Family Gauge Symmetry as an Origin of Koide's Mass Formula and Charged Lepton Spectrum

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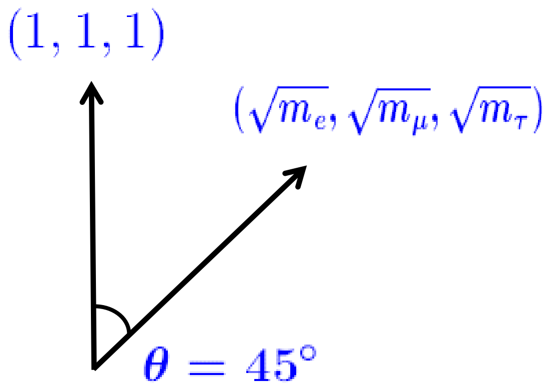
1. Introduction

Koide's mass formula

Koide '82



$$\frac{\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}}{\sqrt{3\,(m_e+m_\mu+m_\tau)}} = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Charged lepton on-shell (pole) masses:

$$m_e = 0.510998910 \pm 0.000000013 \text{ MeV}$$
$$m_\mu = 105.658367 \pm 0.000004 \text{ MeV}$$
$$m_\tau = 1776.82 \pm 0.16 \text{ MeV}$$

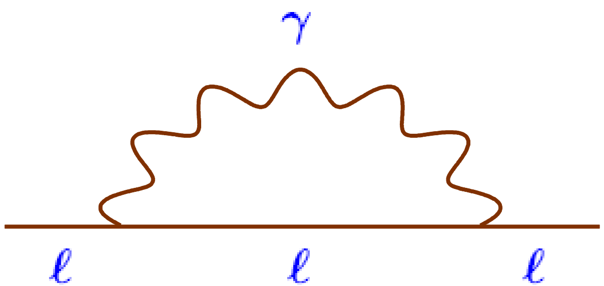
PDG '11

$$\Rightarrow \quad \sqrt{2}\left[\frac{\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}}{\sqrt{3\,(m_e+m_\mu+m_\tau)}}\right] = 1.000006 \pm 0.000007$$

7×10^{-6} accuracy !

Problem of QED correction

If *running masses* $\bar{m}_i(\mu)$ at $\mu \gg M_W$ satisfy Koide's relation,
QED corr. violates the relation among the *pole masses*:



$$m_i^{\text{pole}} = \left[1 + \frac{\alpha}{\pi}\left\{\frac{3}{4}\log\left(\frac{\mu^2}{\bar{m}_i(\mu)^2}\right) + 1\right\}\right] \bar{m}_i(\mu)$$

$\delta\left(\sqrt{2}\cos\theta\right) \sim 0.1\%$ 130 times larger than exp. error !

Problem against an idea, that a UV theory (beyond SM) predicts Koide's relation.

Rem: Corr. of the form $\text{const.} \times \bar{m}_i$ does not affect Koide's relation.

⇒ QED corr. is indep. of μ

Other corr. (W,Z, Higgs, would-be Goldstone) are smaller than exp. error.

Interesting scenario to generate Koide's formula

Koide, et al.

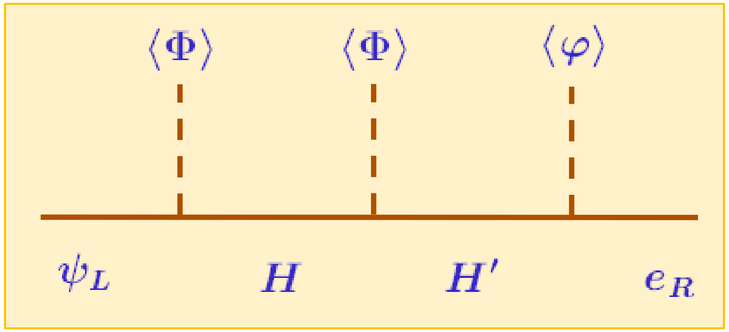
Charged lepton mass matrix

$$\mathcal{M}_\ell \propto \langle\Phi\rangle\langle\Phi\rangle \quad \text{with} \quad \langle\Phi\rangle = \begin{pmatrix} v_1(\mu) & 0 & 0 \\ 0 & v_2(\mu) & 0 \\ 0 & 0 & v_3(\mu) \end{pmatrix}$$

originating from effective higher-dimensional op.

$$\mathcal{O} = \frac{\kappa(\mu)}{\Lambda^2}\,\bar{\psi}_{Li}\,\Phi_{ik}\,\Phi_{kj}\,\varphi\,e_{Rj}$$

e.g. via see-saw mechanism ⇨



Minimize potential $V(\Phi)$ ⇨
$$\frac{v_1(\mu)+v_2(\mu)+v_3(\mu)}{\sqrt{3\,[v_1(\mu)^2+v_2(\mu)^2+v_3(\mu)^2]}} = \frac{1}{\sqrt{2}}$$

(However, no potential in previous models is protected by sym.)

