Holographic QCD Integrated back to Hidden Local Symmetry

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Based on

Phys.Rev.D82, 076010 (2010) [arXiv:1007.4715 [hep-ph]]

In collaboration with

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K. Yamawaki (Kobayahi Maskawa Institute)

Seminar @ Maskawa Institute for Science and Culture

05/21/11

Plan of talk:

1. Introduction

2. A gauge invariant way to integrate out HQCD

3. Application to Sakai-Sugimoto model

4. Summary

1. Introduction

1. Introduction

- Holography (based on AdS/CFT, gauge-gravity duality)

  → utilized to reveal some features in strong coupling gauge theory

- Holographic QCD (HQCD): application to QCD

  → check validity of the duality

So far some models of HQCD:

PTP113(2005);114(2006)

-- achieve realistic chiral symmetry breaking (e.g. Sakai-Sugimoto model)

-- show consistency with Exp. (within several % errors ~30%)

- Types of HQCD :

-- “top-down” approach: starting with 10d-stringy setting
--- > 5d-gauge theory (w/ induced background)

-- “bottom-up” approach: 5d-gauge theory (on AdS background)

One eventually employs a 5d-gauge theory

w/ characteristic metric
boundary conditions

- Holographic recipe:

Large Nc limit

HQCD

dual QCD

4D

$$S_{5D}[\phi] \longrightarrow S_{\mathrm{boundary}}[\phi_c] \equiv W_{4\mathrm{D}}[J]$$

ϕ_c: classical sol. $\leftrightarrow$ source of J

Green functions in QCD :
straightforwardly calculated as $\langle T J(x) J(0) \rangle = \frac{\delta^2 S_{\mathrm{boundary}}}{\delta\phi_c(x)\delta\phi_c(0)}$

- Equivalent approach (we will follow):

KK decomposition

$$S_{5D}[\phi] = S_{4D}[\phi^{(n)}(x)] \qquad \phi(x.z) = \sum_n f_n(z)\phi^{(n)}(x)$$

e.g. Sakai-Sugimoto model

PTP113(2005);114(2006)

4D effective "hadron" model dual to QCD

w/ infinite tower of vector&axial-vector mesons

Identified as $\phi^{(n)} = \rho, \omega, a_1, \rho', \rho'', \cdots$

4D effective "hadron" model dual to QCD

- Nambu-Goldstone (NG) bosons (Wilson line) $U = e^{2i\pi/f_\pi}$
- ∞ vector/axial-vector mesons (KK modes)
- $\equiv$ ∞ Hidden Local Symmetry (HLS) gauge bosons

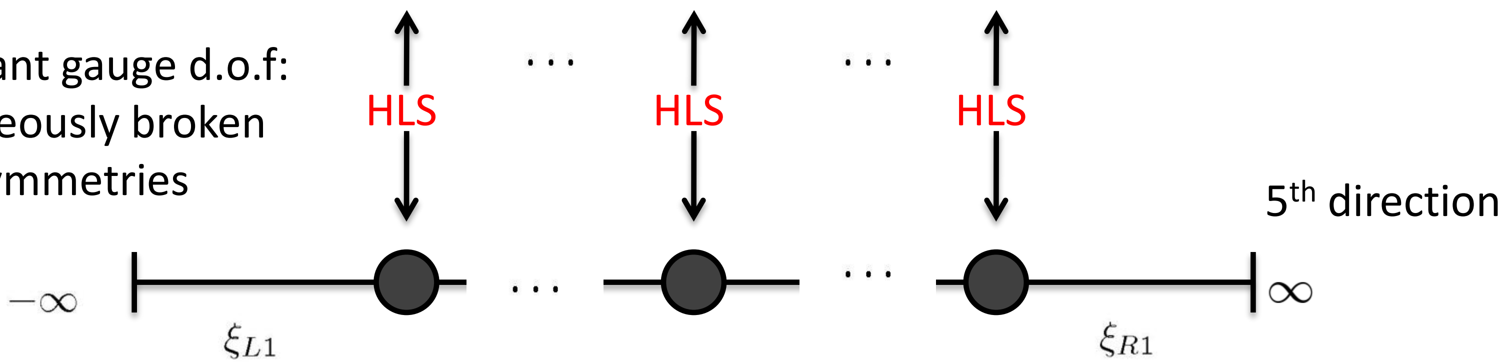
M.Bando, et al, PRL54(1985);
M.Bando, et al, NPB259(1985);
M.Bando, et al, PTP79(1988);
M.Bando, et al, PR164(1988)

