Hidden sector renormalization in minimal supersymmetric standard model

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Talk based on arXiv: 1001.1509 and 1011.3998 with
S. Kawai (Sungkyunkwan U.) and N. Okada (U. of Alabama)

Where is Czech ?

首都：プラハ
公用語：チェコ語
人口：1千43万人

Where is Czech ?

首都：プラハ
公用語：チェコ語
人口：1千43万人
ビール消費量159リットル/人/年
（日本の約3倍）

Czech food

Czech food

Utopenec
(水死体)

Prague

日本人人口： 462人(2000年) → 1530人（2009年）
企業数：58（2000年） → 241（2009年）
日本人研究者：4人以上(物理１，生物２，機械１）

Prague

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チェコで働いた科学者：Albert Einstein, Johannes Kepler, Tycho de Brahe, Ernst Mach, Christian Doppler, Kurt Godel

Introduction - Purpose

Supersymmetry: Expected to be observed at LHC.

Studying RG flows of masses of superparticles

Introduction - Purpose

Supersymmetry: Expected to be observed at LHC.

Studying RG flows of masses of superparticles

1. Contribution from SUSY breaking sector
2. Spontaneous breaking of SUSY as a consequence of strong gauge dynamics

(Theoretical interests rather than phenomenology)

Introduction

- Standard Model
  - Successful theory up to O(100) GeV

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- Problems
  - Gauge hierarchy problem
  - No candidate for dark matter

Introduction

- Standard Model
  - Successful theory up to O(100) GeV
- Problems
  - Gauge hierarchy problem
  - No candidate for dark matter
- Beyond the Standard Model
  - Supersymmetry

$$Q|\mathrm{boson}>=|\mathrm{fermion}>$$

$$m_{\mathrm{boson}}=m_{\mathrm{fermion}}$$

Introduction

- Minimal Supersymmetric Standard Model (MSSM)

$$u_L,\ d_L \Rightarrow \tilde{u}_L,\ \tilde{d}_L$$

Superpartner

Introduction

- Minimal Supersymmetric Standard Model (MSSM)

Multiplets		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks (×3 families)	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\mathbf{3},\ \mathbf{2},\ \frac{1}{6})$
	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons (×3 families)	L	$(\widetilde{\nu}\ \ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$
	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H^+_u\ \ H^0_u)$	$(\widetilde{H}^+_u\ \ \widetilde{H}^0_u)$	$(\mathbf{1},\ \mathbf{2},\ +\frac{1}{2})$
	H_d	$(H^0_d\ \ H^-_d)$	$(\widetilde{H}^0_d\ \ \widetilde{H}^-_d)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$

Multiplets	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8},\ \mathbf{1},\ 0)$
winos, W bosons	$\widetilde{W}^\pm\ \ \widetilde{W}^0$	$W^\pm\ \ W^0$	$(\mathbf{1},\ \mathbf{3},\ 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\mathbf{1},\ \mathbf{1},\ 0)$

Introduction

- Minimal Supersymmetric Standard Model (MSSM)

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Superpartner

New Physics

Introduction

- Minimal Supersymmetric Standard Model (MSSM)

$$u_L\,,\ d_L \Rightarrow \tilde{u}_L\,,\ \tilde{d}_L$$

Superpartner

New Physics

No observation of superpartner with the same mass

⇩

SUSY must be broken.

Introduction

MSSM

Not break SUSY.
Need to extend.

Introduction

MSSM

Not break SUSY.
Need to extend.

SUSY breaking (hidden) sector

- Various SUSY breaking models

Introduction

MSSM ⇔ Messenger sector ⇔ SUSY breaking (hidden) sector

Not break SUSY.
Need to extend.

- Various SUSY breaking models
- (Direct) Gauge mediation
- Gravity mediation
- Etc.

⇓

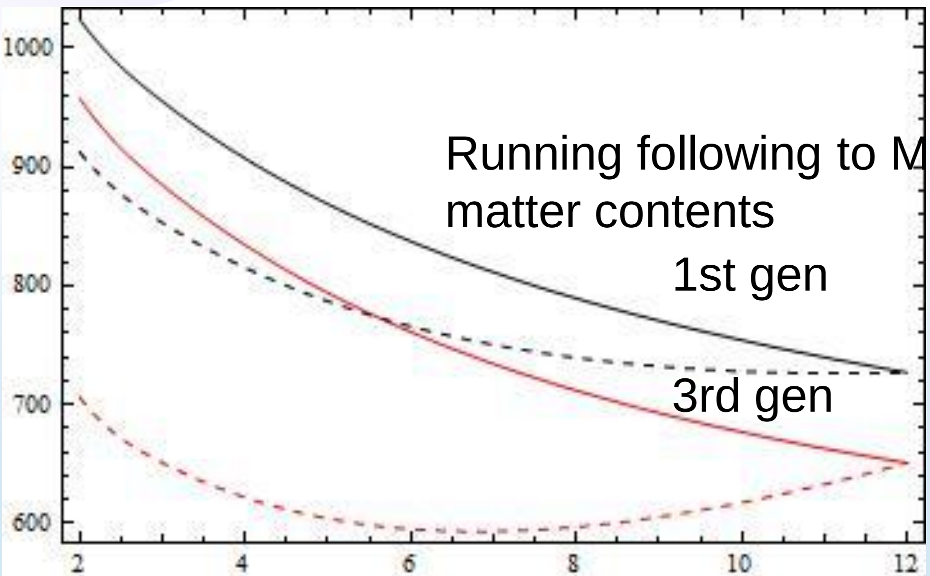
These structures may be revealed by experimental data such as *masses of superpartners*.

Introduction

- Mass & Renormalization group equation (RGE)
  - Minimal gauge mediated SUSY breaking (GMSB)

Squark masses

GeV



Running following to MSSM matter contents

1st gen

3rd gen

Left handed squark

Right handed squark

EW

Log10[μ /GeV)]

Messenger scale

Introduction

- Hidden sector effects are not considered
  - Hidden sector may affect on RGE if $M_{\rm hid} < M_{\rm mes}$

[Cohen Roy Schmaltz (2007)]

Introduction

- Hidden sector effects are not considered
  - Hidden sector may affect on RGE if $M_{\rm hid} < M_{\rm mes}$

    [Cohen Roy Schmaltz (2007)]

    - RG study for constrained MSSM

      [Campbell Ellis Maybury (2008)]

    - RG study for minimal GMSB

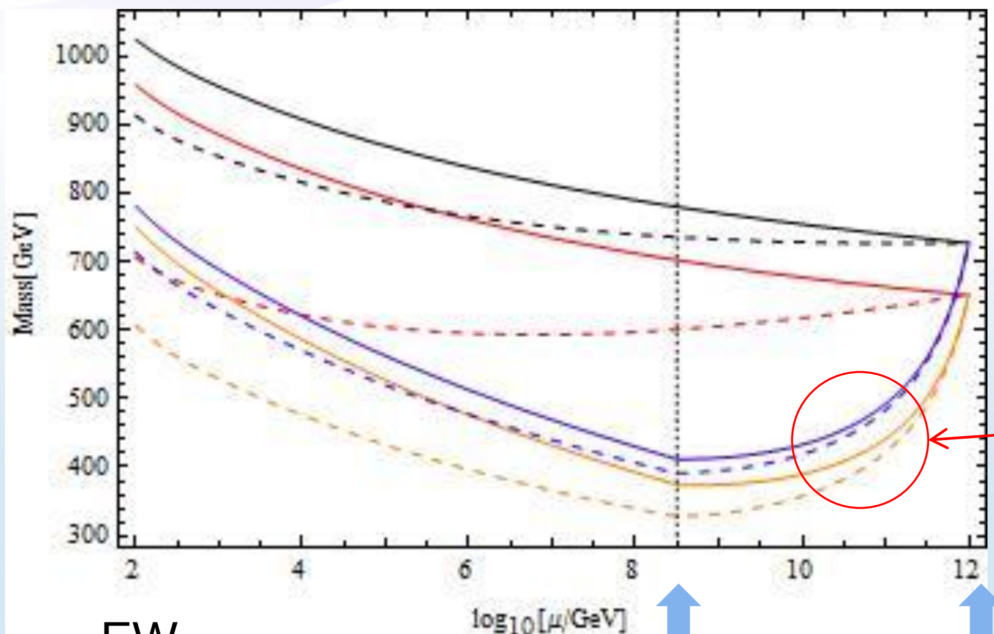
      [MA Kawai Okada (2010)]

Introduction

- RGE in minimal gauge mediation scenario
  - Hidden sector (toy): $\mathcal{W} = \frac{\lambda}{3} X^3 \quad \langle F_X \rangle \neq 0$

MA, N. Okada, S. kawai

Squark masses



Left handed

Right handed

MSSM+hidden flow

$\lambda = 3.8$

EW

Hidden scale Messenger scale

Introduction

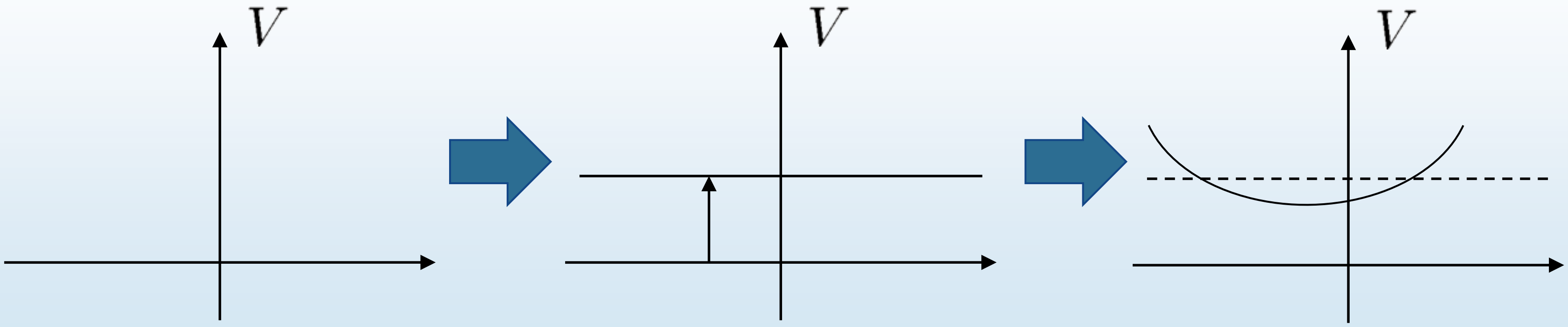
- Prediction changes
  - Next lightest superparticle

Without hidden sector effects	With hidden sector effects
Bino (superpartner of $U(1)_Y$ gauge boson)	Scalar tau

  - Scalar tau may have long lifetime.
    - Ex. 100 sec for $m_{\tilde{\tau}} \sim 130\ \mathrm{GeV},\ m_{3/2} = 0.1\ \mathrm{GeV}$
    - Maybe possible to trap it outside detector.

Purpose of work

- Consider a more desirable hidden sector
  - Spontaneous SUSY breaking sector
    - SUSY breaking vacuum appears as a consequence of strong gauge dynamics



SUSY is broken, but vacuum is degenrate.

One vacuum is selected, included quantum corrections

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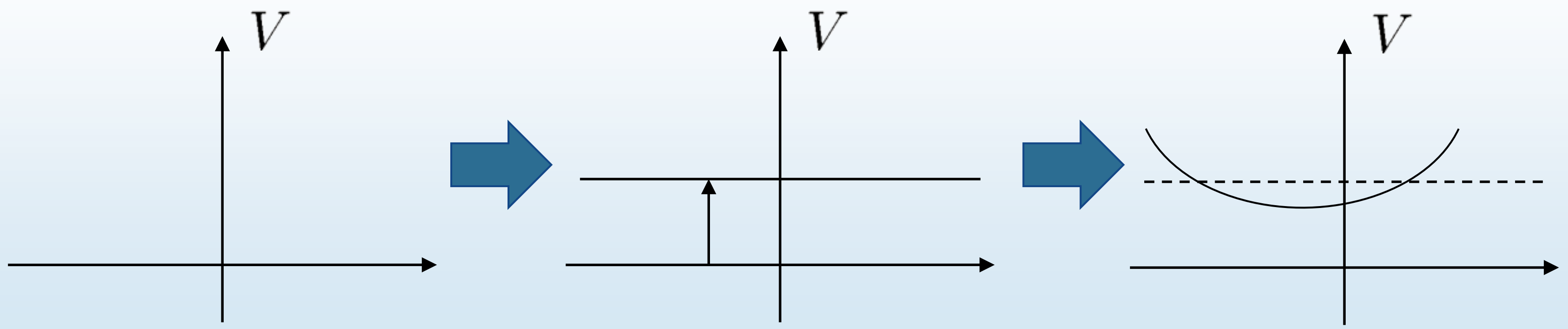

  - Perturbed N=2 SUSY QCD

[MA Okada (2001)] [Ooguri Ookouchi Park (2007)]
[Pastras (2007)][Marsano Ooguri Ookouchi Park (2007)]

Purpose of work

- Studying mass RG flow including hidden sector effects of the following system

  - Visible sector: MSSM
  - Hidden sector: Perturbed N=2 SUSY QCD

in the GMSB scenario.