In a similar scenario and EFT valid at $\mu < \Lambda$, we examine indication of Koide's formula and charged lepton spectrum.

We arrive at a specific family gauge symmetry.

☆ Plan of Talk

- Introduction
- Cancellation of QED corrections
- Charged lepton spectrum and family sym.
- A Model: EFT and Potential

  predicting Koide's formula and charged lepton spectrum
- Summary and discussion

2. Cancellation of QED correction

We consider $U(3) \simeq SU(3) \times U(1)$ family gauge sym.

- Generators

$$\mathrm{tr}\left(T^\alpha T^\beta\right) = \frac{1}{2}\delta^{\alpha\beta}, \quad T^\alpha = T^{\alpha\dagger} \qquad T^0 = \frac{1}{\sqrt{6}}\mathbf{1}, \quad T^a = \frac{\lambda^a}{2}$$

$0 \le \alpha, \beta \le 8$ $\qquad$ $1 \le a \le 8$

- Reps. $\quad \psi_L : (3, 1), \quad e_R : (\bar{3}, -1), \quad \Phi : (3, 1), \quad \varphi : (1, 0)$

[transf. $\psi_L \to U\,\psi_L\,, \quad e_R \to U^*\,e_R\,, \quad \Phi \to U\,\Phi \;\; ; \quad U = \exp\left(i\theta^\alpha T^\alpha\right)$]

- Higher-dimensional op.

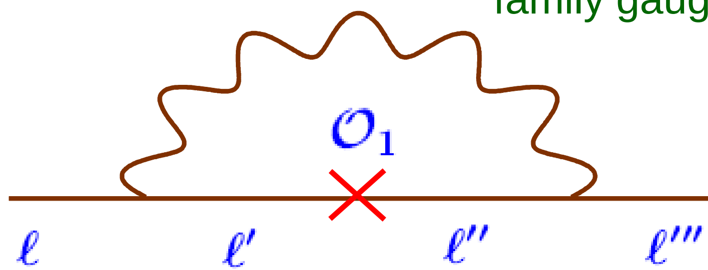
$$\mathcal{O}_1 = \frac{\kappa}{\Lambda^2}\,\bar{\psi}_L\,\Phi\,\Phi^T\,\varphi\,e_R$$

(Larger sym. $U(3) \times O(3)$: $\Phi \to U\Phi O^T, \quad O\,O^T = 1$)

$\langle\Phi\rangle$ breaks $U(3)$ symmetry completely, and the spectrum of family gauge bosons is determined by it.

$$\langle\Phi\rangle = \begin{pmatrix} v_1(\mu) & 0 & 0 \\ 0 & v_2(\mu) & 0 \\ 0 & 0 & v_3(\mu) \end{pmatrix}$$

Rad. corr. by family gauge bosons has the same form as the QED corr. but with opposite sign:

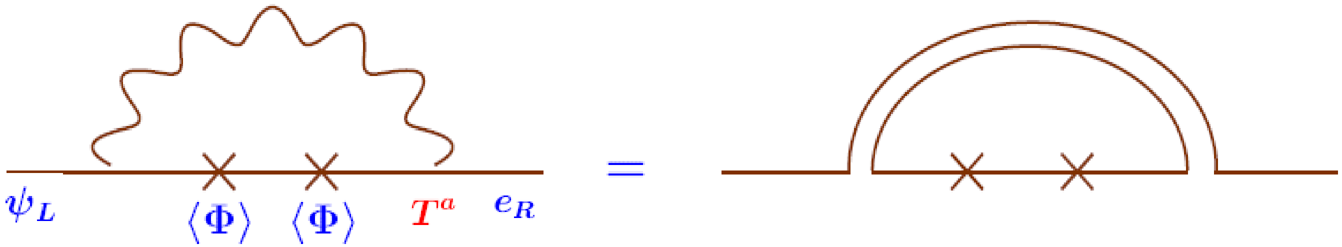


$$\delta m_i^{\rm pole} = -\frac{3\,\alpha_F}{8\,\pi}\left[\log\left(\frac{\mu^2}{v_i(\mu)^2}\right)+c\right]\, m_i(\mu)\,, \qquad m_i(\mu)=\frac{\kappa(\mu)\,v_{\rm EW}}{\Lambda^2}\,v_i(\mu)^2$$

$v_i(\mu)$ defined from the minimum of 1-loop effective pot. of Φ in Landau gauge.

