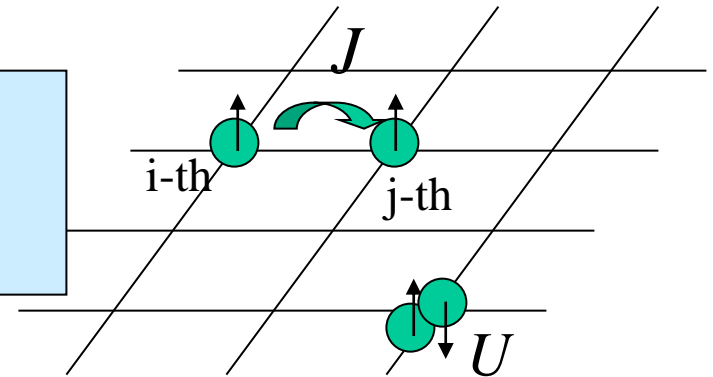
DMRG(密度行列繰り込み群)

Exact Diagonalization (厳密対角化)

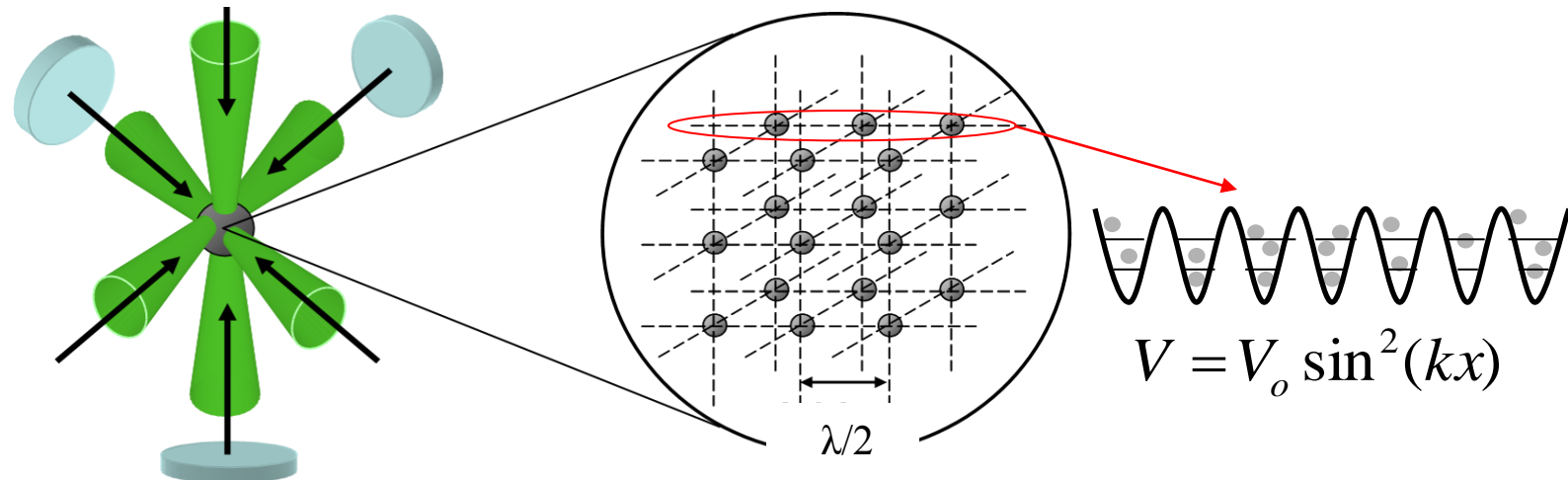
Quantum Simulation

Hubbard Model:

$$H = -J \sum_{\langle i,j \rangle} c_i^\dagger c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



→ Cold Atoms in Optical Lattice



Outline

Atom Manipulation Technique

Laser Cooling and Trapping

Optical Lattice

Tuning Interatomic Interaction

Quantum Simulation of Hubbard Model Using Alkali Atoms in an Optical Lattice

1) Bose-Hubbard Model

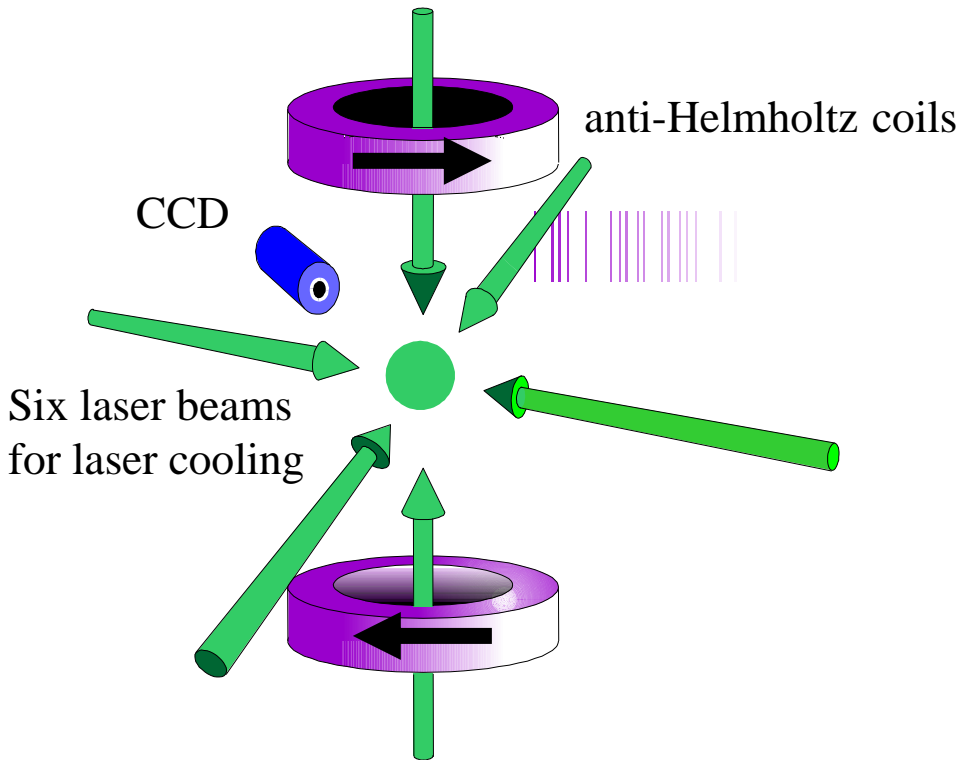
Superfluid-Mott Insulator Transition

Quantum Gas Microscope

2) Fermi-Hubbard Model

Mott insulator

Laser Cooling and Trapping



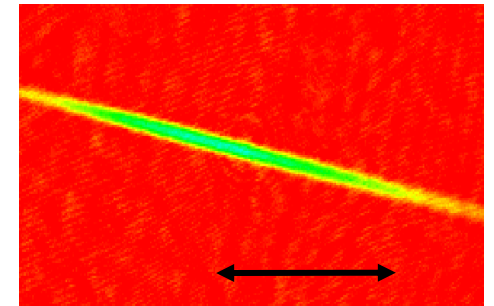
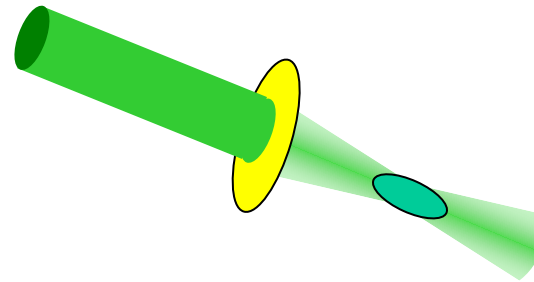
- Number: 10^7
- Density: $10^{11}/\text{cm}^3$
- Temperature: $10\mu\text{K}$

“Magneto-optical Trap”

“optical trap”

$$V_{\text{int}} = -p \cdot E$$

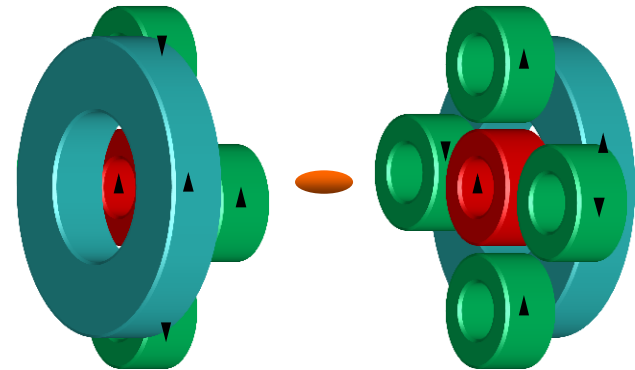
$$U_{\text{pot}}(r) = -\frac{\chi E(r)^2}{2}$$



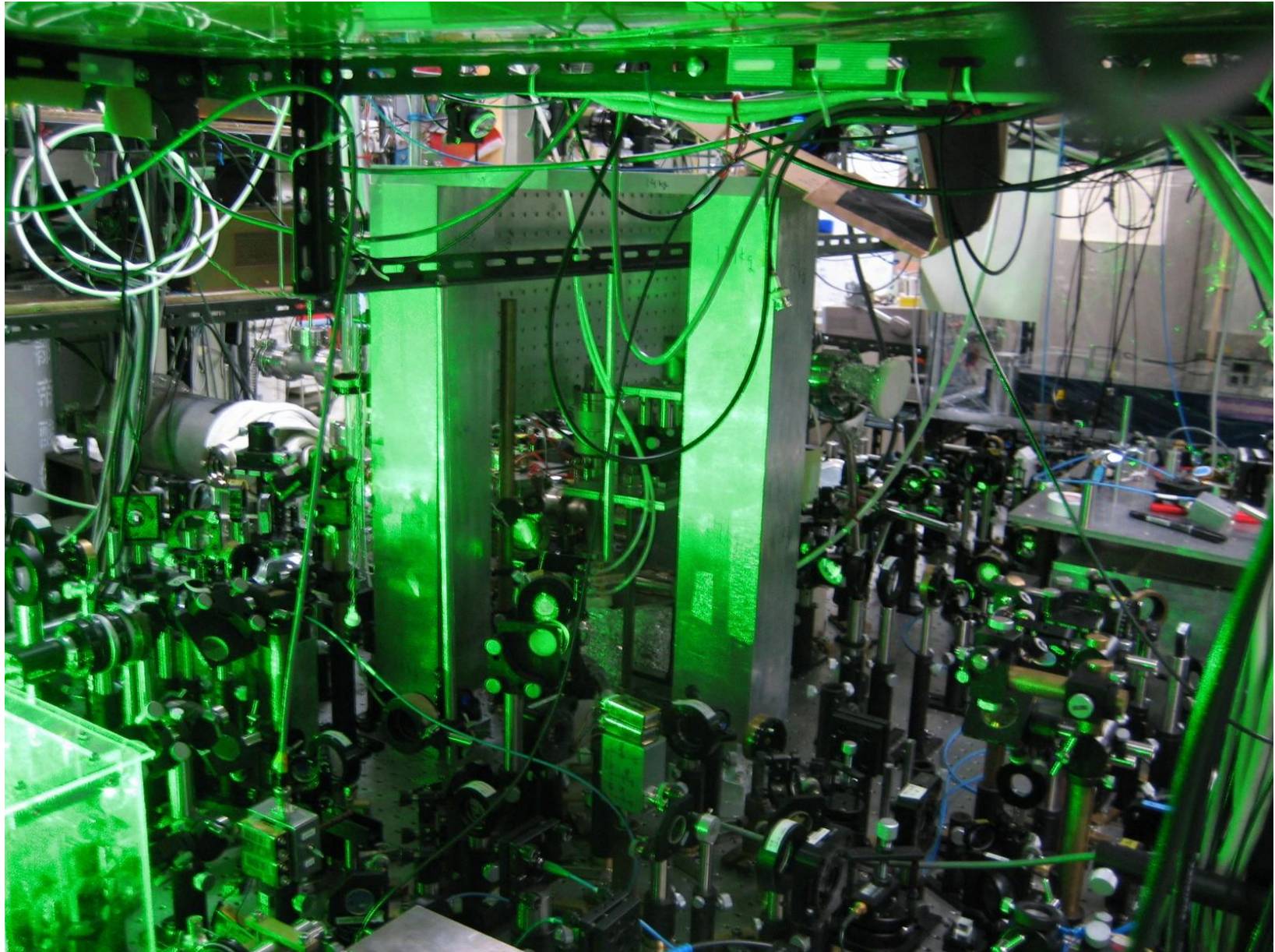
500 μm

“magnetic trap”

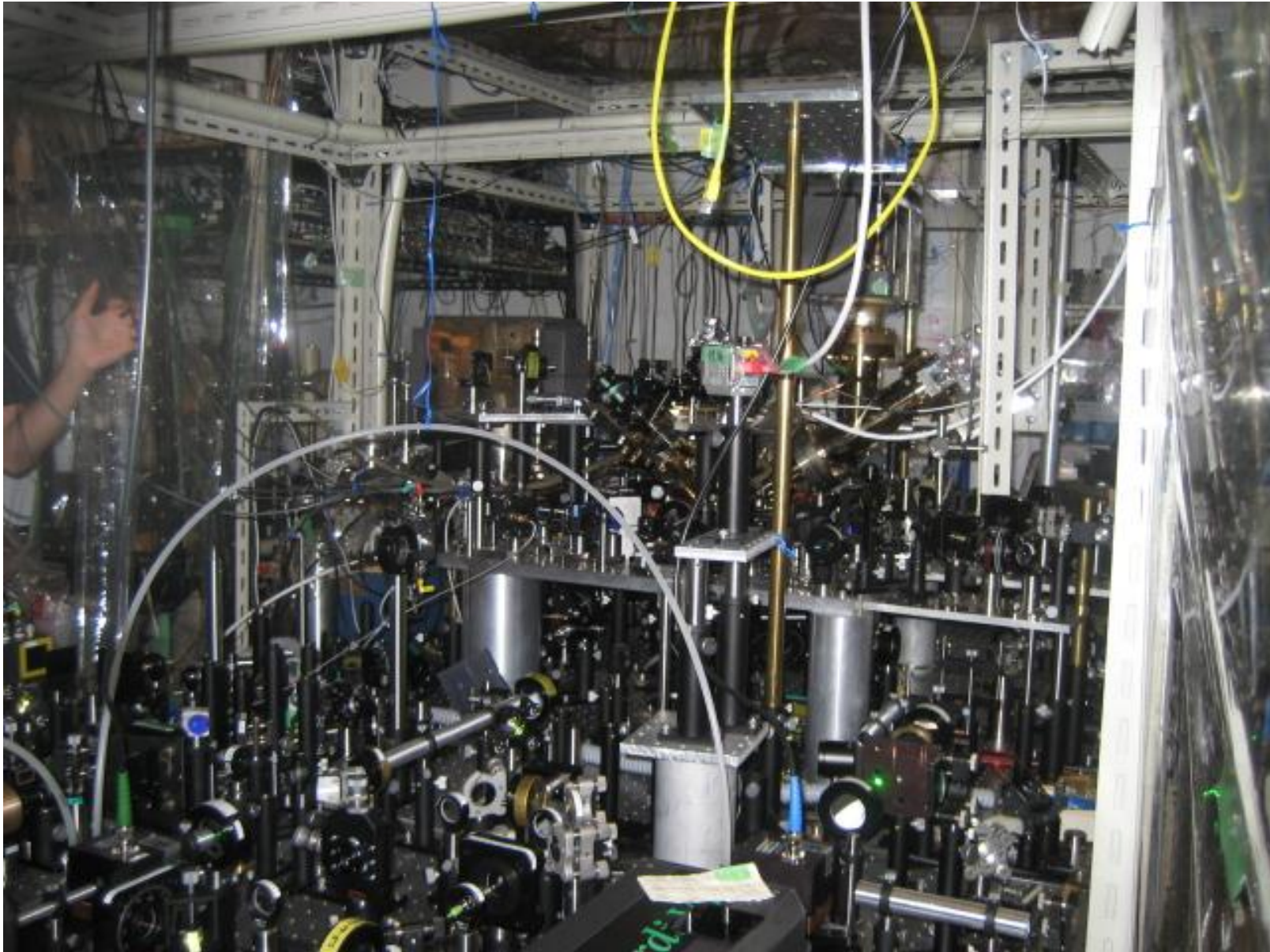
$$V_{\text{int}} = -\mu \cdot B$$



Experimental Setup for Cold Atom

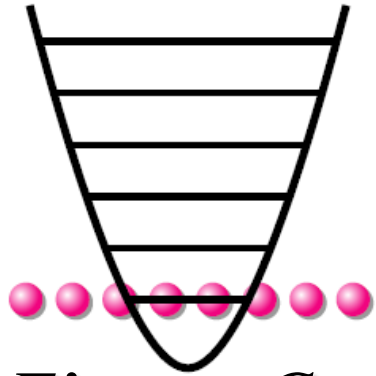


Experimental Setup for Cold Atom

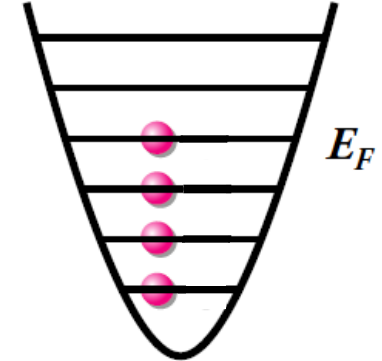
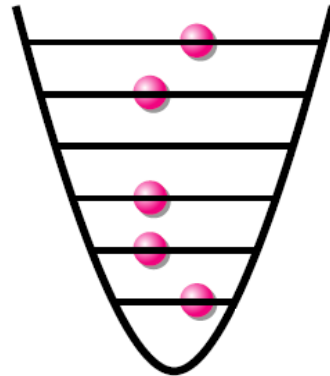


Atomic Gases Reach the Quantum Degenerate Regime

“Boson versus Fermion”

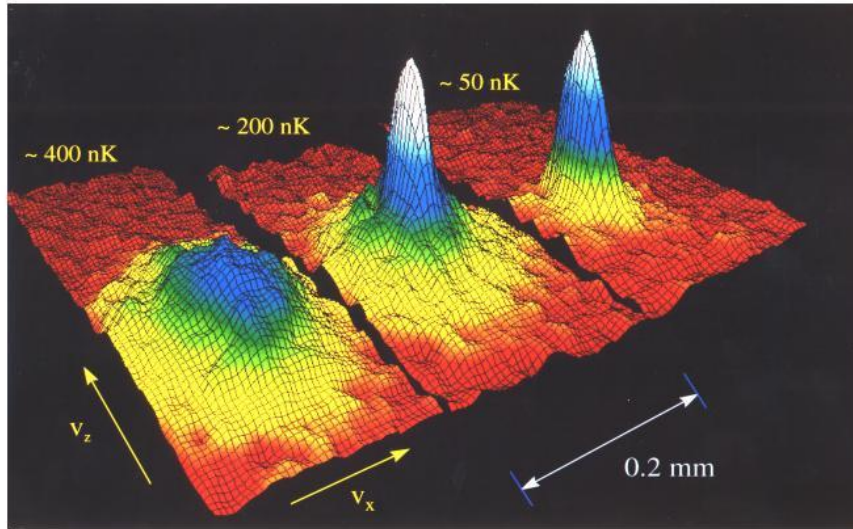


“Bose-Einstein Condensation”



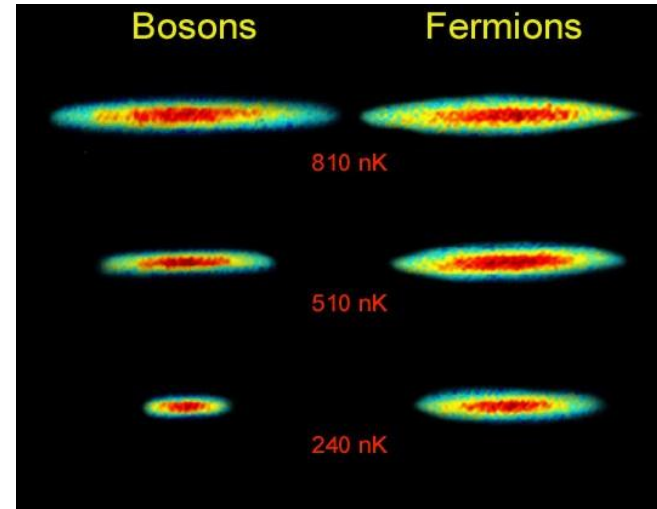
“Fermi Degeneracy”

^{87}Rb



Momentum Distribution

[E. Cornell et al, (1995)]

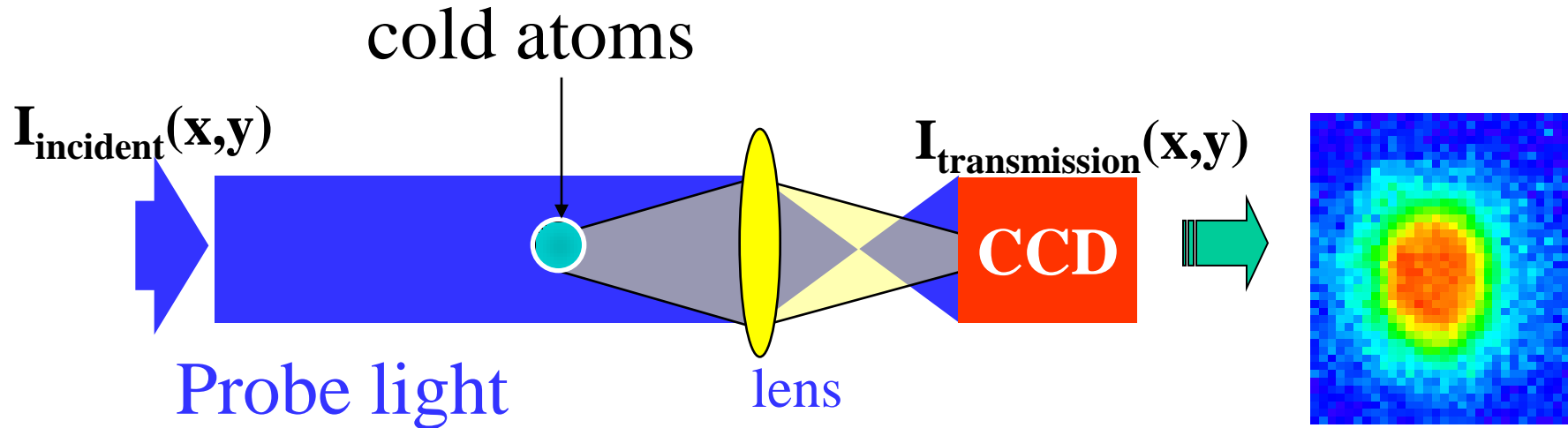


^6Li and ^7Li

Spatial Distribution

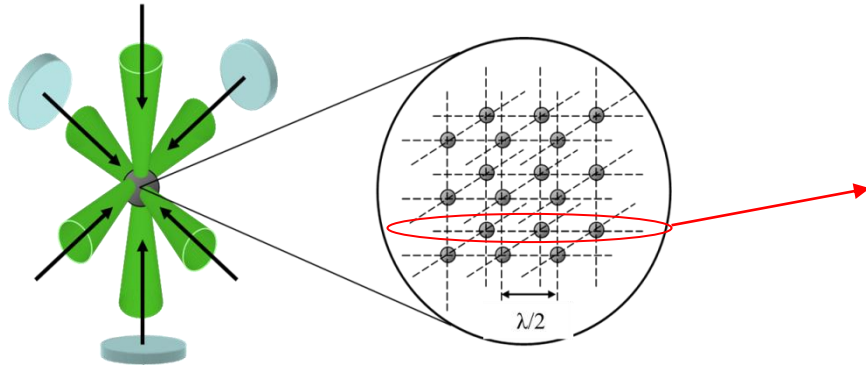
[R. Hulet et al, (2000)]

Optical Absorption Imaging of Atoms

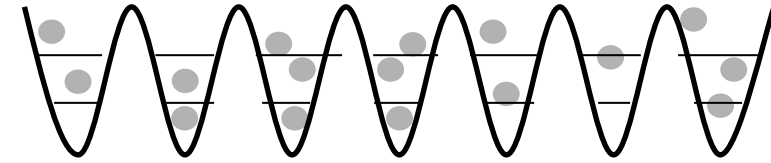


- *In-Situ* Image: \longrightarrow Reflect “**density**” distribution in a trap
- Time-of-Flight Image: \longrightarrow Reflect “**momentum**” distribution in a trap
 $t=0$ release atoms from a trap
 $t=t_{\text{TOF}}$ observe atom density distribution
$$x = p / M \cdot t_{\text{TOF}}$$

Optical Lattice

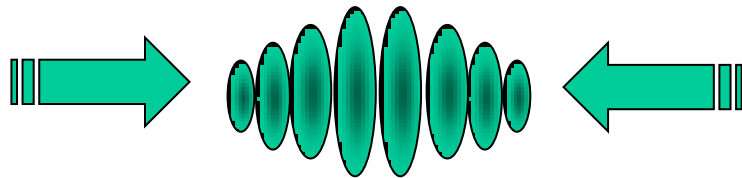


$$V_o(x) = V_o \sin^2(k_L x)$$

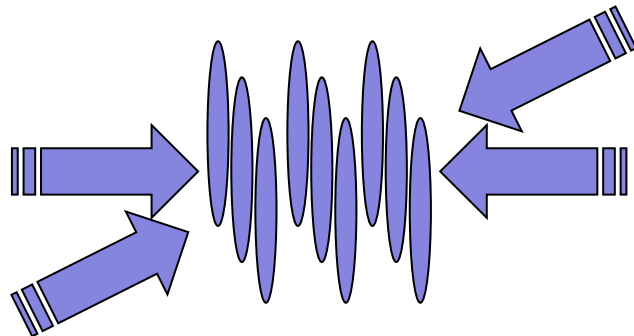


$$V_o(\mathbf{x}) = \sum_{j=1}^3 V_{oj} \sin^2(k_L x_j) = V_o \sum_{j=1}^3 \sin^2(k_L x_j)$$

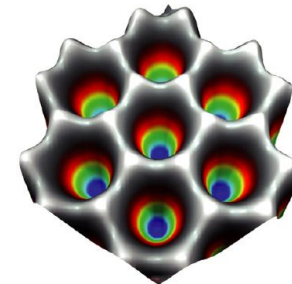
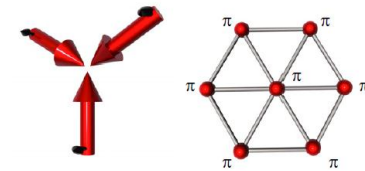
$$E_R = \frac{(\hbar k_L)^2}{2m}, s = \frac{V_0}{E_R}$$



2D gas
(pancake)



1D gas
(tube)

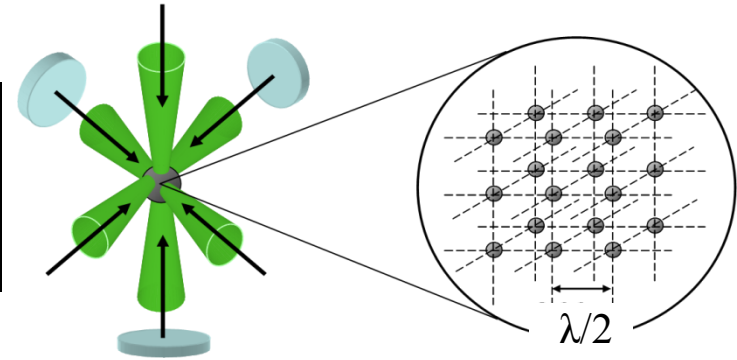


[C. Becker *et al.*,
New J. Phys. **12** 065025(2010)]

Quantum Simulation of Hubbard Model using “Cold Atoms in Optical Lattice”

[D. Jaksch *et al.*, PRL, **81**, 3108(1998)]

$$H = -J \sum_{\langle i,j \rangle} c_i^+ c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$




$$J = E_R (2 / \sqrt{\pi}) s^{3/4} \exp(-2\sqrt{s})$$

$$U = E_R a_s k_L \sqrt{8 / \pi} s^{3/4}$$

$s \equiv V_o / E_R$, $E_R \equiv (\hbar k_L)^2 / 2m$, a_s : scattering length

Controllable Parameters

hopping between lattice sites	: J		lattice potential	: V_o
On-site interaction	: U		Feshbach Resonance	: a_s
filling factor (e- or h-doping)	: n		atom density	: n

Various geometry

Feshbach Resonance:

ability to tune an inter-atomic interaction

Collision is in Quantum Regime

It is described by s-wave scattering length a_s

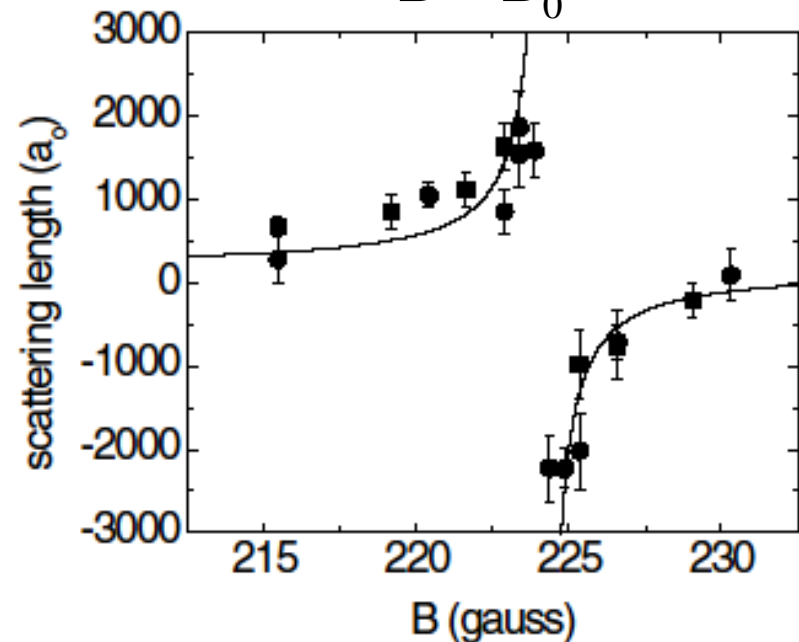
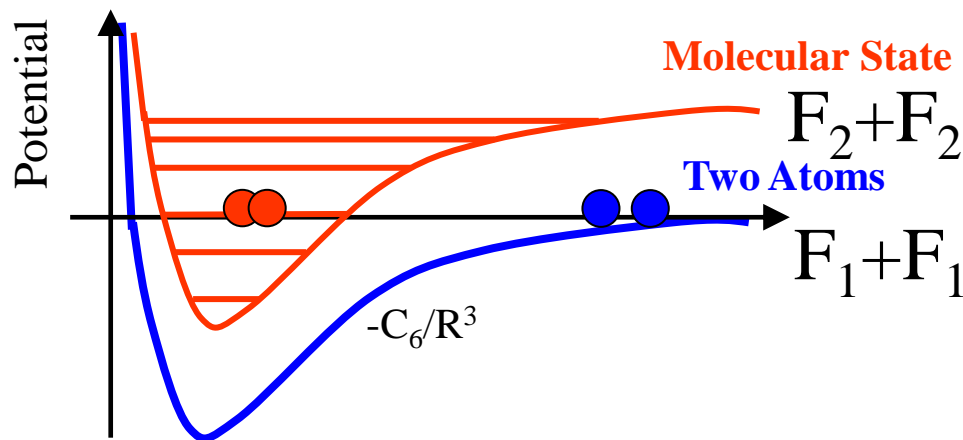
$$a_s = -\delta_l / k$$

$$\sigma_0 = 4\pi |f_0|^2 = 4\pi |a_s|^2$$

Coupling between “Open Channel” and “Closed Channel”