HLS & "deconstruction"

C.Hill, et al, PRD64 (2001)
N.Arkani-Hamed et al, PRL86 (2001)

$$U = \xi_{L1} \cdot \xi_{L2} \cdots \xi_{L\infty} \cdot \xi_{R\infty} \cdots \xi_{R2} \cdot \xi_{R1}$$

Redundant gauge d.o.f: spontaneously broken gauge symmetries



"HLS-KK modes"

KK modes as gauge bosons of HLSs
→not necessarily mass-eigenstates!
→ can mix with photon in a gauge-invarinant manner

- Holographic QCD: typical cutoff

e.g. Sakai-Sugimoto model

M_{KK}

PTP113(2005);114(2006)

$$\Lambda \sim 1 \text{ GeV}$$

Higher KK resonances **should not contribute** in the low-energy physics.

- Needs **"integrating out"** higher KK modes $a_1(1260\,\text{MeV}), \rho'(1450\,\text{MeV}), \cdots$
  In a gauge-invariant way:

Before proceeding ....

Let’s briefly review the HLS formalism!

M.Bando et al, PRL54 (1985);
M.Bando et al, Phys.Rept. 164, (1988);

- Hidden Local Symmetry (HLS) formalism

$$U(N_f)_L \times U(N_f)_R \xrightarrow{\text{extended}} U(N_f)_L \times U(N_f)_R \times [U(N_f)_H]_{\text{local}}$$

HLS Lagrangian with WZW term and HLS-invariant (intrinsic parity-odd (IP-odd))terms

---

- chiral field $U \to g_L \cdot U \cdot g_R^\dagger$

$$U = \xi_L^\dagger \cdot \xi_R$$

gauge-redundancy = “Hidden Local Symmetry” (HLS)

$$\xi_{L,R} \to h(x) \cdot \xi_{L,R} \cdot g_{L,R}^\dagger$$

- V: introduced as HLS dynamical gauge fields $V_\mu \to h(x) V_\mu h^\dagger(x) + i h(x) \partial_\mu h^\dagger(x)$

V masses m_V associated with SSB of HLS (“Higgs mechanism”)

$$U(N_f)_L \times U(N_f)_R \times [U(N_f)_H]_{\text{local}} \to U(N_f)_V$$

$$m_V = g f_\sigma$$

g: HLS gauge coupling

$$\xi_{L,R} = \exp i[\tfrac{\sigma}{f_\sigma} \mp \tfrac{\pi}{f_\pi}]$$

$\sigma \cdots$ NGBs eaten by HLS gauge bosons

$\pi \cdots$ exact (massless) (p)NGBs

- HLS Lagrangian = chiral & HLS-invariant in terms of non-linear realization

$$[U(N_f)_L \times U(N_f)_R]_{\text{gauged}} \times [U(N_f)_H]_{\text{local}} \to U(N_f)_V$$

$$\mathcal{L}_{\text{HLS}} = \mathcal{L}(\rho, \omega, \pi, \cdots)$$ w/ external photon, W, Z

→ ρ-γ, ω-γ mixings automatically introduced by HLS

→ Thanks to the manifest gauge symmetry (HLS),
HLS theory: renormalizable in derivative expansion (order by order)

M.Harada et al, Phys.Rept. 381 (2003)

- Fixing HLS-gauge ($\sigma = 0$)
& integrating out V $= (\rho, \omega, \cdots)$ ········→ Chiral Lagrangian

"gauge-equivalent"

$$E < m_\rho \, : \quad \mathcal{L}_{\text{HLS}}(\rho, \pi, \cdots, A_L, A_R) \to \mathcal{L}_{\text{ChL}}(\pi, A_L, A_R)$$

- Anomalous sector = WZW terms + chiral & HLS-invariant but *intrinsic parity-odd (IP-odd)* terms

$$\Gamma^{\text{anomaly}}_{\text{HLS}}[\rho,\cdots,U,A_L,A_R] = \Gamma_{\text{WZW}}[U,A_L,A_R] + \Delta\Gamma_{\text{HLS}}[\rho,\cdots,U,A_L,A_R]$$

s.t.