Setup

How hidden sector affects
on masses of RG flow?

- Sfermion mass

$$\mu \frac{dm_i^2}{d\mu} = (\mathrm{MSSM\ RGE\ part}) + \gamma m_i^2$$

Anomalous dimension: $\gamma = -\mu \frac{d}{d\mu} \ln Z_X$

- Gaugino mass RGE; the same as MSSM
- Mainly explained by superfield

Setup

$$\mathcal{L} = \mathcal{L}_{\rm MSSM}(\Phi, W_\alpha) + \mathcal{L}_{\rm mes}(X, Q, \tilde{Q}) + \mathcal{L}_{\rm hid}(X)$$

Setup

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- MSSM sector

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squarks, quarks	Q	$(\widetilde{u}_L \ \ \widetilde{d}_L)$	$(u_L \ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
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Setup

$$\mathcal{L} = \mathcal{L}_{\rm MSSM}(\Phi, W_\alpha) + \mathcal{L}_{\rm mes}(X, Q, \tilde{Q}) + \mathcal{L}_{\rm hid}(X)$$

- $Q, \tilde{Q}$: Messenger field charged under 5 & $\bar{5}$ rep. of SU(5) ($\supset SU(3)_C \times SU(2)_L \times U(1)_Y$)

$$\mathcal{L}_{\rm mes} = \int d^2\theta W_{\rm mes} + h.c. \quad W_{\rm mes} = XQ\tilde{Q}$$

Setup

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$$\mathcal{L}_{\rm mes} = \int d^2\theta W_{\rm mes} + h.c. \quad W_{\rm mes} = XQ\tilde{Q}$$

- $\mathcal{L}_{\rm hid}(X)$: SUSY breaking sector
  - perturbed N=2 SUSY QCD

➡ $\langle X \rangle \neq 0 \quad \langle F_X \rangle \neq 0$

$$W_{\rm mes} = (X + \langle X \rangle)Q\tilde{Q} = (X + M_{\rm mes})Q\tilde{Q}$$

Mass of messenger

Integrating out messenger fields

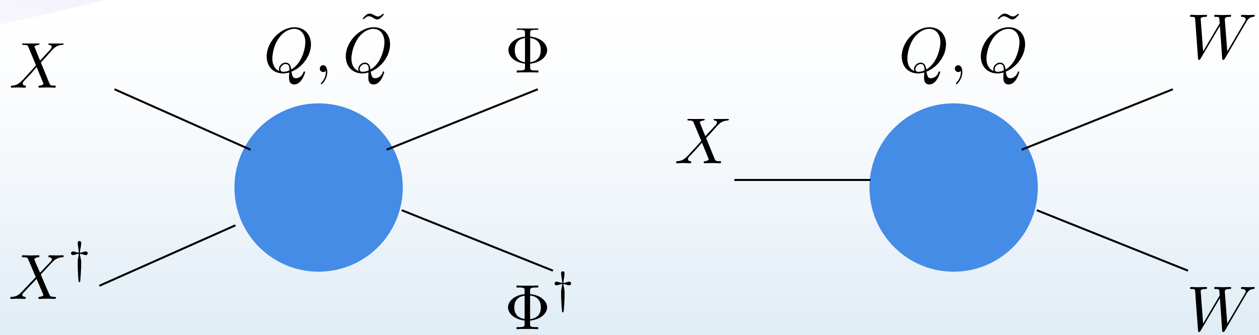
- Consider $M_{\rm hid} < M_{\rm mes}$ ($M_{\rm hid}$ mass of hidden field)

$$\mathcal{L} = \mathcal{L}_{\rm MSSM}(\Phi, W_\alpha) + \mathcal{L}_{\rm mes}(X, Q, \tilde{Q}) + \mathcal{L}_{\rm hid}(X)$$

Integrating out messenger fields

❖ Consider $M_{\rm hid} < M_{\rm mes}$ ($M_{\rm hid}$ mass of hidden field)

$$\mathcal{L} = \mathcal{L}_{\rm MSSM}(\Phi, W_\alpha) + \mathcal{L}_{\rm mes}(X, Q, \tilde{Q}) + \mathcal{L}_{\rm hid}(X)$$



$$\mathcal{L} = \mathcal{L}_{\rm MSSM}(\Phi, W_\alpha) + \mathcal{L}_{\rm int}(X, \Phi) + \mathcal{L}_{\rm hid}(X)$$

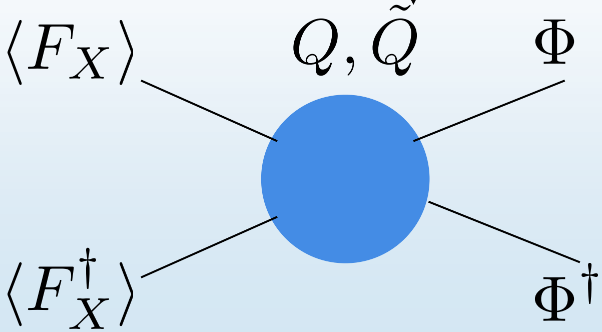
$$\mathcal{L}_{\rm int} = k_i \int d^4\theta \frac{XX^\dagger}{M_{\rm mes}^2} \Phi_i \Phi_i^\dagger + \left(w_a \int d^2\theta \frac{X}{M_{\rm mes}} W^{a\alpha} W_\alpha^a + h.c. \right)$$

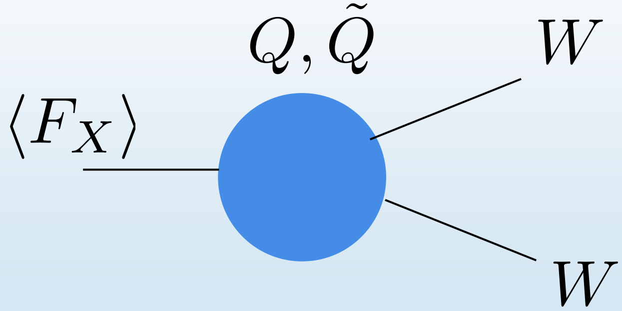
Effective Lagrangian

❖ Sfermion and gaugino masses

$$\mathcal{L}_{\rm int} = k_i \int d^4\theta \frac{XX^\dagger}{M_{\rm mes}^2}\Phi_i\Phi_i^\dagger + \left(w_a \int d^2\theta \frac{X}{M_{\rm mes}} W^{a\alpha}W^a_\alpha + h.c.\right)$$

$\langle X\rangle \neq 0$ $\langle F_X\rangle \neq 0$





Sfermion masses $m_i^2 = k_i\Lambda^2$

Gaugino masses $M_a = w_a\Lambda$

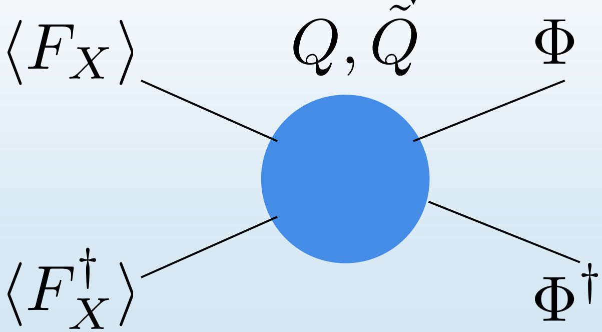
$$\Lambda = \frac{F_X}{M_{\rm mes}}$$

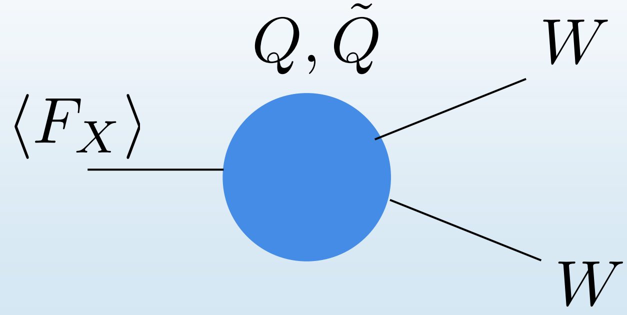
Effective Lagrangian

❖ Sfermion and gaugino masses

$$\mathcal{L}_{\rm int} = k_i \int d^4\theta \frac{XX^\dagger}{M_{\rm mes}^2}\Phi_i\Phi_i^\dagger + \left(w_a \int d^2\theta \frac{X}{M_{\rm mes}} W^{a\alpha}W^a_\alpha + h.c. \right)$$

$\downarrow \quad \langle X \rangle \neq 0 \quad \langle F_X \rangle \neq 0 \quad \downarrow$





Sfermion masses $m_i^2 = k_i \Lambda^2$

$$k_i = 2\sum_{a=1}^{3} C_2^a(R_i)\left(\frac{\alpha_a}{4\pi}\right)^2$$

Gaugino masses $M_a = w_a \Lambda$

$$w_a = \frac{\alpha_a}{4\pi} \qquad \Lambda = \frac{F_X}{M_{\rm mes}}$$

Hidden sector effects and RGE

❖ How hidden sector affects mass RGE?

$$\mathcal{L}_{\rm int} = k_i \int d^4\theta \frac{XX^\dagger}{M_{\rm mes}^2}\Phi_i\Phi_i^\dagger + \left(w_a \int d^2\theta \frac{X}{M_{\rm mes}} W^{a\alpha}W^a_\alpha + h.c. \right)$$

$$X \to Z_X^{-1/2}X \qquad \Phi \to Z_\Phi^{-1/2}\Phi \qquad W \to Z_W^{-1/2}W$$

$$\mathcal{L}_{\rm int} = k_i \int d^4\theta Z_X^{-1} Z_\Phi^{-1} \frac{XX^\dagger}{M_{\rm mes}^2}\Phi_i\Phi_i^\dagger + \left(w_a \int d^2\theta Z_X^{-1/2} Z_W^{-1} \frac{X}{M_{\rm mes}} W^{a\alpha}W^a_\alpha + h.c. \right)$$

k_i and w_a are renormalized.

➡ RG equations of k_i and w_a are derived.

Hidden sector effects and RGE

- How hidden sector affects mass RGE?
  - Mass RGE of sfermion (from RGE of k_i)

$$\mu \frac{dm_i^2}{d\mu} = (\text{MSSM RGE part}) + \gamma m_i^2$$

Anomalous dimension: $\gamma = -\mu \frac{d}{d\mu} \ln Z_X$

  - Mass RGE of gaugino (from RGE of w_a)
    - The same as one of MSSM (no effect from hidden sector)

Extracting hidden sector effects

- Information of hidden sector is encoded in Z_X
  - Z_X: Kähler metric of hidden sector

$$\mathcal{L}_{\rm hid} = g_{XX^\dagger}\partial X \partial X^\dagger + \cdots \to \partial X \partial X^\dagger + \cdots$$

$$X \to Z_X^{-1/2} X$$

- Obtained it at most perturbatively in N=1 SUSY
- Possible to derive it exactly in (perturbed) N=2 SUSY gauge theory

  ⇒ Our hidden sector model

Short summary - 1

Hidden sector affects on masses of RG flow

- Sfermion mass RGE

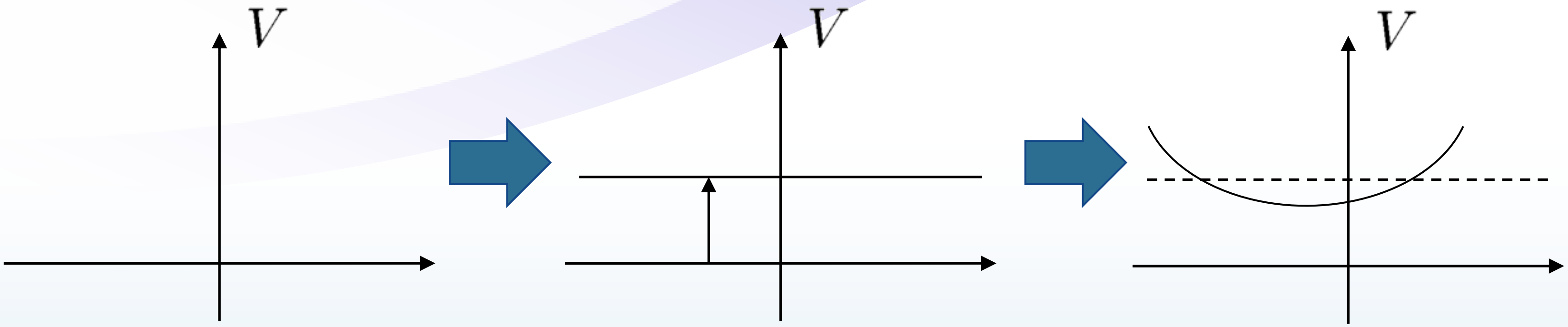
$$\mu \frac{dm_i^2}{d\mu} = (\text{MSSM RGE part}) + \gamma m_i^2$$

Anomalous dimension: $\gamma = -\mu \frac{d}{d\mu} \ln Z_X$

- Gaugino mass RGE; the same as MSSM
- Z_X can be derived exactly (as will be seen).