Control of Interaction(a_s)

$$a_s(B) = a_{bg} \left(1 - \frac{\Delta B}{B - B_0}\right)$$

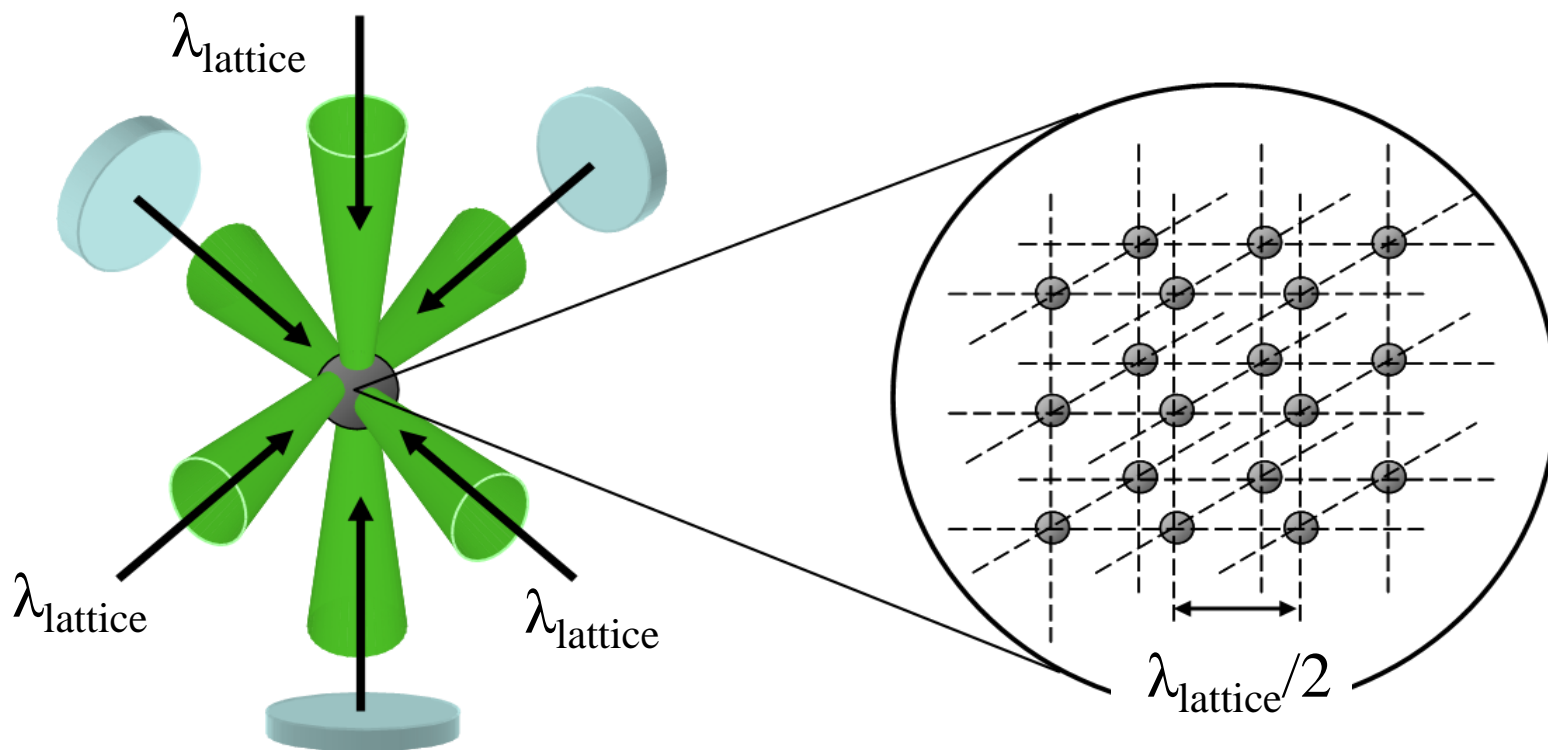


**Quantum Simulation of Hubbard Model
Using Ultracold **Alkali Atoms**
in an Optical Lattice**

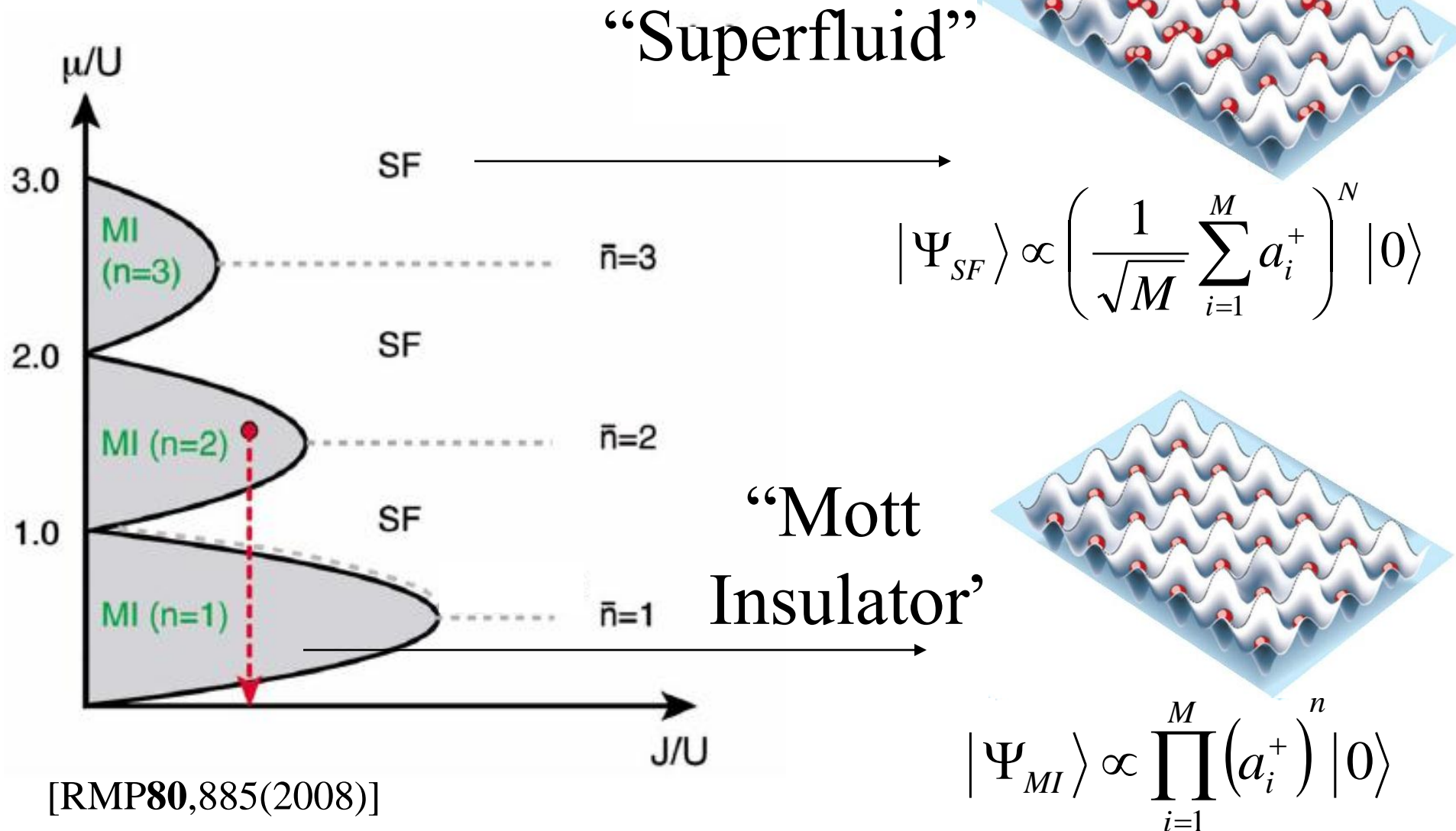
Bosons in a 3D optical lattice

$$H = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1) + \sum_i \varepsilon_i n_i$$

“Bose-Hubbard Model”

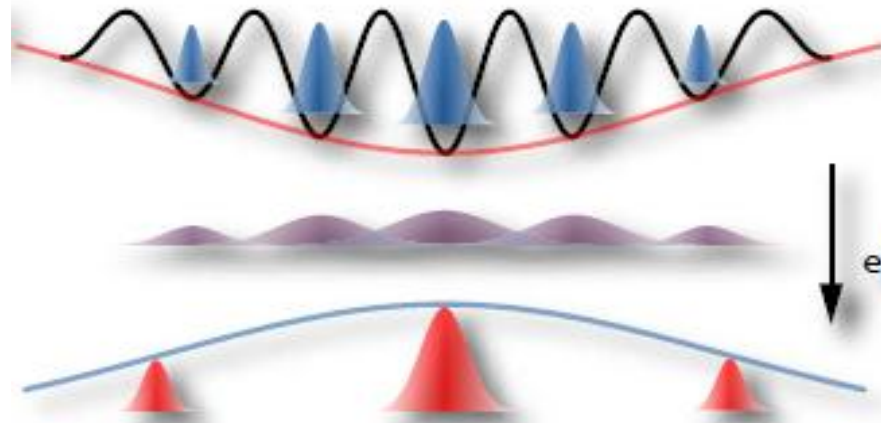


Phase Diagram of Repulsively Interacting Bosons



Interference Fringe : the direct signature of the phase coherence

“Sudden Release”



free
expansion

t_{TOF}

$$x \leftrightarrow \hbar k$$

$$x = (\hbar k / M) t_{TOF}$$

$$n(k) \propto |\tilde{w}(k)|^2 G(k)$$

Fourier Transform of the Wannier function

$$G(k) = \sum_{R,R'} \exp(ik \cdot (R - R')) \langle \hat{a}_R^+ \hat{a}_{R'} \rangle$$

no long-range order:

$$\langle \hat{a}_R^+ \hat{a}_{R'} \rangle = \delta_{R,R'} \rightarrow G(k) = N$$

uniform long-range order:

$$\langle \hat{a}_R^+ \hat{a}_{R'} \rangle = 1 \rightarrow G(k) = \frac{\sin^2(kdN/2)}{\sin^2(kd/2)}$$

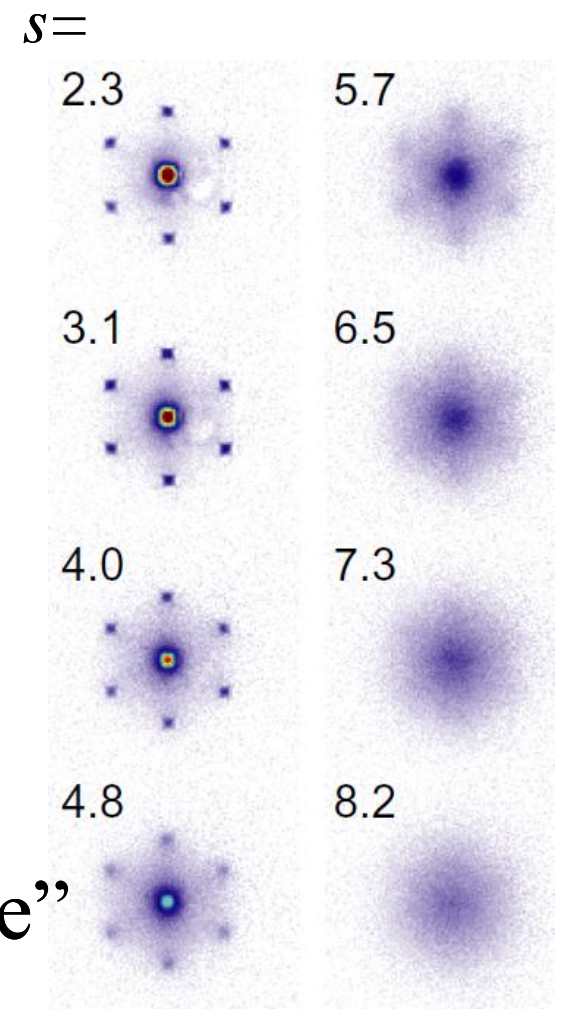
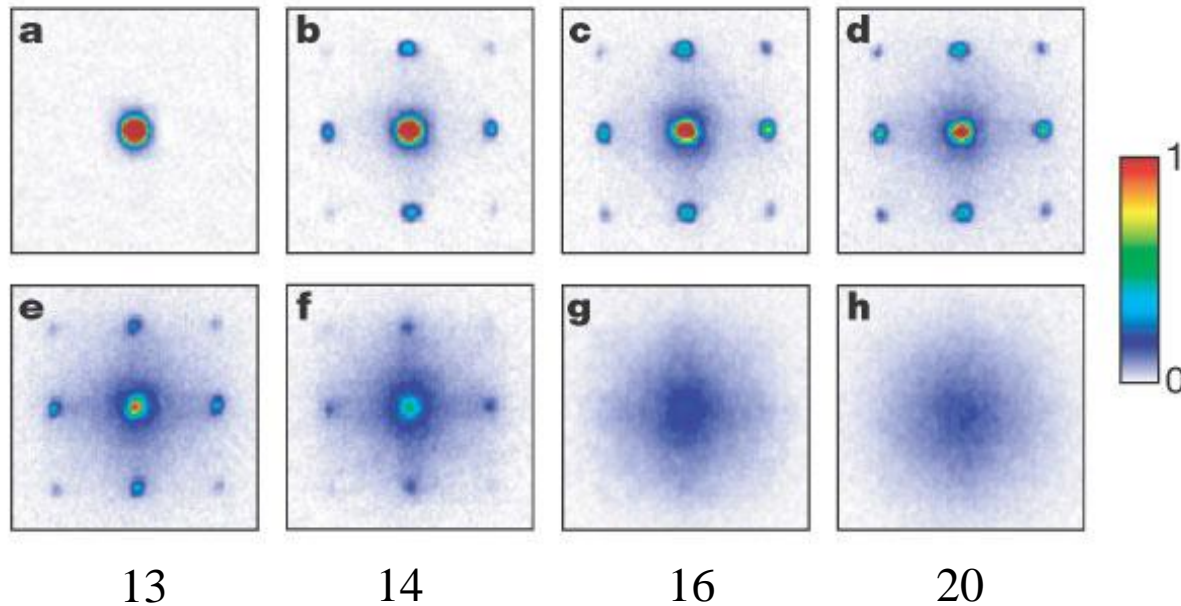
peaks at $\pm 2n\hbar k_L$ ($n=0,1,2,\dots$)

Bose-Hubbard Model:

“Superfluid - Mott-insulator Transition”

[M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415,39 (2002)]

No lattice $V_0/E_R = 3$ 7 10

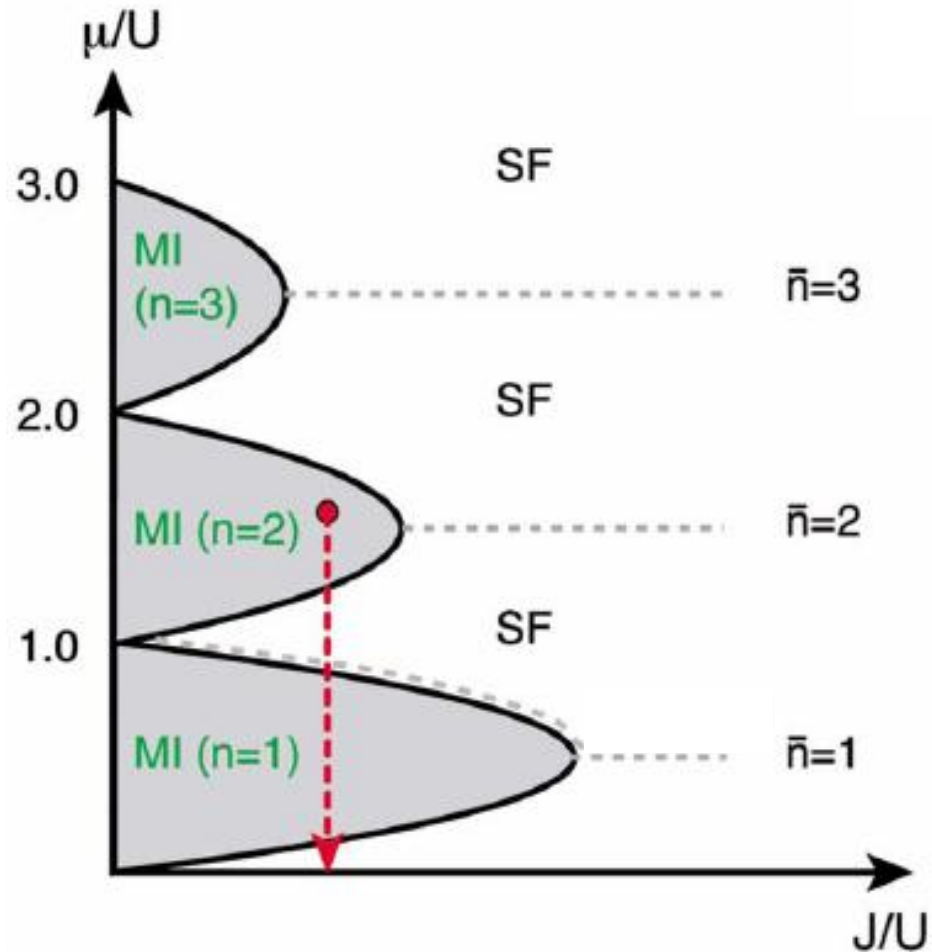


“cubic lattice”

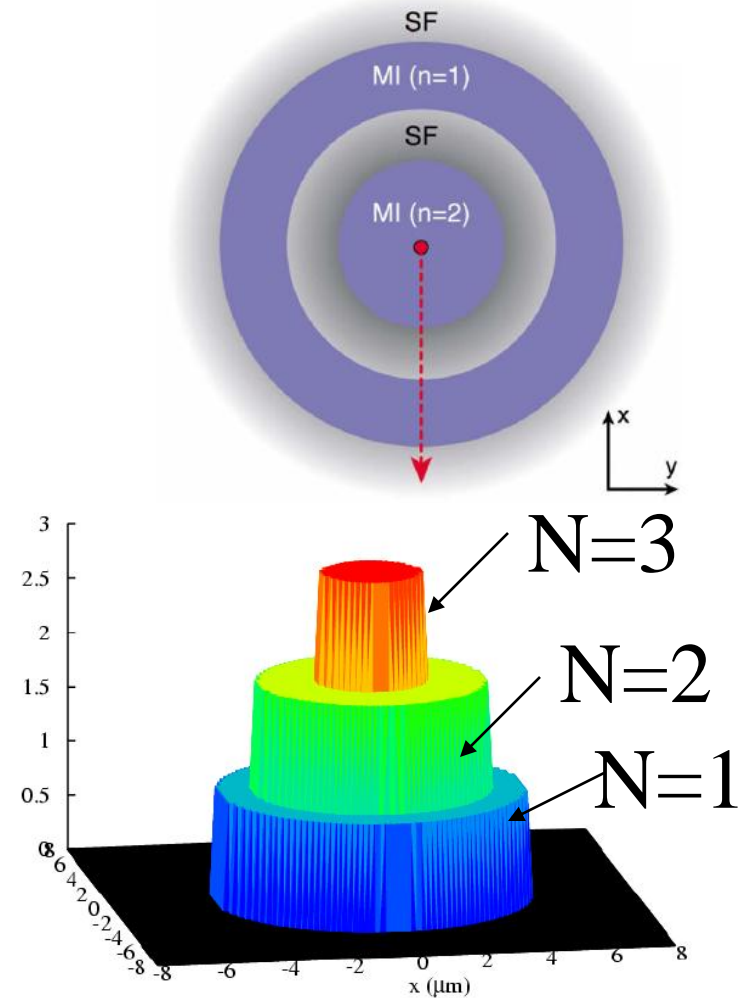
“triangular lattice”

[C. Becker *et al.*, New J. Phys. **12** 065025(2010)]

Phase Diagram of Repulsively Interacting Bosons



[RMP80,885(2008)]



Shell Structure of Mott States

High-Resolution RF Spectroscopy: Observation of Mott Shell Structure

[G. K. Campbell et al., Science 313, 649 (2006)]

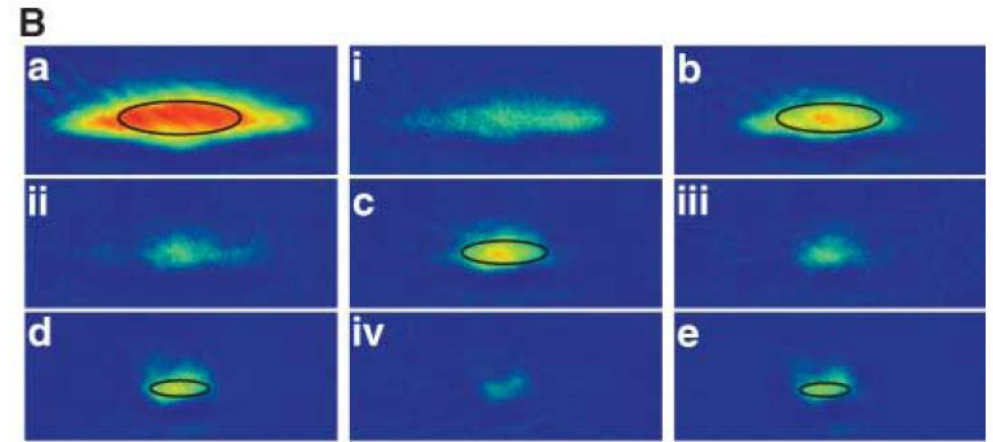
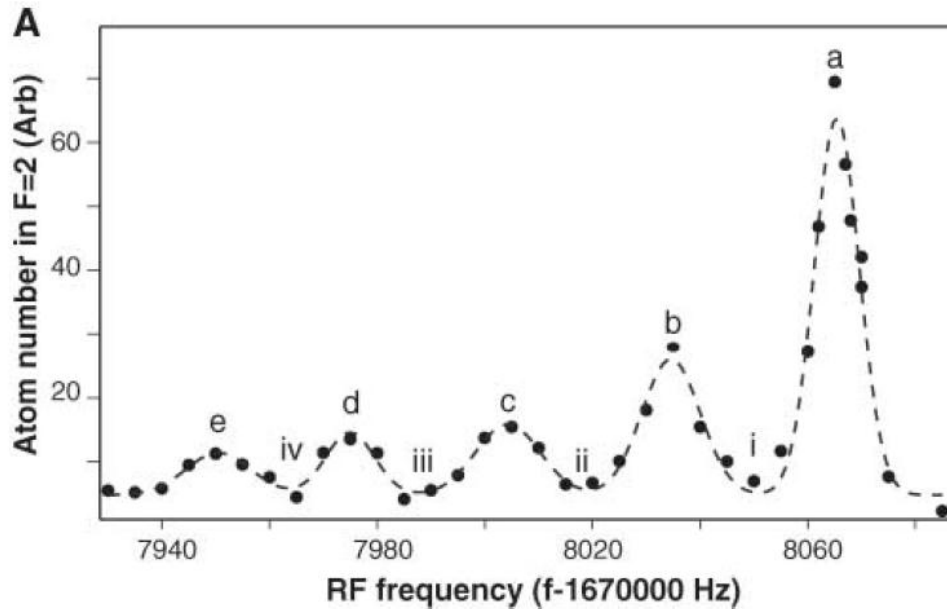


Fig. 3. Imaging the shell structure of the MI. **(A)** Spectrum of the MI at $V = 35E_{\text{rec}}$. **(B)** Absorption images for decreasing rf frequencies. Images a to e were taken on resonance with the peaks shown in (A) and display the spatial distribution of the $n = 1$ to $n = 5$ shells. The solid lines show the predicted contours of the shells.

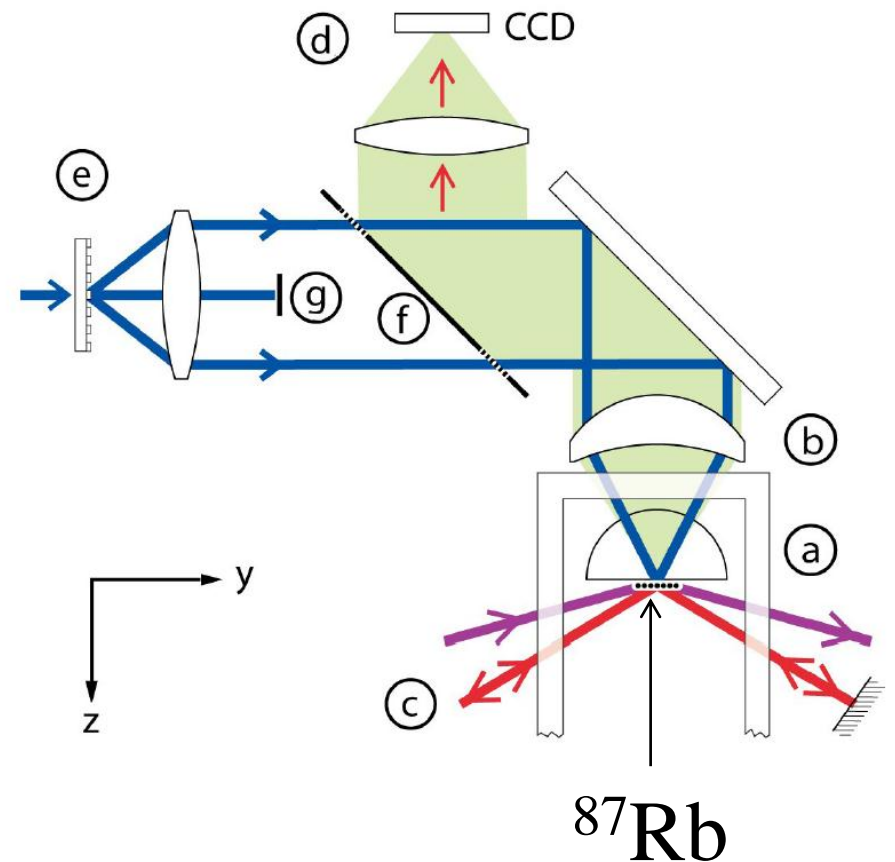
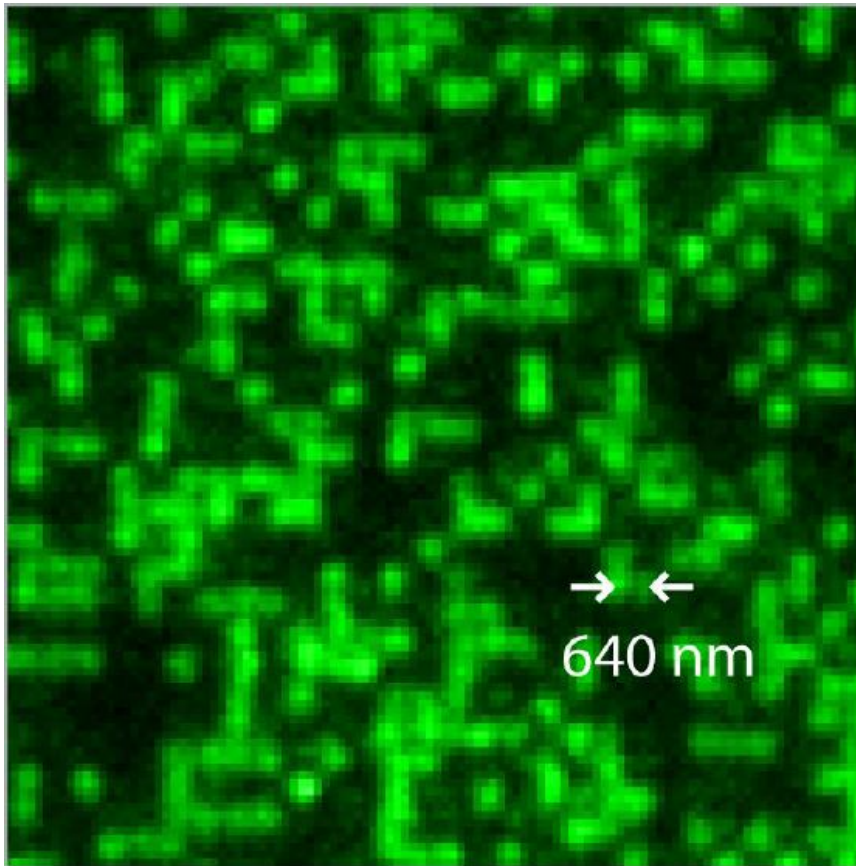
Absorption images taken for rf frequencies between the peaks (images i to iv) show a much smaller signal. The field of view was $185 \mu\text{m}$ by $80 \mu\text{m}$.

$$h\nu_n = \frac{U}{a_{11}} (a_{12} - a_{11})(n-1)$$

New Technique: Single Site Observation

[WS. Bakr, I. Gillen, A. Peng, S. Folling, and M. Greiner, Nature 462(426), 74-77(2009)]

Fluorescence Imaging



Single Site Resolved Detection of MI

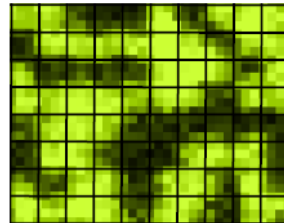
[WS Bakr, et al., Science 329, 547–550 (2010)]

SF

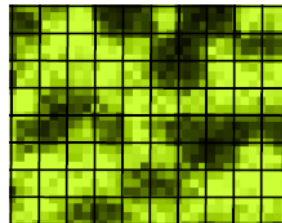


MI

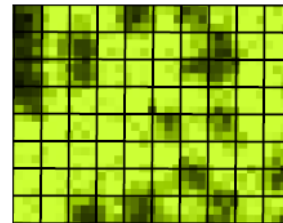
A $6E_r$



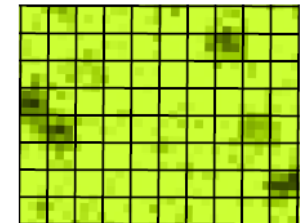
B $10E_r$



C $12E_r$



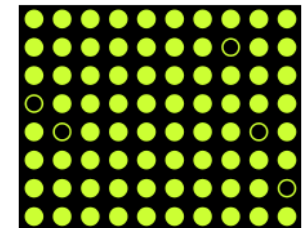
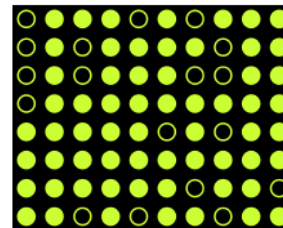
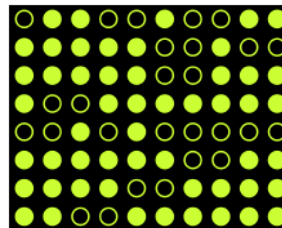
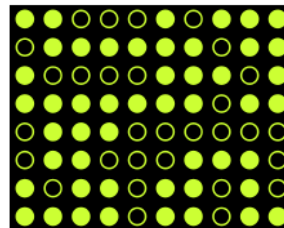
D $16E_r$ \rightarrow 680nm