- If $\frac{v_1(\mu)+v_2(\mu)+v_3(\mu)}{\sqrt{3\,[v_1(\mu)^2+v_2(\mu)^2+v_3(\mu)^2]}}=\frac{1}{\sqrt{2}}$ is satisfied at tree-level, there is no $\mathcal{O}(\alpha_F)$ correction to this relation.
- The form of $\delta m_i^{\rm pole}$ is determined by multiplicative renormalization of $\mathcal{O}_1$ and by the sym. breaking pattern: $U(3)\to U(2)\to U(1)\to \text{nothing}\,.$
- If $\alpha=\frac{1}{4}\alpha_F$, $\delta m_i^{\rm pole}$ cancels QED corr. for arbitrary $\mu\,(> M_F)$

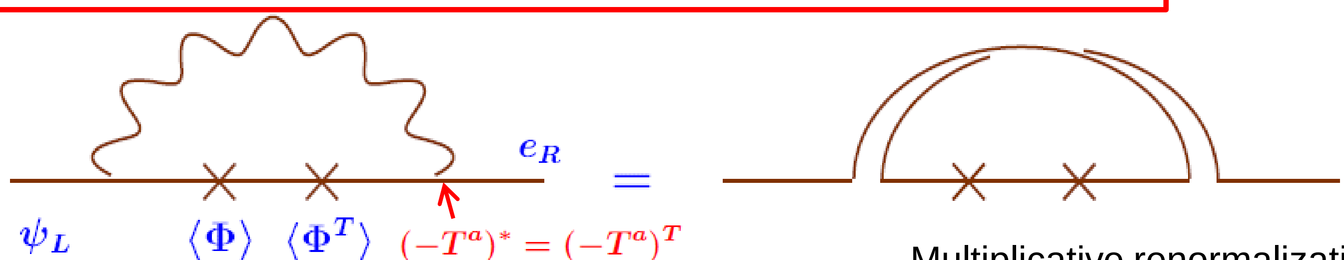
ψ_L and e_R in same rep. of SU(3) or O(3): $\psi_L : (3, Q), \quad e_R : (3, Q')$



$+\ \mathcal{O}(\frac{1}{N_F})$

$$\mathcal{O}' \sim \frac{\alpha_F}{\pi} \times \kappa\, \bar{\psi}_{Li}\, \varphi\, e_{Ri} \times \frac{\langle\Phi\rangle_{jk}\langle\Phi\rangle_{kj}}{\Lambda^2} \ \Rightarrow\ (\delta m_e, \delta m_\mu, \delta m_\tau) \ \propto\ (1,1,1)$$

ψ_L and e_R in conjugate reps of SU(3): $\psi_L : (3, Q), \quad e_R : (\bar{3}, Q')$



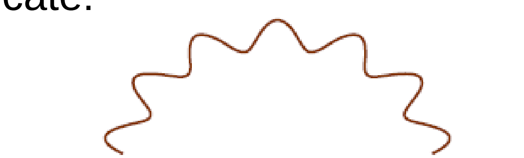
$$\frac{\kappa}{\Lambda^2}\, \bar{\psi}_L\, \Phi\, \Phi^T\, \varphi\, e_R$$

Multiplicative renormalization

Other than in Landau gauge, each rad. corr. becomes quite intricate:

$\bar{m}_i \log \bar{m}_i$ corrections in $\bar{\psi}_L \not{p}\, Z_\psi\, \psi_L$ and $\bar{e}_R \not{p}\, Z_e\, e_R$

$\mathcal{O}(\alpha_F)$ corrections to the relation among v_i's.



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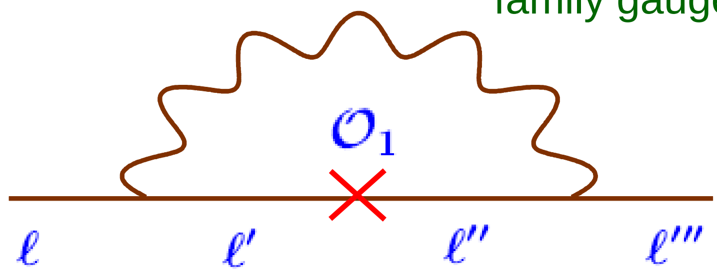

$$\delta m_i^{\rm pole} = -\frac{3\,\alpha_F}{8\,\pi}\left[\log\left(\frac{\mu^2}{v_i(\mu)^2}\right)+c\right]\,m_i(\mu)\,, \qquad m_i(\mu) = \frac{\kappa(\mu)\,v_{\rm EW}}{\Lambda^2}\,v_i(\mu)^2$$

$v_i(\mu)$ defined from the minimum of 1-loop effective pot. of Φ in Landau gauge.