$$\begin{cases} \delta\Gamma^{\text{anomaly}}_{\text{HLS}} = \delta\Gamma_{\text{WZW}} = \delta\Gamma_{\text{QCD}} \\ \delta(\Delta\Gamma_{\text{HLS}}) = 0 \end{cases}$$

T.Fujiwara et al, Prog.Theor.Phys.73, (1985);
M.Harada et al, Phys.Rept. 381 (2003)

$$\Delta\Gamma_{\text{HLS}} = \frac{N_c}{24\pi^2}\int d^4x \sum_{i=1}^4 c_i\,\mathcal{L}_i(\rho,\omega,\cdots,U,A_R,A_L)$$

4 gauge-(C&P) invariant terms
w/ coefficients c1, c2, c3, c4 ← determined by Exp.

Note: LET(low-energy theorem) → satisfied automatically

In low-energy limit: $E \ll m_\rho$

$$\Gamma^{\text{anomaly}}_{\text{HLS}} \to \Gamma_{\text{WZW}}$$

by construction of HLS!

Now, come back to HQCD
and again look at
from the HLS view

4D effective "hadron" model dual to QCD

Nambu-Goldstone (NG) bosons (Wilson line)

$$U = e^{2i\pi/f_\pi}$$

∞ vector/axial-vector mesons (KK modes)

$\equiv$ ∞ Hidden Local Symmetry (HLS) gauge bosons

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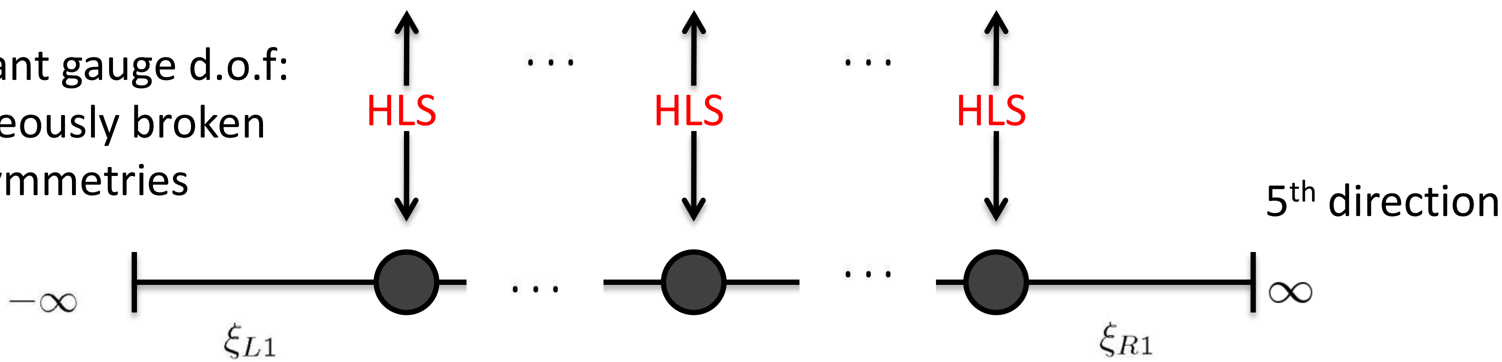

"HLS-KK modes"

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→not necessarily mass-eigenstates!
→ can mix with photon in a gauge-invarinant manner

2. A gauge invariant way to integrate out HQCD

M.Harada, S.M. and K.Yamawaki,
PRD82(2010)

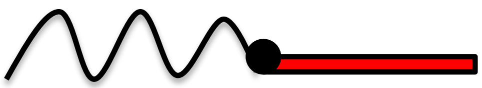
2. A gauge invariant way to integrate out HQCD

- ***CONCEPTUALLY QUITE SIMPLE!***

M.Harada, S.M. and K.Yamawaki, PRD82(2010)

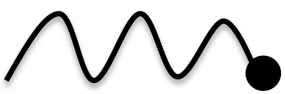
DO NOT TRUNCATE NAIVELY!