Our model

Scalar Potential



SUSY is broken, but vacuum is degenerate.

One vacuum is selected, included quantum corrections

Effective theory of hidden sector

$$\mathcal{L}_{\rm hid}(X) \to \mathcal{L}_{\rm hid}(A_2, A_1, M, \tilde{M})$$

Our model

- Our hidden sector model [MA, Okada (2001)]
  - N=2 SUSY SU(2)xU(1) coupled to 2 massless hypermultiplets perturbed by Fayet-Iliopoulos (FI) term.
  - SUSY is spontaneously broken at classical level.
    - Pseudo flat direction (moduli)
  - Degeneracy of vacua removed by taking quantum corrections into account.

Our model

- Hidden sector Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm VM} + \mathcal{L}_{\rm HM} + \mathcal{L}_{\rm FI}$$

$$\begin{aligned}\mathcal{L}_{\rm VM} &= \frac{1}{2\pi}{\rm Im}\left[{\rm Tr}\left\{\tau_{22}\left(\int d^4\theta\; A_2^\dagger e^{2V_2} A_2 e^{-2V_2} + \frac{1}{2}\int d^2\theta\; W_2^2\right)\right\}\right] \\ &\quad + \frac{1}{4\pi}{\rm Im}\left[\tau_{11}\left(\int d^4\theta\; A_1^\dagger A_1 + \frac{1}{2}\int d^2\theta\; W_1^2\right)\right]\end{aligned}$$

Coupling constants: $\tau_{22} = i\dfrac{8\pi}{g^2} + \dfrac{\theta}{\pi}, \qquad \tau_{11} = i\dfrac{8\pi}{e^2}$

$$\begin{aligned}\mathcal{L}_{\rm HM} &= \int d^4\theta\left(q_r^\dagger e^{2V_2+2V_1} q^r + \tilde{q}_r e^{-2V_2-2V_1}\tilde{q}^{r\dagger}\right) \\ &\quad + \sqrt{2}\left(\int d^2\theta\; \tilde{q}_r(A_2 + A_1)q^r + h.c.\right).\end{aligned}$$

$$\mathcal{L}_{\rm FI} = \int d^2\theta \lambda_{\rm FI} A_1 + h.c.$$

Fayet-Iliopoulos term

Classical vacua

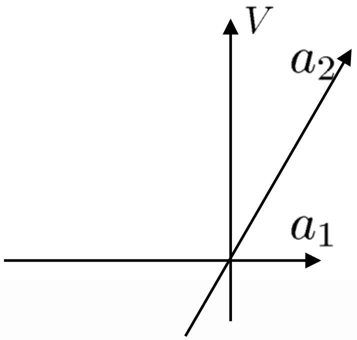
- Potential analysis

$$V = V(A_2,\ A_1,\ q,\ \tilde{q},\ \lambda_{\mathrm{FI}})$$

  - $\lambda_{\mathrm{FI}} = 0$

$$q = \tilde{q} = 0, \quad A_2 = \begin{pmatrix} a_2 & 0 \\ 0 & -a_2 \end{pmatrix}, \quad A_1 = a_1$$

$$SU(2) \times U(1) \Rightarrow U(1)_c \times U(1)$$



Classical vacua

❖ Potential analysis

$$V = V(A_2,\ A_1,\ q,\ \tilde{q},\ \lambda_{\rm FI})$$

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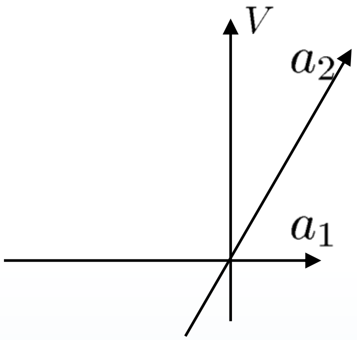


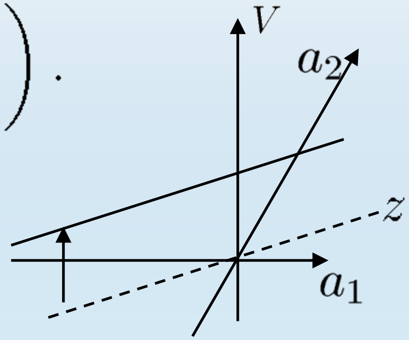
$$SU(2) \times U(1) \Rightarrow U(1)_c \times U(1)$$

- $\lambda_{\rm FI} \neq 0$

$$A_2 + A_1 = \begin{pmatrix} \frac{a_2}{2} & 0 \\ 0 & -\frac{a_2}{2} \end{pmatrix} + \begin{pmatrix} a_1 & 0 \\ 0 & a_1 \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 \\ 0 & z \end{pmatrix}.$$

$$q^1 = \tilde{q}_1 = \begin{pmatrix} v \\ 0 \end{pmatrix}, \quad q^2 = \tilde{q}_2 = 0,$$

$$V = \frac{|\lambda|^2 e^2 g^2}{4e^2 + g^2}$$

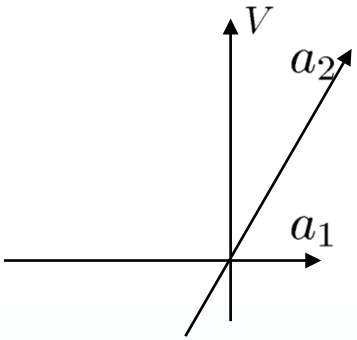


~~SUSY~~ Pseudo flat direction

Classical vacua

❖ Potential analysis

$$V = V(A_2,\ A_1,\ q,\ \tilde{q},\ \lambda_{\rm FI})$$



- $\lambda_{\rm FI} = 0$

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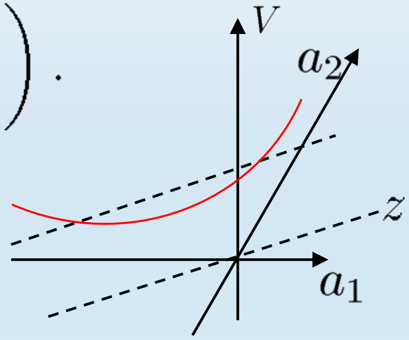
- $\lambda_{\rm FI} \neq 0$

$$A_2 + A_1 = \begin{pmatrix} \frac{a_2}{2} & 0 \\ 0 & -\frac{a_2}{2} \end{pmatrix} + \begin{pmatrix} a_1 & 0 \\ 0 & a_1 \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 \\ 0 & z \end{pmatrix}.$$

$$q^1 = \tilde{q}_1 = \begin{pmatrix} v \\ 0 \end{pmatrix}, \quad q^2 = \tilde{q}_2 = 0,$$

~~SUSY~~



Pseudo flat direction

Quantum theory

❖ Low energy Wilsonian effective action

- Integrating out heavy fields

$$S = \int_{|k|>\Lambda} \Pi_i \mathcal{D}\phi_i e^{i\int \mathcal{L}(\phi_i)}$$

- Effective action

$$\mathcal{L}_{\rm exact} = \mathcal{L}(\phi_i, \lambda_{\rm FI}, \Lambda)$$ ⇦ Difficult task

- Assuming that $\lambda_{\rm FI} \ll \Lambda^2$

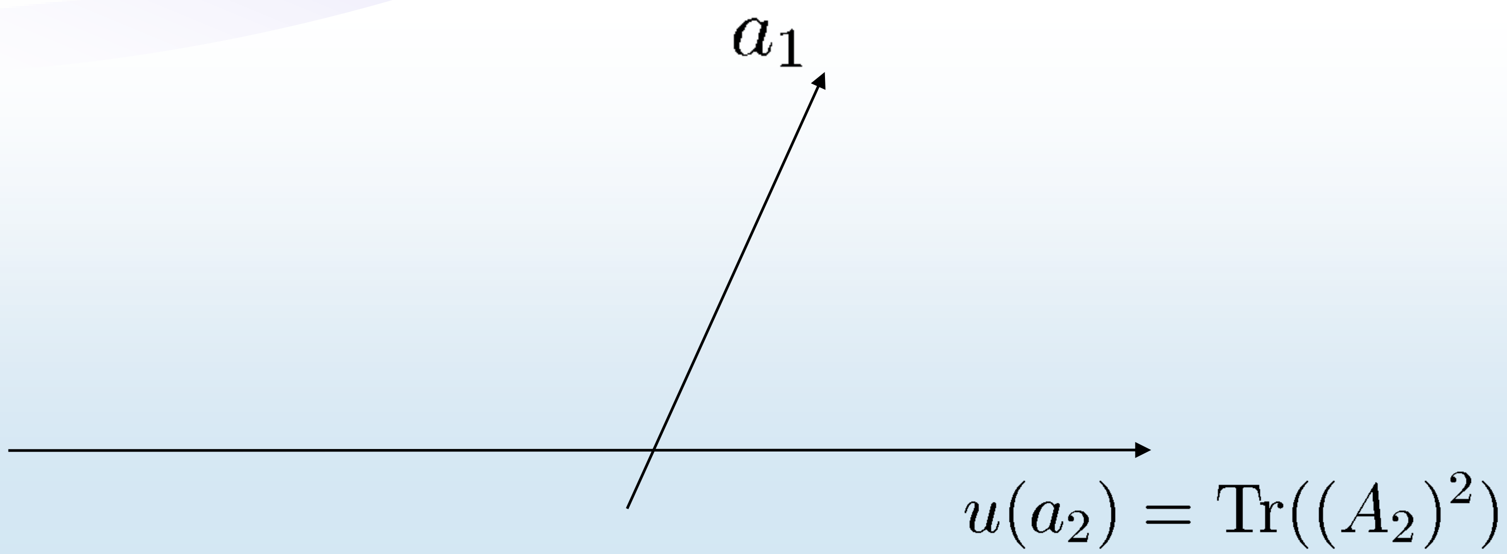
⇨ $$\mathcal{L}_{\rm exact} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm FI} + \mathcal{O}(\lambda_{\rm FI})$$

N=2 SUSY part $(\lambda_{\rm FI} = 0)$ | Leading order in $\lambda_{\rm FI}$ ⇦ Breaking SUSY

Quantum theory

- Low energy effective action – N=2 part: $\mathcal{L}_{\mathrm{SUSY}}$

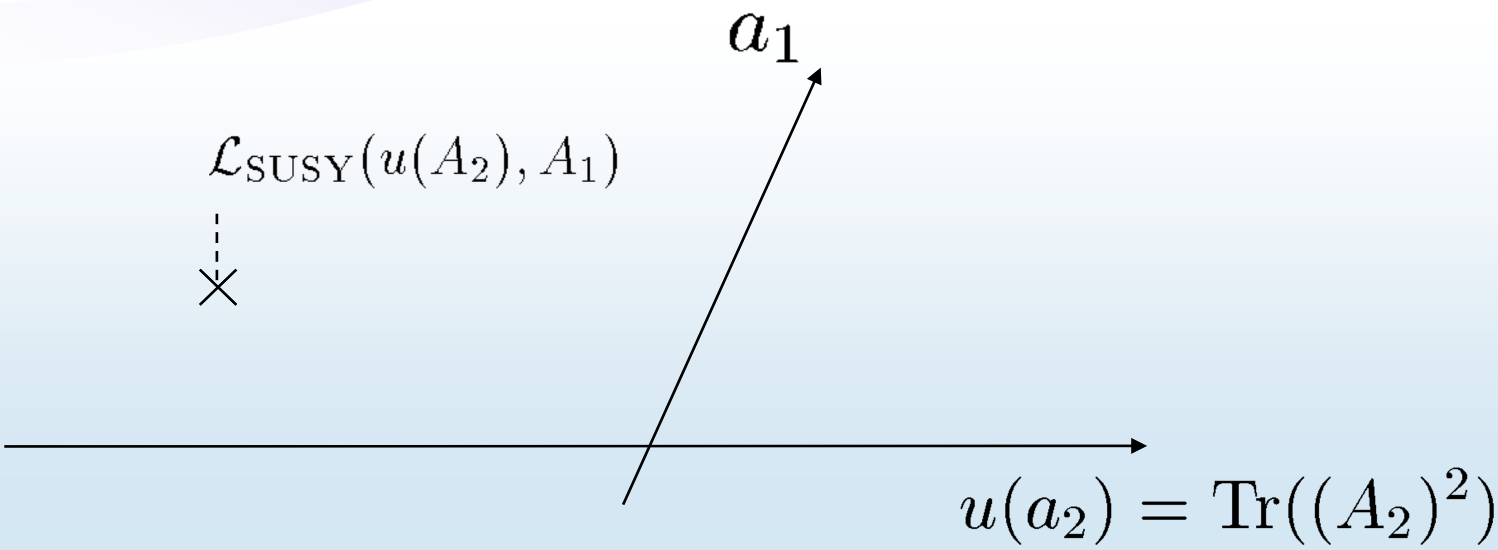
$$\mathcal{L}_{\mathrm{SUSY}} = \mathcal{L}_{\mathrm{VM}} + \mathcal{L}_{\mathrm{HM}} \qquad V = 0$$



