非局所場理論・・・の本音

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§1. Introduction

Why non-local fields?

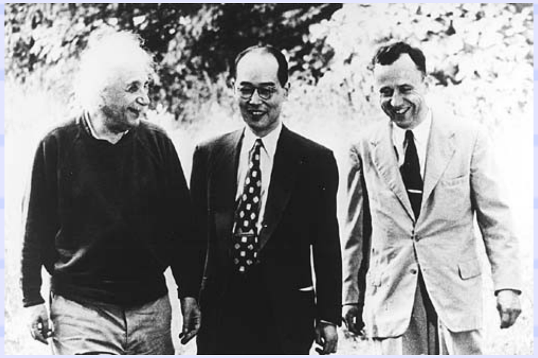
Early Papers:

▷ M. A. Markov (1940), ▷ P. A. M. Dirac (1942)

▷ H.S. Snyder (1947), ▷ C.N. Yang (1947)

Possible Types of Nonlocalizable Fields (Yukawa,1948)

"Firstly, characteristic properties of elementary particles are to be described from their internal structure. It may be too much to hope that all properties of all kinds of particles are to be ascribed to their internal structure."



"Secondly, the theory which we endeavor to attain is to be such that the interaction between particles so not gives rise to any kind of divergence difficulty, which is inherent in the usual local field theory with local interaction. "

(Y. Katayama and H. Yukawa, Field Theory of Elementary Domain and particles I, 1968)

Influence of Yukawa's attempt

- Who's paper ?

Having in mind relativistic invariance, we shall write instead:

$$q_\mu|x\rangle = x_\mu|x\rangle\,, \tag{5}$$

$$\langle x'|x\rangle = \delta^{(4)}(x-x')\,, \tag{6}$$

$$\langle x|U|x'\rangle = \delta^{(4)}(x-x')\phi(x)\,, \tag{7}$$

$$\phi(x) = \frac{\langle x|U|x'\rangle}{\langle x|x'\rangle}\,. \tag{8}$$

Yukawa identifies thus $\phi(x)$ as the local c-number field which then undergoes the usual second quantization procedure. $\phi(x)$ satisfies a wave equation, e.g. a Klein–Gordon equation

$$\left(\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x^\mu} - m^2\right)\phi(x) = 0\,. \tag{9}$$

This follows from the equation of motion at the U-operator level

$$[p_\mu, [p^\mu, U]] = m^2c^2U\,. \tag{10}$$

Equations (2) and (10) hence characterize, in Yukawa's scheme, a local field theory of a spinless particle of mass m and zero size.

Non-local field theories are then introduced by Yukawa through a modification of (2) to read

$$[q, U] \neq 0\,. \tag{11}$$

As a consequence of (11) we can no longer extract a $\delta^{(4)}(x-x')$ from (2) and we shall have

$$\langle x'|U|x\rangle = U(x', x)\,. \tag{12}$$

At this point, in order to restrict the possible choices of NLFT, Yukawa took inspiration from Born reciprocity principle and specified (11) to read:

$$[q_\mu, [q^\mu, U]\,] = \lambda^2 U\,, \tag{19}$$

where λ has obviously dimensions of a length. Notice the close similarity with (10). Notice that, as a consequence of (10) and (19),

$$[q, [q, U]\,] = \frac{\lambda^2}{m^2c^2}[p, [p, U]] = \frac{(\bar\lambda)^4}{\hbar^2}[p, [p, U]]\,, \tag{20}$$

where $\bar\lambda = (\lambda^2\hbar^2 m^{-2}c^{-2})^{1/4}$ has also dimensions of a length. Hence, $[q, [q, U]]$ and $[p, [p, U]]$ are proportional with an assigned constant of proportionality. An immediate consequence of (19) is

$$(r^2 - \lambda^2)U(x, r) = 0 \Rightarrow U(x, r) = \delta(r^2 - \lambda^2)\,\phi(x, r)\,, \tag{21}$$

Non-local Field Theory Suggested by Dual Models, (1973)

Zero Slope Limit

$\Downarrow$

Smeared Fields

$$\frac{\Delta r}{\Delta p} \approx \lambda^2$$

- Study in Japan of Theory of Elementary Particle Extended in Space-Time
  - Katayama, Umemura, Tanaka, Sogami $\cdots$
  - Takabayashi, Ohnuki, Morita $\cdots$
  - Hara, Goto, Ishida, Naka, Mamiki, $\cdots$

Symposiums in Nihon Univ.