A naive truncation



SM current (SU(2), U(1))

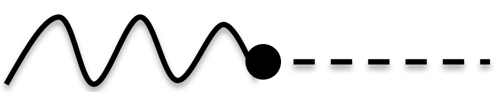
HLS-KK modes $V_\mu^{(n)}$



Breaks chiral symm. & HLS!!

REPLACE IT WITH "PION CLOUD"!

Keeps chiral symm. & HLS!!



$V_\mu^{(n)}, \ \alpha_{\mu||}$

transform like "gauge field" (inhomogeneously) under HLS

$$\alpha_{\mu||} = \frac{i}{f_\pi^2}\pi \overleftrightarrow{\partial}_\mu \pi + \cdots$$

- ***Via E.O.M. at Lagrangian level***

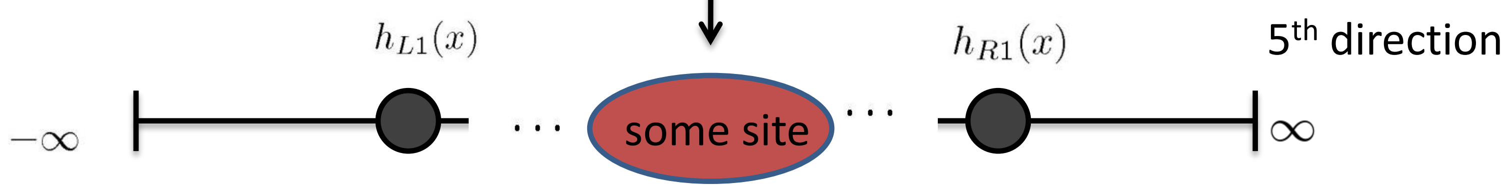
For details, see *M.Harada, S.M. and K.Yamawaki, PRD82(2010)*

Setup of HLSs

$$\overbrace{U = \xi_{L1}}^{h_{L1}(x)} \cdot \xi_{L2} \cdots \text{(some point)} \cdots \xi_{R2} \cdot \overbrace{\xi_{R1}}^{h_{R1}(x)}$$

$$\underbrace{\xi_{L1} \cdot \xi_{L2} \cdots}_{\xi_L} \quad \underbrace{\cdots \xi_{R2} \cdot \xi_{R1}}_{\xi_R}$$

Explicitize only single HLS: ***h(x)***



∞ ***HLS-KK fields associated with each "site"*** $V_{L1}, V_{R1}, \cdots$

transformed into → ∞ ***"dressed" HLS gauge fields transforming with respect to only single h(x)***

$$V_\mu^{(n)}(x) \to h(x) \cdot V_\mu^{(n)}(x) \cdot h^\dagger(x) + i h(x) \cdot \partial_\mu h^\dagger(x)$$

external vector gauge field

The gauge-inv. mass terms must arise as a mixing form:

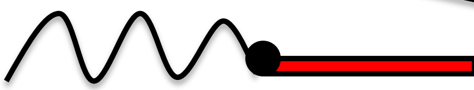
$$\mathcal{L}_{4D} = \sum_n f_{Vn}^2 \mathrm{tr}[\alpha_{\mu||} - V_\mu^{(n)}]^2$$

$$\alpha_{\mu||} = \mathcal{V}_\mu - \frac{i}{2f_\pi^2}[\pi, \partial_\mu \pi] + \cdots$$

which transforms in the same way as HLS-KK

$$V_\mu^{(n)}(x) \to h(x) \cdot V_\mu^{(n)}(x) \cdot h^\dagger(x) + i h(x) \cdot \partial_\mu h^\dagger(x)$$

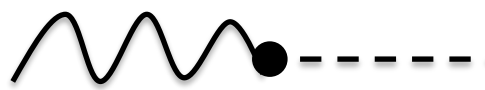
SM current (SU(2), U(1))



HLS-KK modes $V_\mu^{(n)}$

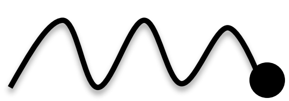
E.O.M for HLS-KK
(ignoring the kinetic term consistently with derivative expansion)

$$(V_\mu^{(n)} - \alpha_{\mu||}) = 0$$



A naive truncation $V_\mu^{(n)}(x) = 0$

$\longrightarrow \mathcal{L}_{4D} = \sum_n f_{Vn}^2 \mathrm{tr}[\alpha_{\mu||} \qquad]^2$ **obviously breaks HLS, accordingly chiral symmetry!**



$$S_5^{\rm HQCD} \sim$$

-- NG bosons $(\pi, K, \cdots)$

-- vector mesons/HLS bosons (ρ, ω)