Quantum theory

- Low energy effective action – N=2 part: $\mathcal{L}_{\mathrm{SUSY}}$

$$\mathcal{L}_{\mathrm{SUSY}} = \mathcal{L}_{\mathrm{VM}} + \mathcal{L}_{\mathrm{HM}} \qquad V = 0$$



Quantum theory

- Low energy effective action – N=2 part: $\mathcal{L}_{\rm SUSY}$

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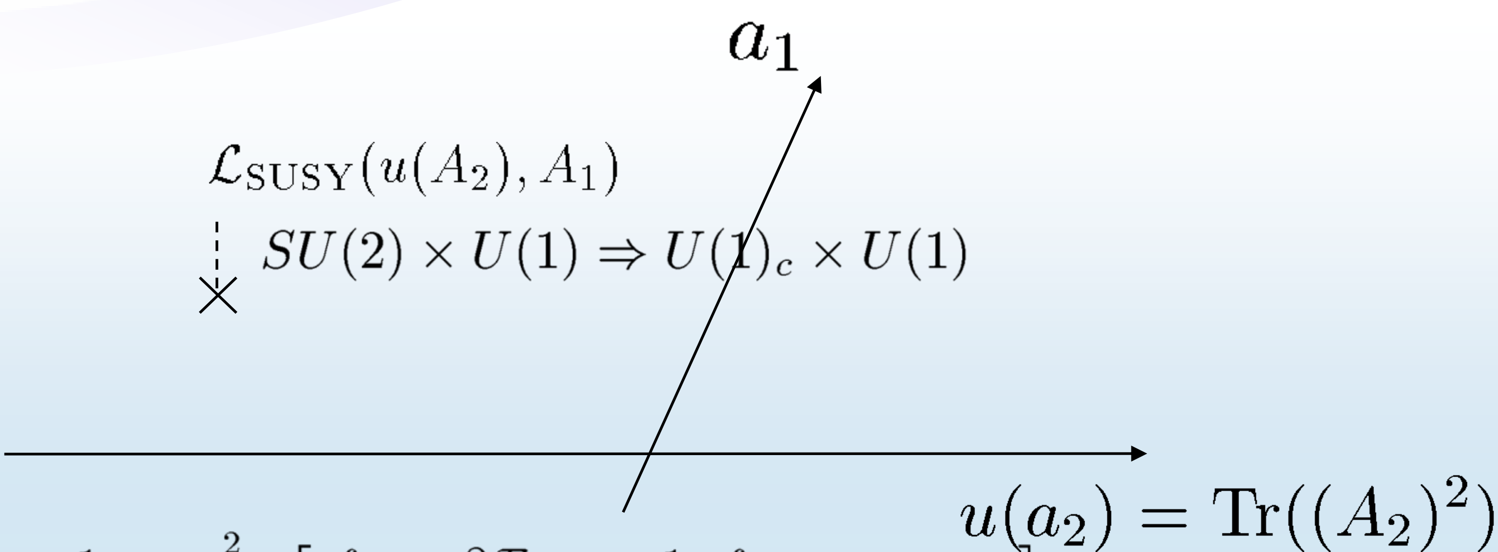


$$\mathcal{L}_{\rm VM} = \frac{1}{4\pi} {\rm Im} \sum_{i,j=1}^{2} \left[\int d^4\theta \, \frac{\partial \mathcal{F}}{\partial A_i} A_i^\dagger + \frac{1}{2} \int d^2\theta \, \tau_{ij} W_i W_j \right]$$

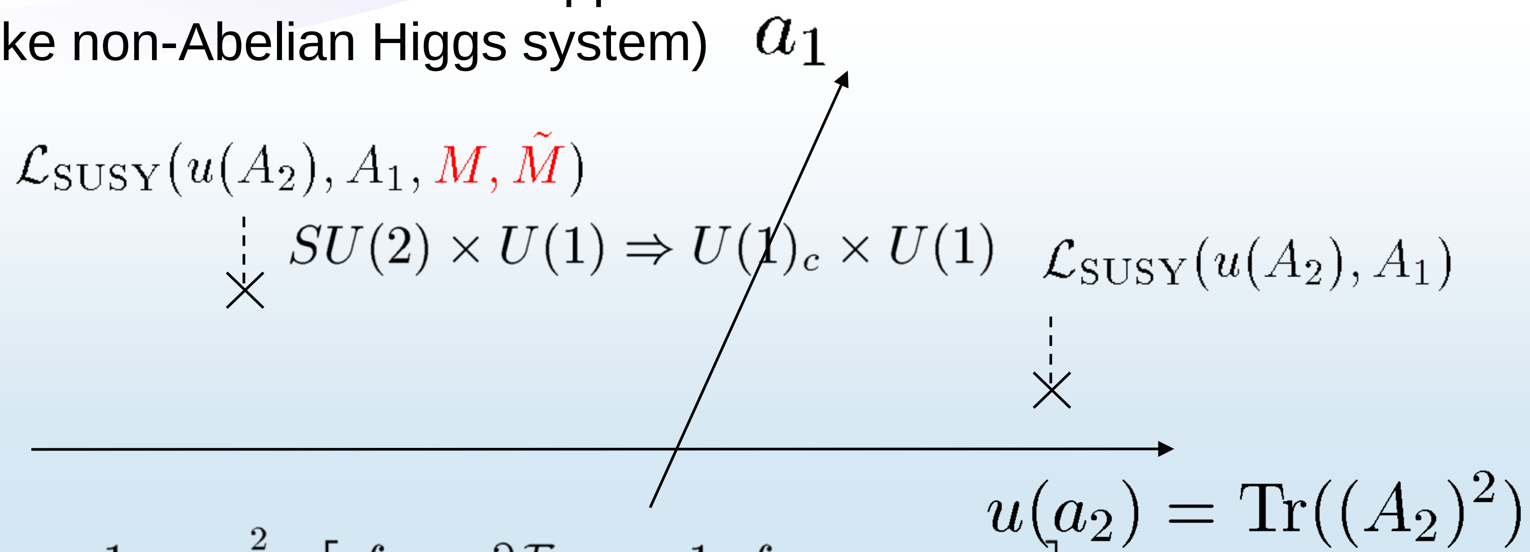
Prepotential: $\mathcal{F} = \mathcal{F}(A_1, A_2, \Lambda, \Lambda_{\rm Landau})$ written by Elliptic function

Quantum theory

❖ Low energy effective action – N=2 part: $\mathcal{L}_{\mathrm{SUSY}}$

$$\mathcal{L}_{\mathrm{SUSY}} = \mathcal{L}_{\mathrm{VM}} + \mathcal{L}_{\mathrm{HM}} \qquad V = 0$$

Massless solitonic state appears
(like non-Abelian Higgs system)



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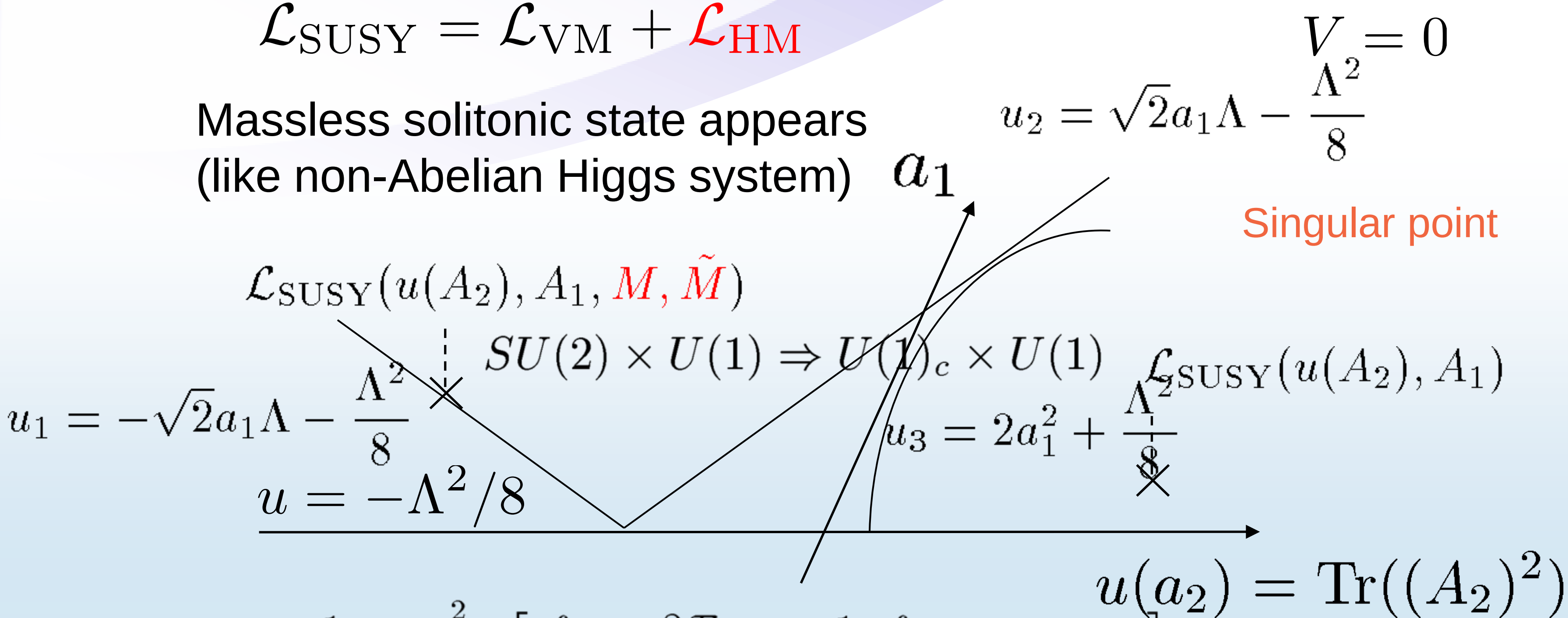

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Quantum theory

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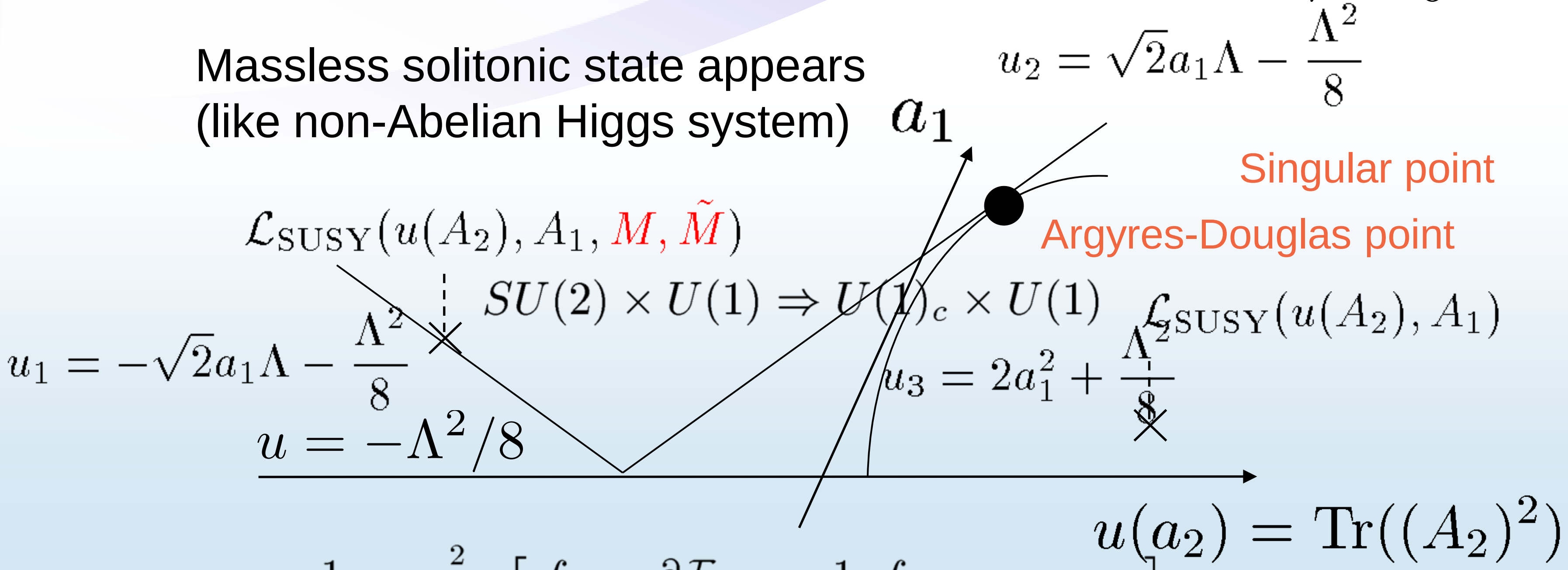