- If $\dfrac{v_1(\mu)+v_2(\mu)+v_3(\mu)}{\sqrt{3\,[v_1(\mu)^2+v_2(\mu)^2+v_3(\mu)^2]}} = \dfrac{1}{\sqrt{2}}$ is satisfied at tree-level, there is no $\mathcal{O}(\alpha_F)$ correction to this relation.
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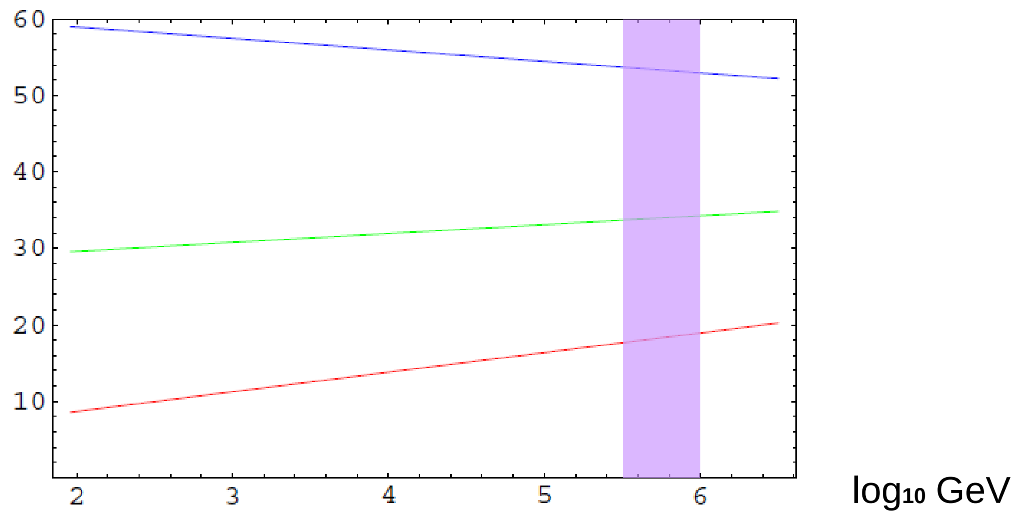
- If $\alpha = \dfrac{1}{4}\alpha_F$, $\delta m_i^{\rm pole}$ cancels QED corr. for arbitrary $\mu\,(> M_F)$.

Speculation

$$\alpha(m_\tau) \approx \frac{1}{4}\alpha_F(M_F)$$ within 1% accuracy **?**

If $SU(2)_L$ and $U(3)_F$ are *unified* at around 10^3 TeV, the above relation would be satisfied, since $\sin^2\theta_W \simeq \frac{1}{4}$.



Tuning required for the unification scale is about a factor of 3.

Charged lepton spectrum and $U(3)\times O(3)$ sym.

Rem: $$\mathcal{O}_1 = \frac{\kappa}{\Lambda^2}\,\bar{\psi}_L\,\Phi\,\Phi^T\,\varphi\,e_R \qquad \left(\begin{array}{c} U(3)\times O(3) \\ \Phi \to U\Phi O^T, \quad O\,O^T = 1 \end{array} \right)$$

Usually difficult to obtain, on top of Koide's relation, a hierarchical spectrum without fine tuning.

$$v_1 : v_2 : v_3 = \sqrt{m_e} : \sqrt{m_\mu} : \sqrt{m_\tau}$$

$$\frac{\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}}{\sqrt{3\,(m_e+m_\mu+m_\tau)}} = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Nevertheless, assuming that Koide's relation is protected, and mimimizing a $U(3)\times O(3)$-inv. potential of Φ, realistic lepton spectrum consistent with experimental data is obtained.

A relation for Φ representing Koide's formula:

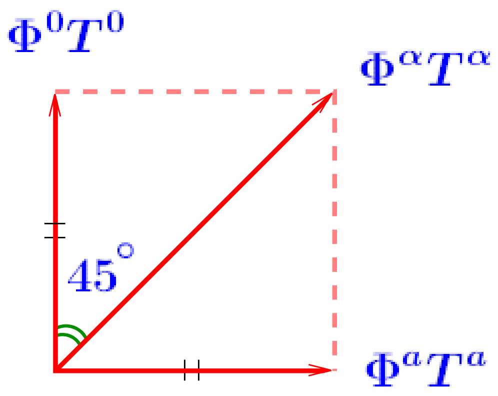
Let $\Phi = \Phi^\alpha T^\alpha$. $\mathrm{tr}\left(T^\alpha T^\beta\right) = \frac{1}{2}\delta^{\alpha\beta}$

Then, if the condition

$$(\Phi^0)^2 = \Phi^a \Phi^a \quad ; \quad \Phi^\alpha \in \mathrm{R}$$

is satisfied, Koide's relation is satisfied by the eigenvalues of Φ .