-- external SM gauge bosons (γ, W, Z)

Integrate out

higher KK $(a_1, \rho', \rho'', \cdots)$

$E < M_{a_1}$ (1 GeV)

$$S_4 = \int d^4x \mathcal{L}_{\rm HLS}^{\mathcal{O}(p^2,p^4)}[\pi, \rho, \omega, \cdots]$$

HLS-ChPT (chiral perturbation theory)

$$\mathcal{L}_{\rm HLS}^{\mathcal{O}(p^2)} = f_\pi^2 {\rm tr}[\alpha_{\perp\mu}^2] + f_\sigma^2 {\rm tr}[\hat{\alpha}_{||\mu}^2] - \frac{1}{2g^2}{\rm tr}[V_{\mu\nu}^2]$$

In Large Nc lmit

--- consistent w/ hoplographic setup

$$\mathcal{L}_{\rm HLS}^{\mathcal{O}(p^4)} = \mathcal{O}(p^4) \text{ terms}$$ *M.Harada, et al, PR381(2003)*

w/ chiral-HLS coefficients determined by the holographic model

where effects from higher KK modes are **fully** included

- **meson loop corrections** $\equiv$ **1/Nc corrections**

Calculable!!!

For SS model, see
M.Harada, S.M. and K.Yamawaki, PRD74(2006)

3. Application to Sakai-Sugimoto model

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M.Harada, S.M. and K.Yamawaki, PRD82(2010)

SS model (Dirac-Born-Infeld (DBI) & Chern-Simons (CS) parts)

PTP113(2005);114(2006)

integrated out Higher KK →

$$S_4 = \int d^4x \mathcal{L}_{\text{HLS}}^{\mathcal{O}(p^2,p^4)}[\pi, \rho, \omega, \cdots]$$

$$S_{CS} = \frac{N_c}{16\pi^2} \int_{M^4} \sum_i c_i \mathcal{L}_i^{\text{HLS:IP-odd}}$$

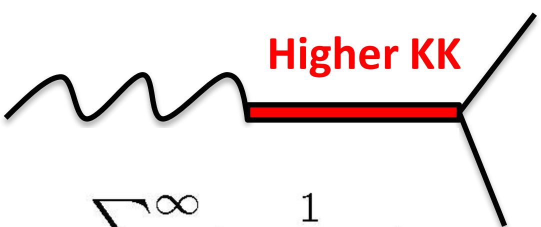
w/ chiral-HLS coefficients determined fully including higher KK-effects.

Physical quantities are written in terms of the generic HLS model

-- instead of dealing with the incalculable infinite sums

-- calculable definitely to be compared with Exp.

In particluar: ***momentum dependence of form factors*** **(not calculated before)**

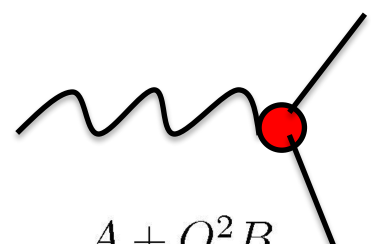


$$\sum_n^{\infty} \frac{1}{Q^2+M_n^2}$$

SS model: VMD form

PTP113(2005);114(2006)

integrated out →

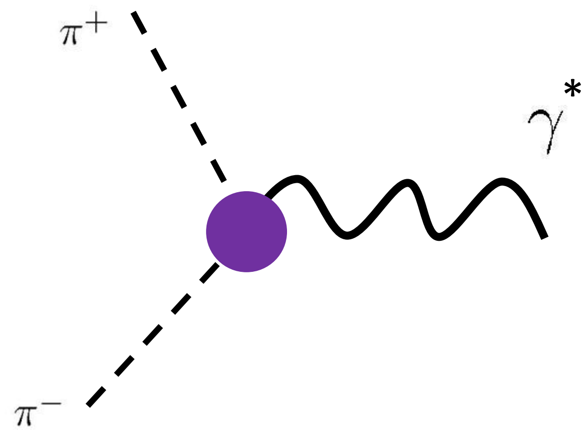


$$A + Q^2 B$$

Pion Electromagnetic form factor (from DBI part)

generic HLS model

$$F_V^{\pi^\pm}(Q^2) = \left(1 - \frac{1}{2}\tilde{a}\right) + \tilde{z}\frac{Q^2}{m_\rho^2} + \frac{\tilde{a}}{2}\frac{m_\rho^2}{m_\rho^2 + Q^2}$$