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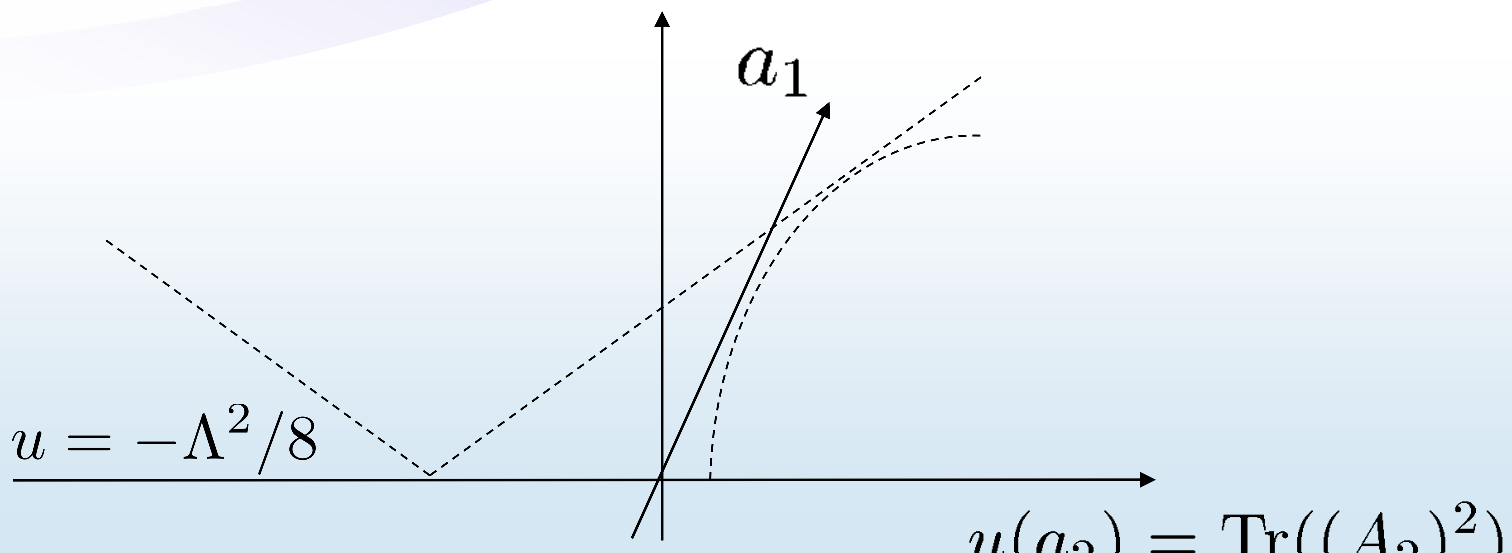
Prepotential: $\mathcal{F} = \mathcal{F}(A_1, A_2, \Lambda, \Lambda_{\rm Landau})$ written by Elliptic function

Quantum theory

- Low energy effective action – N=2 part: $\mathcal{L}_{\mathrm{SUSY}}$

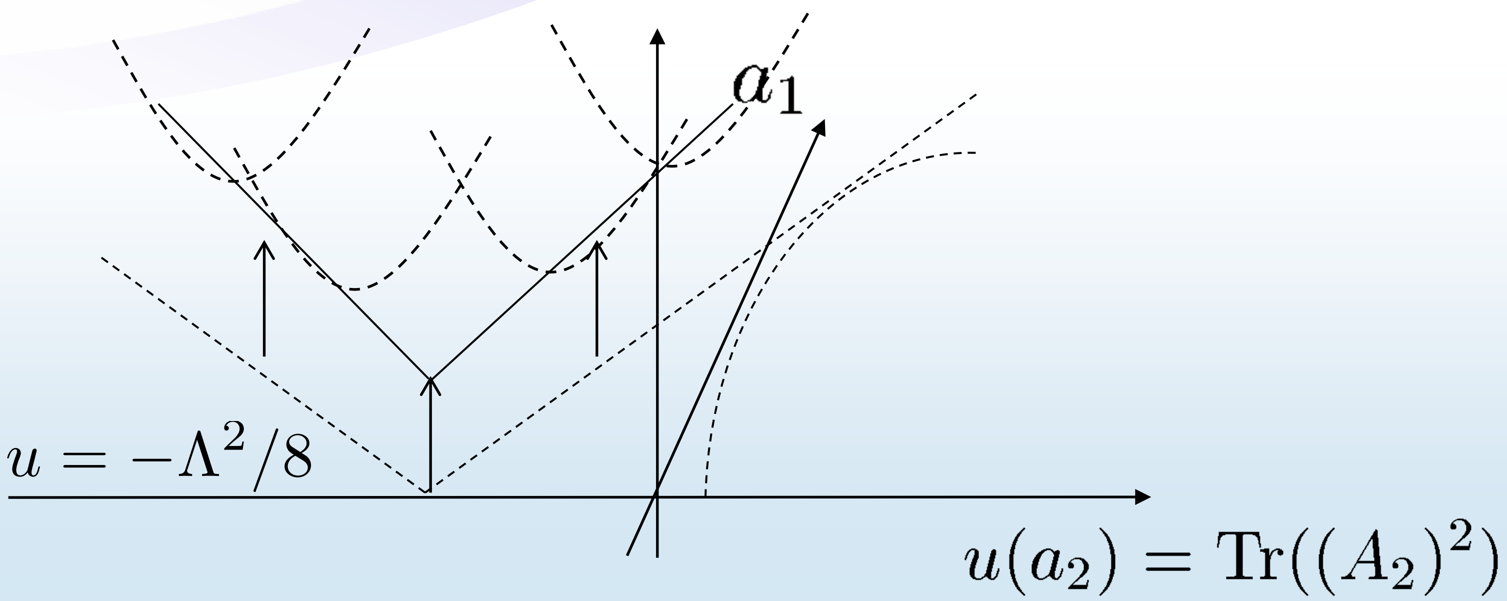
$$\mathcal{L} = \mathcal{L}_{\mathrm{VM}} + \mathcal{L}_{\mathrm{HM}} + \mathcal{L}_{\mathrm{FI}}$$



Quantum theory

❖ Low energy effective action – N=2 part: $\mathcal{L}_{\mathrm{SUSY}}$

$$\mathcal{L} = \mathcal{L}_{\mathrm{VM}} + \mathcal{L}_{\mathrm{HM}} + \mathcal{L}_{\mathrm{FI}}$$

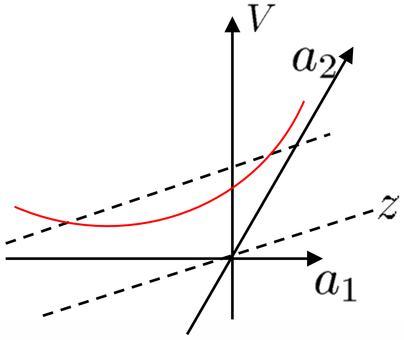


Potential analysis

- Effective scalar potential

$$\mathcal{L} = \mathcal{L}_{\rm VM} + \mathcal{L}_{\rm HM} + \mathcal{L}_{\rm FI}$$

$$\Rightarrow \quad V = V(a_2(u), a_1, M, \tilde{M})$$



- Solving stationary condition with respect to $M,\ \tilde{M}$

$$0 = \frac{\partial V}{\partial M} = \frac{\partial V}{\partial \tilde{M}}$$

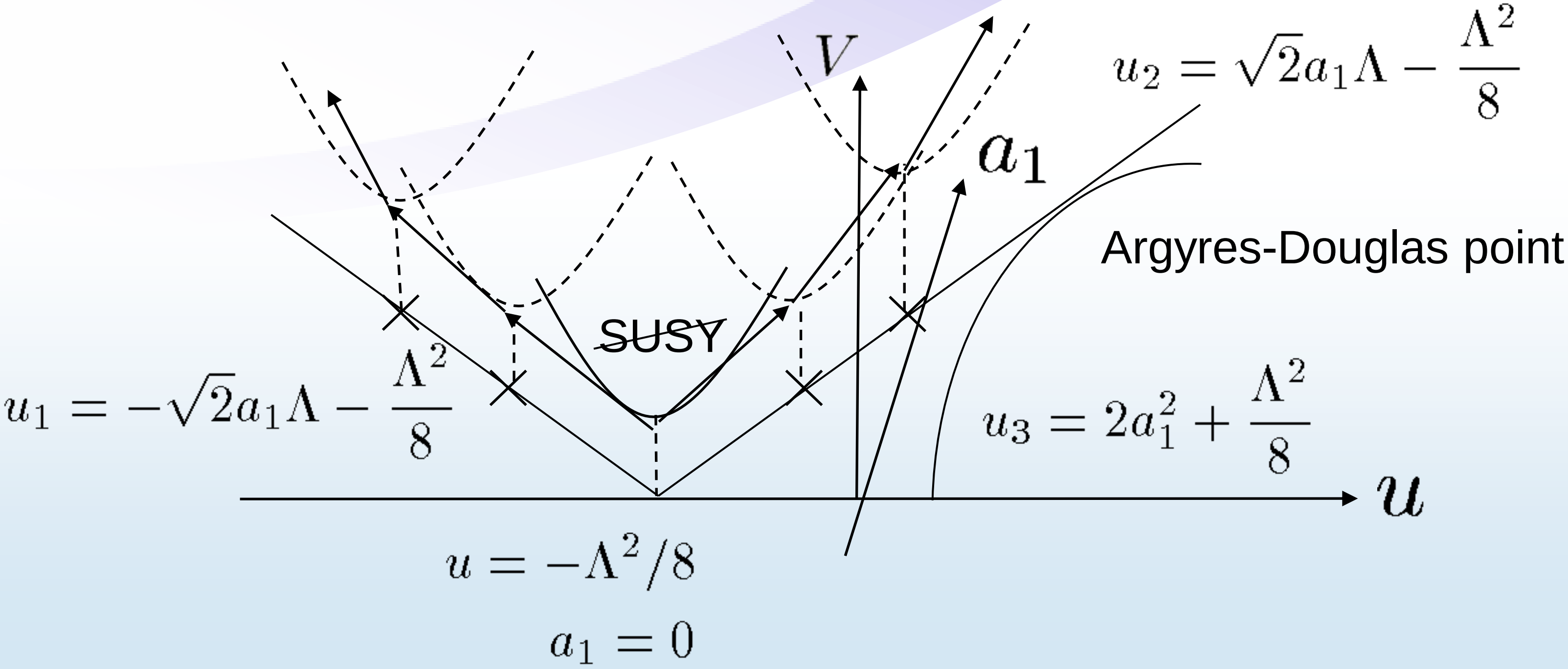
$$\Rightarrow \begin{cases} 1)\ V(a_2(u), a_1) = Y(a_2, a_1)\,, \\ 2)\ V(a_2(u), a_1) = Y(a_2, a_1) - 4S(a_2, a_1)\mathcal{M}(a_2, a_1)^4 \end{cases}$$

$$S(a_2, a_1) > 0 \qquad \mathcal{M} \equiv M = \tilde{M}$$

- Potential minimum is energetically favored if light matter acquires VEV (along only singular points).

Potential analysis

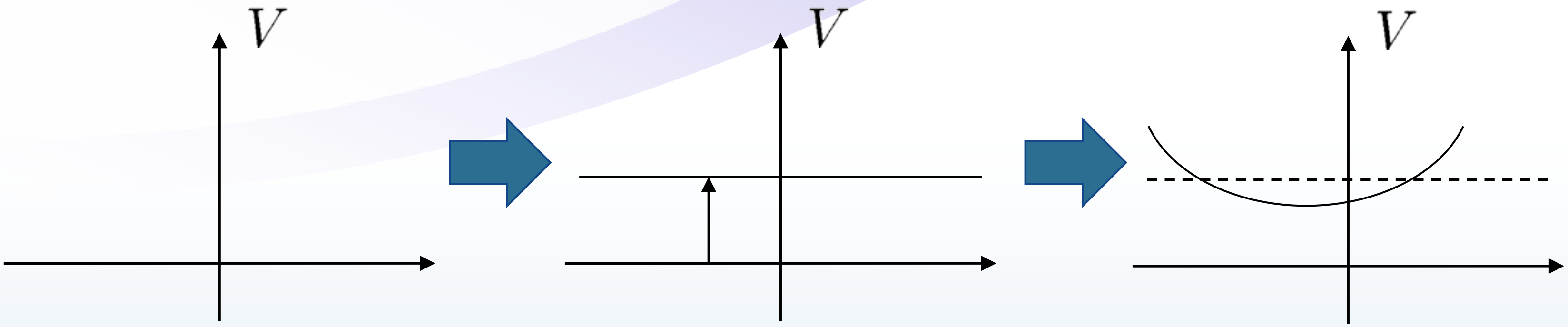
❖ Potential parameterized by 2 moduli parameters



- Local vacua develop along flows of singular points of the theory (N=2 massive SQCD) and make troughs.