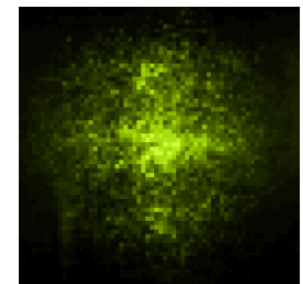
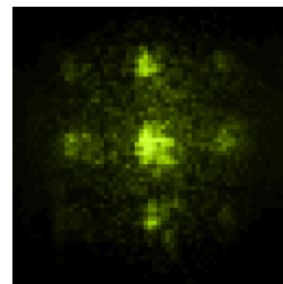
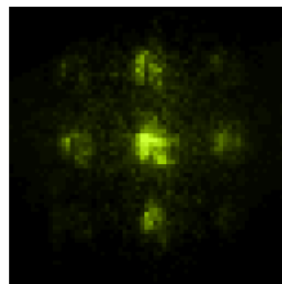
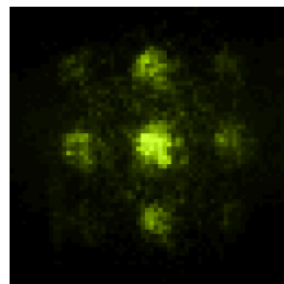
x1000 counts

In Situ-image

after analysis

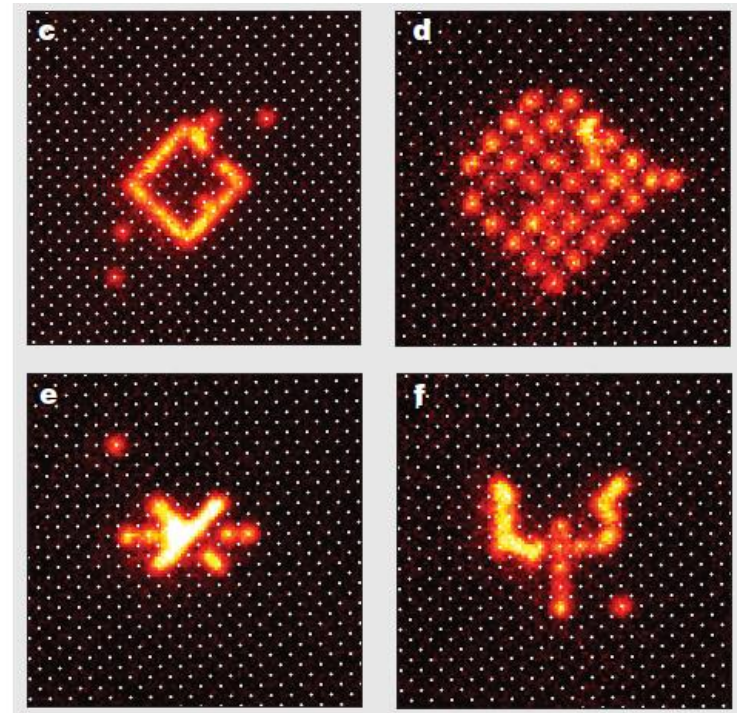
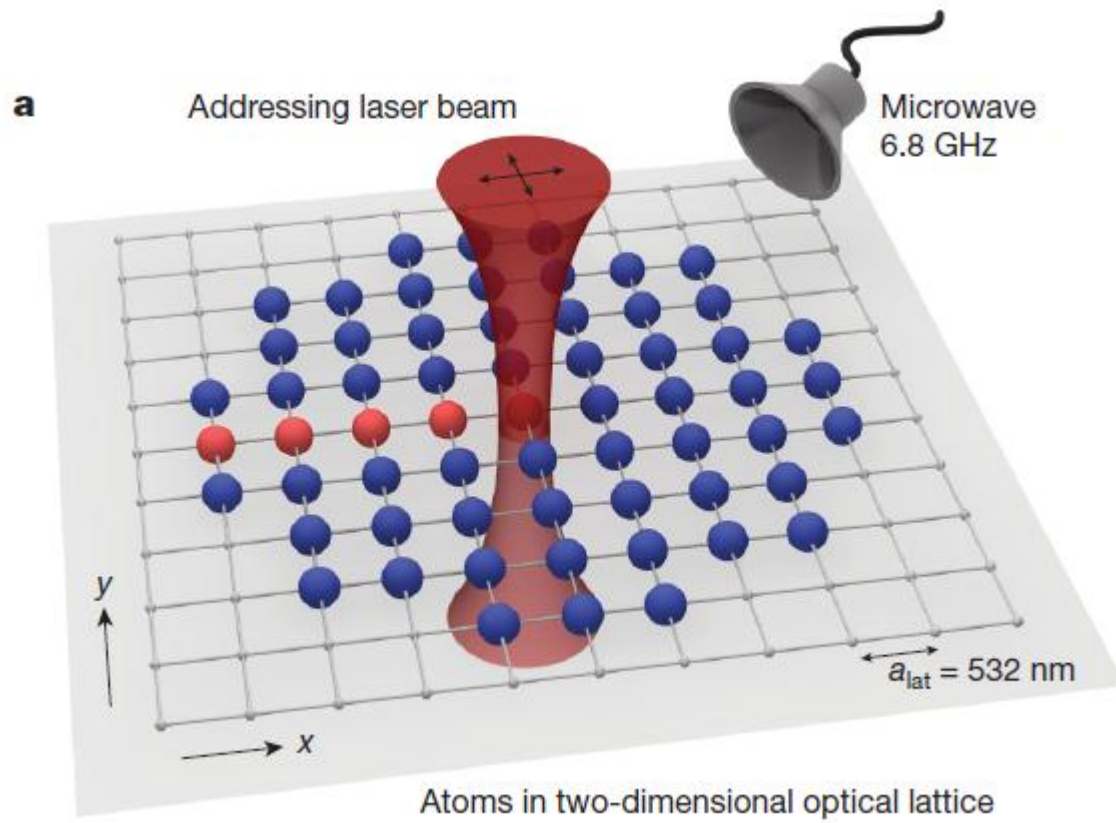


TOF-image



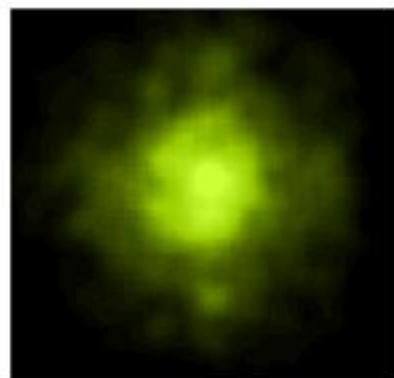
New Technique: Single Site Manipulation

[C. Ewitenberg *et al*, Nature 471, 319(2011)]

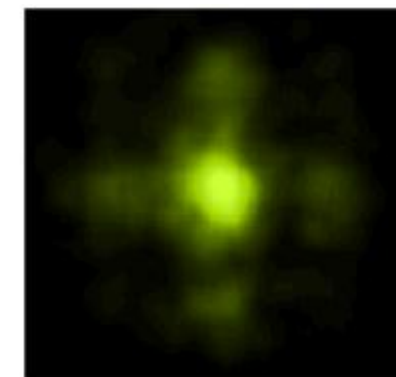
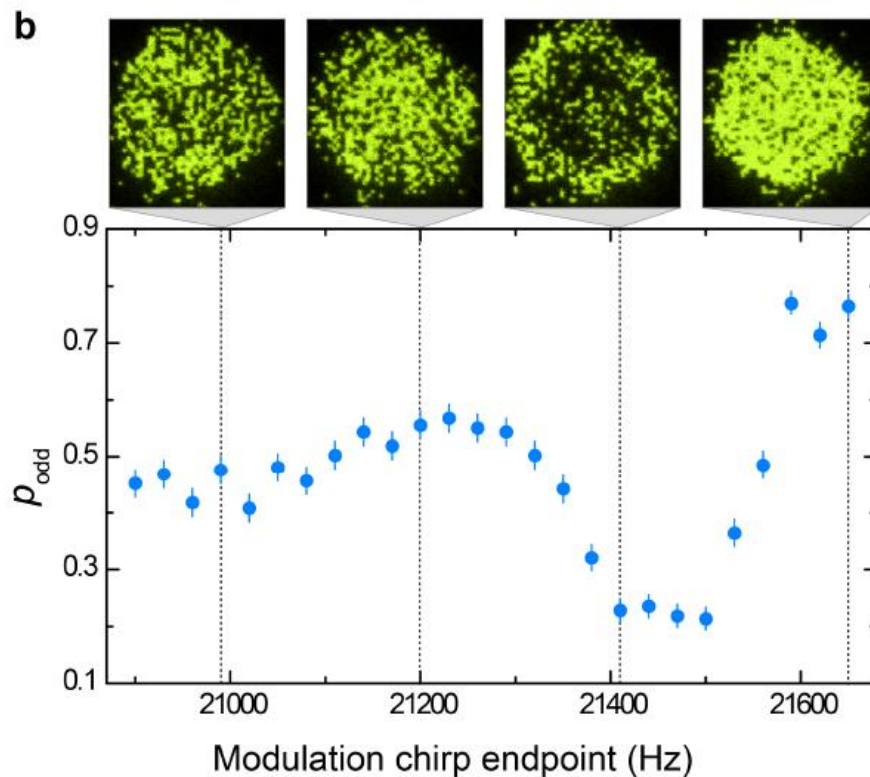


Manipulation of Mott Shell / Filter Cooling (Maxwell Demon)

[arXiv:1105.5834v1, W. S. Bakr, *et al.*,]



Dephased cloud

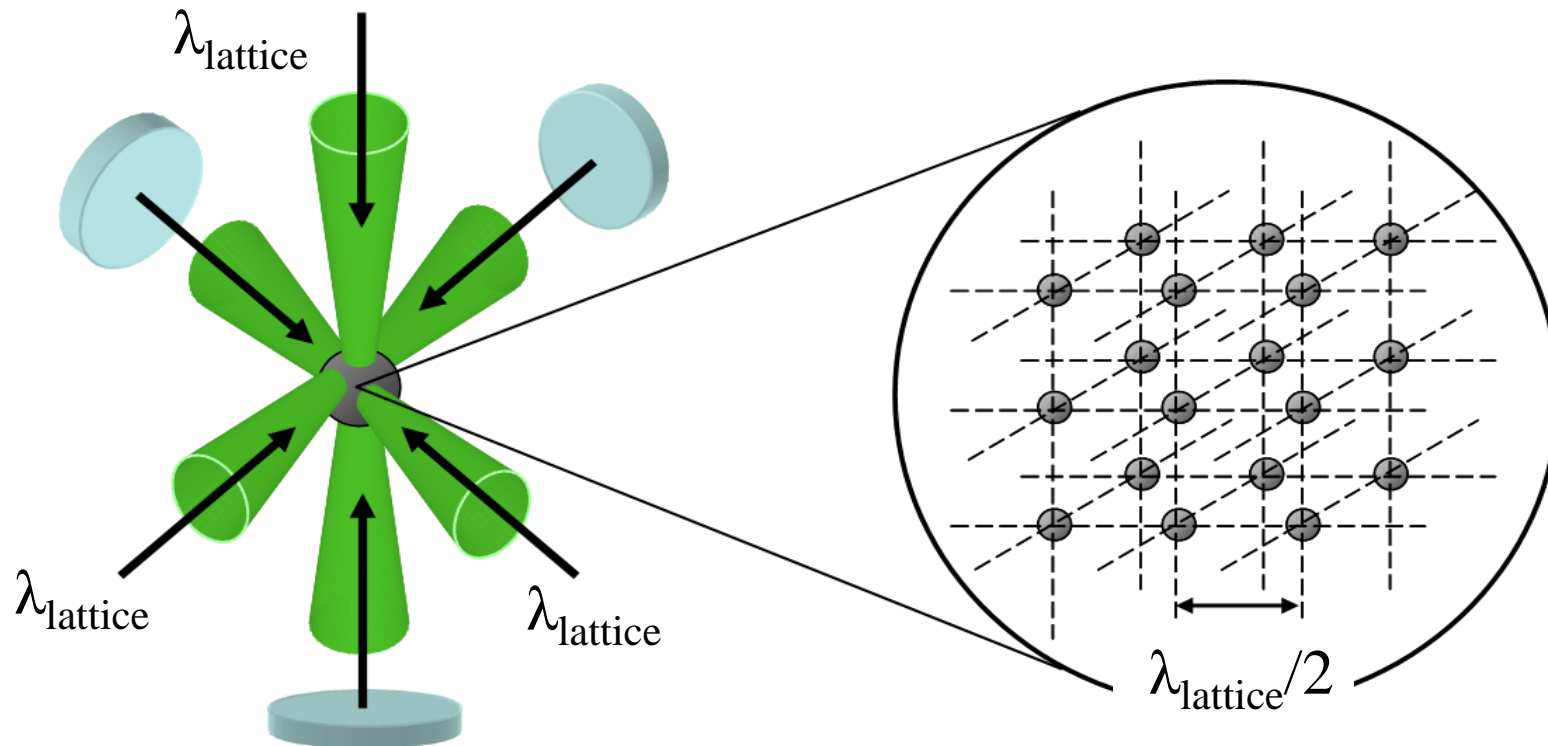


Recooled superfluid

Fermions in a 3D optical lattice

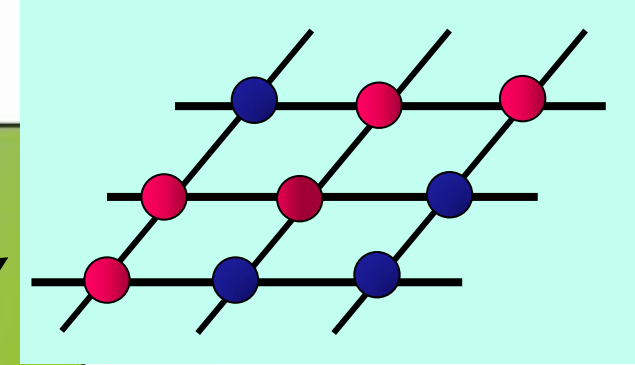
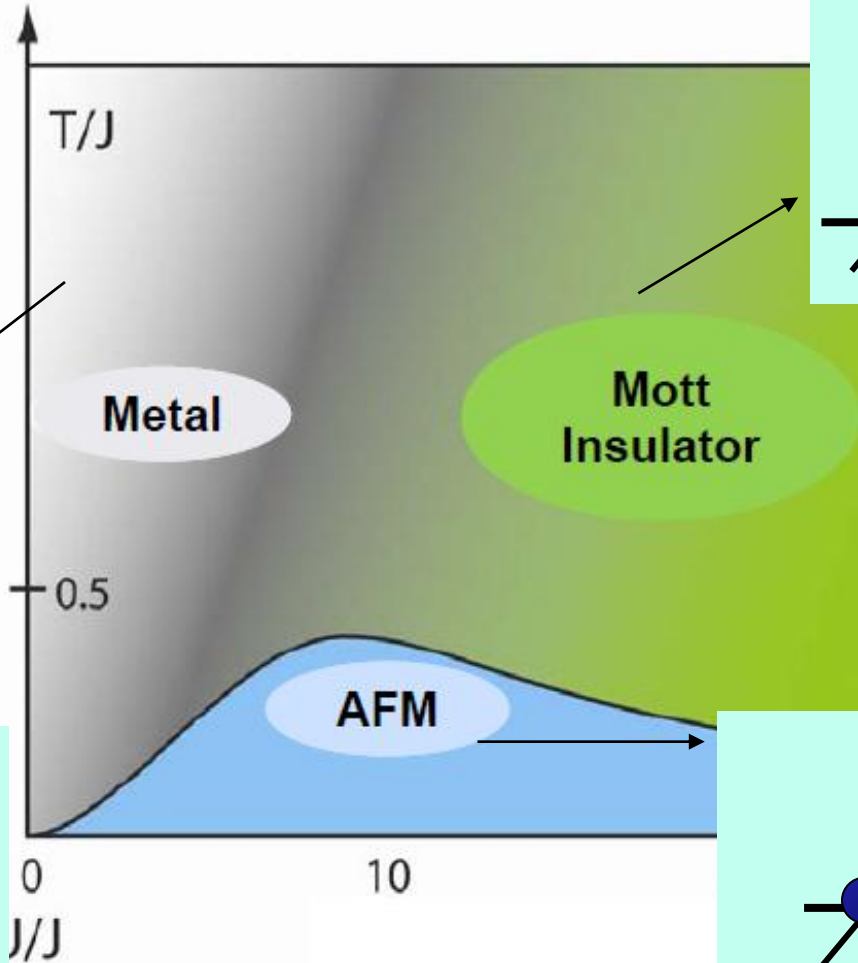
$$H = -J \sum_{\langle i,j \rangle} c_i^\dagger c_j + U \sum_i n_{i,\uparrow} n_{i,\downarrow} + \sum_i \varepsilon_i n_i$$

“Fermi-Hubbard Model”



Phase Diagram of Repulsive Fermi-Hubbard Model

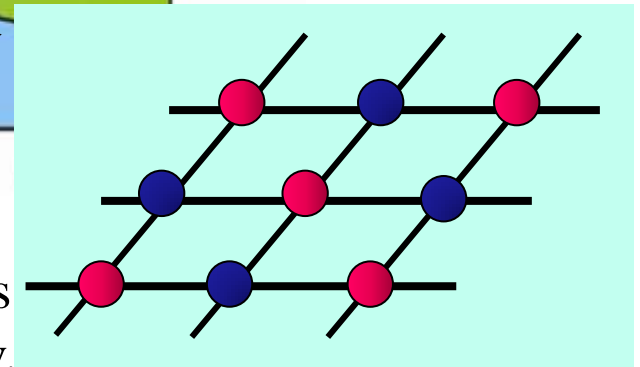
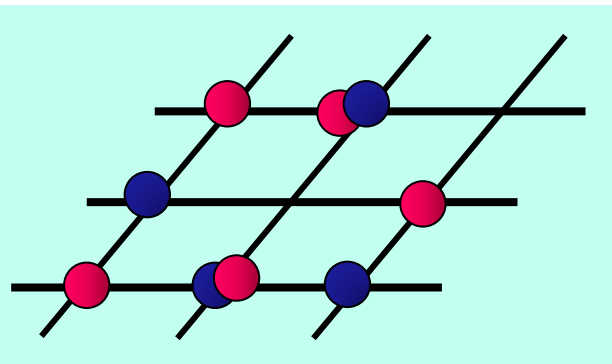
Spin UP Spin DOWN



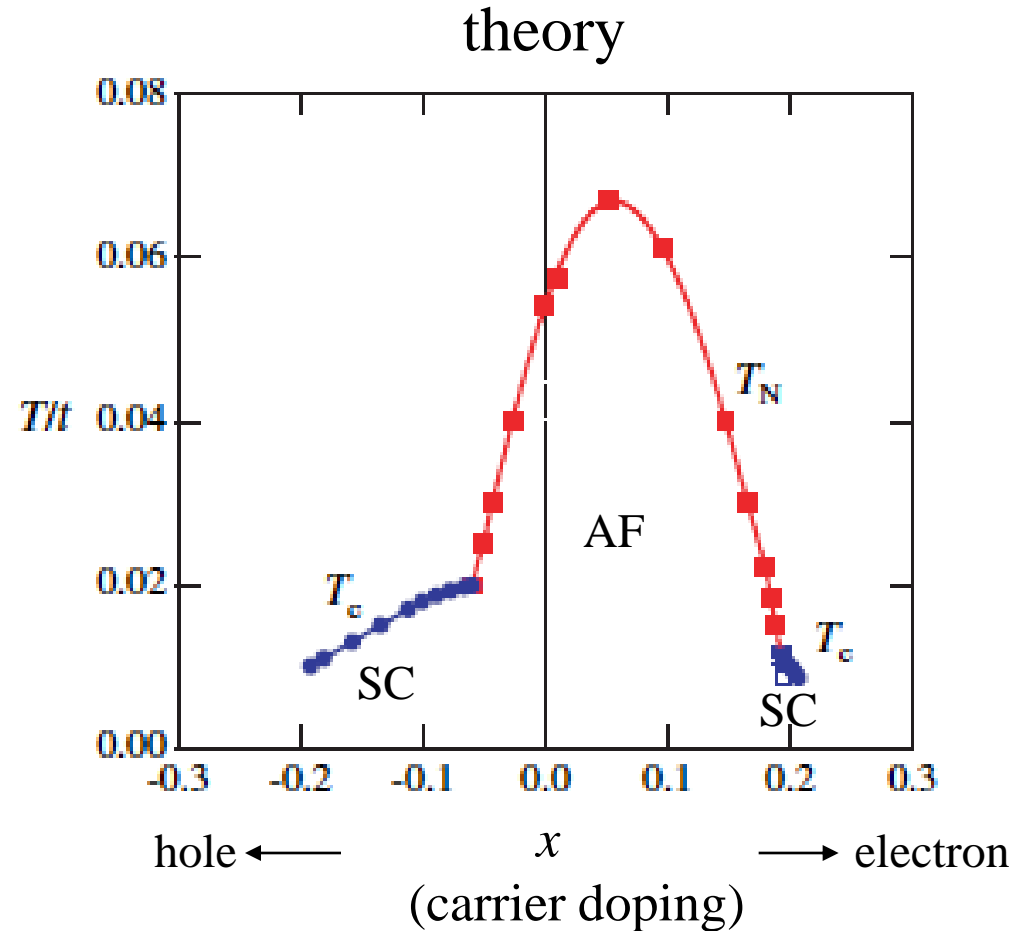
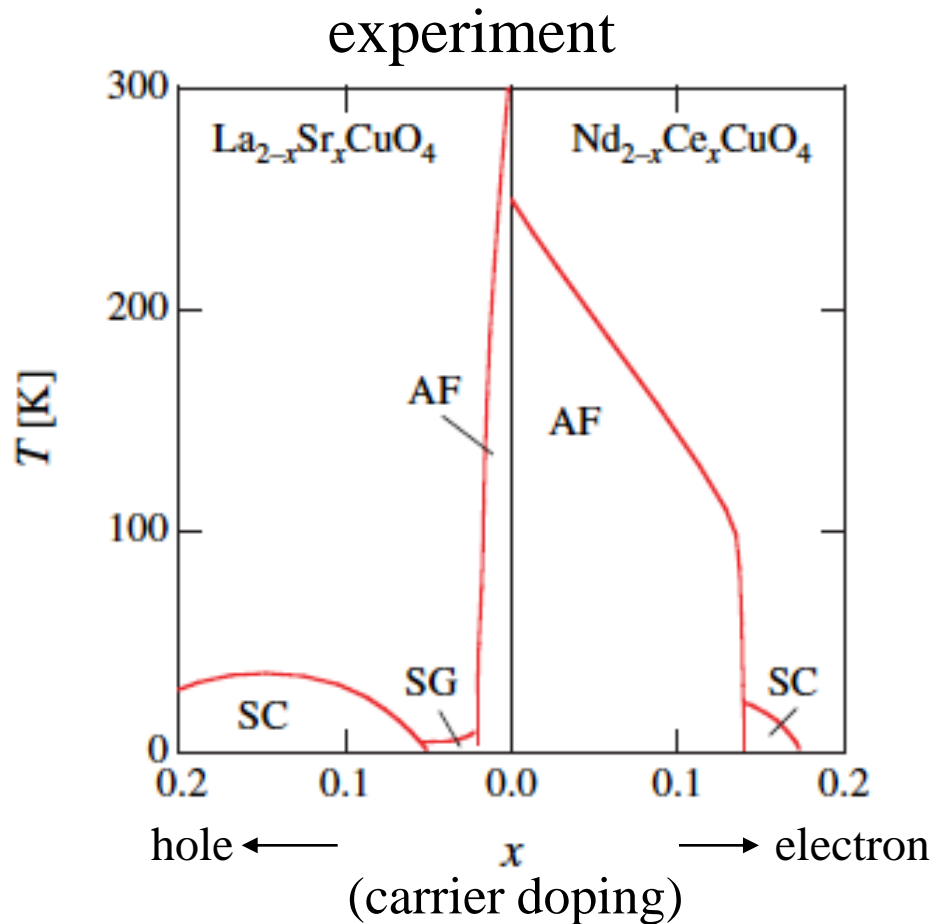
Mott Insulator

**Anti-Ferro
Magnetism**

Metal



Phase Diagram of High- T_c Cuprate Superconductor



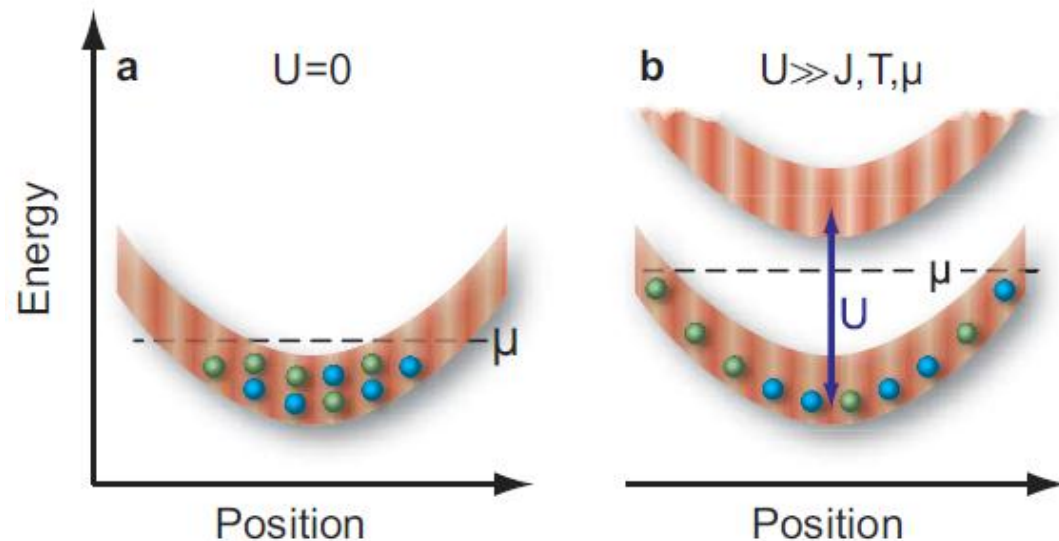
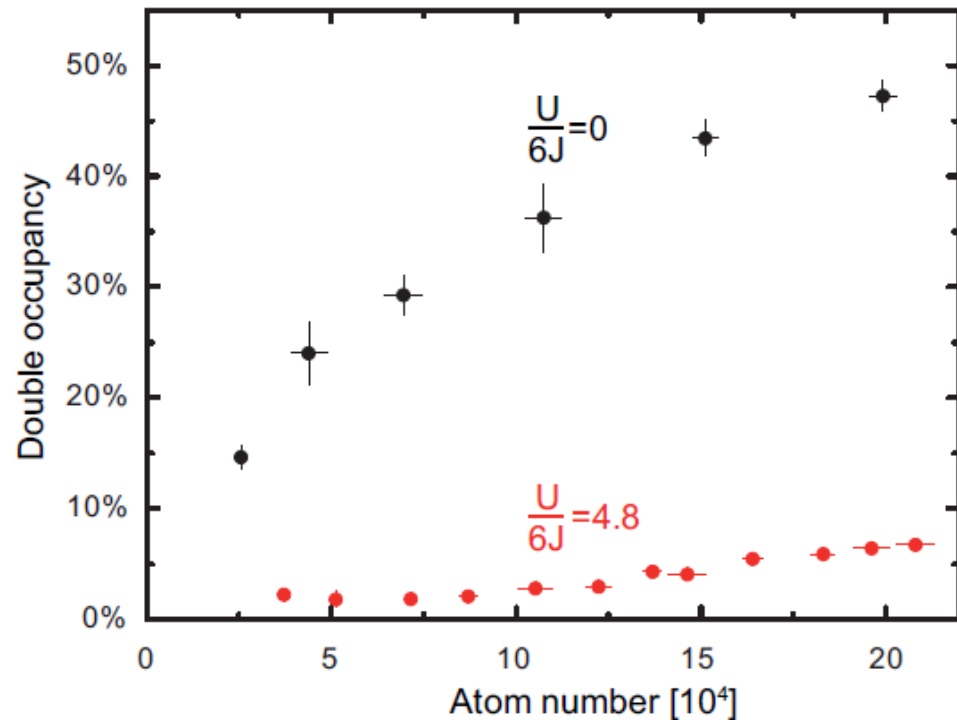
[in T. Moriya and K. Ueda, Rep. Prog.Phys.66(2003)1299]

There is controversy in the under-dope region

Current Status of Quantum Simulation of Fermi Hubbard Model: “Formation of (paramagnetic) Mott insulator”

“A Mott insulator of ^{40}K atoms (2-component)”

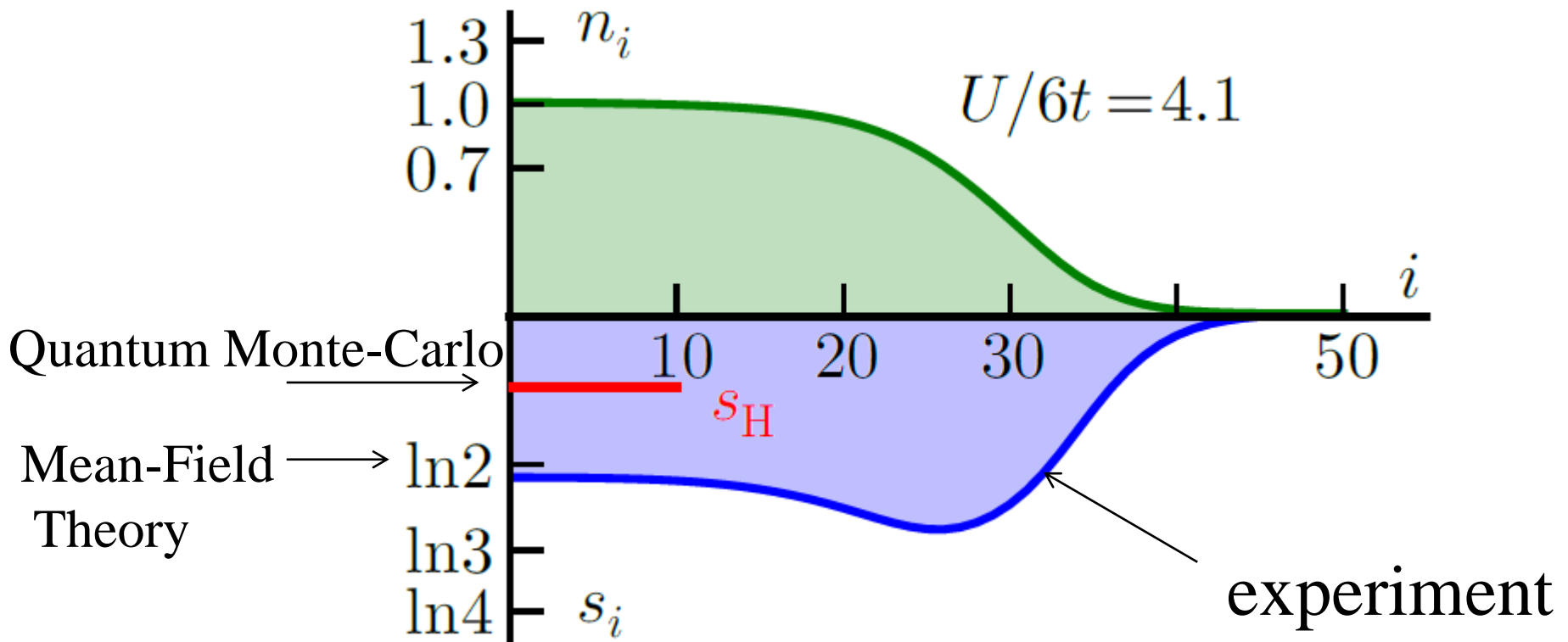
[R. Jördens *et al.*, Nature **455**, 204 (2008)] [U. Schneider, *et al.*, Science **322**,1520(2008)]