SS model gives

$$\tilde{a}_{SS} \simeq 2.62$$
$$\tilde{z}_{SS} \simeq 0.08$$

Space-like momentum region < 1 GeV

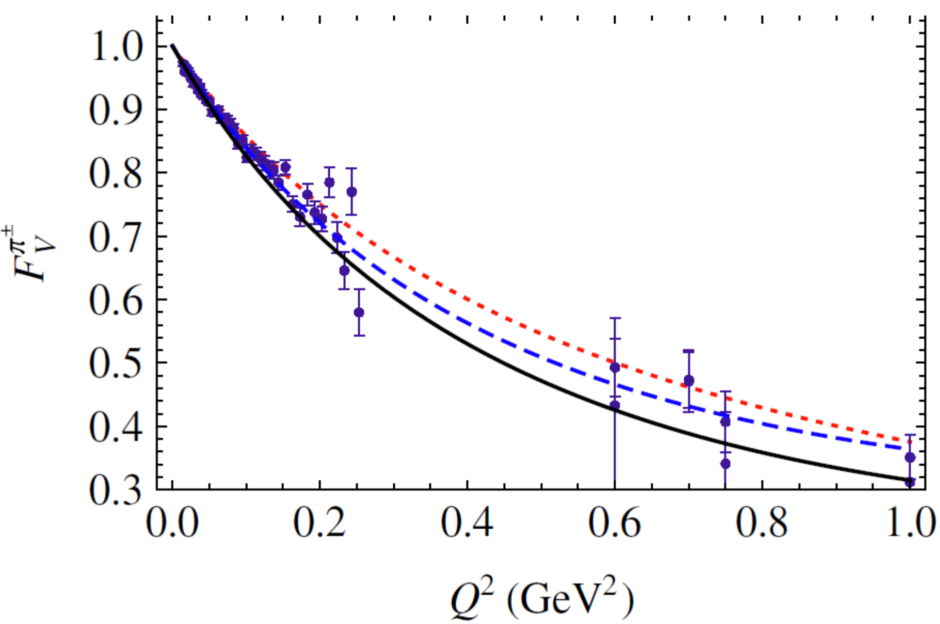


FIG. 2 (color online). The prediction (black solid curve) of the pion EM form factor $F_V^{\pi^\pm}$ fitting the experimental data [24–27] with $\chi^2/\text{d.o.f} = 147/53 = 2.8$. The red dotted curve corresponds to the form factor in the ρ-meson dominance hypothesis with $\tilde{a} = 2$ and $\tilde{z} = 0$ ($\chi^2/\text{d.o.f} = 226/53 = 4.3$). The blue dashed curve is the best fit to experimental data with $\tilde{a}|_{\text{best}} = 2.44$, $\tilde{z}|_{\text{best}} = 0.08$ ($\chi^2/\text{d.o.f} = 81/51 = 1.6$).

SS

HLS-best fit

rho meson dominance

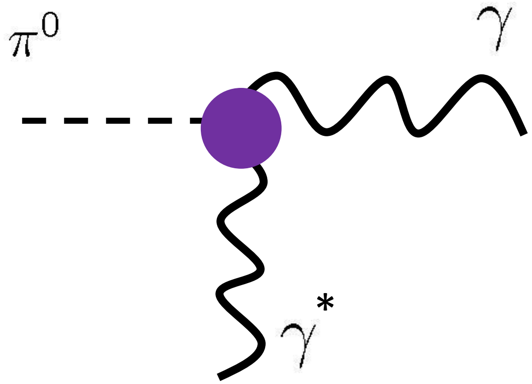
good agreement

Pi0- photon transition form factor (from CS part)

generic HLS model

$$F_{\pi^0\gamma}(Q^2) = (1-\tilde{c}) + \frac{\tilde{c}}{2}[D_\rho(Q^2) + D_\omega(Q^2)],$$