Short summary 2

Scalar Potential



SUSY is broken, but
vacuum is degenerate.

One vacuum is selected,
included quantum corrections

Effective theory of hidden sector

$$\mathcal{L}_{\rm hid}(X) \to \mathcal{L}_{\rm hid}(A_2, A_1, M, \tilde{M})$$

RGE analysis

Mass RGE

$$\mu \frac{dm_i^2}{d\mu} = (\text{MSSM RGE part}) + \gamma m_i^2 \qquad \gamma = -\mu \frac{d}{d\mu} \ln Z_X$$

Hidden sector

$$\mathcal{L}_{\text{hid}}(A_2, A_1, M, \tilde{M})$$

$$X \to Z_X^{-1/2} X \qquad X = A_1,\ A_2$$

(Numerical evaluation)

Possible coupling to the messenger fields

- 2 possible messenger - hidden couplings

$$\mathcal{L} = \mathcal{L}_{\rm MSSM}(\Phi, W_\alpha) + \mathcal{L}_{\rm mes}(X, \Phi) + \mathcal{L}_{\rm hid}(X)$$

$$W_{\rm mes} = XQ\tilde{Q} + m_{\rm mes}Q\tilde{Q}$$

- Possibility 1: $X = A_1$ $\quad A_1 \to Z_{A_1}^{-1/2} A_1$
- Possibility 2: $X = u/\tilde{M}$ $\quad A_2 \to Z_{A_2}^{-1/2} A_2$

$$\mu \frac{dm_i^2}{d\mu} = (\text{MSSM RGE part}) + \gamma m_i^2 \qquad \gamma = -\mu \frac{d}{d\mu} \ln Z_X$$

Wave function renormalization

$$\mathcal{L}_{\rm VM} = \frac{1}{4\pi}{\rm Im}\sum_{i,j=1}^{2}\left[\int d^4\theta\,\frac{\partial\mathcal{F}}{\partial A_i}A_i^\dagger + \frac{1}{2}\int d^2\theta\,\tau_{ij}W_iW_j\right]$$
$$= g_{A_1A_1^\dagger}\partial A_1\partial A_1^\dagger + g_{A_2A_2^\dagger}\partial A_2\partial A_2^\dagger + \cdots$$

Effective couplings of A_1 & A_2

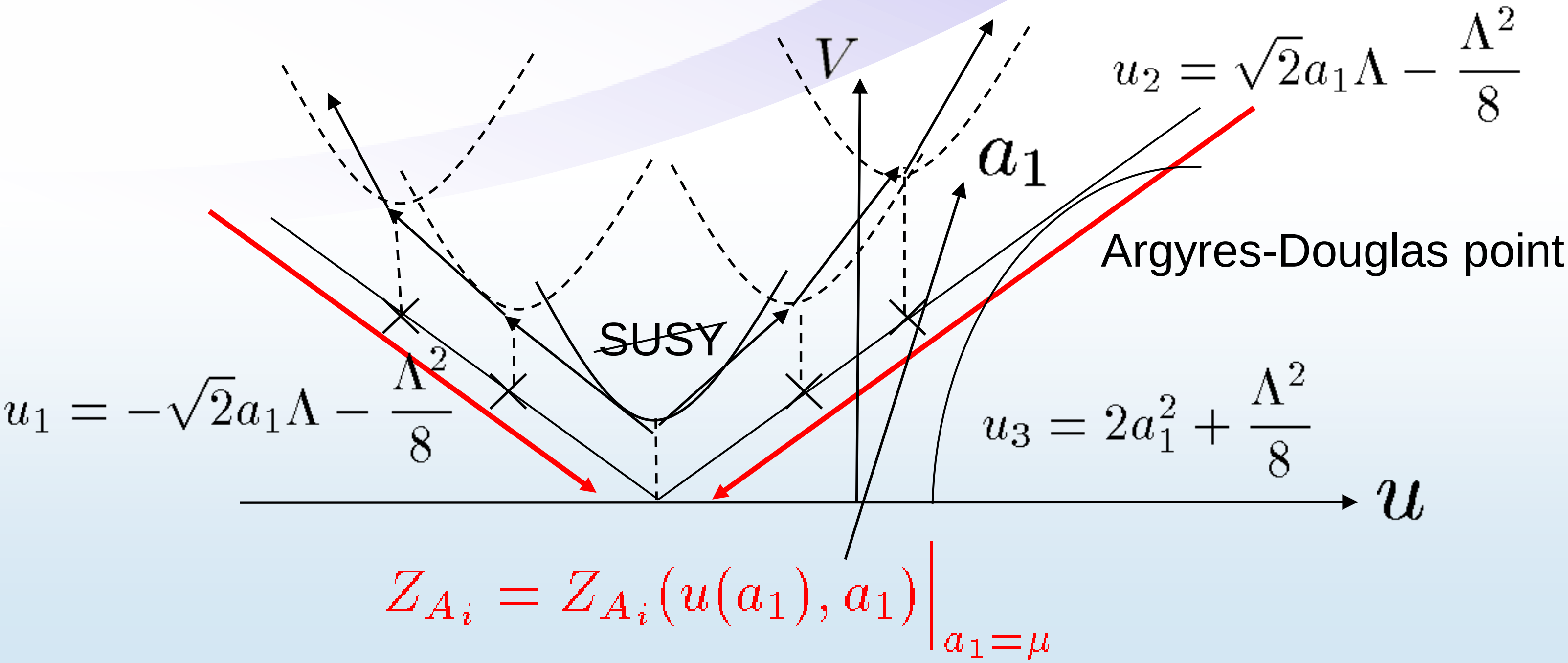
$$g_{A_1A_1^\dagger}(u, a_1) = Z_{A_1} = \frac{\partial^2 K}{\partial A_1\partial A_1^\dagger} = \frac{1}{8\pi}{\rm Im}\frac{\partial^2}{\partial A_1\partial A_1^\dagger}\left(\frac{\partial\mathcal{F}}{\partial A_1}A_1^\dagger\right)$$
$$g_{A_2A_2^\dagger}(u, a_1) = Z_{A_2} = \frac{\partial^2 K}{\partial A_2\partial A_2^\dagger} = \frac{1}{8\pi}{\rm Im}\frac{\partial^2}{\partial A_2\partial A_2^\dagger}\left(\frac{\partial\mathcal{F}}{\partial A_2}A_2^\dagger\right)$$

Identify moduli parameters as a renormalization scale in RGE

➡ How to identify two moduli parameters as a scale?

RGE flow

❖ Potential parameterized by 2 moduli parameters



- Choosing troughs of the potential (flow of the singular points) [cf. Sher (1989)]

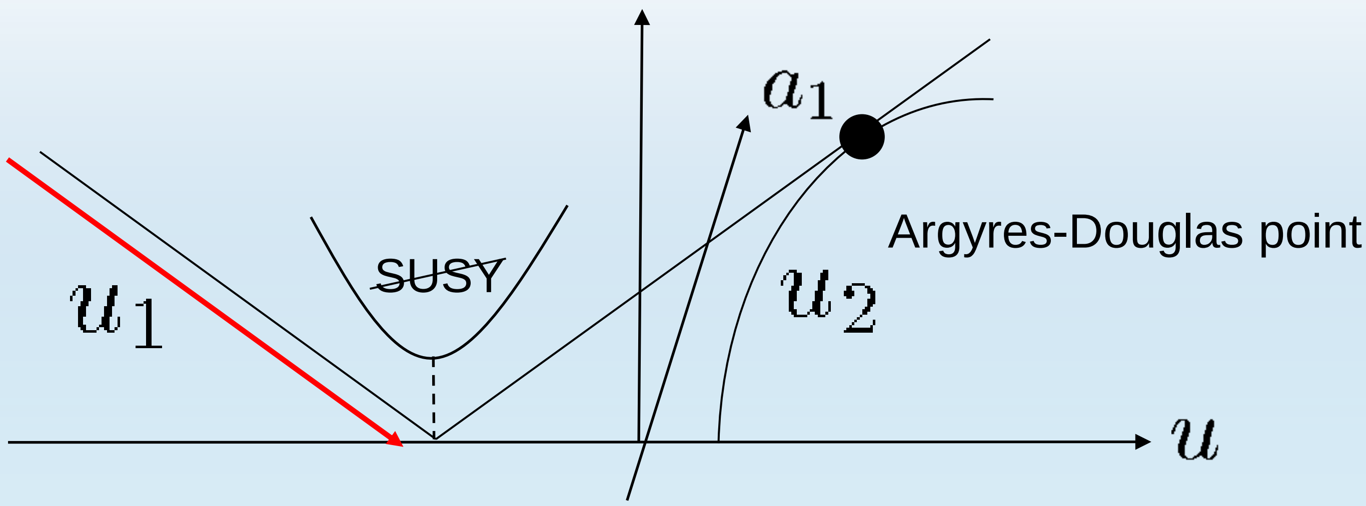
Possible coupling to the messenger fields

- 2 possible messenger - hidden couplings

$$\mathcal{L} = \mathcal{L}_{\rm MSSM}(\Phi, W_\alpha) + \mathcal{L}_{\rm mes}(X, \Phi) + \mathcal{L}_{\rm hid}(X)$$

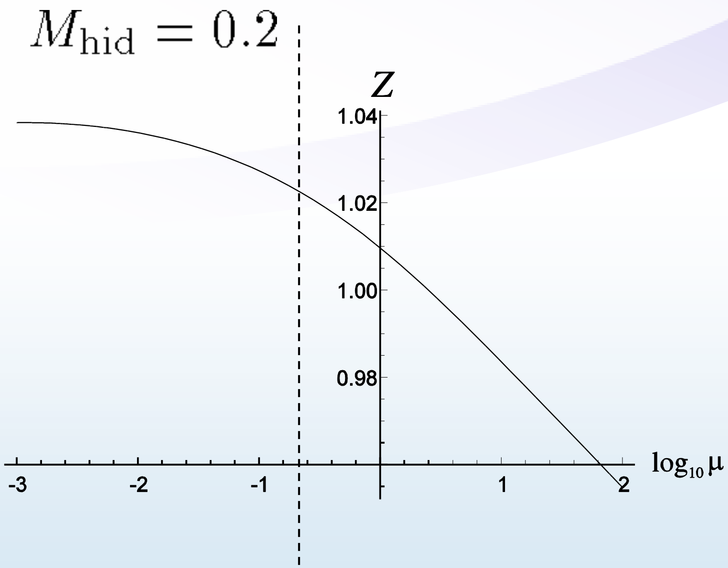
  - Model 1 - U(1) field coupled: $X = A_1$

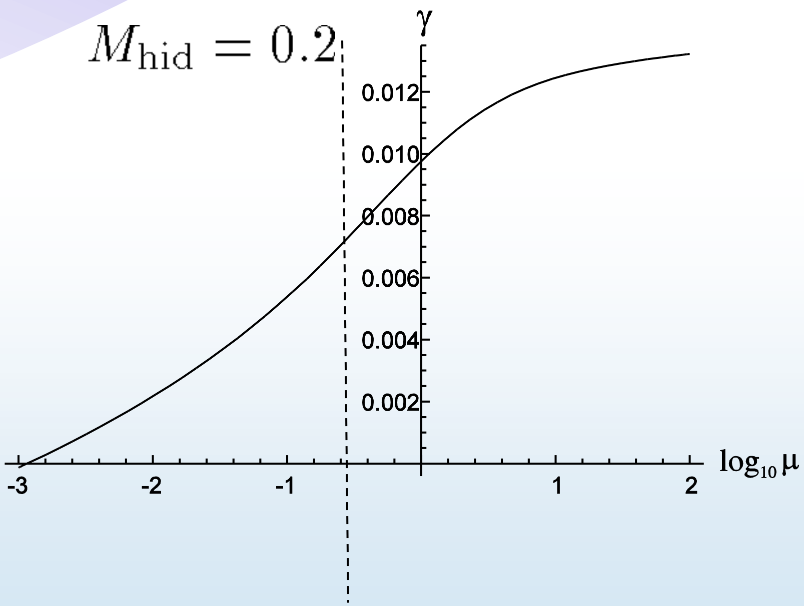
$$W_{\rm mes} = A_1 Q\tilde{Q} + m_{\rm mes} Q\tilde{Q} \qquad A_1 \to Z_{A_1}^{-1/2} A_1$$



Wave function renormalization

- Z_{A_1} & γ_{A_1} along u_1



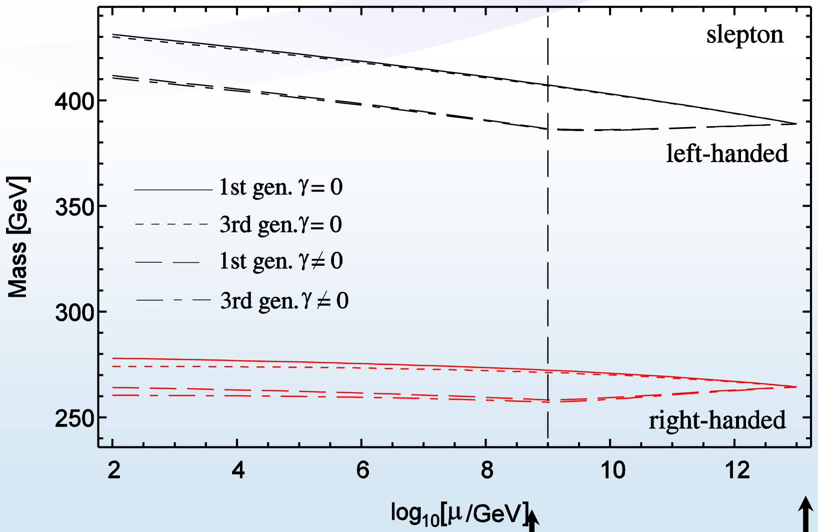


$$\mu \frac{dm_i^2}{d\mu} = (\text{MSSM RGE part}) + \gamma m_i^2 \qquad \gamma = -\mu \frac{d}{d\mu} \ln Z_X$$

Mass RG along u1 – model 1

- Mass RG flow of sleptons $(\mu \to (10^9/M_{\rm hid})\mu,\ M_{\rm hid} = 0.2)$

MSSM flow | MSSM + hidden flow



Hidden sector effects decrease soft masses.