Current Status of Quantum Simulation of Fermi Hubbard Model: “Formation of (paramagnetic) Mott insulator”

[R. Jördens *et al.*, PRL **104**, 180401 (2010)]

^{40}K atoms (2-component)

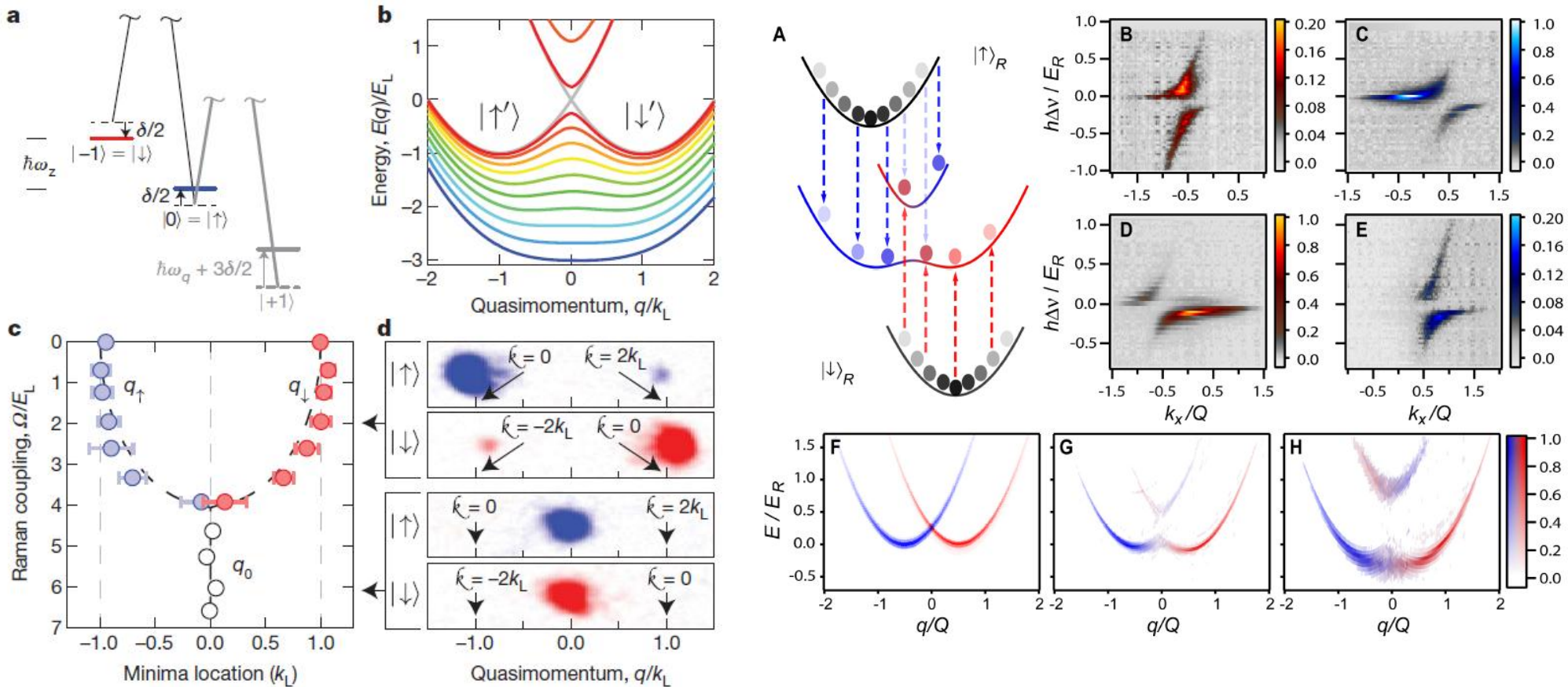


Other Progress

Spin-Orbit Interaction in Cold Atoms:

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} - \frac{g\mu_B}{\hbar} \mathbf{S} \cdot (\mathbf{B}^{(D)} + \mathbf{B}^{(R)} + \mathbf{B}^{(Z)}), \quad \mathbf{B}^{(R)} = \alpha(-k_y, k_x, 0)$$

$$\mathbf{B}^{(D)} = \beta(k_y, k_x, 0)$$



“Boson: ^{87}Rb ”

“Fermion: ^6Li ”

Summary1

Quantum Simulation of Hubbard Model Using **Alkali Atoms** in an Optical Lattice

Tuning Interatomic Interaction:

magnetic Feshbach resonance

Superfluid-Mott Insulator Transition

matter-wave interference, spectroscopy

Quantum Gas Microscope

*SF-Mott insulator transition, Single-site manipulation,
entropy reduction by Maxwell demon*

Fermi Mott Insulator

SU(2) Mott insulator

Spin-Orbit Interaction

BEC, Fermi gas

Quantum Simulation of Hubbard Model Using Ultracold **Ytterbium Atoms** in an Optical Lattice

1) Bose-Hubbard Model:

SF-Mott Insulator Transition by Laser spectroscopy

2) Fermi-Hubbard Model: *Fermi Mott Insulator*

SU(6) Mott insulator, Pomeranchuk Cooling,

3) Bose-Fermi-Hubbard Model:

Mixed Mott Insulator

4) Plan

PERIODIC TABLE

Atomic Properties of the Elements

NIST

National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

18
VIII A

Frequently used fundamental physical constants
For the most accurate values of these and other constants, visit physics.nist.gov/constants
1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ¹³³Cs

Ytterbium atom

$R_{\infty}c$ 3.289 842 × 10¹⁵ Hz
 $R_{\infty}hc$ 13.6057 eV
 Boltzmann constant k 1.3807 × 10⁻²³ J K⁻¹

Physics Laboratory physics.nist.gov **Standard Reference Data Group** www.nist.gov/srd

Group 1 IA
1 H Hydrogen 1.00794 1s 13.5984
2 IIA
3 IIIB
4 IVB
5 VB
6 VIB
7 VIIB
8 VIII
9 VIII
10 VIII
11 IB
12 IIB
13 IIIA
14 IVA
15 VA
16 VIA
17 VIIA
18 VIII A

3 Li Lithium 6.941 1s ² 2s 5.3917	4 Be Beryllium 9.012182 1s ² 2s ² 9.3227	5 B Boron 10.811 1s ² 2s ² 2p 8.2980	6 C Carbon 12.0107 1s ² 2s ² 2p ² 11.2603	7 N Nitrogen 14.0067 1s ² 2s ² 2p ³ 14.5341	8 O Oxygen 15.9994 1s ² 2s ² 2p ⁴ 13.6181	9 F Fluorine 18.9984032 1s ² 2s ² 2p ⁵ 17.4228	10 Ne Neon 20.1797 1s ² 2s ² 2p ⁶ 21.5645	11 Na Sodium 22.989770 [Ne]3s 5.1391	12 Mg Magnesium 24.3050 [Ne]3s ² 7.6462	13 Al Aluminum 26.981538 [Ne]3s ² 3p 5.9858	14 Si Silicon 28.0855 [Ne]3s ² 3p ² 8.1517	15 P Phosphorus 30.973761 [Ne]3s ² 3p ³ 10.4867	16 S Sulfur 32.065 [Ne]3s ² 3p ⁴ 10.3680	17 Cl Chlorine 35.453 [Ne]3s ² 3p ⁵ 12.9676	18 Ar Argon 39.948 [Ne]3s ² 3p ⁶ 15.7596	19 K Potassium 39.0983 [Ar]4s 4.3407	20 Ca Calcium 40.078 [Ar]4s 6.1132	21 Sc Scandium 44.955910 [Ar]3d ¹ 4s 6.5615	22 Ti Titanium 47.867 [Ar]3d ² 4s 6.8281	23 V Vanadium 50.9415 [Ar]3d ³ 4s 6.7462	24 Cr Chromium 51.9961 [Ar]3d ⁵ 4s 6.7665	25 Mn Manganese 54.938049 [Ar]3d ⁵ 4s 7.4340	26 Fe Iron 55.845 [Ar]3d ⁶ 4s 7.9024	27 Co Cobalt 58.933200 [Ar]3d ⁷ 4s 7.8810	28 Ni Nickel 58.6934 [Ar]3d ⁸ 4s 7.6398	29 Cu Copper 63.546 [Ar]3d ¹⁰ 4s 7.7264	30 Zn Zinc 65.409 [Ar]3d ¹⁰ 4s 9.942	31 Ga Gallium 69.723 [Ar]3d ¹⁰ 4s ² 4p 5.9993	32 Ge Germanium 72.64 [Ar]3d ¹⁰ 4s ² 4p ² 7.8994	33 As Arsenic 74.92160 [Ar]3d ¹⁰ 4s ² 4p ³ 7.8986	34 Se Selenium 78.96 [Ar]3d ¹⁰ 4s ² 4p ⁴ 9.7524	35 Br Bromine 79.904 [Ar]3d ¹⁰ 4s ² 4p ⁵ 11.8138	36 Kr Krypton 83.798 [Ar]3d ¹⁰ 4s ² 4p ⁶ 13.9996	37 Rb Rubidium 85.4678 [Kr]5s 4.1771	38 Sr Strontium 87.62 [Kr]5s ² 6.6399	39 Y Yttrium 88.90585 [Kr]4d ⁵ 5s 6.2173	40 Zr Zirconium 91.224 [Kr]4d ⁵ 5s 6.8399	41 Nb Niobium 92.90638 [Kr]4d ⁵ 5s 6.7589	42 Mo Molybdenum 95.94 [Kr]4d ⁵ 5s 7.28	43 Tc Technetium (98)	44 Ru Ruthenium 101.07 [Kr]4d ⁸ 5s 7.3605	45 Rh Rhodium 102.90550 [Kr]4d ⁸ 5s 7.4589	46 Pd Palladium 106.42 [Kr]4d ¹⁰ 8.3369	47 Ag Silver 107.8682 [Kr]4d ¹⁰ 5s 7.5762	48 Cd Cadmium 112.411 [Kr]4d ¹⁰ 5s 8.9938	49 In Indium 114.818 [Kr]4d ¹⁰ 5s ² 5p 5.7864	50 Sn Tin 118.710 [Kr]4d ¹⁰ 5s ² 5p ² 7.3439	51 Sb Antimony 121.760 [Kr]4d ¹⁰ 5s ² 5p ³ 8.6084	52 Te Tellurium 127.60 [Kr]4d ¹⁰ 5s ² 5p ⁴ 9.0096	53 I Iodine 126.90447 10.4513	54 Xe Xenon 131.293 [Kr]4d ¹⁰ 5s ² 5p ⁶ 12.1298	55 Cs Cesium 132.90545 [Xe]6s 3.8939	56 Ba Barium 137.327 [Xe]6s ² 5.2117	57 La Lanthanum 138.9055 [Xe]5d ¹ 6s ² 5.5769	58 Ce Cerium 140.116 [Xe]4f ^{5d} 6s ² 5.5387	59 Pr Praseodymium 140.90765 [Xe]4f ³ 6s 5.473	60 Nd Neodymium 144.24 [Xe]4f ⁴ 6s ² 5.5250	61 Pm Promethium (145) [Xe]4f ⁵ 6s ² 5.582	62 Sm Samarium 150.36 [Xe]4f ⁶ 6s ² 5.6437	63 Eu Europium 151.964 [Xe]4f ⁷ 6s ² 5.6704	64 Gd Gadolinium 157.25 [Xe]4f ⁷ 5d ¹ 6s ² 6.1496	65 Tb Terbium 158.92534 [Xe]4f ⁹ 6s ² 5.8638	66 Dy Dysprosium 162.500 [Xe]4f ¹⁰ 6s ² 5.9389	67 Ho Holmium 162.50332 [Xe]4f ¹¹ 6s ² 6.0215	68 Er Erbium 167.259 [Xe]4f ¹² 6s ² 6.1077	69 Tm Thulium 168.93421 [Xe]4f ¹³ 6s ² 6.1843	70 Yb Ytterbium 173.04 [Xe]4f ¹⁴ 6s ² 6.2542	71 Lu Lutetium 174.967 [Xe]4f ¹⁴ 5d ¹ 6s ² 5.4259	72 Hf Hafnium 178.49 [Xe]4f ¹⁴ 5d ² 6s ² 6.8251	73 Ta Tantalum 180.9479 [Xe]4f ¹⁴ 5d ³ 6s ² 7.5496	74 W Tungsten 183.84 [Xe]4f ¹⁴ 5d ⁴ 6s ² 7.8640	75 Re Rhenium 186.207 [Xe]4f ¹⁴ 5d ⁵ 6s ² 7.8335	76 Os Osmium 190.23 [Xe]4f ¹⁴ 5d ⁶ 6s ² 8.4382	77 Ir Iridium 192.217 [Xe]4f ¹⁴ 5d ⁷ 6s ² 8.9670	78 Pt Platinum 195.078 [Xe]4f ¹⁴ 5d ⁹ 6s ¹ 8.9588	79 Au Gold 196.96655 [Xe]4f ¹⁴ 5d ¹⁰ 6s ¹ 9.2255	80 Hg Mercury 200.59 10.4375	81 Tl Thallium 204.3833 [Hg]6p 6.1082	82 Pb Lead 207.2 [Hg]6p ² 7.4167	83 Bi Bismuth 208.98038 [Hg]6p ³ 7.2855	84 Po Polonium (209) [Hg]6p ⁴ 8.414	85 At Astatine (210) [Hg]6p ⁵ 8.414	86 Rn Radon (222) [Hg]6p ⁶ 10.7485	87 Fr Francium (223) [Rn]7s 4.0727	88 Ra Radium (226) [Rn]7s ² 5.2784	89 Ac Actinium (227) [Rn]5f ¹ 6d ¹ 7s ² 5.17	90 Th Thorium 232.0381 [Rn]6d ² 7s ² 6.3067	91 Pa Protactinium 231.03588 [Rn]5f ² 6d ¹ 7s ² 5.89	92 U Uranium 238.02891 [Rn]5f ³ 6d ¹ 7s ² 6.1941	93 Np Neptunium (237) [Rn]5f ⁴ 6d ¹ 7s ² 6.2657	94 Pu Plutonium (244) [Rn]5f ⁶ 7s ² 6.0260	95 Am Americium (243) [Rn]5f ⁷ 7s ² 5.9738	96 Cm Curium (247) [Rn]5f ⁸ 6d ¹ 7s ² 5.9914	97 Bk Berkelium (247) [Rn]5f ⁹ 7s ² 6.1979	98 Cf Californium (251) [Rn]5f ¹⁰ 7s ² 6.2817	99 Es Einsteinium (257) [Rn]5f ¹¹ 7s ² 6.42	100 Fm Fermium (257) [Rn]5f ¹² 7s ² 6.50	101 Md Mendelevium (288) [Rn]5f ¹³ 7s ² 6.58	102 No Nobelium (259) [Rn]5f ¹⁴ 7s ² 6.65	103 Lr Lawrencium (262) [Rn]5f ¹⁴ 7s ² 7p ¹ 4.9 ?
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13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIII A																																																																																							
5 B Boron 10.811 1s ² 2s ² 2p 8.2980	6 C Carbon 12.0107 1s ² 2s ² 2p ² 11.2603	7 N Nitrogen 14.0067 1s ² 2s ² 2p ³ 14.5341	8 O Oxygen 15.9994 1s ² 2s ² 2p ⁴ 13.6181	9 F Fluorine 18.9984032 1s ² 2s ² 2p ⁵ 17.4228	10 Ne Neon 20.1797 1s ² 2s ² 2p ⁶ 21.5645	11 Na Sodium 22.989770 [Ne]3s 5.1391	12 Mg Magnesium 24.3050 [Ne]3s ² 7.6462	13 Al Aluminum 26.981538 [Ne]3s ² 3p 5.9858	14 Si Silicon 28.0855 [Ne]3s ² 3p ² 8.1517	15 P Phosphorus 30.973761 [Ne]3s ² 3p ³ 10.4867	16 S Sulfur 32.065 [Ne]3s ² 3p ⁴ 10.3680	17 Cl Chlorine 35.453 [Ne]3s ² 3p ⁵ 12.9676	18 Ar Argon 39.948 [Ne]3s ² 3p ⁶ 15.7596	19 K Potassium 39.0983 [Ar]4s 4.3407	20 Ca Calcium 40.078 [Ar]4s 6.1132	21 Sc Scandium 44.955910 [Ar]3d ¹ 4s 6.5615	22 Ti Titanium 47.867 [Ar]3d ² 4s 6.8281	23 V Vanadium 50.9415 [Ar]3d ³ 4s 6.7462	24 Cr Chromium 51.9961 [Ar]3d ⁵ 4s 6.7665	25 Mn Manganese 54.938049 [Ar]3d ⁵ 4s 7.4340	26 Fe Iron 55.845 [Ar]3d ⁶ 4s 7.9024	27 Co Cobalt 58.933200 [Ar]3d ⁷ 4s 7.8810	28 Ni Nickel 58.6934 [Ar]3d ⁸ 4s 7.6398	29 Cu Copper 63.546 [Ar]3d ¹⁰ 4s 7.7264	30 Zn Zinc 65.409 [Ar]3d ¹⁰ 4s 9.942	31 Ga Gallium 69.723 [Ar]3d ¹⁰ 4s ² 4p 5.9993	32 Ge Germanium 72.64 [Ar]3d ¹⁰ 4s ² 4p ² 7.8994	33 As Arsenic 74.92160 [Ar]3d ¹⁰ 4s ² 4p ³ 7.8986	34 Se Selenium 78.96 [Ar]3d ¹⁰ 4s ² 4p ⁴ 9.7524	35 Br Bromine 79.904 [Ar]3d ¹⁰ 4s ² 4p ⁵ 11.8138	36 Kr Krypton 83.798 [Ar]3d ¹⁰ 4s ² 4p ⁶ 13.9996	37 Rb Rubidium 85.4678 [Kr]5s 4.1771	38 Sr Strontium 87.62 [Kr]5s ² 6.6399	39 Y Yttrium 88.90585 [Kr]4d ⁵ 5s 6.2173	40 Zr Zirconium 91.224 [Kr]4d ⁵ 5s 6.8399	41 Nb Niobium 92.90638 [Kr]4d ⁵ 5s 6.7589	42 Mo Molybdenum 95.94 [Kr]4d ⁵ 5s 7.28	43 Tc Technetium (98)	44 Ru Ruthenium 101.07 [Kr]4d ⁸ 5s 7.3605	45 Rh Rhodium 102.90550 [Kr]4d ⁸ 5s 7.4589	46 Pd Palladium 106.42 [Kr]4d ¹⁰ 8.3369	47 Ag Silver 107.8682 [Kr]4d ¹⁰ 5s 7.5762	48 Cd Cadmium 112.411 [Kr]4d ¹⁰ 5s 8.9938	49 In Indium 114.818 [Kr]4d ¹⁰ 5s ² 5p 5.7864	50 Sn Tin 118.710 [Kr]4d ¹⁰ 5s ² 5p ² 7.3439	51 Sb Antimony 121.760 [Kr]4d ¹⁰ 5s ² 5p ³ 8.6084	52 Te Tellurium 127.60 [Kr]4d ¹⁰ 5s ² 5p ⁴ 9.0096	53 I Iodine 126.90447 10.4513	54 Xe Xenon 131.293 [Kr]4d ¹⁰ 5s ² 5p ⁶ 12.1298	55 Cs Cesium 132.90545 [Xe]6s 3.8939	56 Ba Barium 137.327 [Xe]6s ² 5.2117	57 La Lanthanum 138.9055 [Xe]5d ¹ 6s ² 5.5769	58 Ce Cerium 140.116 [Xe]4f ^{5d} 6s ² 5.5387	59 Pr Praseodymium 140.90765 [Xe]4f ³ 6s 5.473	60 Nd Neodymium 144.24 [Xe]4f ⁴ 6s ² 5.5250	61 Pm Promethium (145) [Xe]4f ⁵ 6s ² 5.582	62 Sm Samarium 150.36 [Xe]4f ⁶ 6s ² 5.6437	63 Eu Europium 151.964 [Xe]4f ⁷ 6s ² 5.6704	64 Gd Gadolinium 157.25 [Xe]4f ⁷ 5d ¹ 6s ² 6.1496	65 Tb Terbium 158.92534 [Xe]4f ⁹ 6s ² 5.8638	66 Dy Dysprosium 162.500 [Xe]4f ¹⁰ 6s ² 5.9389	67 Ho Holmium 162.50332 [Xe]4f ¹¹ 6s ² 6.0215	68 Er Erbium 167.259 [Xe]4f ¹² 6s ² 6.1077	69 Tm Thulium 168.93421 [Xe]4f ¹³ 6s ² 6.1843	70 Yb Ytterbium 173.04 [Xe]4f ¹⁴ 6s ² 6.2542	71 Lu Lutetium 174.967 [Xe]4f ¹⁴ 5d ¹ 6s ² 5.4259	72 Hf Hafnium 178.49 [Xe]4f ¹⁴ 5d ² 6s ² 6.8251	73 Ta Tantalum 180.9479 [Xe]4f ¹⁴ 5d ³ 6s ² 7.5496	74 W Tungsten 183.84 [Xe]4f ¹⁴ 5d ⁴ 6s ² 7.8640	75 Re Rhenium 186.207 [Xe]4f ¹⁴ 5d ⁵ 6s ² 7.8335	76 Os Osmium 190.23 [Xe]4f ¹⁴ 5d ⁶ 6s ² 8.4382	77 Ir Iridium 192.217 [Xe]4f ¹⁴ 5d ⁷ 6s ² 8.9670	78 Pt Platinum 195.078 [Xe]4f ¹⁴ 5d ⁹ 6s ¹ 8.9588	79 Au Gold 196.96655 [Xe]4f ¹⁴ 5d ¹⁰ 6s ¹ 9.2255	80 Hg Mercury 200.59 10.4375	81 Tl Thallium 204.3833 [Hg]6p 6.1082	82 Pb Lead 207.2 [Hg]6p ² 7.4167	83 Bi Bismuth 208.98038 [Hg]6p ³ 7.2855	84 Po Polonium (209) [Hg]6p ⁴ 8.414	85 At Astatine (210) [Hg]6p ⁵ 8.414	86 Rn Radon (222) [Hg]6p ⁶ 10.7485	87 Fr Francium (223) [Rn]7s 4.0727	88 Ra Radium (226) [Rn]7s ² 5.2784	89 Ac Actinium (227) [Rn]5f ¹ 6d ¹ 7s ² 5.17	90 Th Thorium 232.0381 [Rn]6d ² 7s ² 6.3067	91 Pa Protactinium 231.03588 [Rn]5f ² 6d ¹ 7s ² 5.89	92 U Uranium 238.02891 [Rn]5f ³ 6d ¹ 7s ² 6.1941	93 Np Neptunium (237) [Rn]5f ⁴ 6d ¹ 7s ² 6.2657	94 Pu Plutonium (244) [Rn]5f ⁶ 7s ² 6.0260	95 Am Americium (243) [Rn]5f ⁷ 7s ² 5.9738	96 Cm Curium (247) [Rn]5f ⁸ 6d ¹ 7s ² 5.9914	97 Bk Berkelium (247) [Rn]5

Unique Features of Ytterbium Atoms

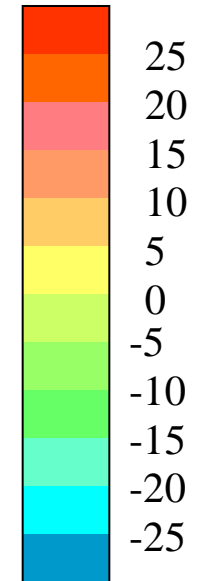
Rich Variety of Isotopes

^{168}Yb (0.13%)	^{170}Yb (3.05%)	^{171}Yb (14.3%)	^{172}Yb (21.9%)	^{173}Yb (16.2%)	^{174}Yb (31.8%)	^{176}Yb (12.7%)
Boson	Boson	Fermion	Boson	Fermion	Boson	Boson

Isotopic Tuning of Interatomic Interaction

Mass number	168	170	171	172	173	174	176
168	13						
170	6.2	3.4					
171	4.7	1.9	-0.1				
172	3.4	-0.1	-4.3	-30			
173	2.0	-4.2	-29	22	11		
174	0.2	-26	23	11	7.3	5.5	
176	-18	11	7.5	5.6	4.2	2.9	-1.2

Scattering Length



(nm)

[M. Kitagawa, *et al*, PRA77, 012719 (2008)]

Collaboration with R. Ciurylo, P. Naidon, P. Julienne

Unique Features of Ytterbium Atoms

Rich Variety of Isotopes

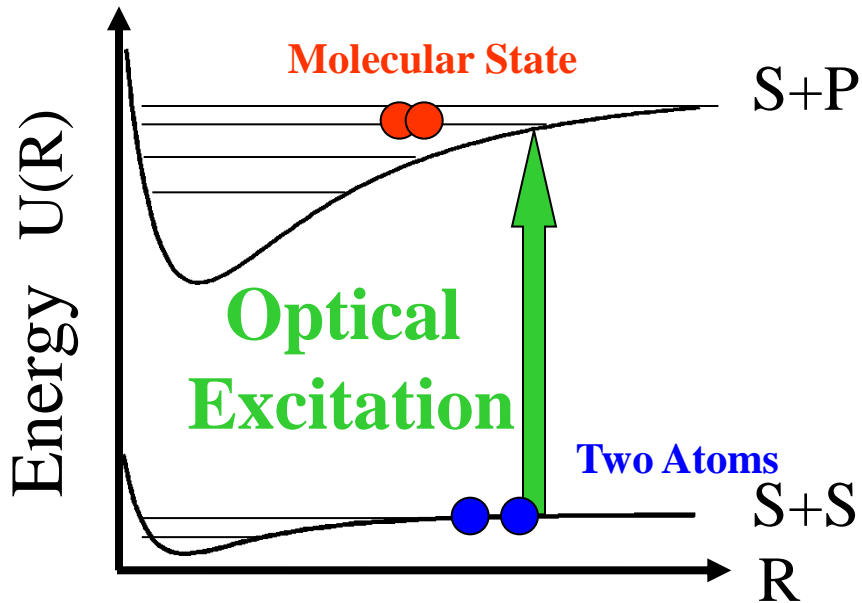
^{168}Yb (0.13%)	^{170}Yb (3.05%)	^{171}Yb (14.3%)	^{172}Yb (21.9%)	^{173}Yb (16.2%)	^{174}Yb (31.8%)	^{176}Yb (12.7%)
Boson	Boson	Fermion	Boson	Fermion	Boson	Boson

$$^{173}\text{Yb} (I=5/2) \quad H_{\text{int}} = \frac{4\pi\hbar^2 a_s}{M} \delta(\vec{r}_1 - \vec{r}_2) \quad \mathbf{SU(6) system}$$

→ novel magnetism

[M. A. Cazalilla, *et al.*, N. J. Phys $\mathbf{11}$, 103033(2009), Hermele, *et al.*, PRL 103, 130351 (2009); A. V. Gorshkov, *et al.*, Nat. Physics, 6, 289(2010)]

Optical Feshbach Resonance



$$S_{00} = \frac{\Delta - i\Gamma_S / 2 + i\gamma / 2}{\Delta + i\Gamma_S / 2 + i\gamma / 2}$$

$$\Gamma_S \propto |\langle b | V_{las} | f \rangle|^2$$

γ :spontaneous decay rate

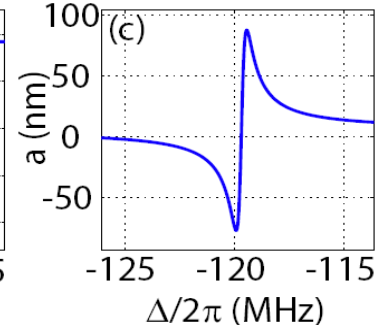
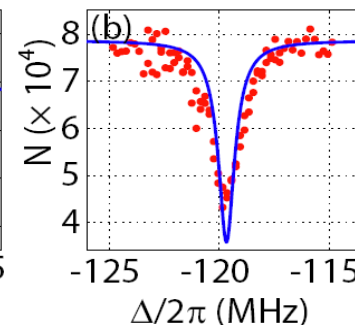
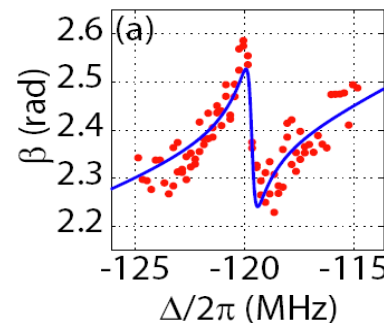
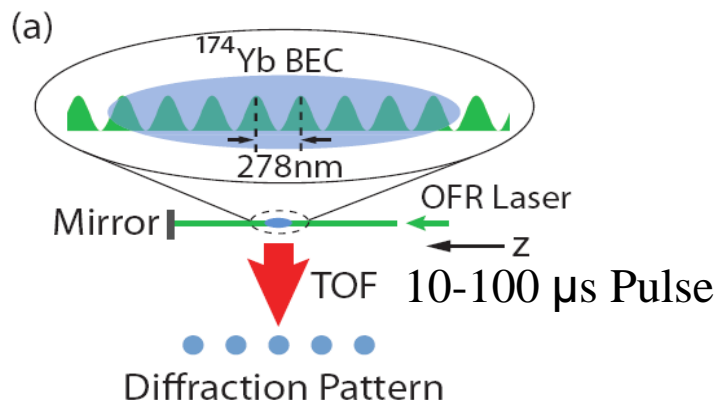
Δ :detuning from the PA resonance

[J. Bohn and P. Julienne PRA(1999)]

Advantages for Intercombination Lines

R. Ciurylo, *et al.* *Phys. Rev. A* **70**. 062710 (2004)

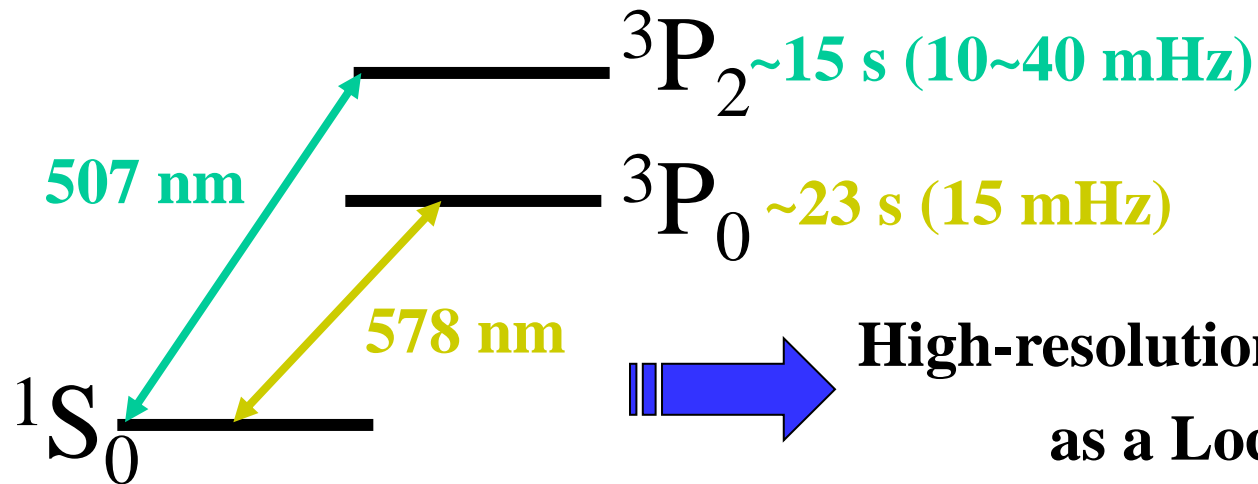
Nanometer-scale Spatial Modulation



[R. Yamazaki *et al.*, PRL**105**, 050405 (2010)]