Karuizawa, Japan March,1992

Extended Objects and Bound Systems

(Atomic Energy Re-search Institute)

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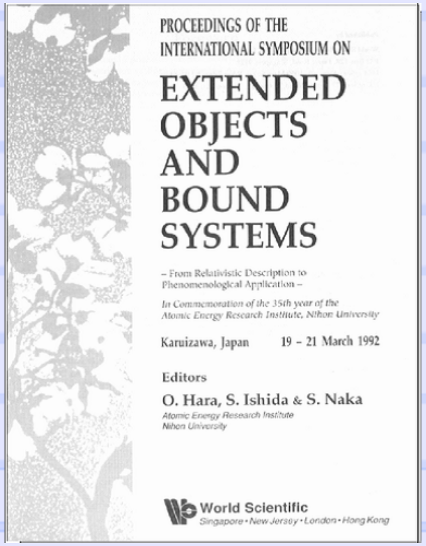
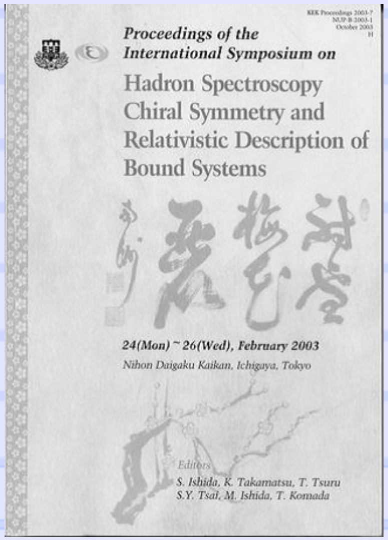
Tokyo, Japan February, 2003

Hadron Spectroscopy, Chiral Symmetry and Relativistic Description of Bound Systems

(Nihon-U & KEK)





§2 Yukawa's Bi-Local Field Theory

- In 1947, Yukawa suggested that if the material particles have a space-like extension, then force fields $\{A_\mu\}$ are not necessarily functions of x^μ alone, but may depend on p^μ also.

$\cdots$ $x_\mu A_\nu \neq A_\nu x_\mu$ ($x_\mu x_\nu \neq x_\nu x_\mu$ noncommutativity ?).

- In 1948, Yukawa introduced bi-local fields:

$$\left.\begin{array}{c} [p^\mu,[p_\mu,U]] - m^2 U = 0 \\ [x^\mu,[x_\mu]] + \lambda^2 = 0 \\ [p^\mu,[x_\mu,U]] = 0 \end{array}\right\} \quad \text{Born's reciprocity} \quad p \leftrightarrow x \text{ symmetry}$$

- x representation

$$\left.\begin{array}{c} (P^\mu P_\mu - m^2)\Phi = 0 \\ (\bar{x}^\mu \bar{x}_\mu + \lambda^2)\Phi = 0 \\ P^\mu \bar{x}_\mu \Phi = 0 \end{array}\right\} \quad \begin{array}{l} \Phi(X,\bar{x}) = \langle x'|U|x''\rangle \\ X = \frac{x'+x''}{2}, \bar{x} = x' - x'' \end{array}$$

Critical comments

- The masses of all spin states are degenerate to m.
- The λ can be removed by a canonical transformation.
  (Hara-Shimazu,1950)

In answer to these claims

"This is certainly not true, whenever we take into account the interaction between two non-local fields". Yukawa extended the master wave equation to the form

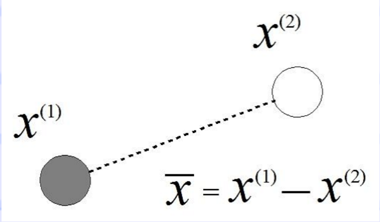
$$(P^{\mu}P_{\mu} - M^2(P, \bar{p}, \bar{x}))\Phi = 0$$
$$((x^2 + \lambda^2)\Phi = 0 \rightarrow \text{discarded } !)$$

In 1953, Yukawa proposed the following form of M^2:

$$M^2(P, \bar{p}, \bar{x}) = \alpha(\bar{p}^{\mu}\bar{p}_{\mu} + \kappa^2\bar{x}^{\mu}\bar{x}_{\mu})$$

▷ $\rightarrow$ linear $J \propto m^2$ relation

▷ non-local fields as four-dimensional oscillator model for the elementary particles.
(i.e., relativistic potential approach to the two-particle bound systems)



Physical state conditions

P.S.C.