Koide '90



With above constraint, and minimize

$$V(\Phi) = -\mu^2 \,\mathrm{tr}(\Phi^\dagger \Phi) + \lambda \left[\mathrm{tr}(\Phi^\dagger \Phi)\right]^2 + g_1 \,\mathrm{tr}(\Phi^\dagger \Phi \Phi^\dagger \Phi) + g_2 \,\mathrm{tr}(\Phi \Phi^T \Phi^* \Phi^\dagger)$$

If $1 \gg g_2 \gg g_1$, a realistic spectrum, consistent with experimental data, is obtained.

Important aspect

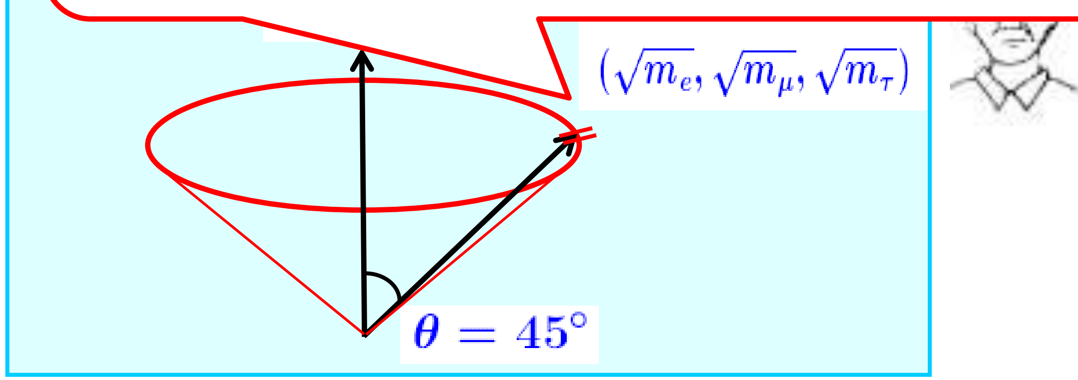
☆ Cancellation of QED corr. with $\frac{\kappa}{\Lambda^2}\bar{\psi}_L \Phi \Phi^T \varphi e_R$, invariant under $U(3)\times O(3)$ family gauge symmetry.

☆ Potential with the same sym. ⇒ realistic $m_e : m_\mu : m_\tau$ for $g_2 \gg g_1$ (provided Koide's relation is protected).

$$V_\Phi = -\mu^2 \Phi^{\alpha *}\Phi^\alpha + \lambda (\Phi^{\alpha *}\Phi^\alpha)^2 + g_1 \mathrm{tr}(\Phi^\dagger \Phi \Phi^\dagger \Phi) + g_2 \mathrm{tr}(\Phi \Phi^T \Phi^* \Phi^\dagger)$$

Exp. values $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}) \propto (0.01647, 0.2369, 0.9714)$

Prediction for $g_1 = 0$ $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}) \propto (0.01775, 0.2352, 0.9718)$



3. A Model: EFT and Potential predicting Koide's formula and charged lepton spectrum

We introduce a model with $U(3)\times O(3)$ sym. as an EFT valid at $\mu < \Lambda$.

Motivated by 9-dim. geometrical picture for Koide's relation and sym. of $V(\Phi)$, we assume $U(9)$ as sym. above the cut-off scale.

$$\mu > \Lambda \qquad U(9) \qquad \Phi^\alpha \to U_9^{\alpha\beta}\,\Phi^\beta$$

$$\downarrow$$

$$\mu = \Lambda \qquad U(3)\times O(3) \qquad \Phi \to U\,\Phi O^T$$

cf. $\mathrm{tr}(\Phi^\dagger\Phi) = \frac{1}{2}\Phi^{\alpha *}\Phi^\alpha$

$$V(\Phi) \;=\; -\mu^2\,\mathrm{tr}(\Phi^\dagger\,\Phi) + \lambda\left[\mathrm{tr}(\Phi^\dagger\,\Phi)\right]^2 + g_1\,\mathrm{tr}(\Phi^\dagger\,\Phi\,\Phi^\dagger\,\Phi) + g_2\,\mathrm{tr}(\Phi\,\Phi^T\,\Phi^*\,\Phi^\dagger)$$

Scalars $U(9)$ $\Phi^\alpha : (9, Q),$ $X^{\alpha\beta} : (45, Q')$

2nd-rank sym. tensor $X \to U_9 \, X \, U_9^T$

unitary $X^\dagger X = 1_9$

$V(\Phi, X)$ with $U(3)\times O(3)$ sym. and sym. enhancement to $U(9)$.

In finite region of param. space, Koide's relation is satisfied by the eigenvalues of $\langle \Phi \rangle$.