w/ $D_{\rho,\omega}(Q^2) = \frac{m^2_{\rho,\omega}}{m^2_{\rho,\omega}+Q^2}$



SS model gives

$$\tilde{c}_{\,\mathrm{SS}} \simeq 1.31$$

Space-like momentum region < 1 GeV

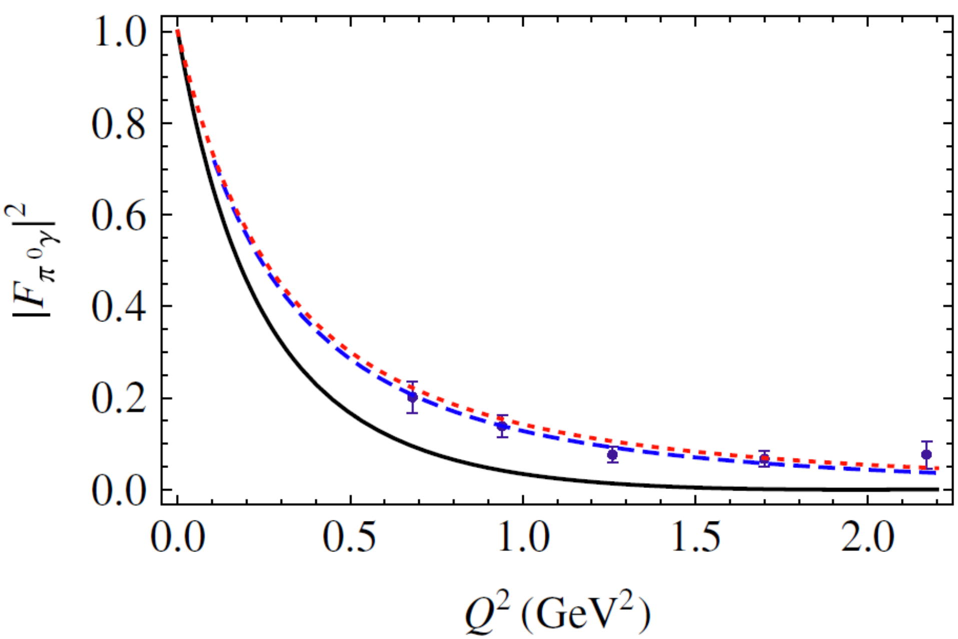


FIG. 3 (color online). The prediction (black solid curve) of the π^0-γ transition form factor $F_{\pi^0\gamma}$ with respect to the spacelike momentum squared Q^2. Comparison with the experimental data [29] yields $\chi^2/\mathrm{d.o.f} = 63/5 = 13$. The blue dashed and red dotted curves, respectively, correspond to the best fit curve with $\tilde{c}_{\mathrm{best}} = 1.03$ ($\chi^2/\mathrm{d.o.f} = 3.0/4 = 0.7$) and the ρ/ω-meson dominance with $\tilde{c} = 1$ ($\chi^2/\mathrm{d.o.f} = 4.8/5 = 1.0$).