Yukawa (1953)

$$P^\mu a_\mu^\dagger |\Phi_{phy}\rangle = 0$$

$\downarrow$

ground state

$$a_0^\dagger|0\rangle = a_i|0\rangle = 0$$

$\downarrow$

definite metric

Takabayasi (1964)

$$P^\mu a_\mu |\Phi_{phy}\rangle = 0$$

$\downarrow$

ground state

$$a_0|0\rangle = a_i|0\rangle = 0$$

$\downarrow$

indefinite metric

$$a_\mu = \sqrt{\frac{1}{2\kappa}}(\kappa\bar{x}_\mu + i\bar{p}_\mu), \quad a_\mu^\dagger = \sqrt{\frac{1}{2\kappa}}(\kappa\bar{x}_\mu - i\bar{p}_\mu)$$

$$[a_\mu, {a_\nu}^\dagger] = -g_{\mu\nu}$$

prototype of bi-local field equation

$$\left(\alpha' P^2 + \frac{1}{2}\{a_\mu^\dagger, a^\mu\} - \omega\right)|\Psi\rangle = 0$$

$$P\cdot a|\Phi\rangle = 0$$

string like ?

Interaction of Bi-Local Fields (1950∼1985)

- Interaction Lagrangian ($\not\rightarrow$ convergent unitary S-matrix)

$$S_{int} = g\int d^4x d^4y d^4z \bar{\psi}(x)\Phi(x,y,z)u(y)\psi(z) \cdots \text{Bloch, kristensen-M}\phi\text{ller}$$

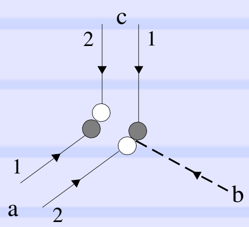
$$(S_{int} = g\int d^4x d^4y d^4z V(x,y)^* U(y,z) V(z,x) \cdots \text{Yukawa-Katayama})$$

▷ Y-F method → Hamiltonian: $H(x/\sigma) = \sum_{n=0}^{\infty} g^n H_n(x/\sigma) \cdots$ Hayashi

▷ second quantization ··· Sogami, Capri-Chang

▷ Regge pole behavior ··· Bando, Inoue, Takeda, Tanaka

- Form factor



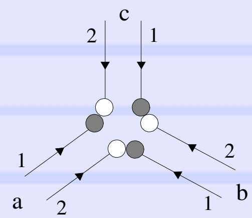
▷ definite metric ··· pole type

(Namiki, Ishida, Kim, ···)

▷ definite metric ··· Gaussian type

(Gotō-Kamimura, Naka, ···)

- Three vertex

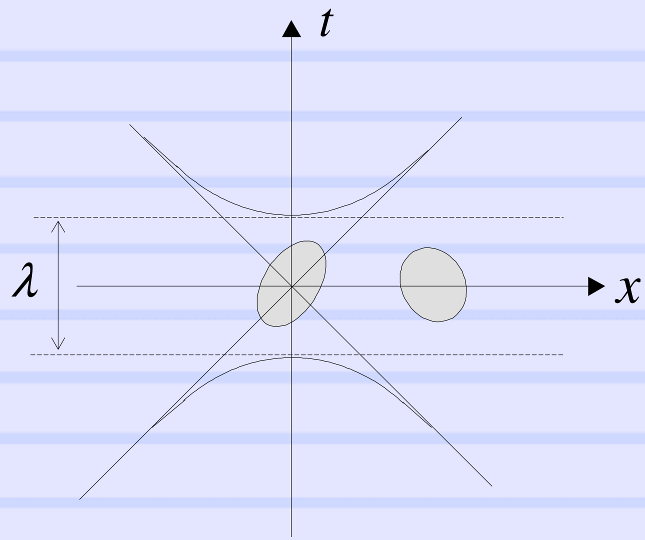


▷ Gotō-Naka

(→ string: Kaku-Kikkawa)