Hidden scale

messenger scale With hidden sector effects

$$M_{\rm hid} = 10^9\ \text{GeV} \qquad M = 10^{13}\ \text{GeV} \qquad \tan\beta = 10$$

Mass RG along u1 – model 1

❖ Mass RG flow of squarks $(\mu \to (10^9/M_{\rm hid})\mu,\ \ M_{\rm hid} = 0.2)$

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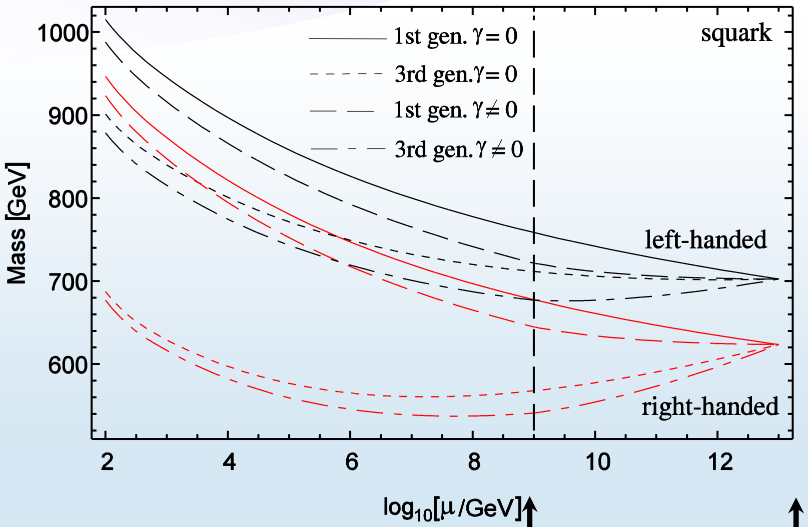
MSSM
+ hidden flow



Hidden sector effects decrease soft masses.

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messenger scale

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  - Model 2 - SU(2) field coupled: $X = u/\tilde{M}$

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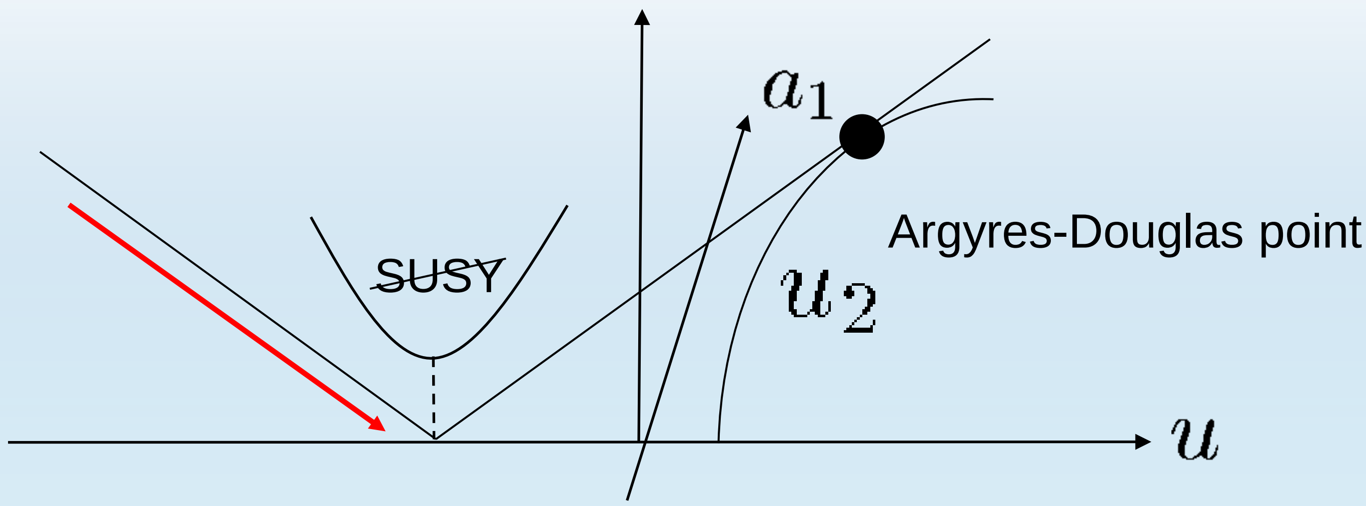
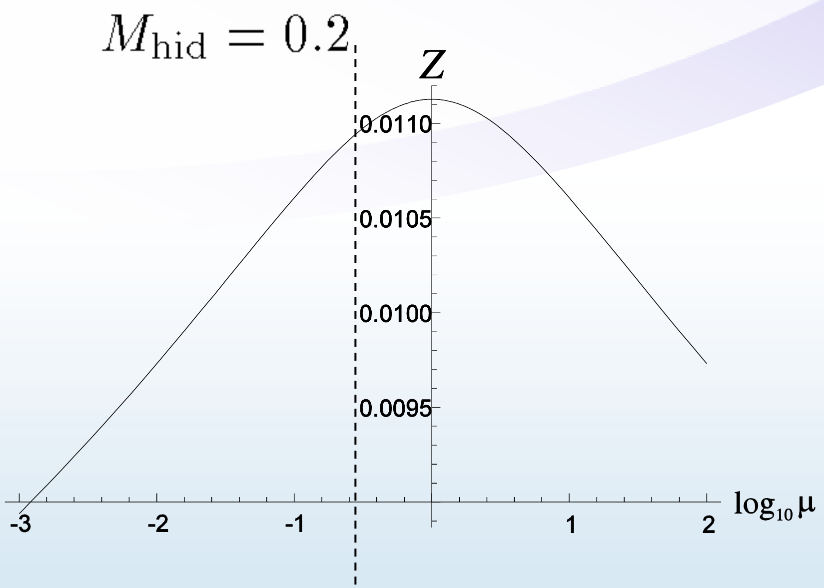
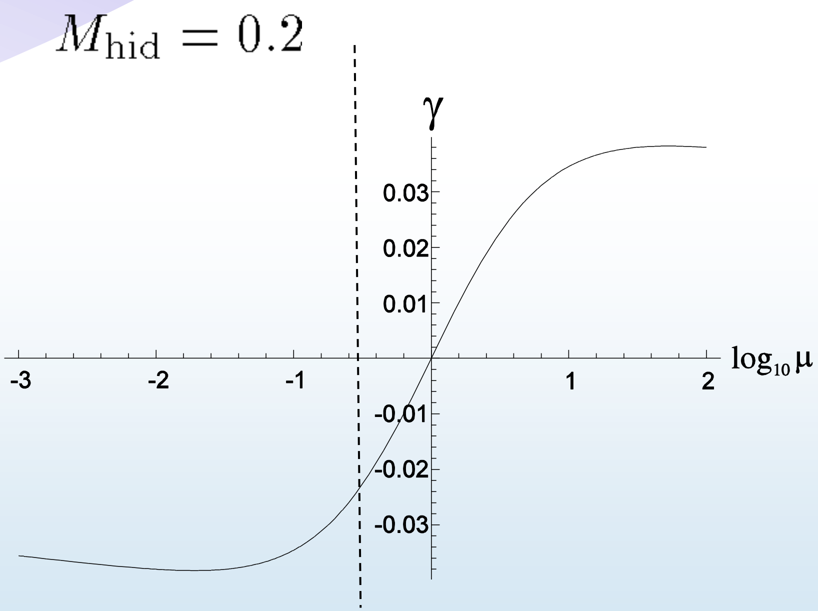


Wave function renormalization

- Z_{A_2} & γ_{A_2} along u_1





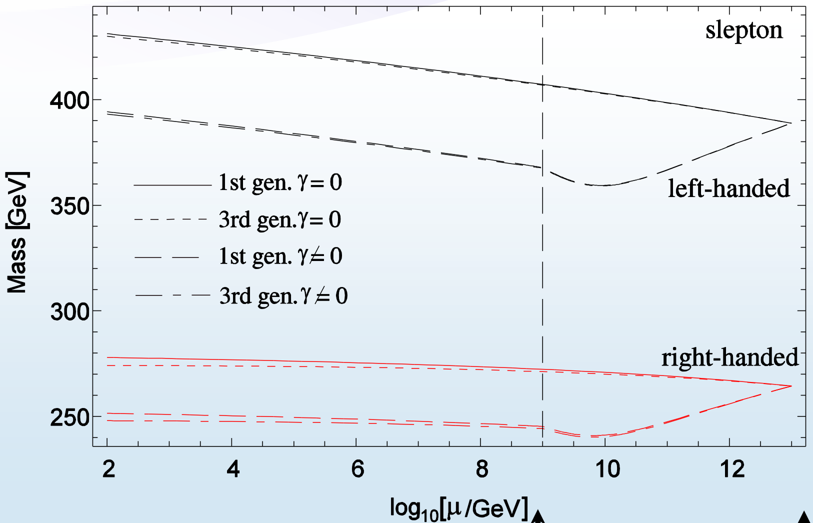
$$\mu \frac{dm_i^2}{d\mu} = (\text{MSSM RGE part}) + \gamma m_i^2 \qquad \gamma = -\mu \frac{d}{d\mu} \ln Z_X$$

Mass RG along u2- model 2

- Mass RG flow of sleptons $(\mu \to (10^9/M_{\rm hid})\mu,\ M_{\rm hid} = 0.2)$

MSSM flow

MSSM
+ hidden flow



Hidden sector effects decrease soft masses.

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messenger scale $M = 10^{13}$ GeV

$\tan\beta = 10$

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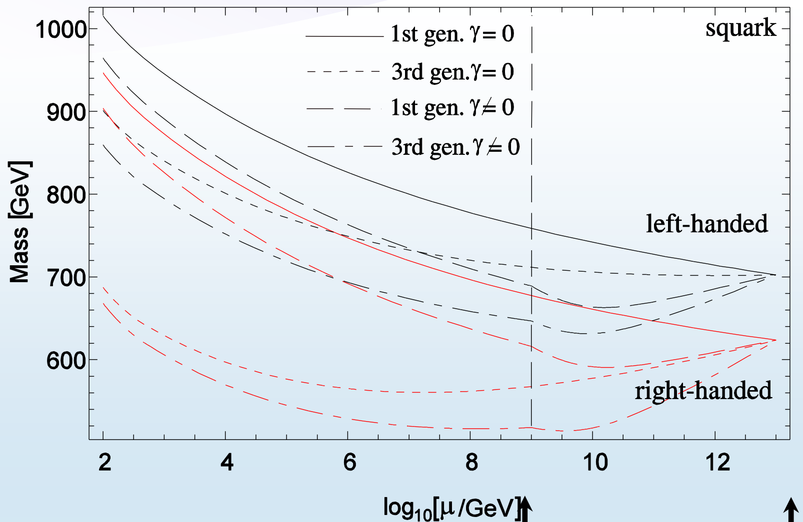
MSSM flow | MSSM + hidden flow