Unique Features of Ytterbium Atoms

Ultra-narrow Optical Transitions



**High-resolution laser spectroscopy
as a Local Probe**

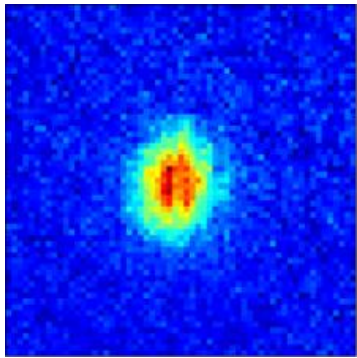
**Another Useful Orbital States with
Different Characters**

Quantum Degenerate Gases of Yb

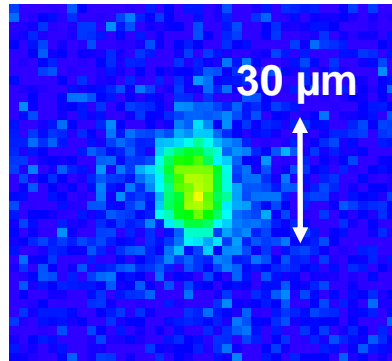
[Y. Takasu *et al.*, PRL **91**, 040404 (2003)] [T. Fukuhara *et al.*, PRA **76**, 051604(R)(2007)]

[S. Sugawa *et al.*, PRA **84**, 011610(R)(2011)]

^{168}Yb (0.13%)

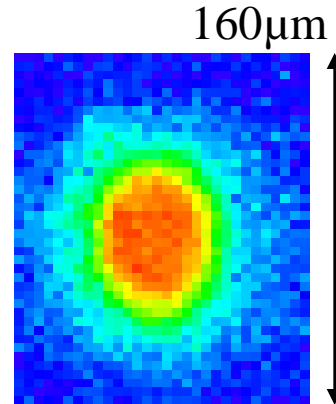


^{170}Yb

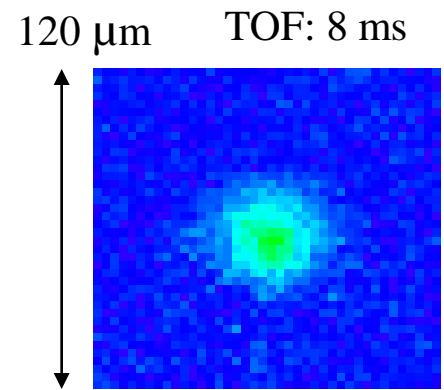


TOF:
10ms

^{174}Yb

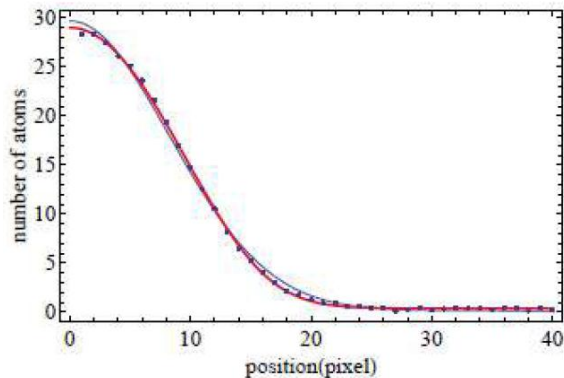
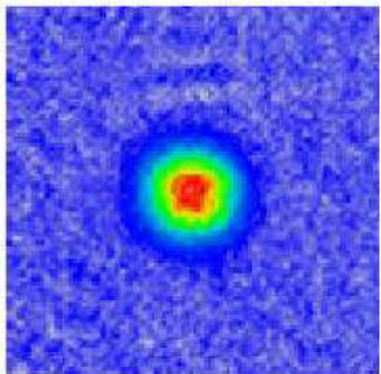


^{176}Yb

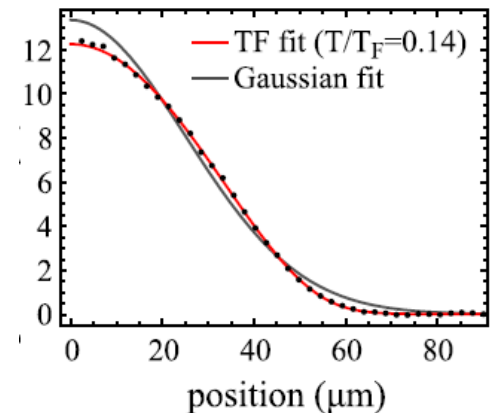
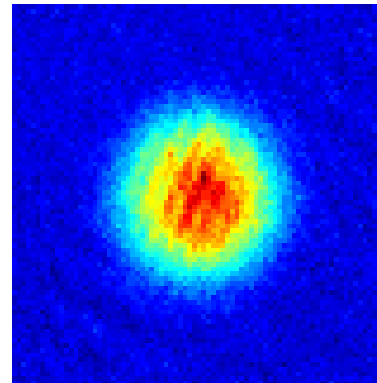


Fermion [T. Fukuhara *et al.*, PRL. **98**, 030401 (2007)] [S. Taie *et al.*, PRL **105**, 190401(2010)]

^{171}Yb ($I=1/2$) $T/T_F = 0.3$
(2-component)



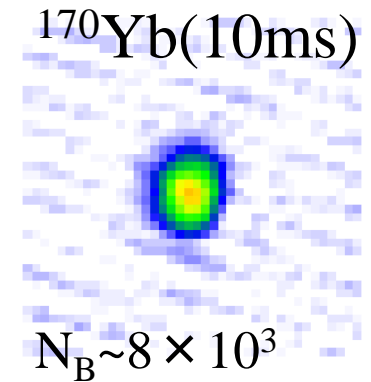
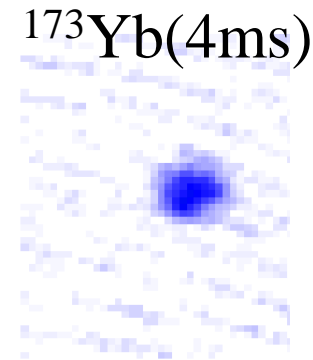
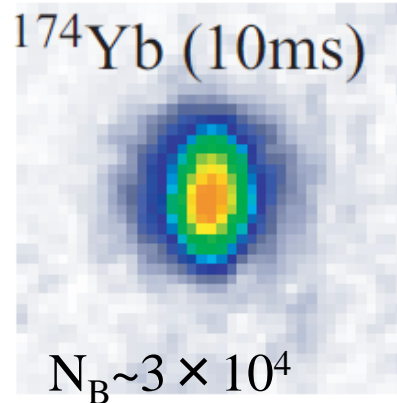
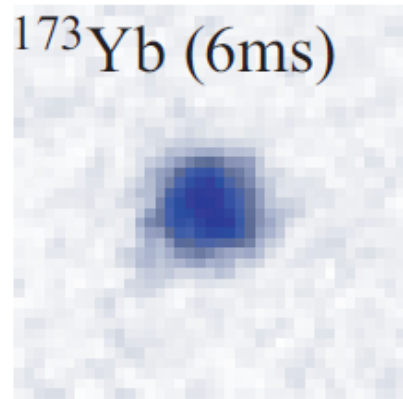
^{173}Yb ($I=5/2$): **SU(6)** ($T/T_F = 0.14$)
(6-component)



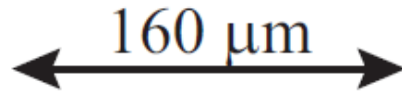
Quantum Degenerate Mixtures of Yb

[T. Fukuhara *et al.*, Phys. Rev. A 79, 021601(R) (2008)] [S. Taie *et al.*, PRL105, 190401(2010)]

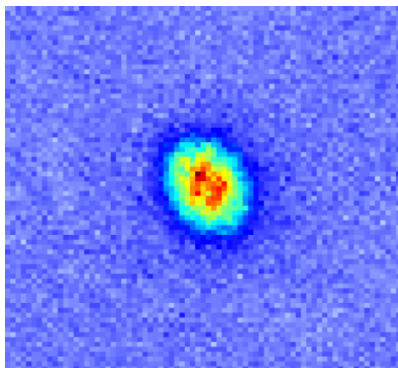
^{173}Yb (Fermion) + ^{174}Yb (Boson) ^{173}Yb (Fermion) + ^{170}Yb (Boson)



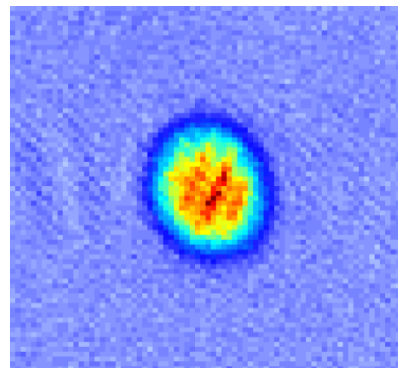
160 μm



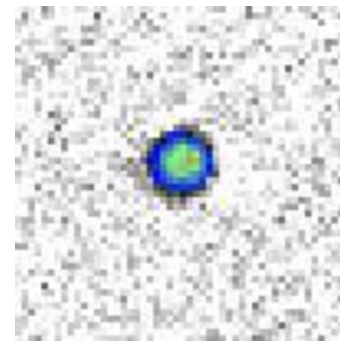
^{168}Yb (Boson) + ^{174}Yb (Boson) ^{171}Yb (Fermion) + ^{173}Yb (Fermion)



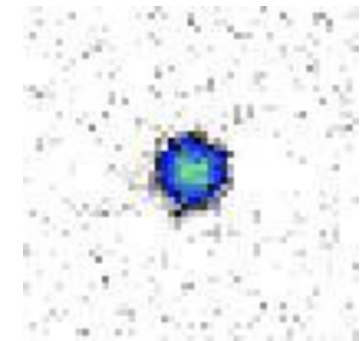
$N_B \sim 6 \times 10^3$



$N_B \sim 4.5 \times 10^4$



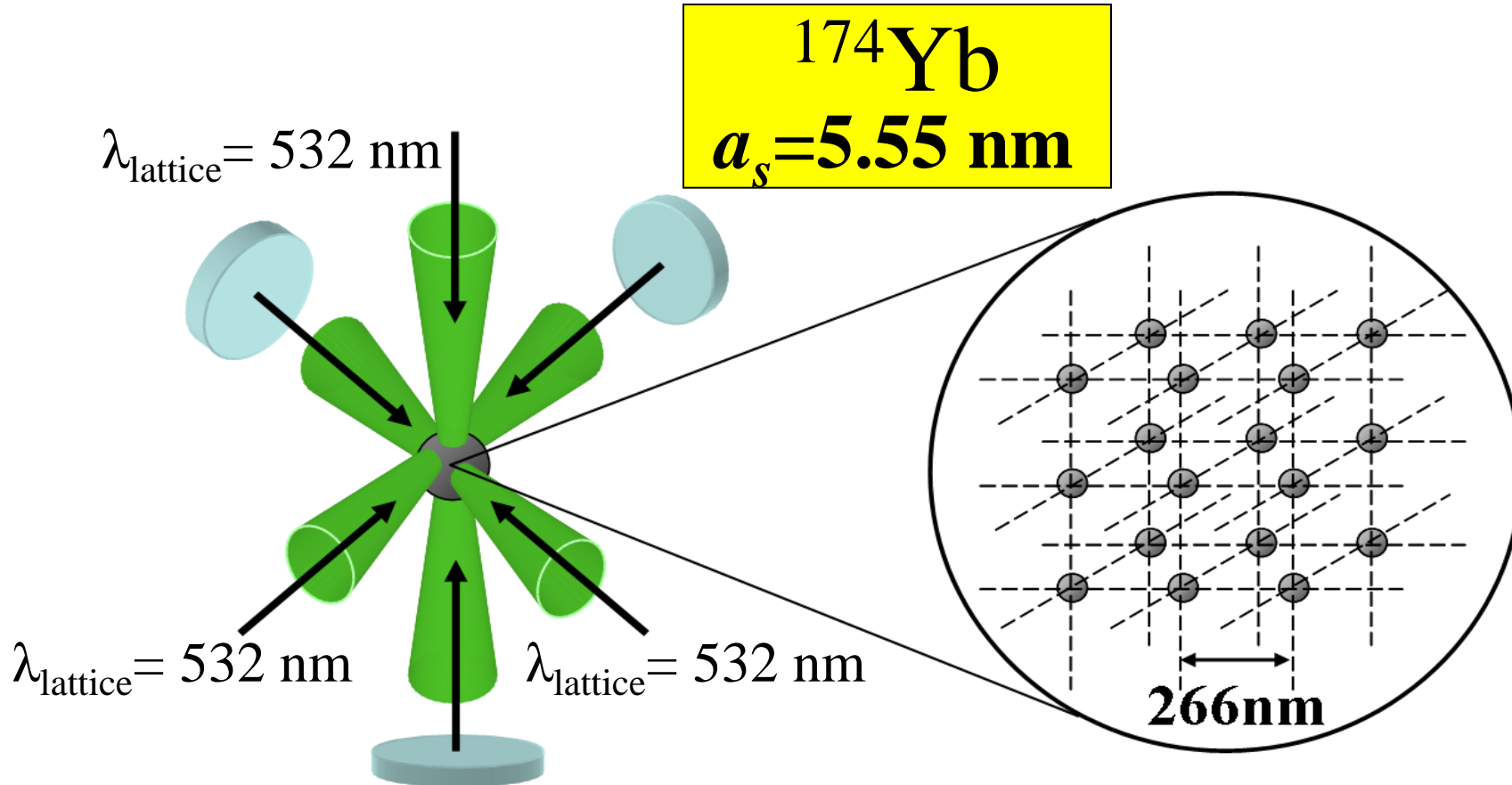
$T/T_F = 0.33$



$T/T_F = 0.3$

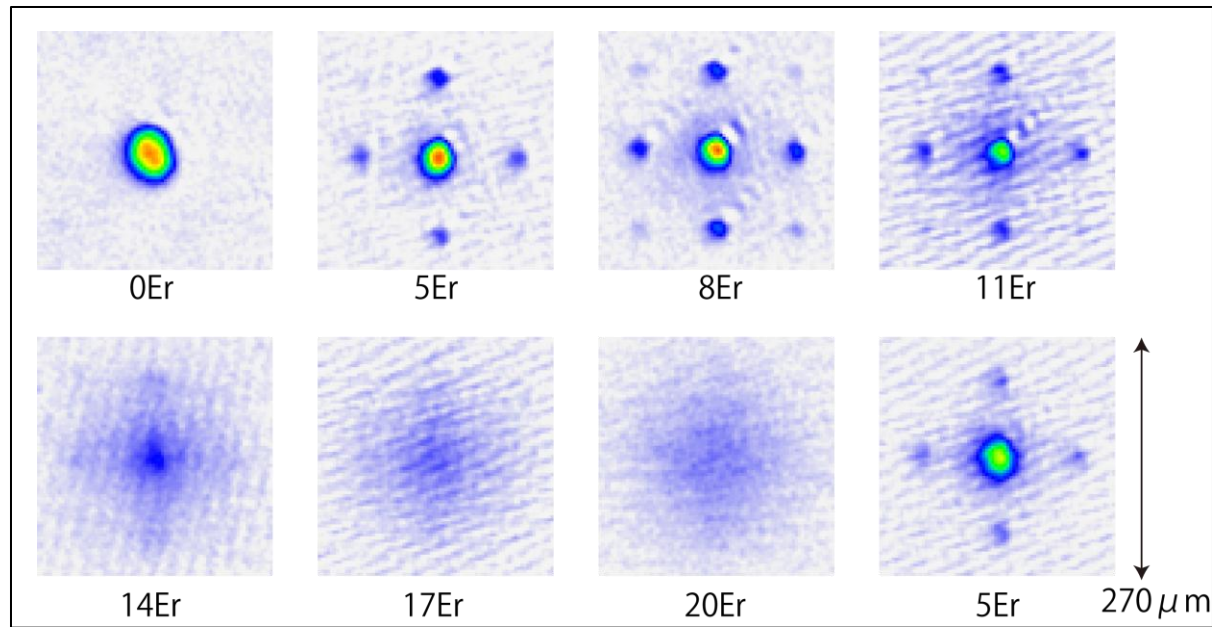
Boson ^{174}Yb in a 3D optical lattice

$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1) + \sum_i \varepsilon_i n_i$$



Superfluid-Mott Transition

T. Fukuhara, *et al.*, *PRA*. **79**, 041604R (2009); H. Moritz and T. Esslinger, *Physics* **2**, 31(2009)(Viewpoint)



→ Unique Applications to Quantum Computing

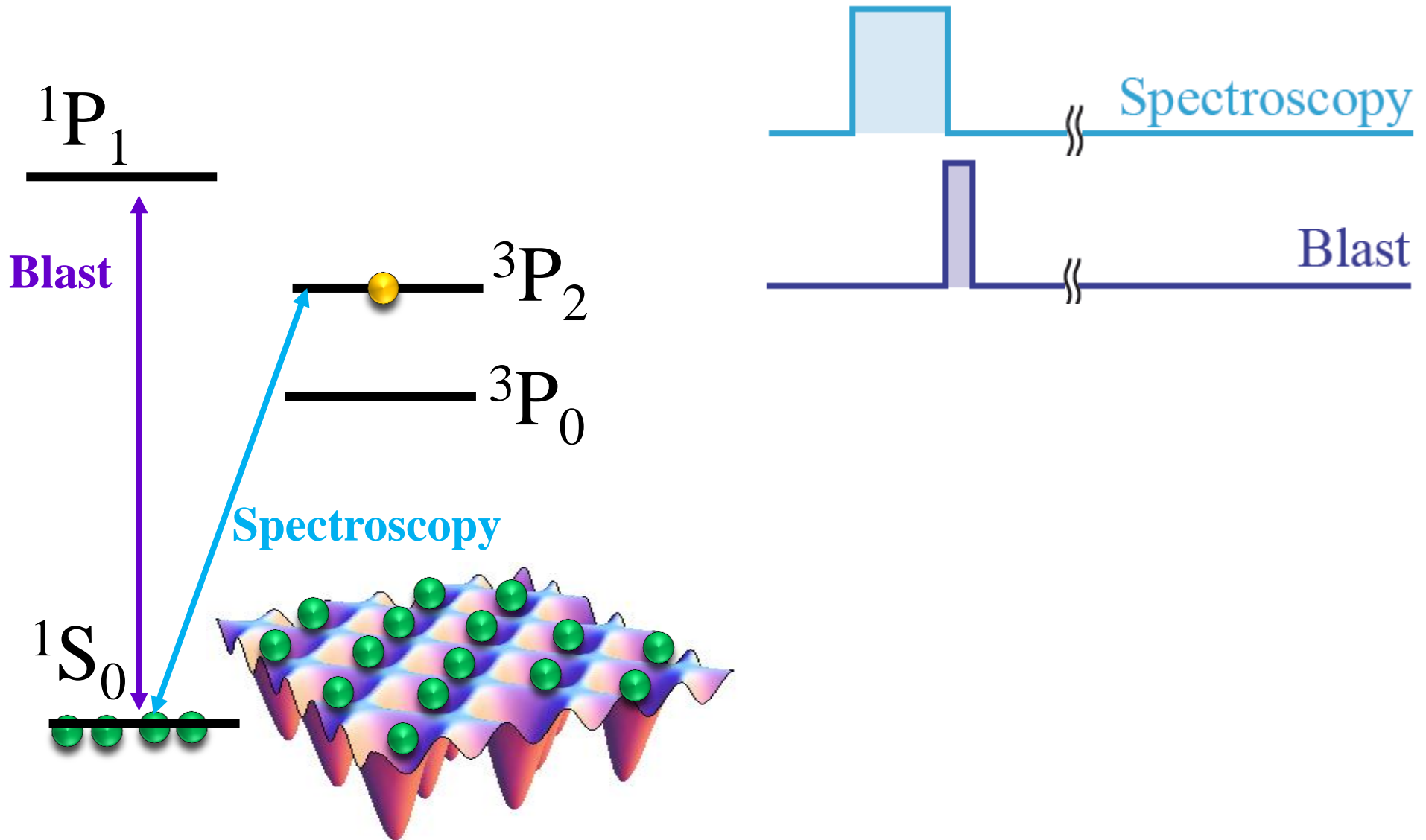
K. Shibata *et al.*, *Appl. Phys. B* **97**, 753(2009). Single-Atom Addressing by MRI

A. J. Daley *et al.*, *PRL*. **101**, 170504(2008). Dual Lattice Configuration

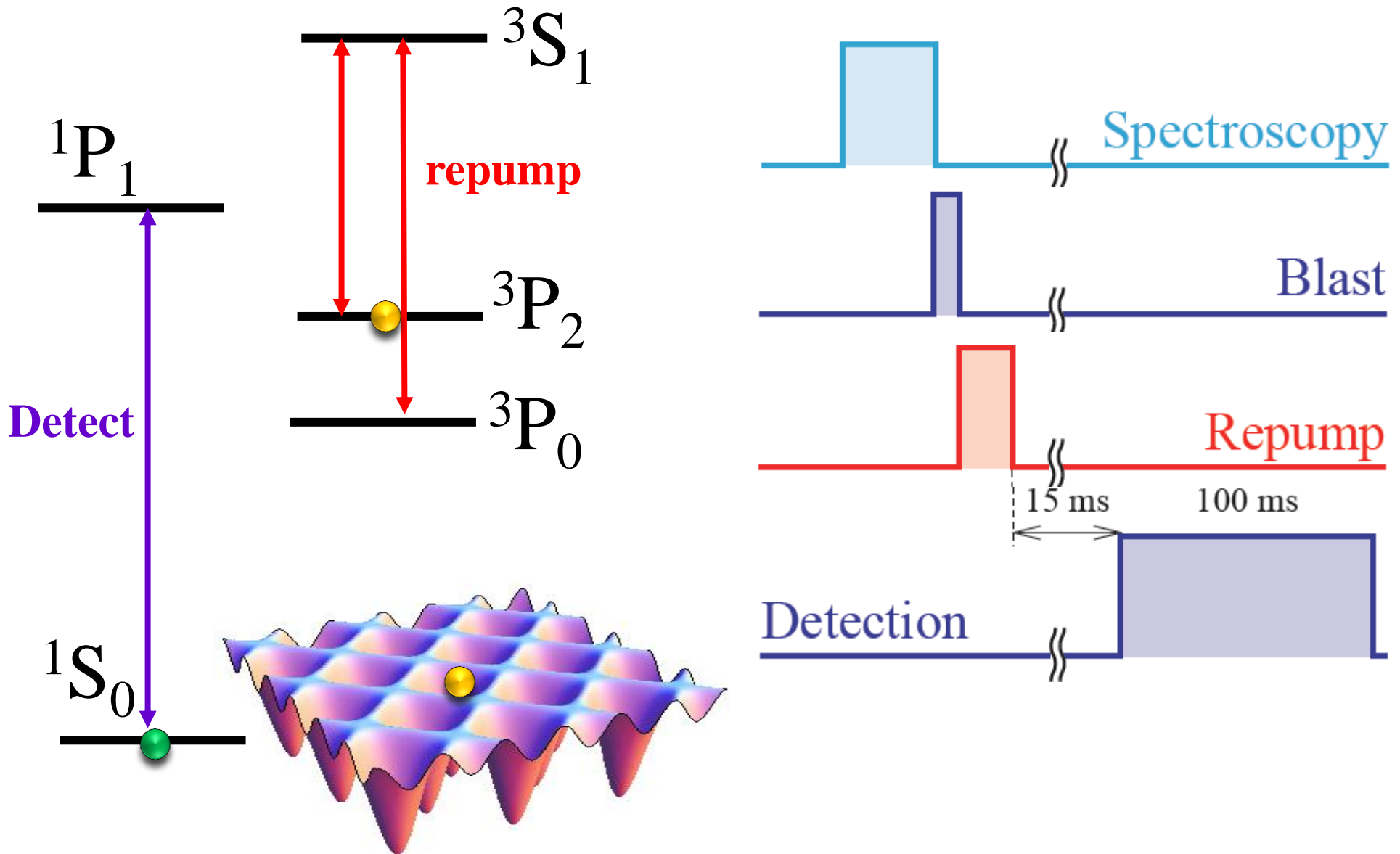
A. V. Gorshkov *et al.*, *PRL*. **102**, 110503(2009). Few-Qubit Quantum Register

F. Gerbier and J. Dalibard, *New J. Physics* **12**, 033007(2010). Gauge fields

Spectroscopy of Atoms in an Optical lattice

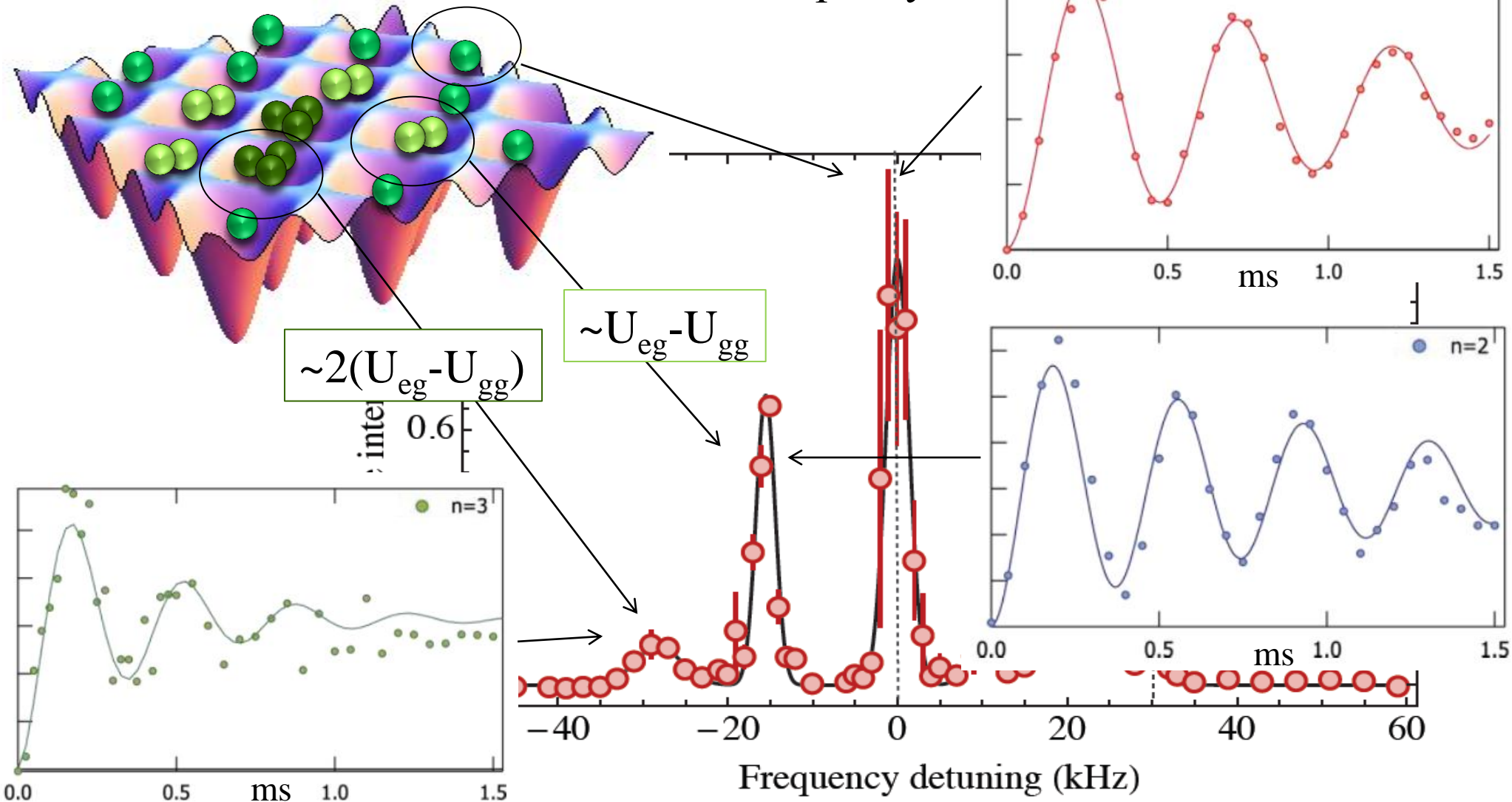


Spectroscopy of Atoms in an Optical lattice

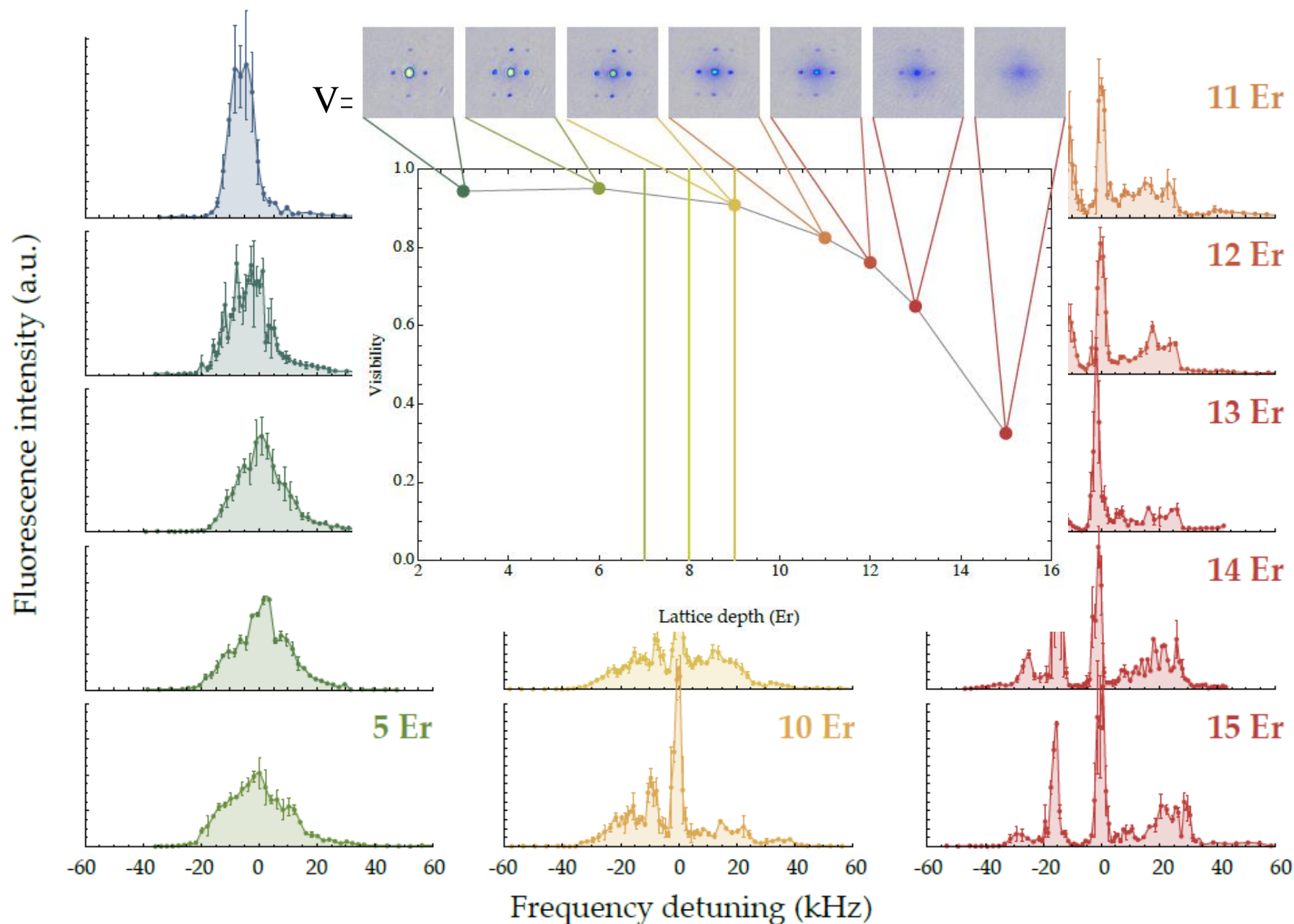


Spectroscopy of Atoms in a Mott Insulating State

“We can spectroscopically resolve and independently control the single, double, and triple occupancy”