Sketch of argument

$U(9)$ inv. potential

$$\begin{cases} V_\Phi = -\mu^2 \, \Phi^{\alpha *}\Phi^\alpha + \lambda \, (\Phi^{\alpha *}\Phi^\alpha)^2 + \cdots \\ V_X = \text{const.} \\ V_{\Phi X} = \varepsilon_K \, |\Phi^\beta \, X^{\beta\gamma *} \, \Phi^\gamma|^2 + \cdots \end{cases}$$

← X is unitary

$U(3)\times O(3)$ inv. potential

$$\begin{cases} \widetilde{V}_\Phi = g_1 \, \mathrm{tr}(\Phi^\dagger \, \Phi \, \Phi^\dagger \, \Phi) + g_2 \, \mathrm{tr}(\Phi \, \Phi^T \, \Phi^* \, \Phi^\dagger) + \cdots \\ \widetilde{V}_X = h_1 \, \mathrm{tr}(T^\alpha \, T^\rho \, T^\beta \, T^\sigma) \, X^{\alpha\beta} \, X^{\rho\sigma *} + \cdots \\ \widetilde{V}_{\Phi X} = \cdots \end{cases}$$

→ global minimum at $\langle X^{\alpha\beta} \rangle = \mathrm{diag.}(-1, \overbrace{+1, \cdots, +1}^{8})$

→ $\Phi^\beta \, X^{\beta\gamma *} \, \Phi^\gamma = 0 \, , \quad (\Phi^\alpha = \Phi^{\alpha *})$ →

Koide's relation $(\Phi^0)^2 = \Phi^a \, \Phi^a$
at $\mu = \Lambda$

$SU(9)\times U(1)$ inv. potential

$$\begin{cases} V_\Phi = -\mu^2\, \Phi^{\alpha *}\Phi^{\alpha} + \lambda\,(\Phi^{\alpha *}\Phi^{\alpha})^2 \\ V_X = \text{const.} \\ V_{\Phi X} = \varepsilon_K \left| \Phi^{\beta}\, X^{\beta\gamma *}\, \Phi^{\gamma}\right|^2 + \cdots \end{cases}$$

← X is unitary

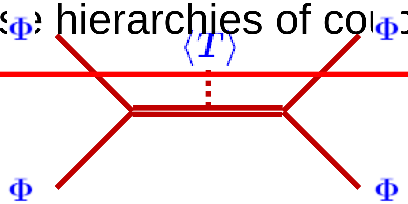
$U(3)\times O(3)$ inv. potential

$$\begin{cases} \widetilde{V}_\Phi = g_1\, \mathrm{tr}(\Phi^\dagger\, \Phi\, \Phi^\dagger\, \Phi) + g_2\, \mathrm{tr}(\Phi\, \Phi^T\, \Phi^*\, \Phi^\dagger) \\ \widetilde{V}_X = h_1\, \mathrm{tr}(T^\alpha\, T^\rho\, T^\beta\, T^\sigma)\, X^{\alpha\beta}\, X^{\rho\sigma *} + \cdots \\ \widetilde{V}_{\Phi X} = \cdots \end{cases}$$

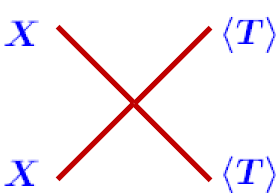
- $h_1 \gg g_1,\, g_2$ ⇨ Koide's relation $\dfrac{v_1(\mu)+v_2(\mu)+v_3(\mu)}{\sqrt{3\,[v_1(\mu)^2+v_2(\mu)^2+v_3(\mu)^2]}} = \dfrac{1}{\sqrt{2}}$
- $h_1 \gg g_2 \gg g_1$ ⇨ Realistic charged lepton spectrum $v_1 : v_2 : v_3 = \sqrt{m_e} : \sqrt{m_\mu} : \sqrt{m_\tau}$

Speculation : $SU(9)\times U(1) \to U(3)\times O(3)$ occurs via $\langle T_{\rho\sigma} \rangle$ ...

These hierarchies of couplings are consistent with assumed sym. and sym. enhancement.



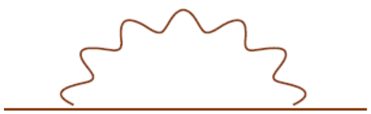
⇨ $\dfrac{\langle T^{\alpha\beta}_{\rho\sigma}\rangle}{M}\, \Phi^\alpha \Phi^\beta \Phi^{\rho *} \Phi^{\sigma *} \sim g_2\, \mathrm{tr}(\Phi\, \Phi^T\, \Phi^*\, \Phi^\dagger)$



⇨ $h_1 \sim \dfrac{\langle T\rangle^2}{\Lambda^2} \gtrsim \mathcal{O}(1)$

4. Summary and discussion

Summary

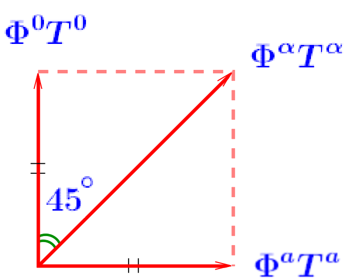


☆ $U(3)\times O(3)$ gauge sym. has a unique property w.r.t. radiative corr. to Koide's mass formula.