§3 Field Theory of Elementary Domains

Y. Katayama, I. Umemura and H. Yukawa (1968)



- $\Psi(D) = \Psi(X_\mu, I_{\mu\nu}, I_{\mu\nu\rho}, \cdots)$

$$X_\mu = \frac{1}{V}\int_D d^4x x_\mu, \quad (V = \int_D d^4x)$$

$$I_{\mu\nu} = \int_D d^4x (x-X)_\mu (x-X)_\nu$$

$$\vdots$$

- $[\Psi(X,\cdots), \Psi(X',\cdots)] = 0$ for $(X-X')^2 < \lambda^2$

- case : $I_{\mu\nu} = \sum_\alpha e_\mu^{(\alpha)} e_\nu^{(\alpha)} (\xi_\alpha)^2 \neq 0, \ I_{\mu\nu\rho} = \cdots = 0$

$$\exp\left(\sum_\alpha \lambda_\alpha e_\mu^{(\alpha)} \frac{\partial}{\partial X_\mu}\right) \Psi(X, e_\mu^{(\alpha)}, \xi_\alpha) = U\Psi(X, e_\mu^{(\alpha)}, \xi_\alpha)$$

$$\xrightarrow{1-\text{dim.}} \Psi(t+\lambda) = e^{-i\kappa}\Psi(t) \quad \to E_n = \frac{-\kappa + 2n\pi}{\lambda}, \ (n = 0, \pm 1, \cdots)$$

$\cdots$ differnce equation

§4 String model

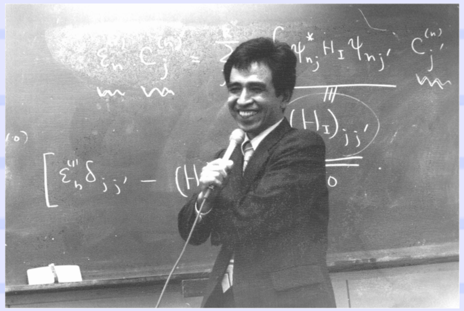
- dual resonance model $\rightarrow$ string $\rightarrow$ Virasoro condition

  Nambu,Susskind, Fubini, Veneziano, Virasoro (1969 ∼1970)

  ▷ detailed wave equation ··· Takabayasi(1970)

  ▷ σ-invariance ··· Hara(1971)

- Nambu-Gotō action



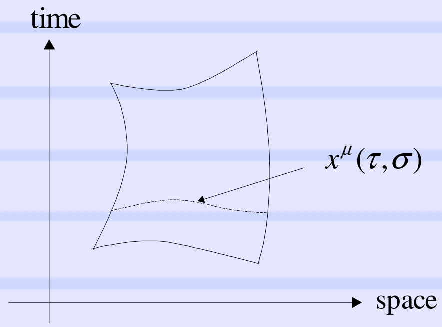
$$S = \kappa_0 \int d\tau d\sigma \sqrt{-g}$$

▷ Nambu, *Lecture at the Copenhagen Summer Symposium* (1970)

▷ Gotō,PTP(1971)

$\Downarrow$ 3-brane

$$S = \kappa \int d\tau d\xi^3 \sqrt{-g}, \ (g_{\alpha\beta} = \frac{\partial x^\mu}{\partial \xi_\alpha}\frac{\partial x_\mu}{\partial \xi_\beta})$$



▷ In this stage, the critical dimension is not yet recognized.

$$ds = \sqrt{\dot{x}^2} d\xi^0, \ dl = \sqrt{-(d_\perp x)^2}$$

$$\rightarrow \mathcal{L} \propto ds dl$$

§5 Miscellaneous

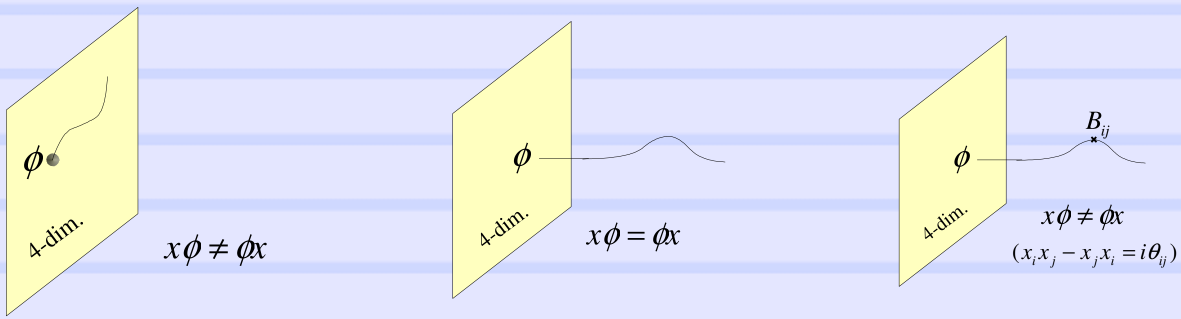
- Rigid body model

spinor: $\xi = i\sqrt{\rho}\begin{pmatrix} \cos\frac{\theta}{2}\exp[-i(\phi+\psi)/2] \\ \sin\frac{\theta}{2}\exp[-i(\phi-\psi)/2] \end{pmatrix}$