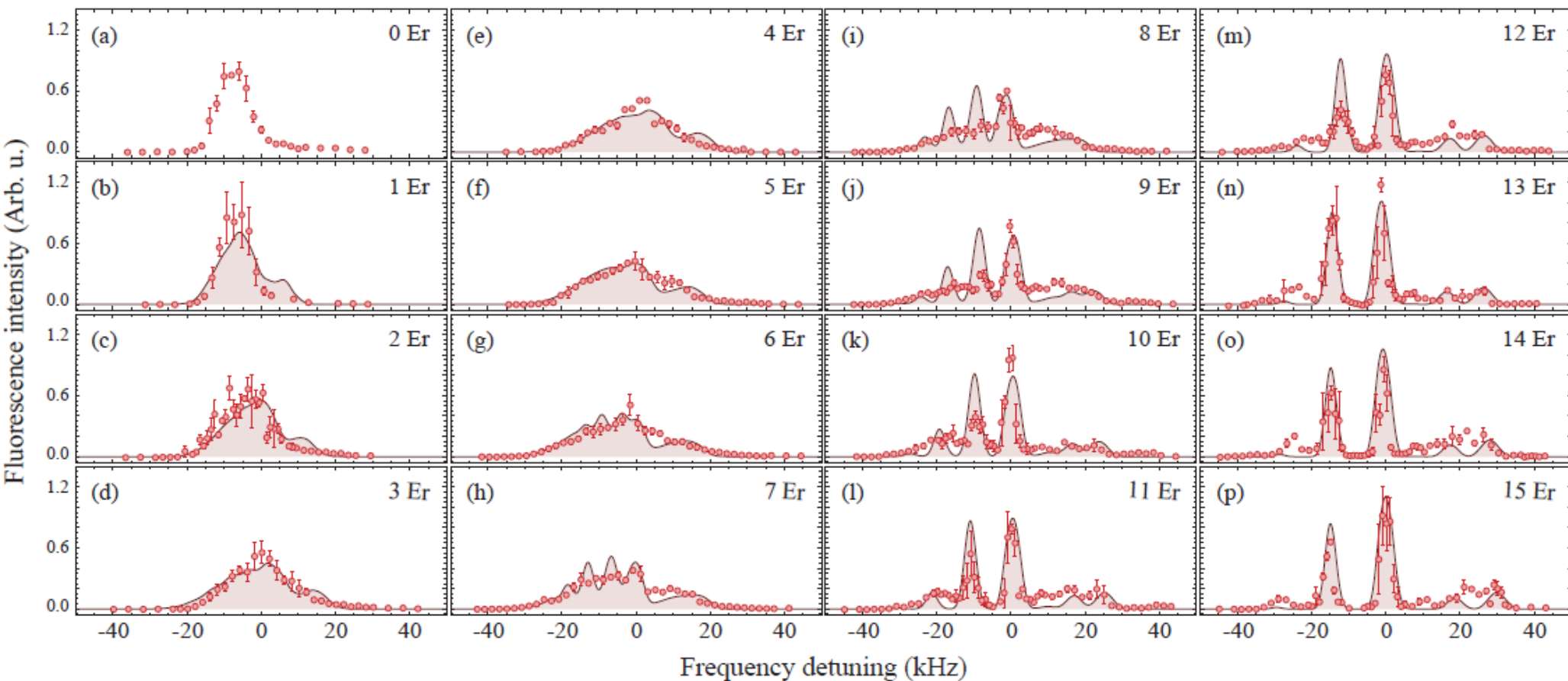


Spectroscopy of Superfluid-Mott Insulator Transition



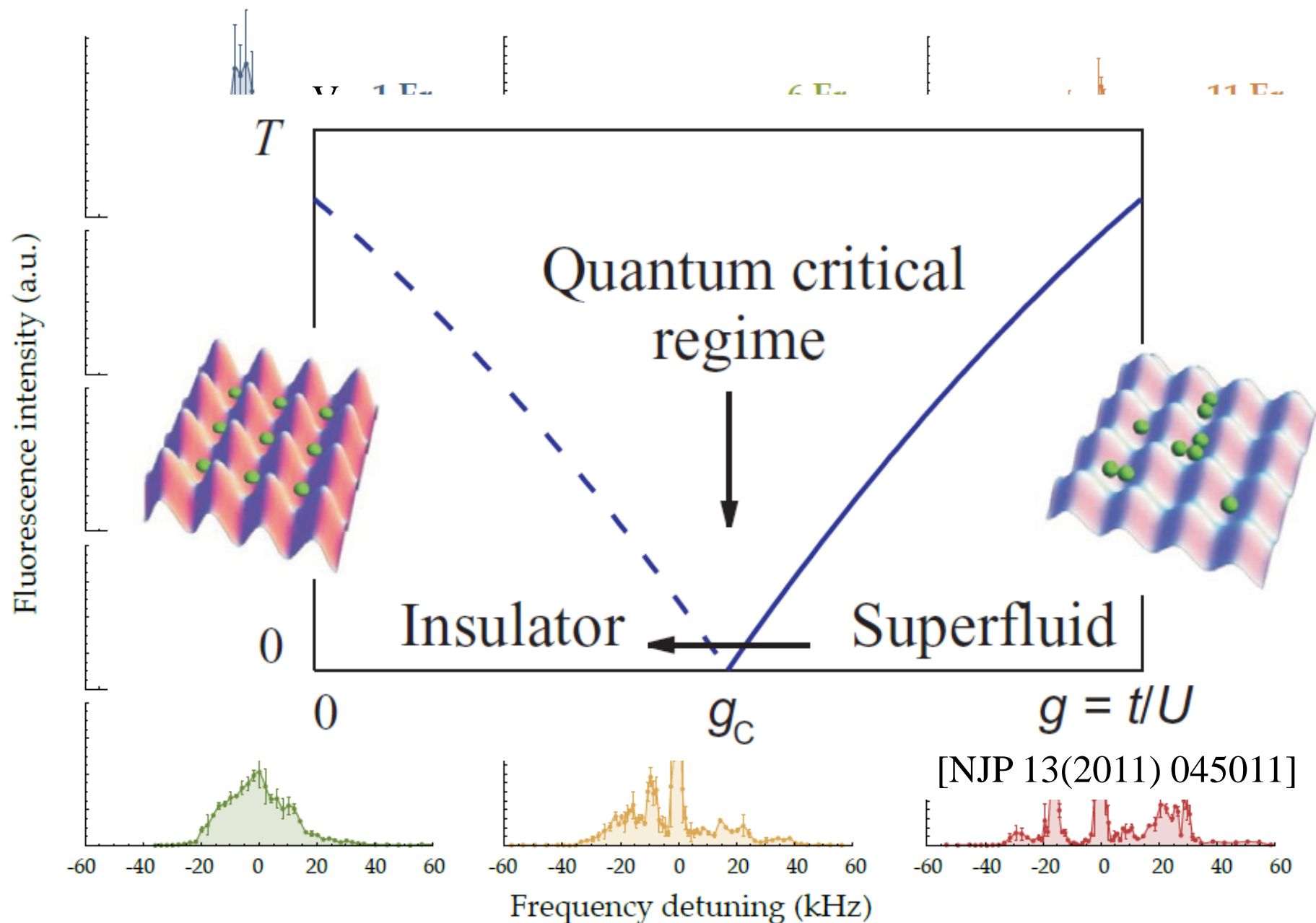
Spectroscopy of Superfluid-Mott Insulator Transition

“Comparison with finite temperature Gutzwiller calculation by Inaba”
(preliminary)



$T_{\text{init}} = 95 \text{ nK}$

Spectroscopy of Superfluid-Mott Insulator Transition

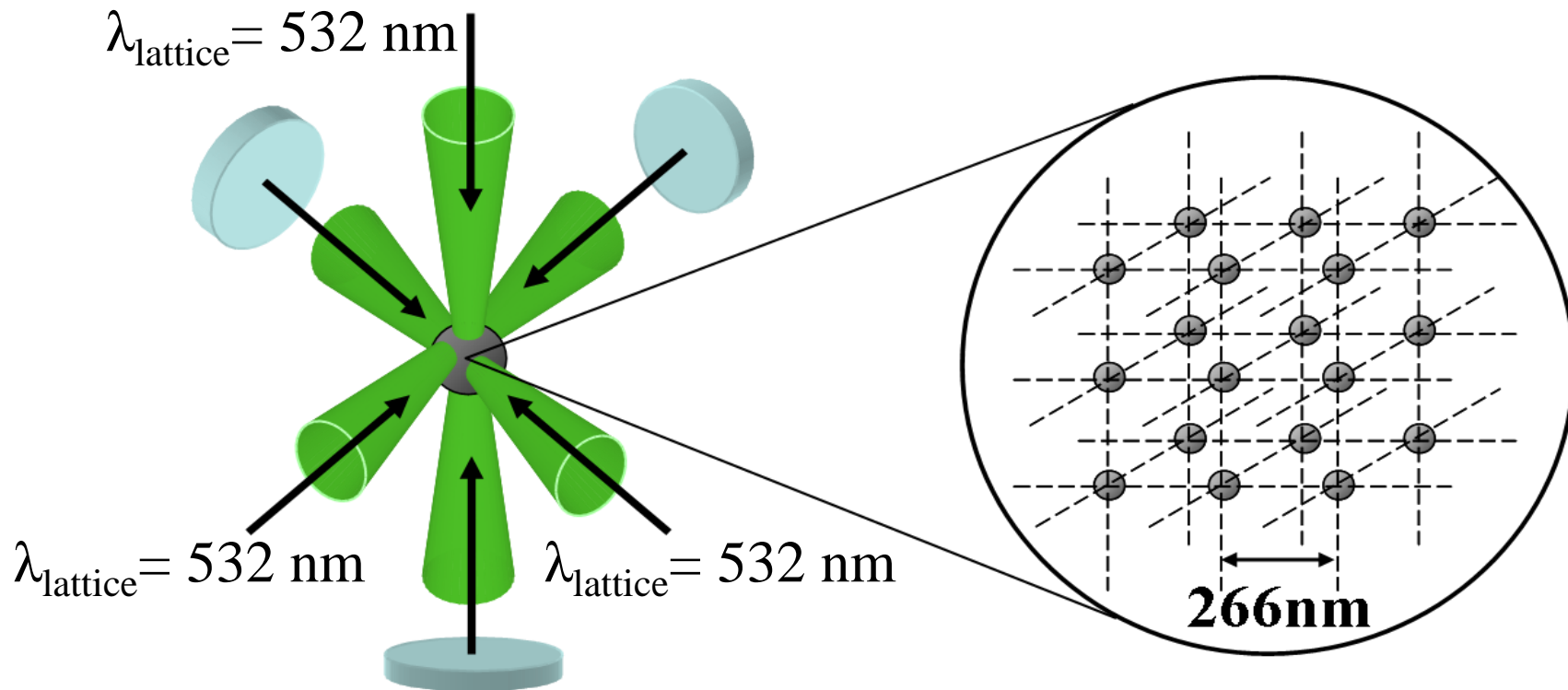


Fermion (^{173}Yb) in a 3D optical lattice

$^{173}\text{Yb}(I=5/2)$
 $a_s=10.5 \text{ nm}$

$$H = -t_F \sum_{\langle i,j \rangle} c_i^\dagger c_j + U_{FF} \sum_{i, m_F \neq m_F'} n_{m_F, i} n_{m_F', i}$$

SU(6)Mott-state



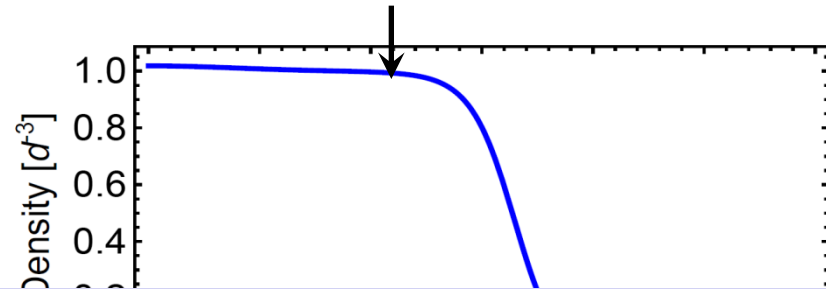
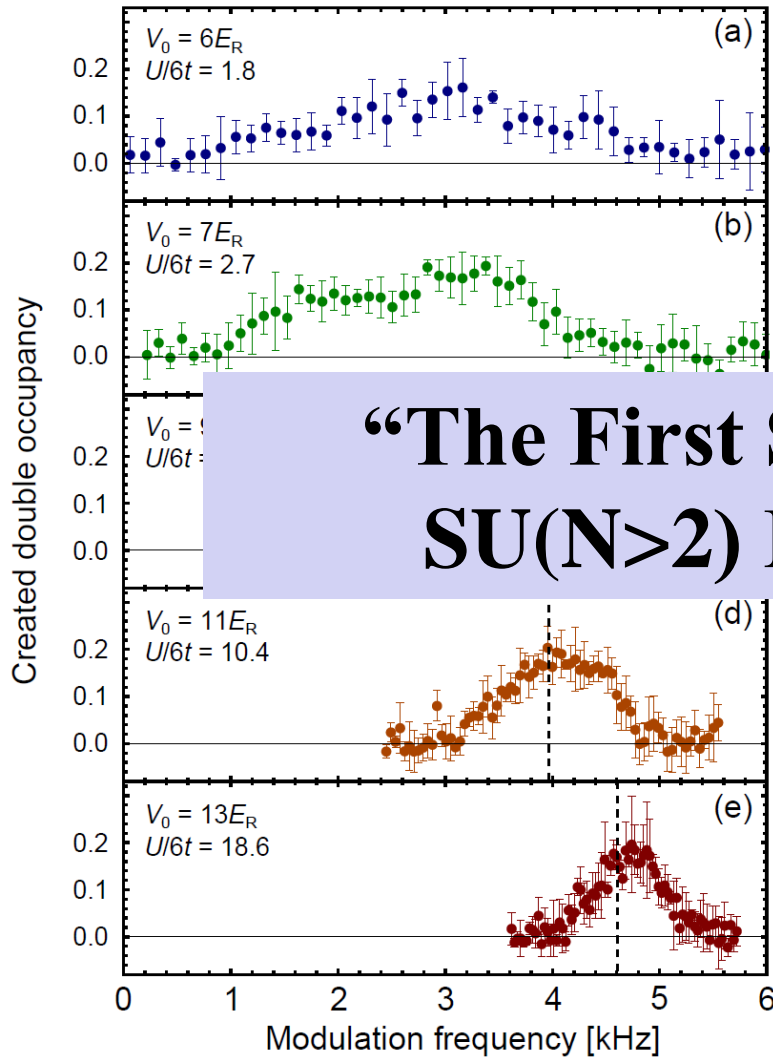
“Formation of SU(6) Mott insulator”

[S. Taie *et al.*,]

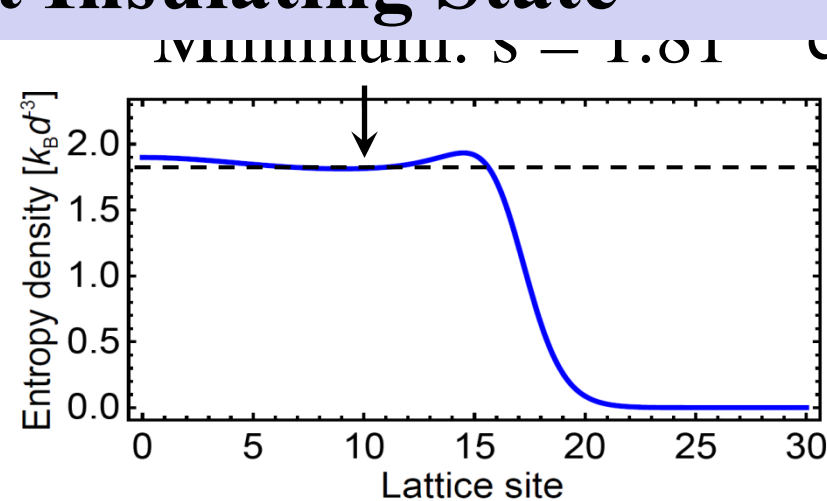
Excitation (Mott) Gap

$T_{\text{lattice}} = 5.1t = 16 \text{ nK}$ $U/t = 62.4$

Mott Plateau ($n=1$)



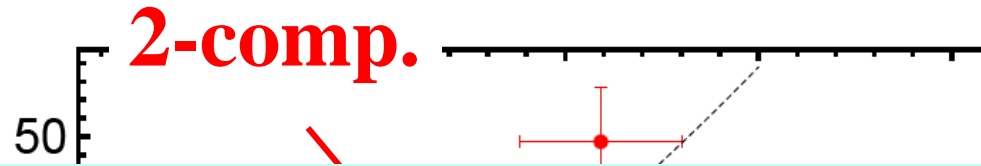
“The First Successful Formation of SU($N > 2$) Mott Insulating State”



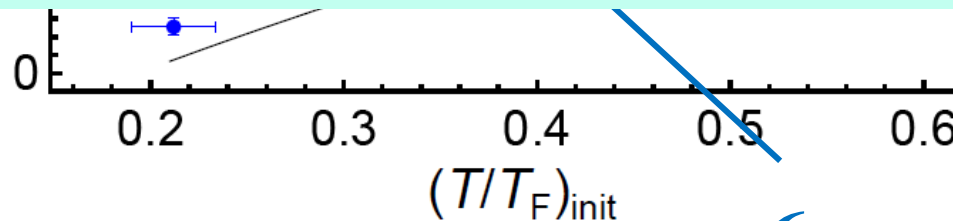
Minimum $S = 1.81$ cf. $\ln(6) = 1.79$

Atomic Pomeranchuk Cooling

[^{173}Yb atoms in optical lattice; Taie *et al.*,]



What is the mechanism of the enhanced cooling ?



6-comp.

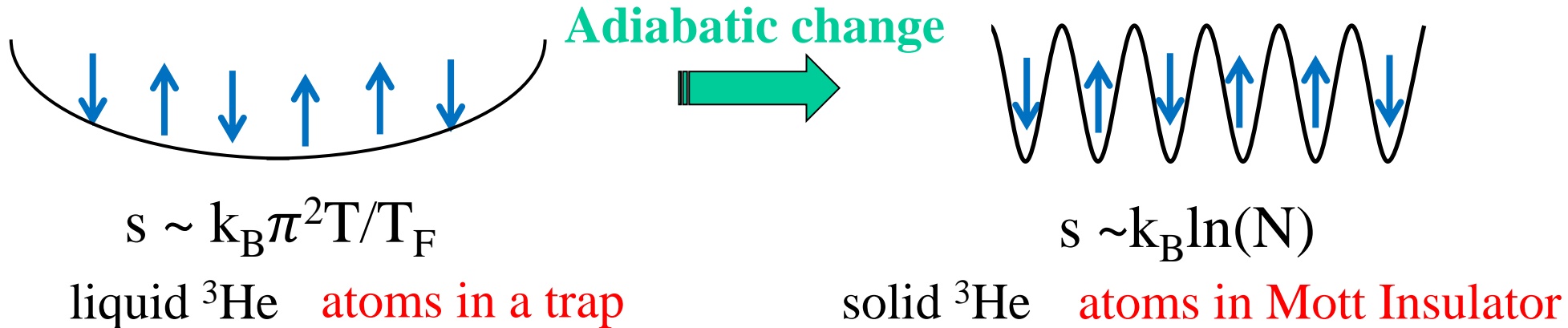
Pomeranchuk Cooling

Pomeranchuk Cooling [Pomeranchuk, (1950)]

—→ Discovery of Superfluid ^3He by Osheroff, Lee, Richardson

**Initial state: Spin *de*polarized
and also with *degeneracy*:**

**Final state: Spin *de*polarized
and also with *localization***



“entropy flows from **motional** degrees of freedom to **spin**,
which results in the low temperature”

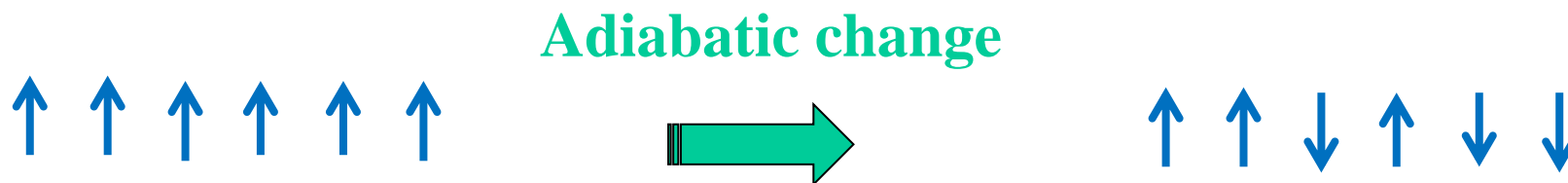
—→ “Pomeranchuk Cooling of an Atomic Gas”

Spin Degrees of Freedom *is Cool*

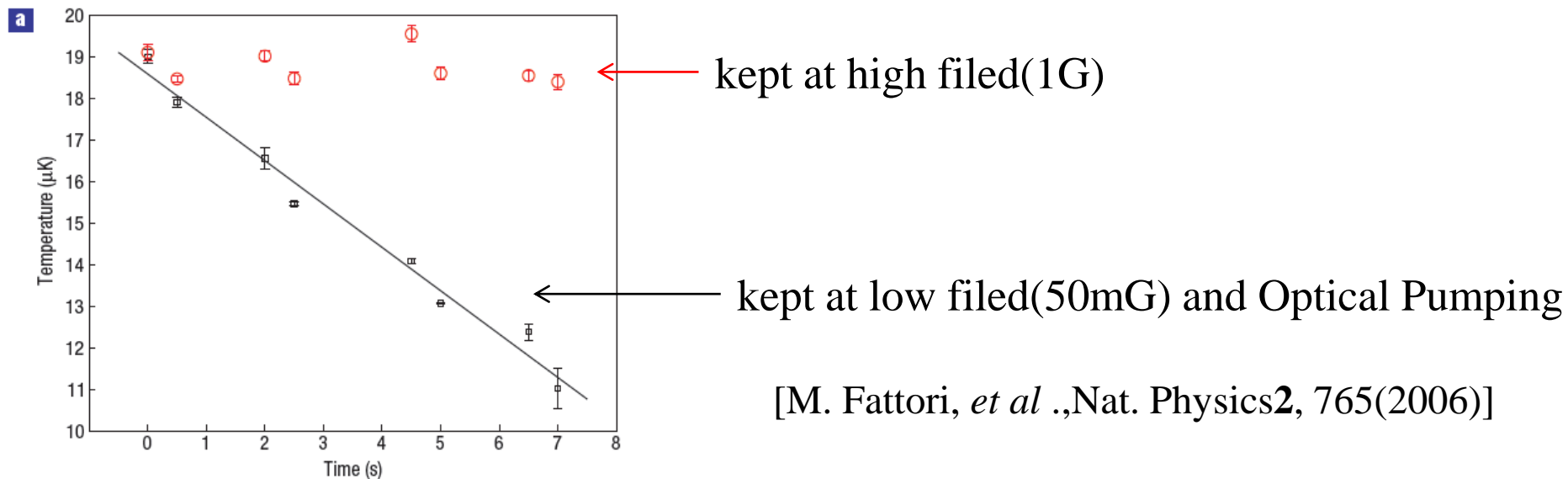
Demagnetization Cooling [W. J. De Haas, *et al.*, (1934)]

Initial state: Spin-polarized:

Final state: Spin-depolarized:



“entropy flows from **motional** degrees of freedom to **spin**, which results in the cooling of the system”



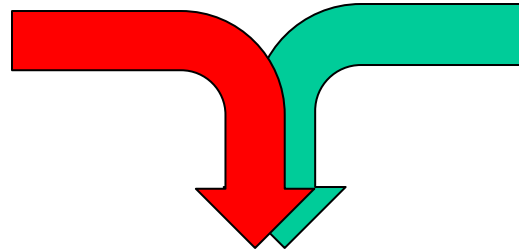
[M. Fattori, *et al.*, Nat. Physics **2**, 765(2006)]

Strongly Interacting Two Different Mott Insulators

[S. Sugawa, K. Inaba, *et al.*, arXiv:1011.4503v2]

Bosonic Mott insulator

Fermionic Mott Insulator



Mixture of Spinless Boson and SU(6) Fermion in a 3D optical lattice

$$^{174}\text{Yb}(\text{Boson}) + ^{173}\text{Yb}(\text{Fermion})$$
$$a_{BB} = +5.6 \text{ nm} \quad a_{FF} = +10.6 \text{ nm}$$

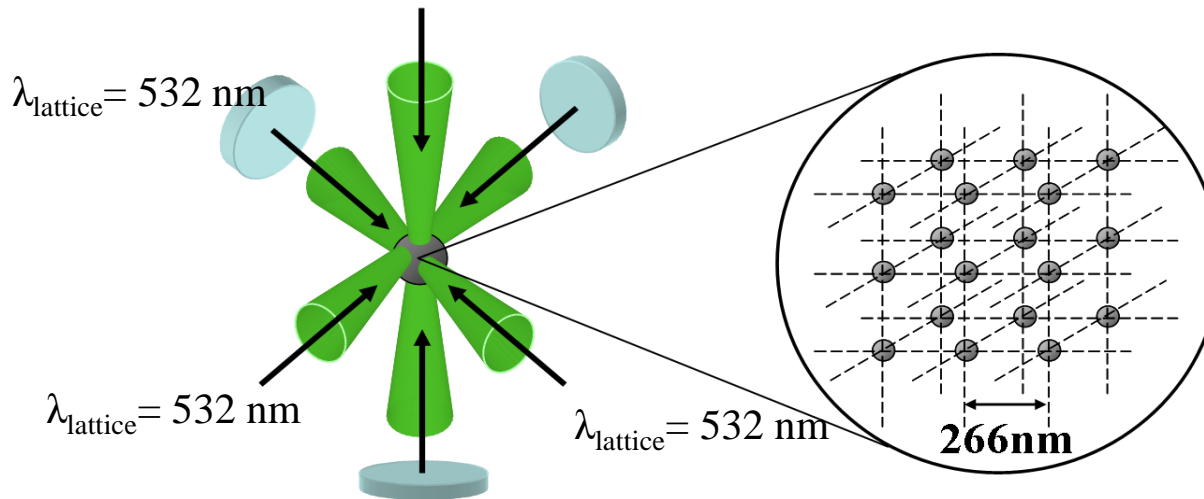
^{173}Yb :
6-spin components

$$a_{BF} = +7.3 \text{ nm}$$

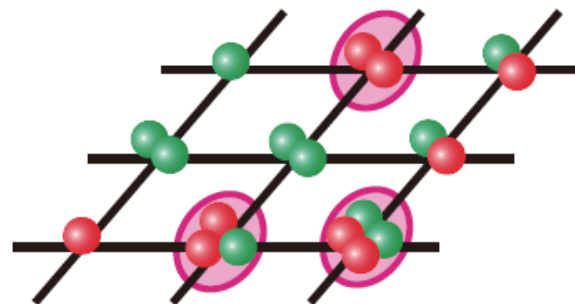
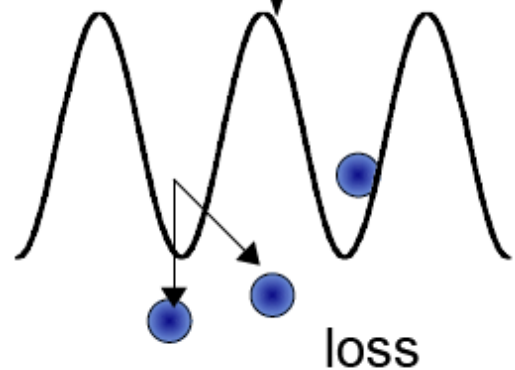
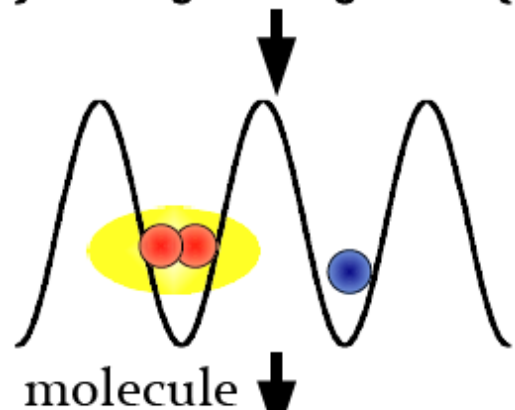
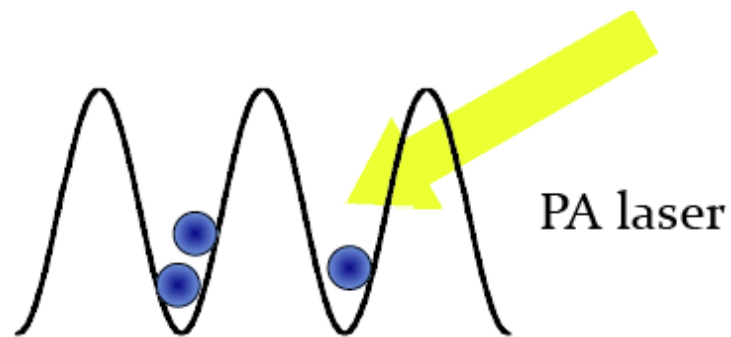
(Repulsive Interaction)