SS

HLS-best fit

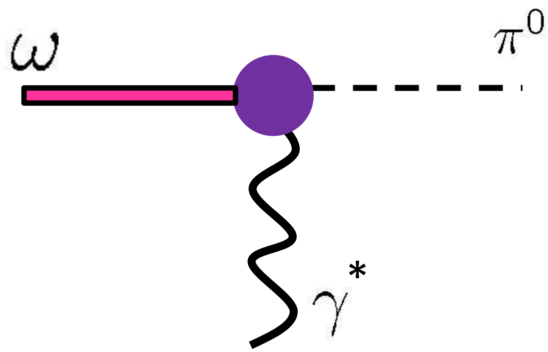
rho/omega meson dominance

disagreement

Pi0 - omega transition form factor

generic HLS model

$$F_{\omega\pi^0}(q^2) = (1 - \tilde{r}) + \tilde{r} D_\rho(q^2)$$



SS model gives

w/ $D_{\rho,\omega}(Q^2) = \frac{m^2_{\rho,\omega}}{m^2_{\rho,\omega} + Q^2}$

$$\tilde{r}_{SS} \simeq 1.53,$$

Tiime-like momentum region<1 GeV

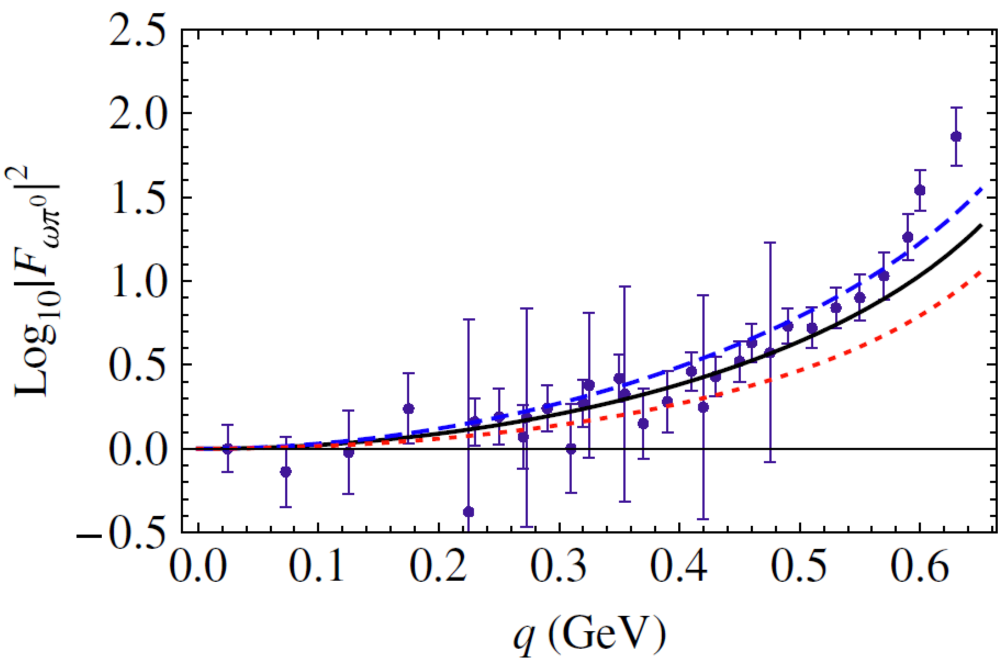
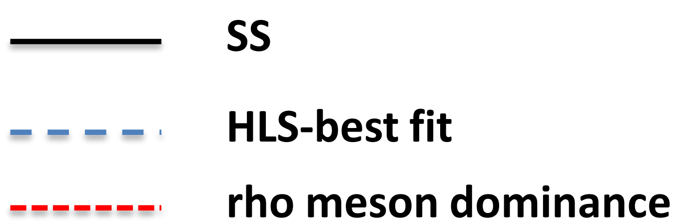


FIG. 4 (color online). The prediction (black solid curve) of the ω-π^0 transition form factor $F_{\omega\pi^0}(q^2)$ with respect to the timelike momentum q. Comparison with the experimental data [30,31] yields $\chi^2/\text{d.o.f} = 45/31 = 1.5$. The best fit curve with $\tilde{r}_{\text{best}} = 2.08$ ($\chi^2/\text{d.o.f} = 24/30 = 0.8$) and the curve corresponding to the ρ-meson dominance with $\tilde{r} = 1$ ($\chi^2/\text{d.o.f} = 124/31 = 4.0$) are drawn by blue dashed and red dotted lines, respectively.



consistent

4. Summary

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- We discussed the gauge-invariant method to integrate out higher KK modes of the HLS gauge bosons identified as vector/axial-vector mesons in HQCD.

- The method is conceptually quite simple:

  Keeps the HLS as well as chiral symmetry.

  Higher HLS-KK modes → "Pion cloud"
  via E.O.M. (integrating out)

- The SS model was taken as an example to show the power of the method.
  Momentum dependence of form factors were definitley calculated for the first time
  --- not calculated before due to complication of handling ∞ sums:

  The SS model →reduced to the generic HLS model (HLS-ChPT)
  w/ chiral-HLS coefficients determined
  including higher KK-effects fully.

  → assessed compared w/ the best-fit of the HLS form.

- The method is applicable to other models of holograhy:
  e.g. models including baryons (*Harada et al, arXiv:1102.5489*),
  holographic walking technicolot, and Higgsless/extradimensional models.