In the case $\alpha(m_\tau)\approx\frac{1}{4}\alpha_F(M_F)$, QED corr. is cancelled.

- Multiplicative renormalization of $\frac{\kappa}{\Lambda^2}\bar{\psi}_L\,\Phi\,\Phi^T\,\varphi\,e_R$ and $\langle\Phi\rangle=\begin{pmatrix} v_1(\mu) & 0 & 0 \\ 0 & v_2(\mu) & 0 \\ 0 & 0 & v_3(\mu)\end{pmatrix}$
- Sym. breaking $U(3)\to U(2)\to U(1)\to \text{nothing}\,.$
- With tree-level Koide's relation $\frac{v_1(\mu)+v_2(\mu)+v_3(\mu)}{\sqrt{3\,[v_1(\mu)^2+v_2(\mu)^2+v_3(\mu)^2]}}=\frac{1}{\sqrt{2}}$ at $\mu\,(>M_F)$

☆ V_Φ with $U(3)\times O(3)$ sym. ⇨ realistic lepton spectrum



☆ A model EFT with enhanced sym. $U(9)$ at $\mu<\Lambda$

$\Phi,\,X$

⇨ Koide's formula

Boundary cond. on potential parameters at $\mu=\Lambda$.

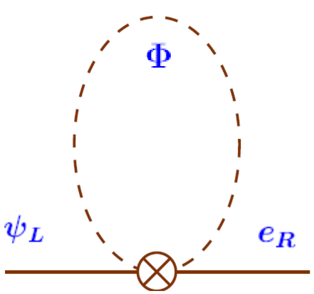
Consistent with sym. and sym. enhancement.

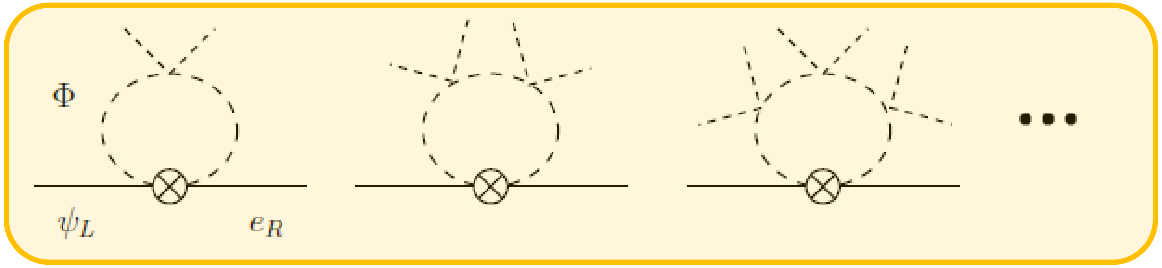
Problems

- Quark+neutrino sectors & anomaly cancellation
- O(3) sym. breaking
- $\langle \Phi \rangle$ cannot be brought to diagonal form by $U(3) \times O(3)$ transf.

  ⇨ e.g. Inclusion of another field $\frac{\kappa}{\Lambda^2} \bar{\psi}_L \Phi \Phi^T \varphi e_R \to \frac{\kappa}{\Lambda^3} \bar{\psi}_L \Phi \Sigma \Phi^T \varphi e_R$
- Unification of $SU(2)_L \times U(3)$ vs. embedding $U(3) \times O(3)$ into $SU(9) \times U(1)$
- $\frac{v_3}{\Lambda} \lesssim 0.1$ ⇨ $\frac{1}{\Lambda^n}$ corr. at tree level.
- Stability of small VEVs: $\langle \varphi \rangle, \langle X \rangle \ll \Lambda$
- Model(s) at $\mu > \Lambda$.

Virtue: a cross-check





⇨ No corr. to Koide's formula

Non-trivial aspects indep. of model details

☆ Cancellation of QED corr. with $\frac{\kappa}{\Lambda^2}\,\bar{\psi}_L\,\Phi\,\Phi^T\,\varphi\,e_R$, invariant under $U(3)\times O(3)$ family gauge