Dual Mott Insulators of Boson and Fermion:

$$J \ll k_B T < U_{BB} < U_{BF} < U_{FF}$$

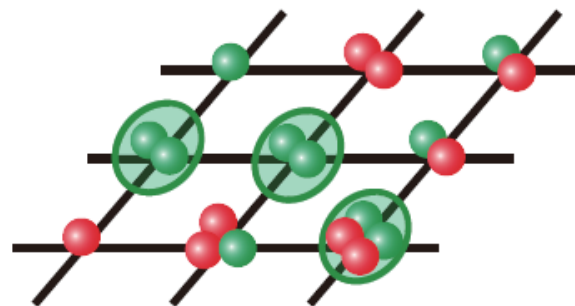


Measurement of Site Occupancy by Photoassociation

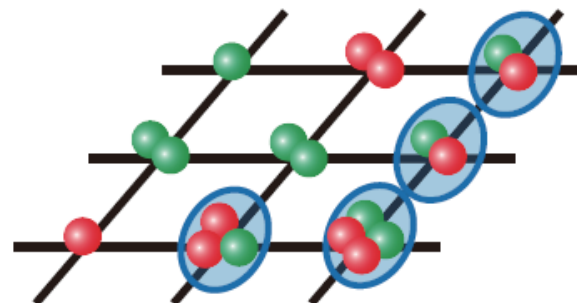


● fermion
● boson

**Bosonic
Double Occupancy**



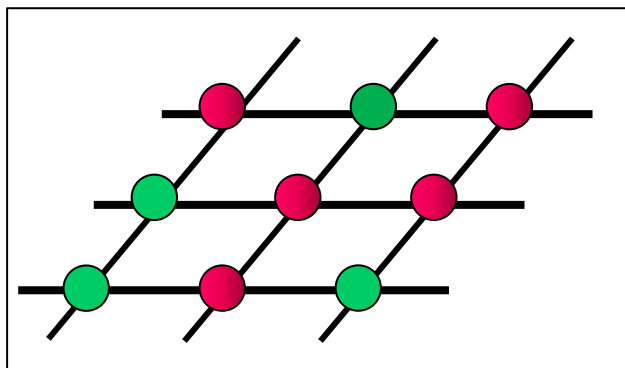
**Fermionic
Double Occupancy**



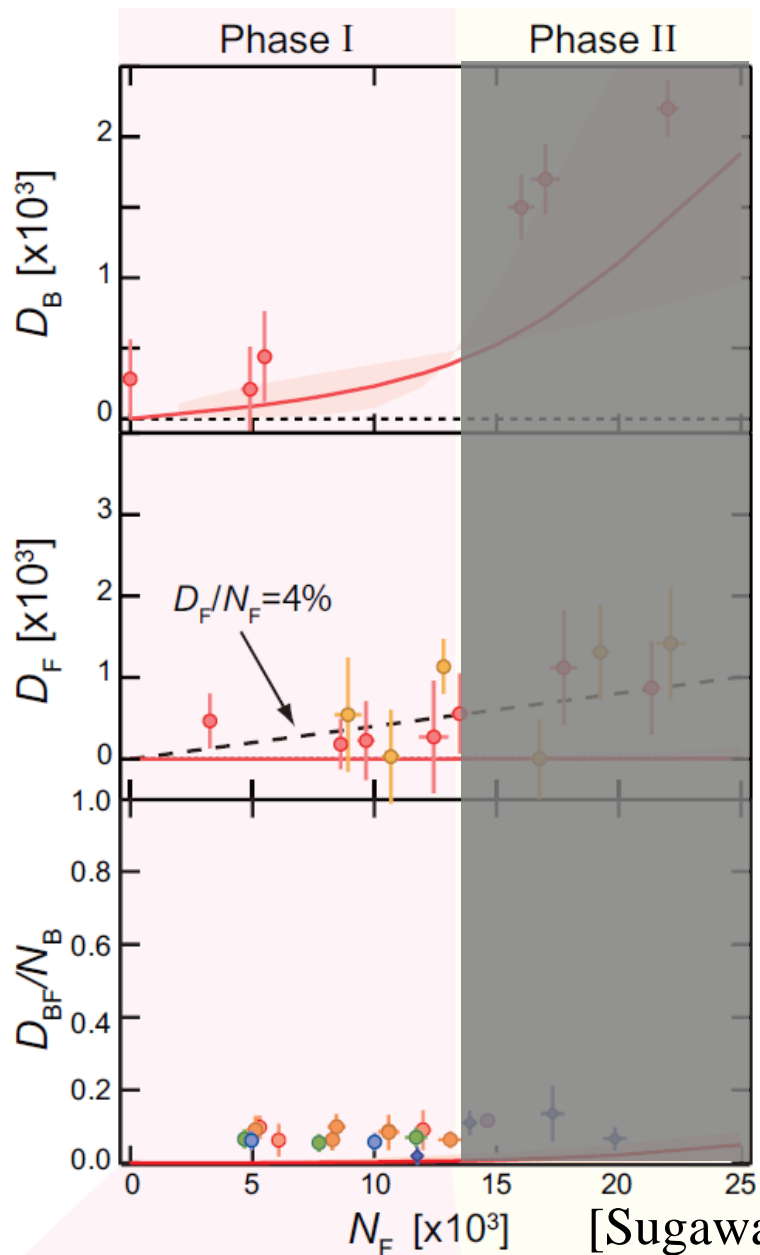
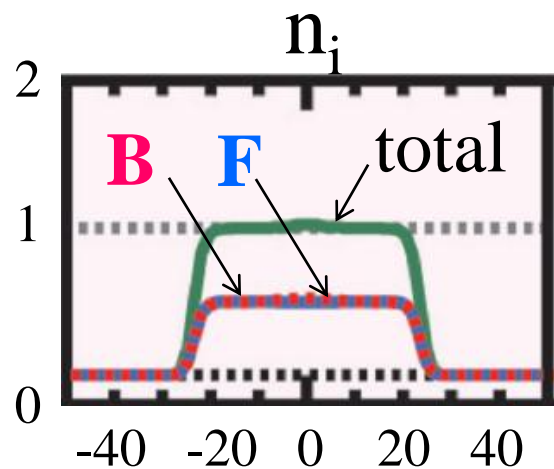
**Bose-Fermi
Pair Occupancy**

Repulsively Interacting Bose-Fermi Mott Insulators

- fermion
- boson

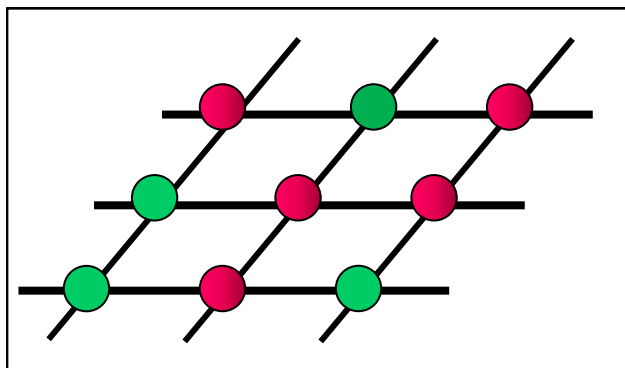


“Mixed Mott Insulator”

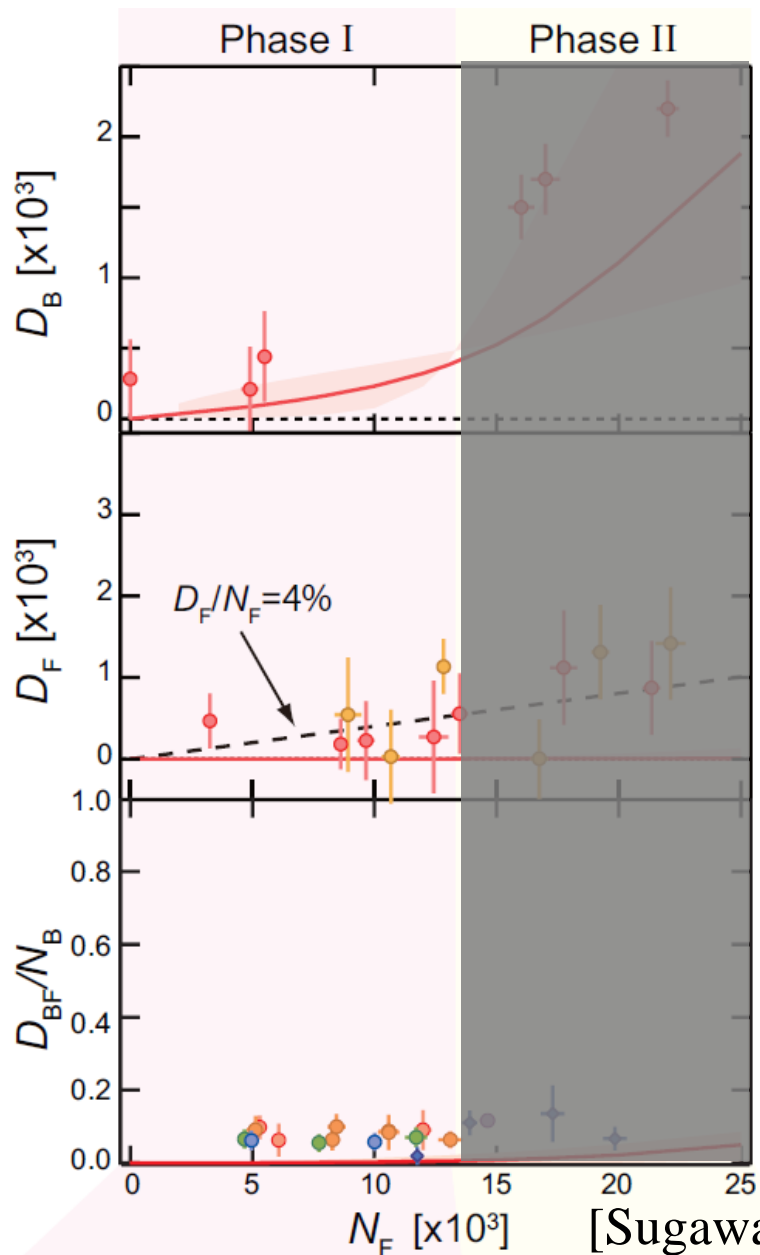
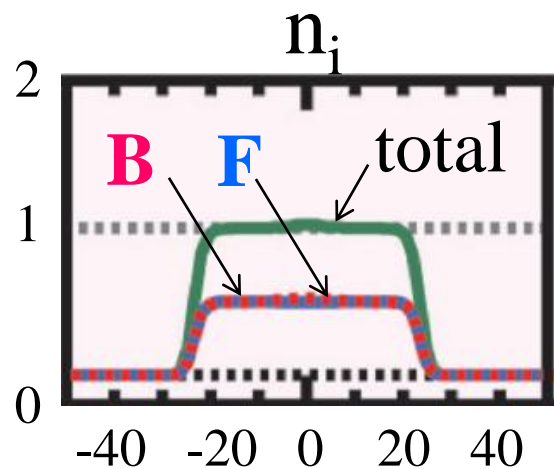


Repulsively Interacting Bose-Fermi Mott Insulators

- fermion
- boson



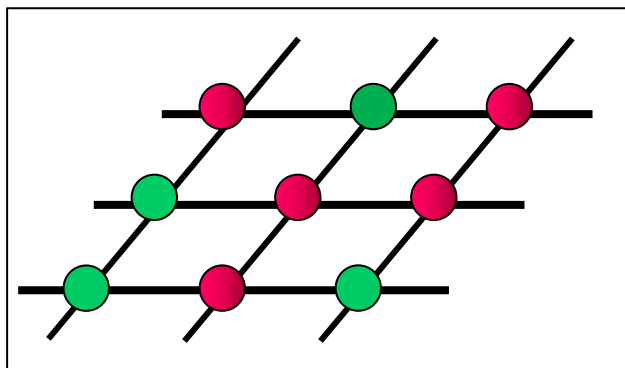
“Mixed Mott Insulator”



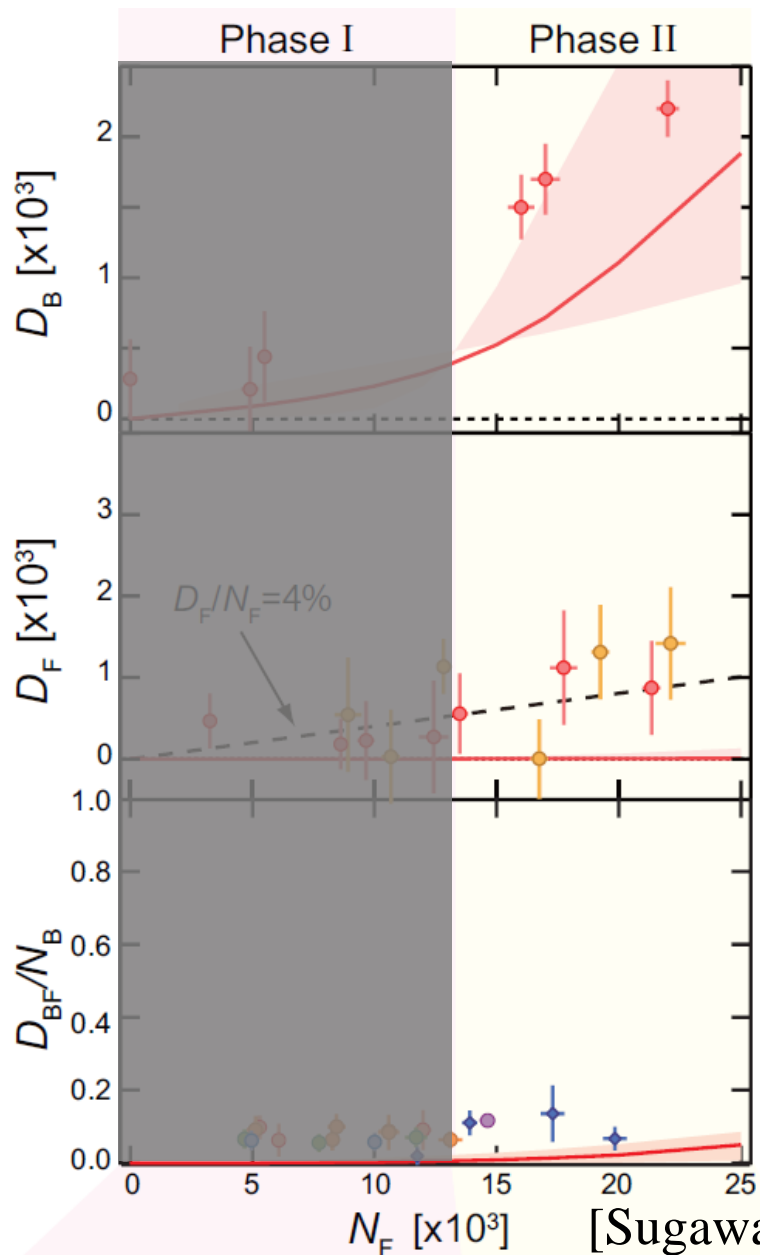
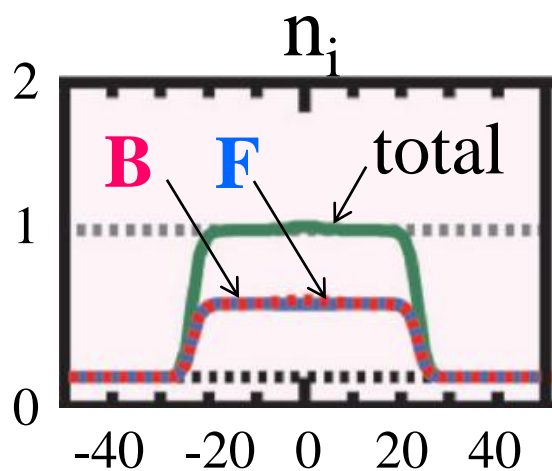
[Sugawa *et al.* NP. 7, 642–648 (2011)]

Repulsively Interacting Bose-Fermi Mott Insulators

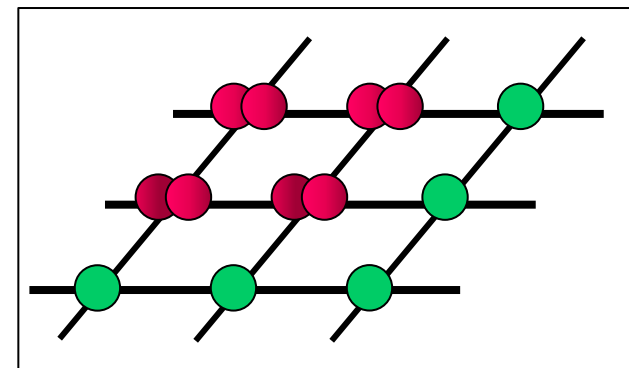
- fermion
- boson



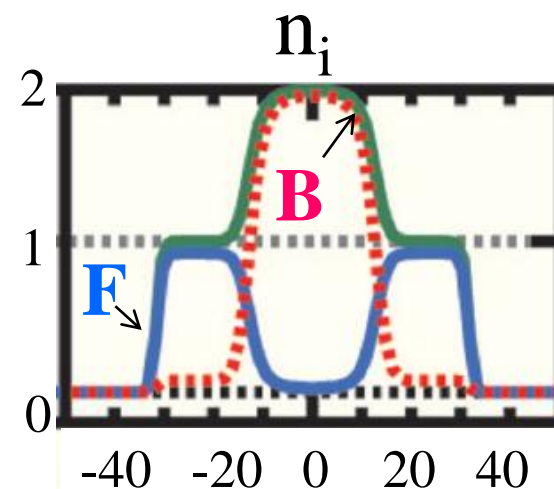
“Mixed Mott Insulator”



- fermion
- boson



“Phase Separation”

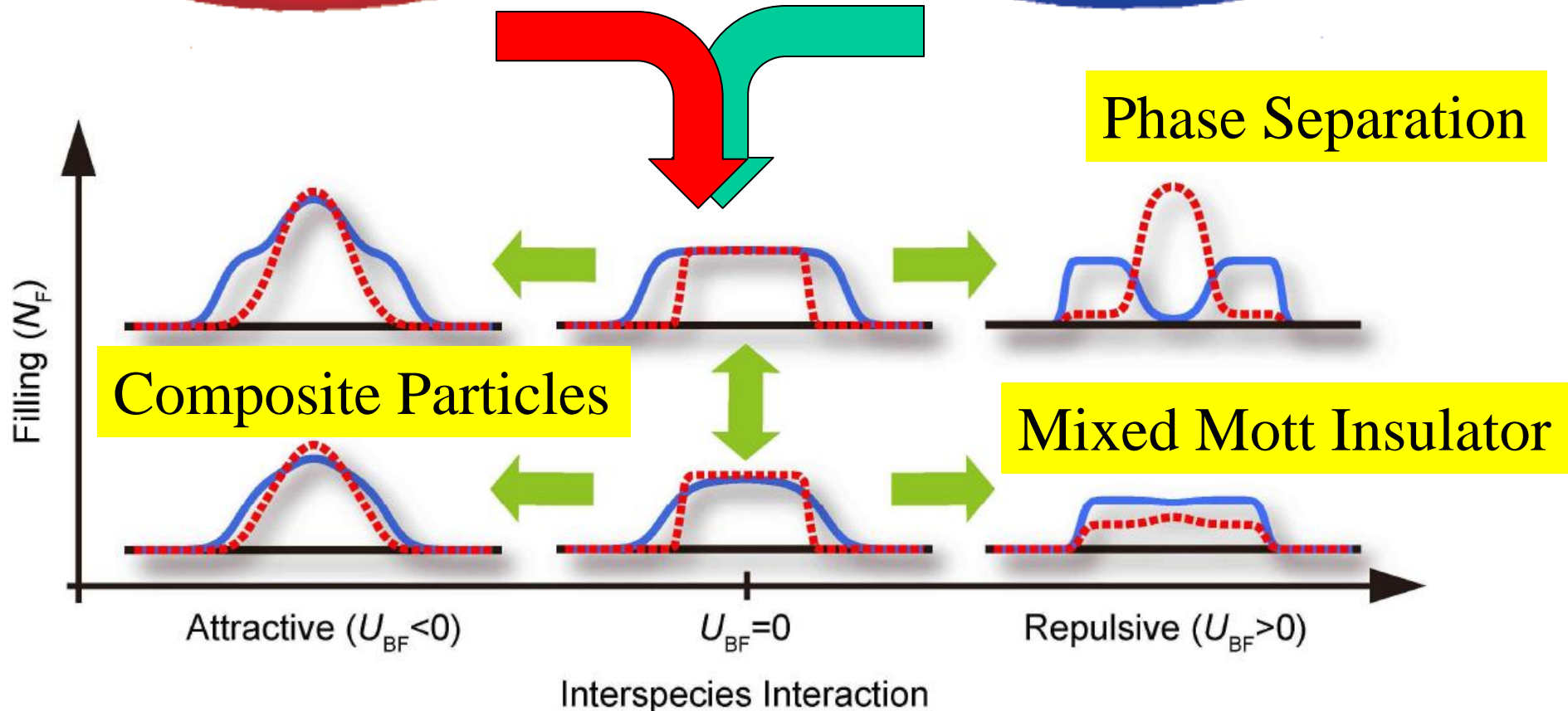


Strongly Interacting Two Different Mott Insulators

[S. Sugawa, K. Inaba, *et al.*, arXiv:1011.4503v2]

Bosonic Mott insulator

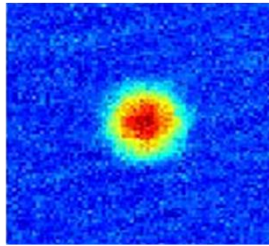
Fermionic Mott Insulator



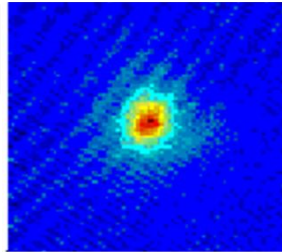
Anderson Hubbard Model with Li-Yb Mixture

Fermion(${}^6\text{Li}$)-Boson(${}^{174}\text{Yb}$)

${}^6\text{Li}$



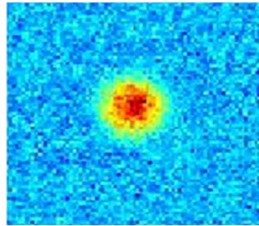
${}^{174}\text{Yb}$



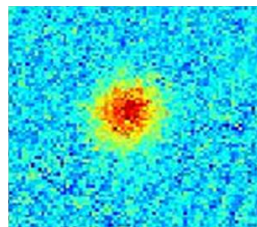
$$T/T_F = 0.08 \pm 0.01$$

Fermion(${}^6\text{Li}$)-Fermion(${}^{173}\text{Yb}$)

${}^6\text{Li}$

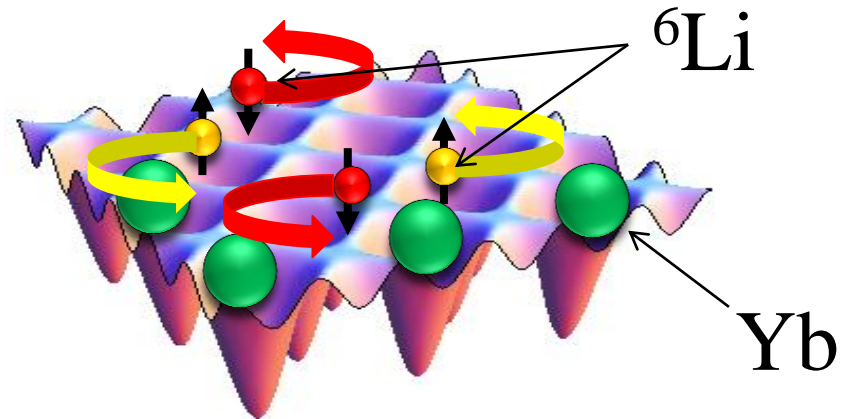


${}^{173}\text{Yb}$



$$T/T_F = 0.07 \pm 0.02$$

[H. Hara *et al.*, PRL **106**, 205304, (2011)]



[D. Semmler, K. Byczuk, and W. Hofstetter, PRB **81**, 115111(2010)]

$$M_{{}^{174}\text{Yb}} / M_{{}^6\text{Li}} \cong 29$$

Summary2

Quantum Simulation of Hubbard Model Using **Yb atoms** in an Optical Lattice

1) Bose-Hubbard Model: *Superfluid-Mott Insulator Transition*

High-Resolution Laser spectroscopy

2) Fermi-Hubbard Model: *Fermi Mott Insulator*

SU(6) Mott insulator

Pomeranchuk Cooling,

Close to quantum magnetism

3) Bose-Fermi-Hubbard Model:

Mixed Mott Insulaotr

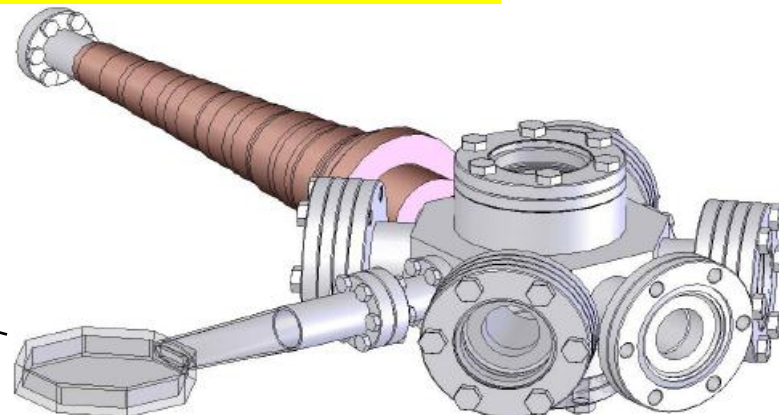
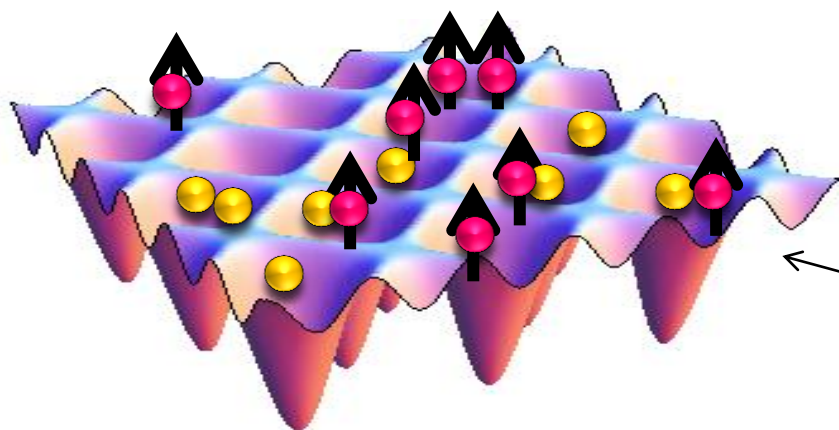
4) Plan

Anderson Localization (Lieb Lattice, Spin-Orbit interaction)

(Optical and magnetic Feshbach resonance, Quantum Gas Microscope)

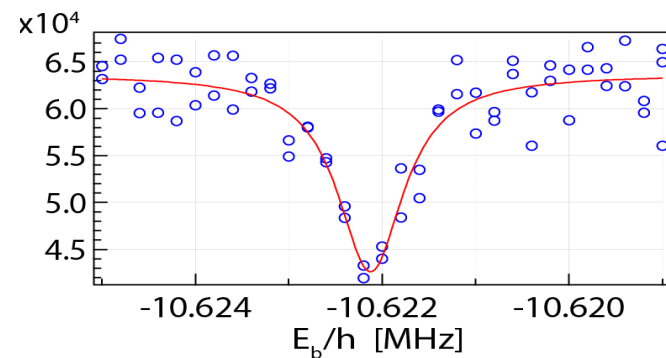
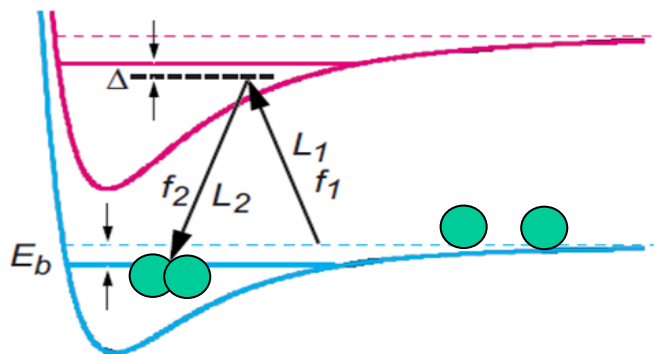
基礎物理学への応用

極低温極性分子を用いた時間反転対称性の検証



極低温原子の超精密分光による近距離重力補正の検証

$$V(r) = -\frac{GM_1M_2}{r} \times \left(1 + \alpha e^{-r/\lambda}\right)$$



Outline

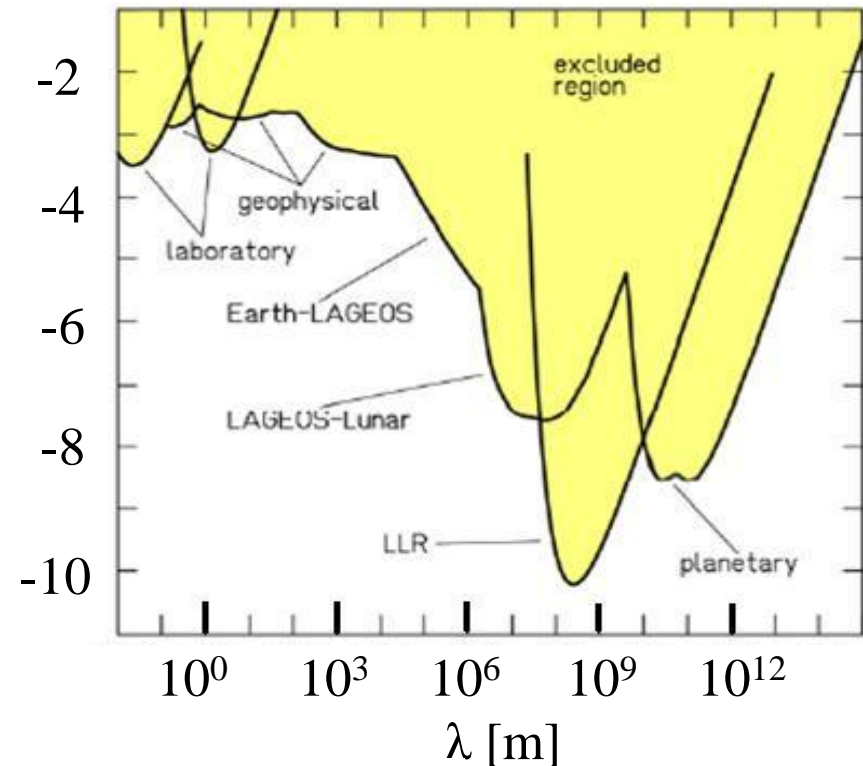
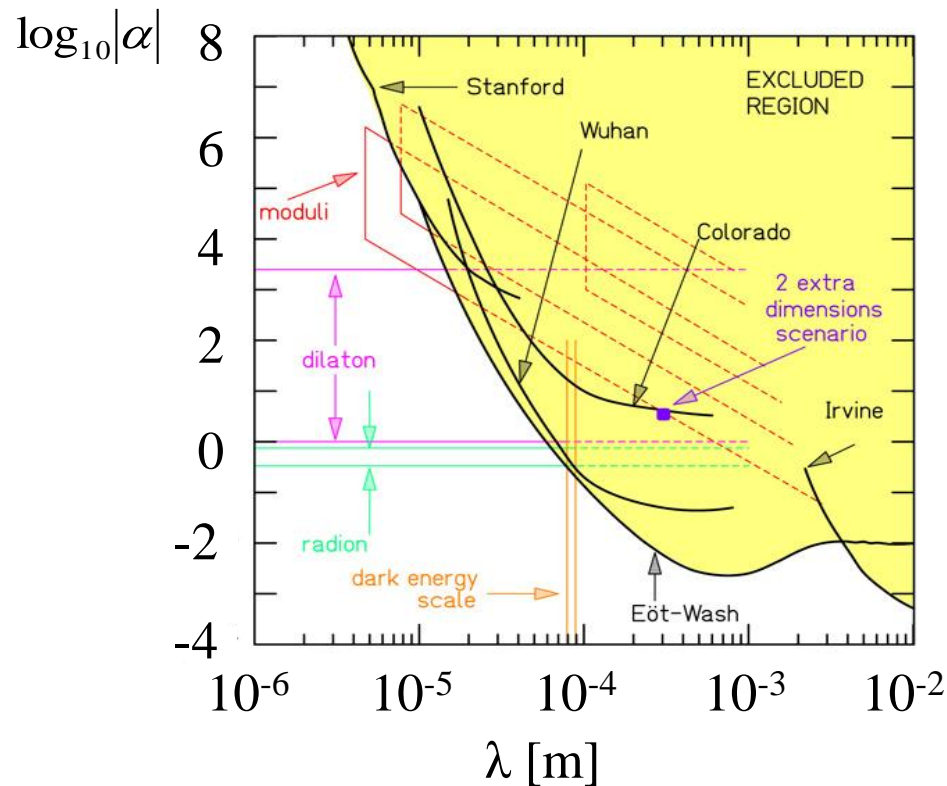
- **Possible Test of Gravity at Short Range
by Photoassociation**
- Possible EDM Search
by using Ultracold HgYb Molecules