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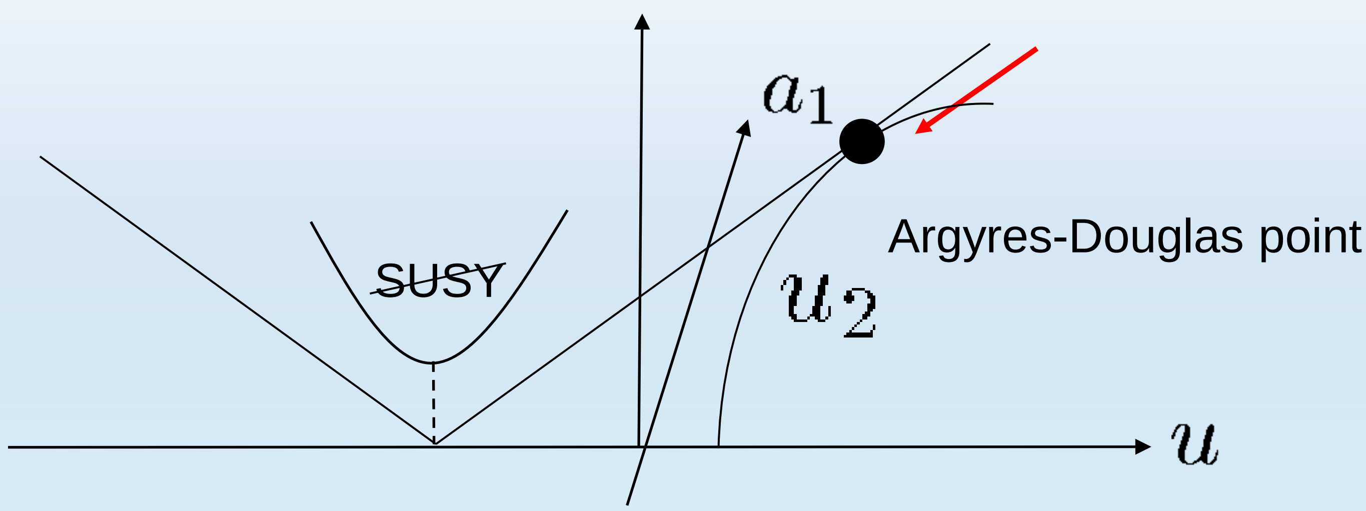
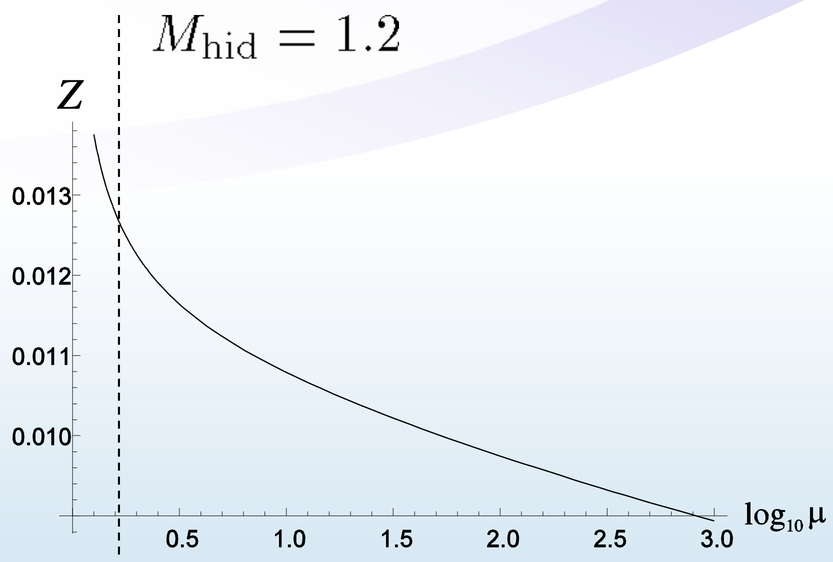
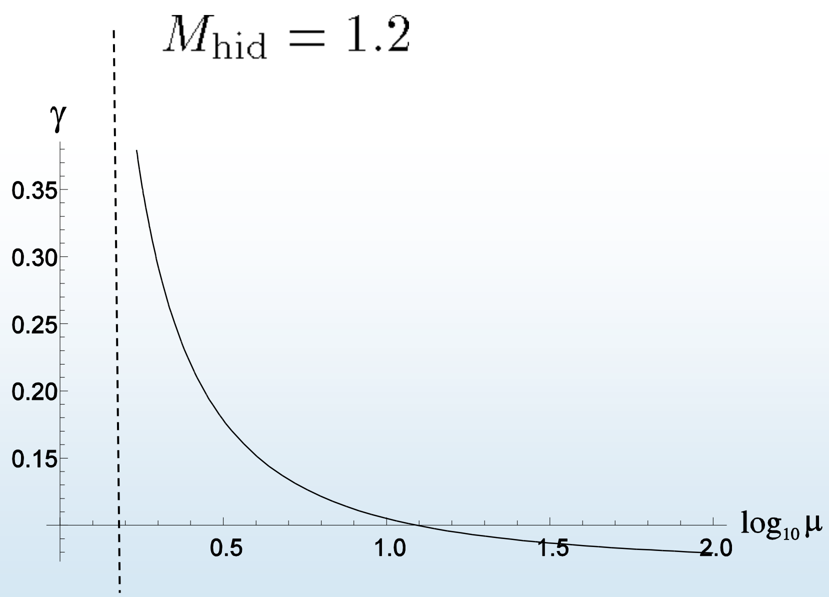


Wave function renormalization

- Z_{A_2} & γ_{A_2} along u_2





AD point

AD point

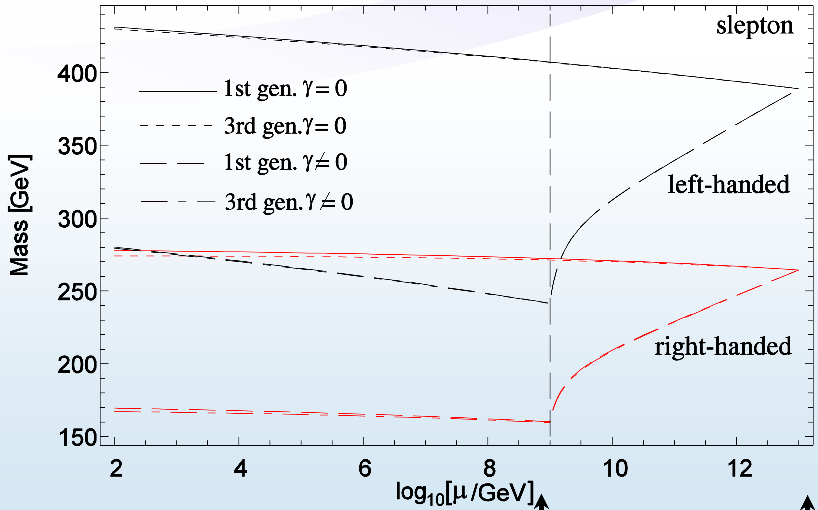
$$\mu \frac{dm_i^2}{d\mu} = (\text{MSSM RGE part}) + \gamma m_i^2 \qquad \gamma = -\mu \frac{d}{d\mu} \ln Z_X$$

Mass RG along u2 – model 2

- Mass RG flow of sleptons $(\mu \to (10^9/M_{\rm hid})\mu,\ M_{\rm hid} = 1.2)$

MSSM flow

MSSM
+ hidden flow



$\tan\beta = 24$

$m_{\tilde{\tau}} = 132.9\ ({\rm GeV})$

$M_1 = 134.9\ ({\rm GeV})$

Stau can be next lightest particle.

Hidden scale $M_{\rm hid} = 10^9\ {\rm GeV}$

messenger scale $M = 10^{13}\ {\rm GeV}$

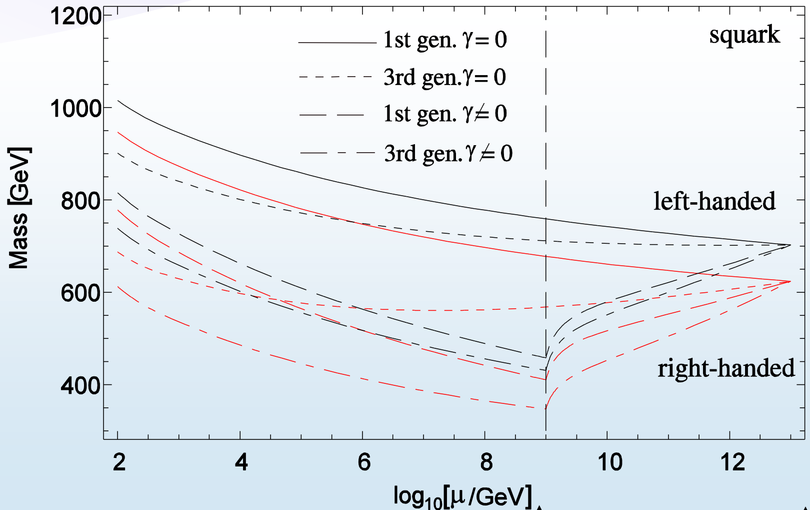
$\tan\beta = 10$

Mass RG along u2 – model 2

❖ Mass RG flow of squarks $(\mu \to (10^9/M_{\rm hid})\mu,\ \ M_{\rm hid} = 1.2)$

MSSM flow

MSSM
+ hidden flow



Hidden scale
$M_{\rm hid} = 10^9\ {\rm GeV}$

messenger scale
$M = 10^{13}\ {\rm GeV}$

$\tan\beta = 10$

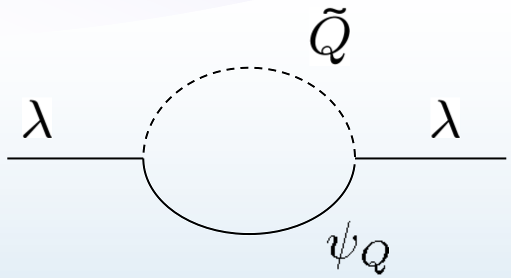
Conclusion

1. We investigated hidden sector contributions to the mass RG flow.
2. We analyzed the strong coupled hidden sector in GMSB.
3. Hidden sector effects make soft masses to decrease.
4. Near Argyres-Douglas point soft masses decrease drastically.

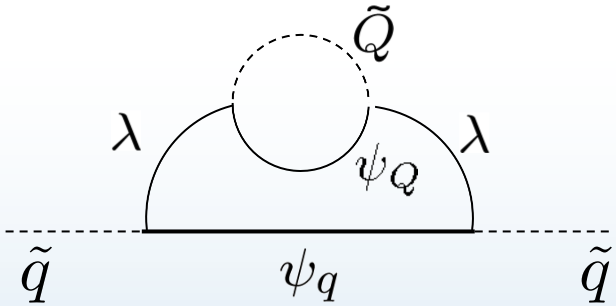
Masses of superpartners

- Gaugino & sfermion masses

$$W_{\rm mes} = XQ\tilde{Q} \qquad M_{\rm mes} = \langle X \rangle \neq 0 \qquad \langle F_X \rangle \neq 0$$



Gaugino mass



Sfermion mass

$$M_a = \frac{\alpha_a}{4\pi}\frac{F_X}{M_{\rm mes}} \qquad m_i^2 = 2\left(\frac{F_X}{M_{\rm mes}}\right)^2 \sum_{a=1}^{3} C_2^a(R_i)\left(\frac{\alpha_a}{4\pi}\right)^2$$

$$(T^a T^a)_i{}^j = C_a(i)\delta_i^j$$

Quantum theory

- Low energy effective action – $\mathcal{L}_{\rm VM}$
  - Effective action respects $U(1)_c \times U(1)$
  - Landau pole $\Lambda_{\rm Landau}$ introduced because of U(1)
    - $a_1 < \Lambda_{\rm Landau}$

$$\mathcal{L}_{\rm VM} = \frac{1}{4\pi}{\rm Im}\sum_{i,j=1}^{2}\left[\int d^4\theta\ \frac{\partial\mathcal{F}}{\partial A_i}A_i^\dagger + \frac{1}{2}\int d^2\theta\ \tau_{ij}W_iW_j\right]$$

Prepotential: $\mathcal{F} = \mathcal{F}(A_1, A_2, \Lambda, \Lambda_{\rm Landau})$

  - Assuming U(1) dynamics does not affect SU(2)

$$\mathcal{F} = \mathcal{F}^{\rm SQCD}_{SU(2)}(A_2, m, \Lambda)\Big|_{m=\sqrt{2}A_1} + cA_1^2$$

N=2 SU(2) SQCD effective action
- written by elliptic curve Seiberg, Witten (1994)

c includes the info of Landau pole.

Quantum theory

- Low energy effective action – $\mathcal{L}_{\rm HM}$
  - Quark, monopole and dyon become light in the vicinity of singular points on (u, a_1)

$$\begin{aligned}\mathcal{L}_{\rm HM} &= \int d^4\theta \Big[M_r^\dagger e^{2n_m V_{2D} + 2n_e V_2 + 2n V_1} M^r \\ &\qquad + \tilde{M}_r e^{-2n_m V_{2D} - 2n_e V_2 - 2n V_1} \tilde{M}^{r\dagger} \Big] \\ &\quad + \sqrt{2} \int d^2\theta \left[\tilde{M}_r (n_m A_{2D} + n_e A_2 + n A_1) M^r + h.c. \right]\end{aligned}$$

$M, \ \tilde{M}$: Quark, monopole, dyon

$(n_m, n_e)_n$: Magnetic, electric charges, U(1) charge.

Ex. Quark: $(n_m, n_e)_n = (0,1)_1$

$$A_{iD} = \frac{\partial \mathcal{F}}{\partial A_i}$$