☆ Potential with s

fine tuning, *pr*

$V_\Phi = -\mu^2$

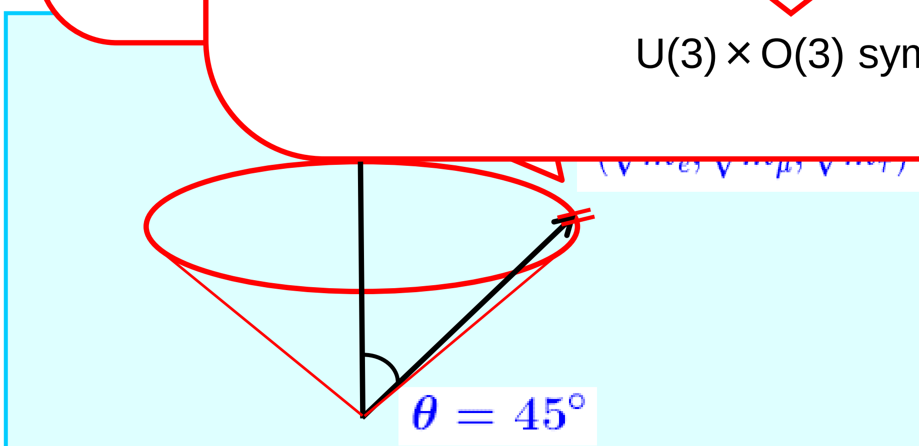
Exp.

Predictio

$g_1 =$

c.f. If t

m_ℓ



Line of argument

High precision of Koide's formula

⇩

Family gauge sym.

(Existence of rad. corr.)

⇩

$\mathcal{M}_\ell \propto \langle\Phi\rangle\langle\Phi\rangle$ + multiplicative renormalization

⇩

U(3) × O(3) sym.

$$SU(4) \to U(3) \simeq SU(3) \times U(1)$$

$$\mathrm{tr}\left(T^{\alpha}T^{\beta}\right) = \frac{1}{2}\delta^{\alpha\beta}$$

$$\frac{1}{2\sqrt{6}}\begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & +3 \end{pmatrix}$$

$$4 \to (3, -1/2)$$

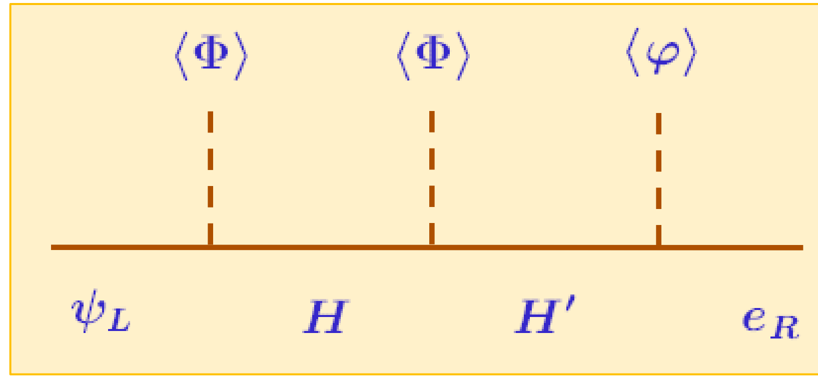
$$6 \to (3, +1) \oplus (\bar{3}, -1)$$

since $T^0 = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$

$$\mathcal{O} = \frac{\kappa(\mu)}{\Lambda^2}\,\bar{\psi}_{Li}\,\Phi_{ik}\,\Phi_{kj}\,\varphi\,e_{Rj}$$

⇩

$$m_i(\mu) = \frac{\kappa(\mu)\,v_{\mathrm{EW}}}{\Lambda^2}\,v_i(\mu)^2$$



$$\mathcal{L} = y_1\,\bar{\psi}_{Li}\Phi_{ij}H_{Rj} + M\,\bar{H}_{Ri}H_{Li} + y_2\,\bar{H}_{Li}\Phi_{ij}H'_{Rj} + M'\,\bar{H}'_{Ri}H'_{Li} + y_3\,\bar{H}'_{Li}\varphi e_{Ri} + (\mathrm{h.c.})$$

For instance, in the case that $v_3/M' \gtrsim 3$, $y_1, y_2, y_3 \approx 1$ and $v_{\mathrm{ew}}/M' < 3\times 10^{-3}$, one finds, by computing the mass eigenvalues,* that the largest correction to the lepton spectrum eq. (7) arises from the operator $-\frac{y_1^3 y_2^3 y_3}{2M^3M'^3}\,\bar{\psi}_L\Phi^6\varphi e_R$; its contribution to the tau mass is $\delta m_\tau/m_\tau = (m_\tau/v_{\mathrm{ew}})^2 \approx 5\times 10^{-5}$. This translates to a correction to Koide's relation of 3×10^{-6},

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* Since the values of m_τ and v_{ew} are known, once we choose the values of $v_3/M'(\gtrsim 3)$ and $y_1, y_2, y_3(\approx 1)$, the value of $v_3/M(\lesssim 0.03)$ will be fixed. Then the mass eigenvalues corresponding to the SM charged leptons can be computed in series expansion in the small parameters v_{ew}/M', v_i/M and $v_i^2/(MM') = \sqrt{2}m_i/v_{\mathrm{ew}}$.