Test of the Gravitational r^{-2} Law at Short Range

$$V(r) = -\frac{GM_1M_2}{r}$$

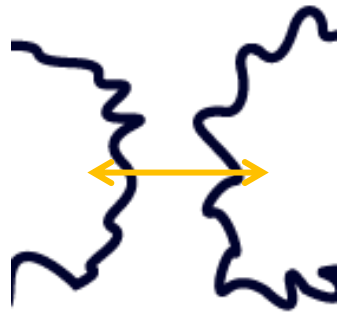


$$V(r) = -\frac{GM_1M_2}{r} \left(1 + \alpha e^{-r/\lambda} \right)$$



Gravity at Short Range

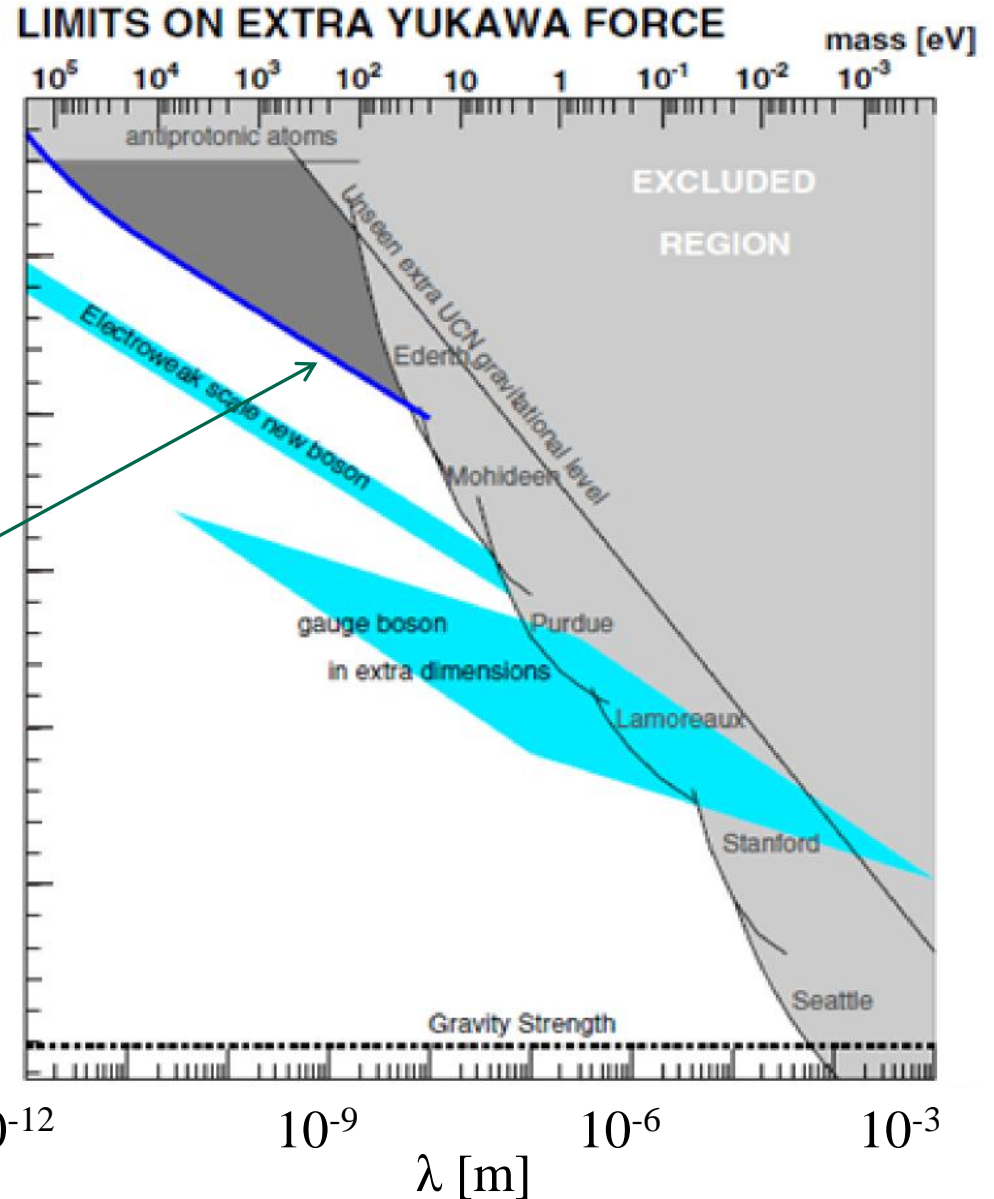
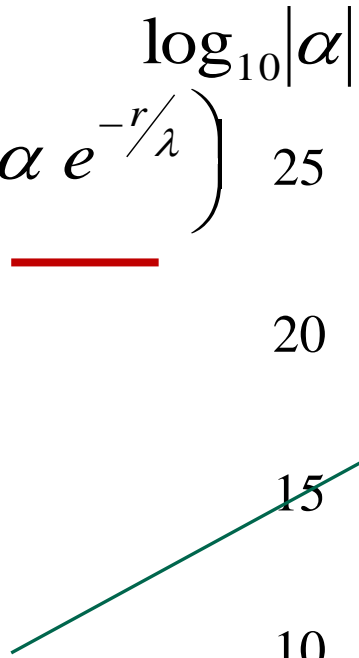
$$V(r) = -\frac{GM_1M_2}{r} \left(1 + \alpha e^{-r/\lambda} \right)$$



$$|\alpha| \sim 10^{22} \text{ @ } 1 \text{ nm}$$

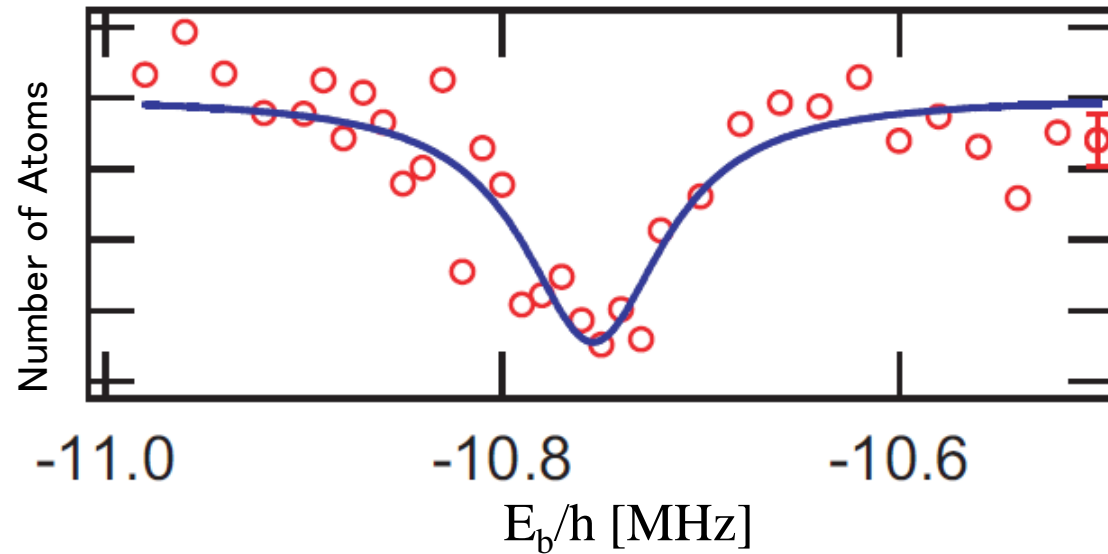
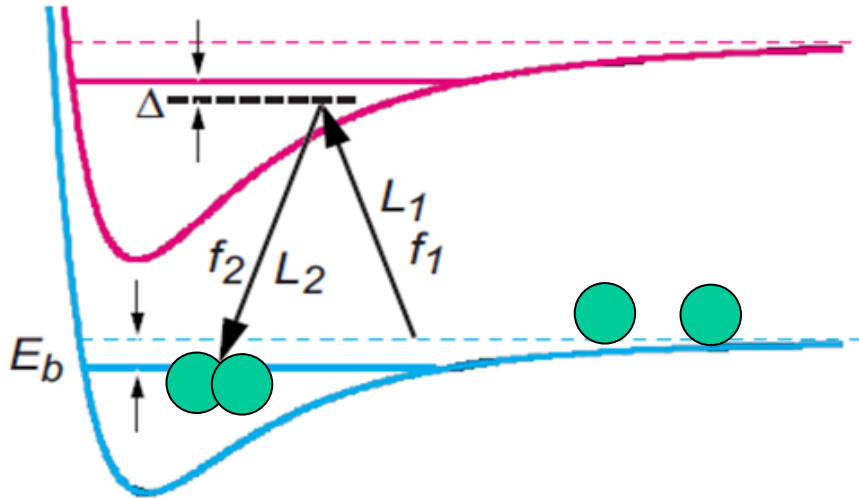
[PRD, 77, 034020 (2008)]

(neutron scattering data)



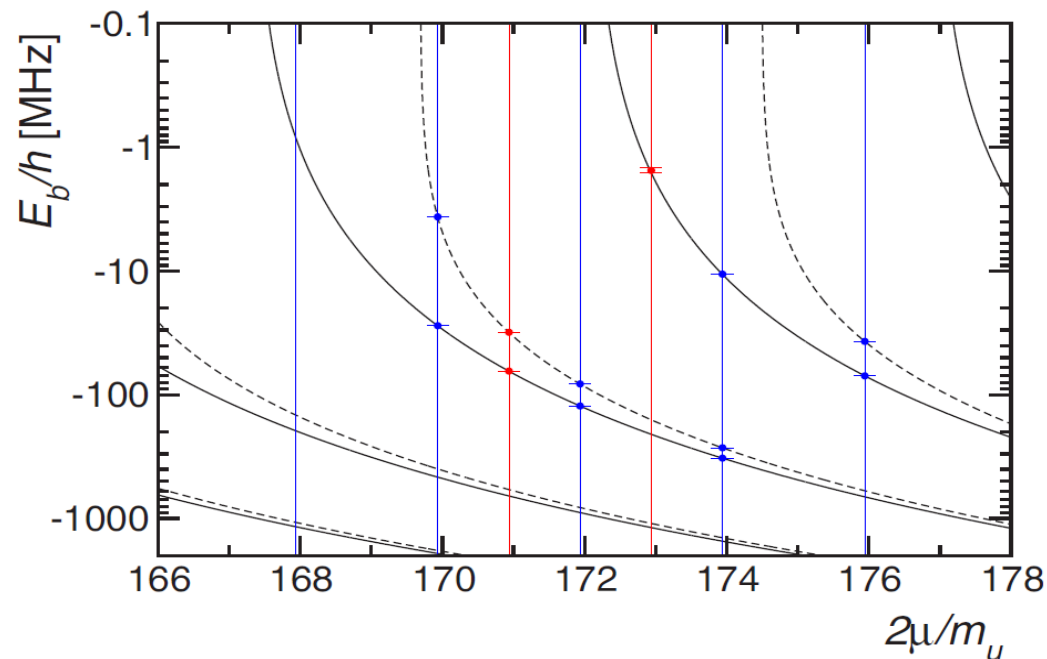
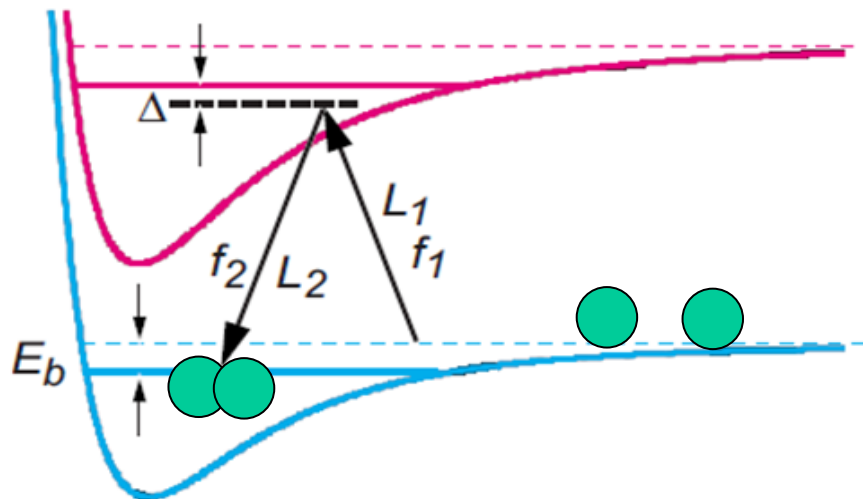
Our Approach : Photoassociation

[M. Kitagawa, et al., PRA 77, 012719(2008)] thermal gas : $\sim 100\text{kHz}$
 $^{174}\text{Yb}:v=1, J=0$



Our Approach : Photoassociation

[M. Kitagawa, et al., PRA 77, 012719(2008)]



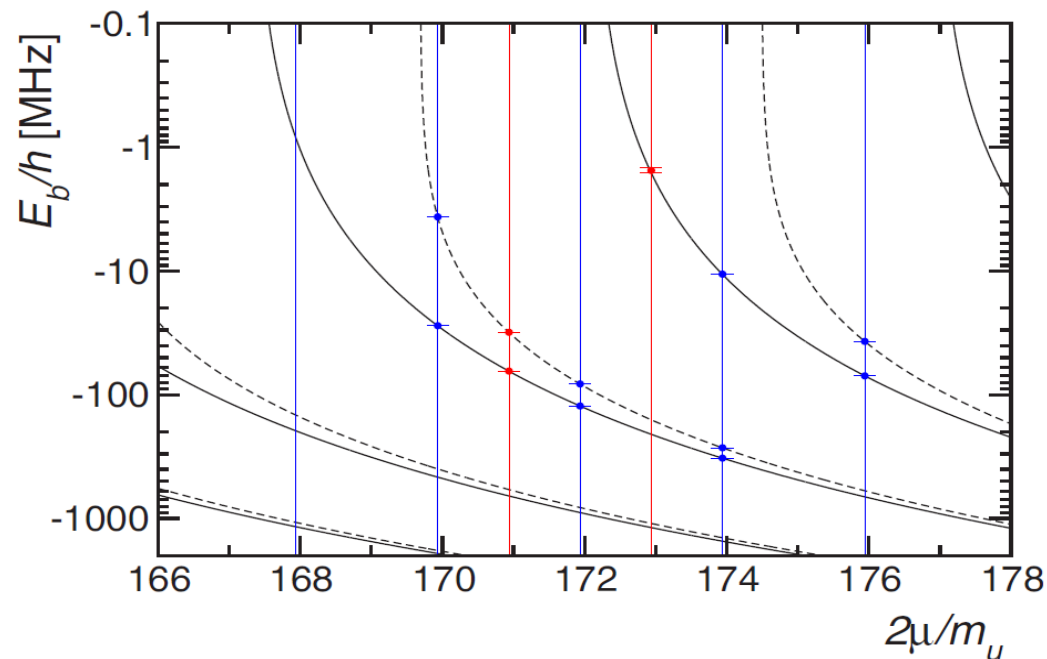
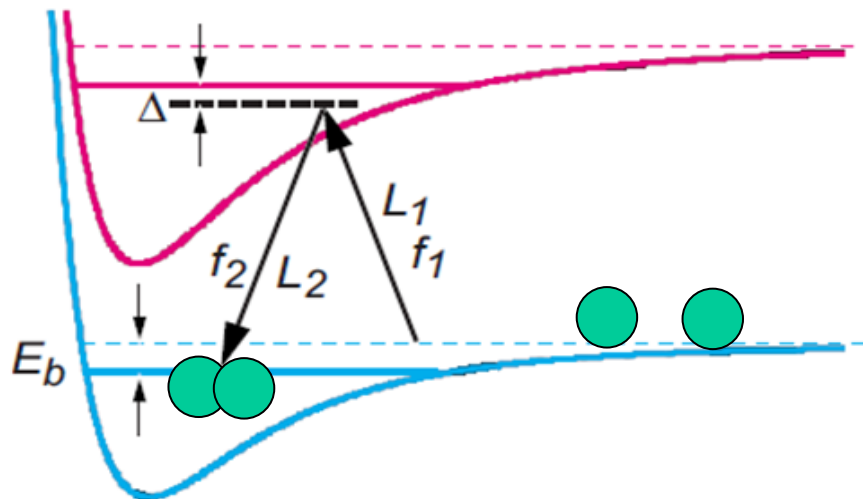
Lenard-Jones-type Potential

$$\Rightarrow V(r) = \frac{C_{12}}{r^{12}} - \frac{C_6}{r^6} - \frac{C_8}{r^8}$$

$$C_6 = 1931.7 E_h a_0^6, \quad C_8 = 1.93 \times 10^6 E_h a_0^8, \quad C_{12} = 1.3041 E_h a_0^{12}$$

Our Approach : Photoassociation

[M. Kitagawa, et al., PRA 77, 012719(2008)]



Lenard-Jones-type Potential

$$\Rightarrow V(r) = \frac{C_{12}}{r^{12}} - \frac{C_6}{r^6} - \frac{C_8}{r^8} - \frac{GM_1M_2}{r} \left(1 + \alpha e^{-r/\lambda} \right)$$

$\Delta f = 1 \text{ kHz}$



$|\alpha| < \sim 10^{20}$

@ 1 nm

$$C_6 = 1931.7 E_h a_0^6, \quad C_8 = 1.93 \times 10^6 E_h a_0^8, \quad C_{12} = 1.3041 E_h a_0^{12}$$

Many Advantages of Ytterbium

Nice Atomic Species for this experiment !

- Heavy (N~174)
- Single Molecular Potential :No Hyperfine Structure

Contrary to Alkali Dimers

Insensitivity to magnetic field

- Many Isotopes:

^{168}Yb , ^{170}Yb , ^{171}Yb , ^{172}Yb , ^{173}Yb , ^{174}Yb , ^{176}Yb

Check the mass dependence

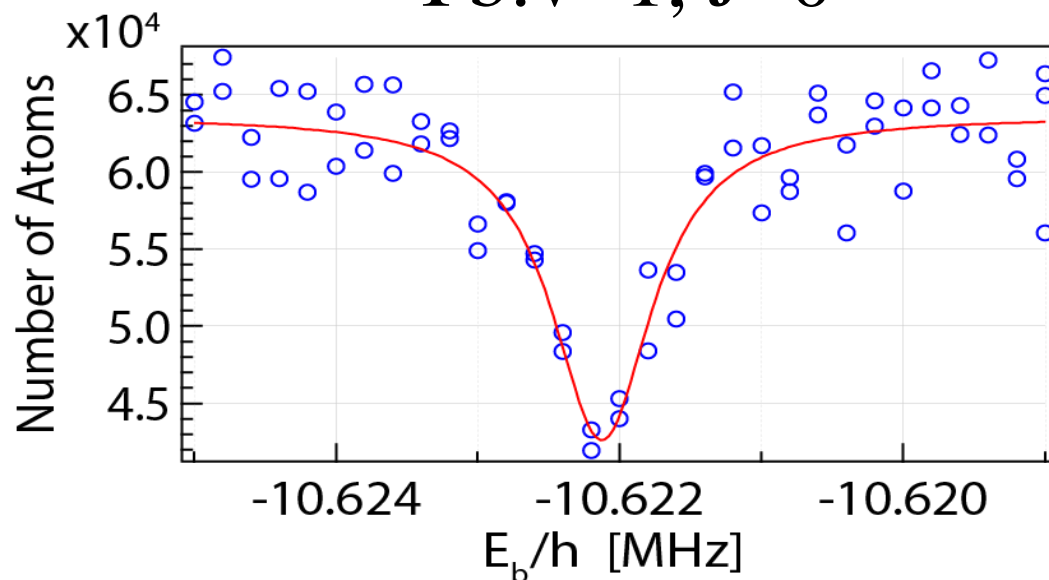
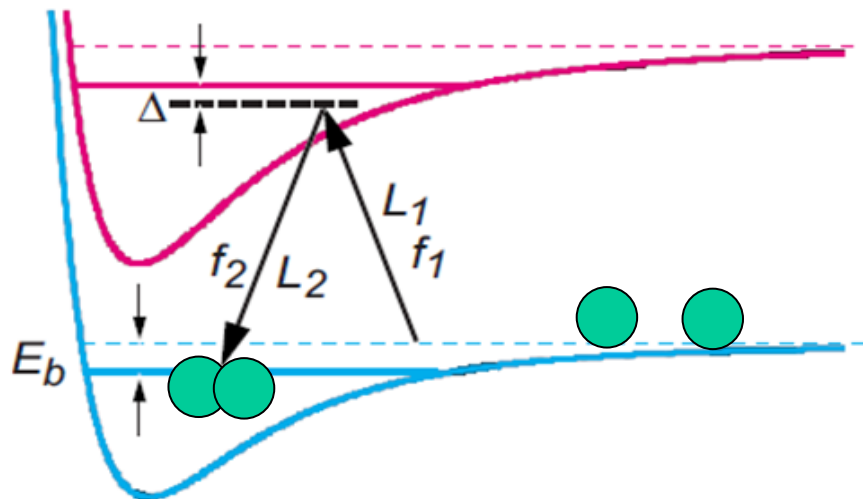
- Ultracold Quantum Gases :

Free from thermal shift and broadening

Our Approach : Photoassociation

BEC : ~1kHz

$^{174}\text{Yb}:v=1, J=0$



Lenard-Jones-type Potential

$$\Rightarrow V(r) = \frac{C_{12}}{r^{12}} - \frac{C_6}{r^6} - \frac{C_8}{r^8} - \frac{GM_1M_2}{r} \left(1 + \alpha e^{-r/\lambda} \right)$$

$\Delta f = 1\text{kHz}$



$|\alpha| < \sim 10^{20}$

@ 1 nm

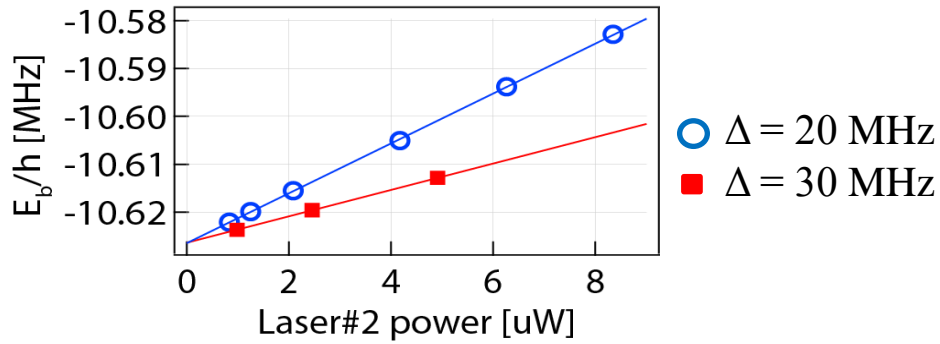
$$C_6 = 1931.7 E_h a_0^6, C_8 = 1.93 \times 10^6 E_h a_0^8, C_{12} = 1.3041 E_h a_0^{12}$$

Evaluation of Systematic Shifts

Light Shift due to Photoassociation Laser

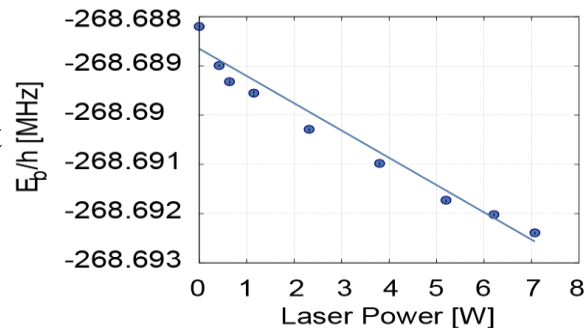
$$\delta_{LS} = \beta \left(\frac{I_1}{\Delta + f_1 - f_2} + \frac{I_2}{\Delta} \right)$$

$$= \alpha_1(\Delta) I_1 + \alpha_2(\Delta) I_2$$



Light Shift due to Optical Trapping Laser

Atoms and Molecules have slightly different polarizabilities

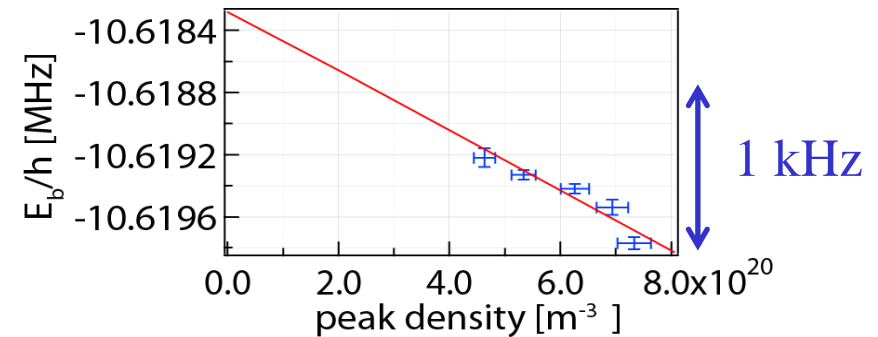
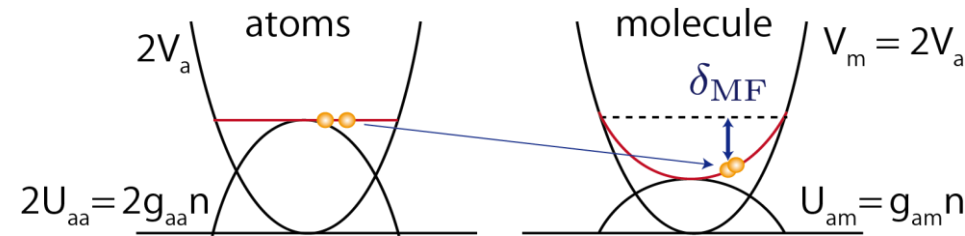


Collision Shift due to Atom-Dimer Collision

$$\delta_{MF} = 2\pi \hbar^2 \left(\frac{2a_{aa}}{\mu_{aa}} - \frac{a_{am}}{\mu_{am}} \right) n_{\text{atom}}(r)$$

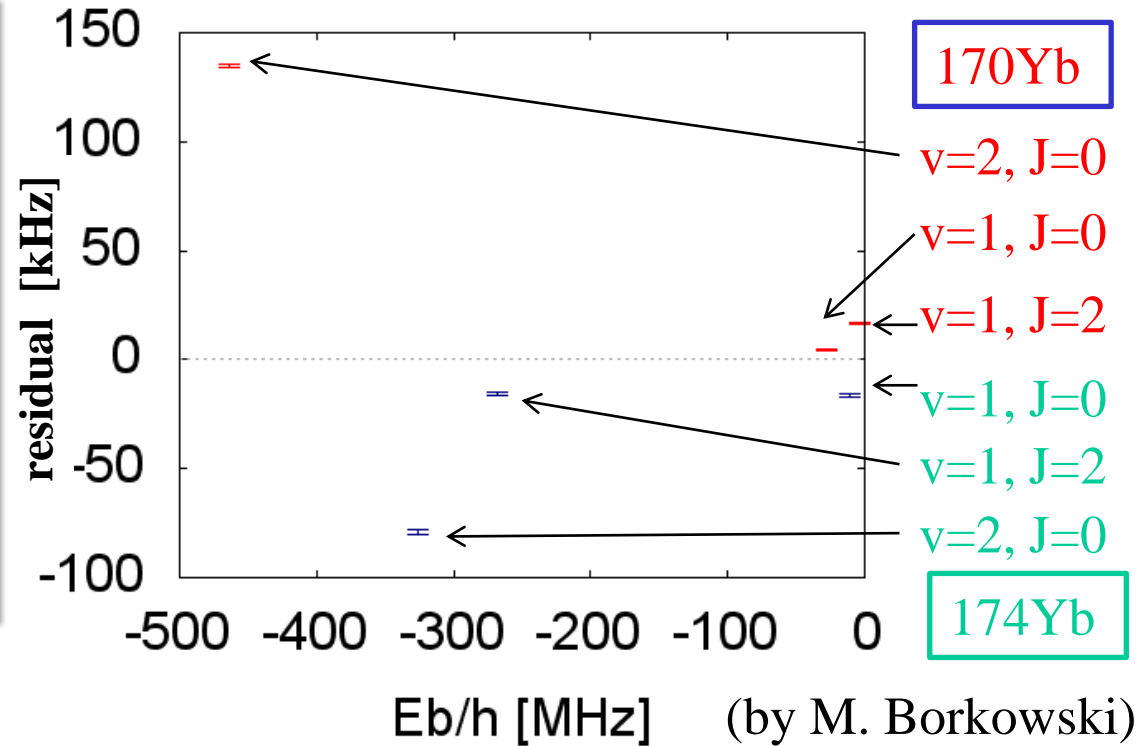
a_{am} : scattering length between atom and molecule

$$\{V(r) + g |\psi(r)|^2\} \psi(r) = \mu \psi(r)$$



Results (Preliminary)

Level	Binding energy [MHz]	
$^{168}\text{Yb}(v=2, J=0)$	-195.18141(46)	
$^{168}\text{Yb}(v=1, J=2)$	-145.53196(48)	
$^{170}\text{Yb}(v=1, J=0)$	-27.70024(44)	#
$^{170}\text{Yb}(v=2, J=0)$	-463.72552(80)	#
$^{170}\text{Yb}(v=1, J=2)$	-3.66831(32)	#
$^{174}\text{Yb}(v=1, J=0)$	-10.62513(53)	#
$^{174}\text{Yb}(v=2, J=0)$	-325.66378(98)	#
$^{174}\text{Yb}(v=3, J=0)$	-1527.88542(40)	
$^{174}\text{Yb}(v=1, J=2)$	-268.63656(56)	#
$^{174}\text{Yb}(v=2, J=2)$	-1432.82493(64)	



$$C_6 = 1933.4 E_h a_0^6, C_8 = 2.086 \times 10^6 E_h a_0^8$$

This does NOT immediately mean the existence of Yukawa force

→ Correction on Born-Oppenheimer Approximation:

$$C_{n,n} = -\frac{\hbar^2}{2\mu} \int \psi_e^n(\vec{r}_i, R) \nabla_R^2 \psi_e^n(\vec{r}_i, R) d\vec{r}_i \propto \frac{1}{\mu}$$