Bopp-Haag(1950), Hara,Gotō and Yabuki(1967)

- scalar field at end point of the string(Veneziano,1973)

$$\left.\begin{array}{l} \bar{\phi}(X) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\tau\phi(X + 2\lambda^2 p\tau + \cdots) \\ X\bar{\phi} \neq \bar{\phi}X \end{array}\right\} \quad \begin{array}{l} \langle x'|\bar{\phi}(X)|x\rangle: \text{ form factor} \\ \alpha' \sim \lambda^2 \sim (2\times 10^{-14}cm) \end{array}$$



- Timelike string (1-dim. domain ?) $\cdots$ Naka(1972)

$$Dx^\mu(\tau+\pi\lambda) - Dx^\mu(\tau-\pi\lambda) = \left(\frac{\sinh\pi\lambda D}{\pi\lambda D}\right) D^2 x^\mu(\tau) = 0, \ (D = \frac{d}{d\tau})$$

Pais-Uhlenbeck expansion → string-like modes

§6 Keywords in Yukawa's non-local field theory

- ▷ 4-dim. Minkowski spacetime
- ▷ spacetime description of elementary particles
- ▷ divergence difficulty
- ▷ timelike extension (difference equation)
- ▷ non-commutative structure

◇ Non-Commutative Geometry, A. Connes, 1990

◇ H.S. Snyder, *Quantized Space-Time*, 1947

$\cdots$ Lorentz invariant discrete spacetime

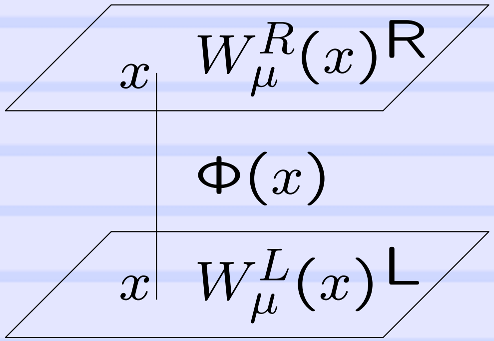
▷ seed of non-local field theories ?

▷ flat space limit of quantum gravity ?

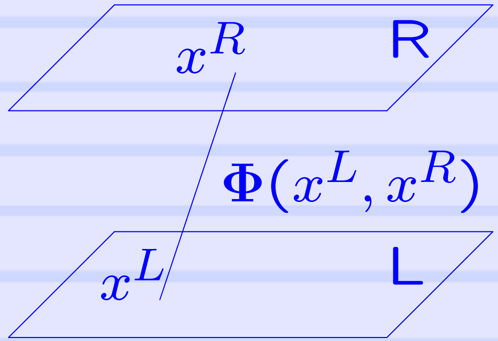
κ-Minkowski Spacetime/ Doubly Special Relativity

$$\lim_{G,\hbar\to 0}\sqrt{\frac{\hbar}{G}} = \kappa \text{ ?} \quad \text{(J.Kowalski-Glikman, hep-th/0405273)}$$

Bi-Local Fields Based on Non-Commutative Geometry



Higgs fields in NCG (Connes,1990)



Higgs-like bi-local field

$\cdots$ Naka et.al. 2000

$M^2(\bar{x}) \propto \bar{x}^2 \Rightarrow 4 - dim.OM$

$$D_\mu = \begin{pmatrix} D_\mu^L & 0 \\ 0 & D_\mu^R \end{pmatrix}$$

$$D_\eta = \partial_\eta - ig \begin{pmatrix} 0 & \phi \\ \phi^* & 0 \end{pmatrix} = ig \begin{pmatrix} 0 & \Phi \\ \Phi^* & 0 \end{pmatrix}$$

$$\eta = \frac{1}{M} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \partial_\eta \eta = -i$$

$$\mathcal{L} = -\frac{1}{4} \mathrm{Tr} F_{AB}^\dagger F^{AB} + \bar{\Psi}(\gamma^\mu i D_\mu + i\kappa D_\eta)\Psi$$

$F_{\mu\nu}^2 \cdots$ kinetic terms for $W_\mu^{L/R}$
$F_{\eta\mu}^2 \cdots$ kinetic terms for ϕ, ϕ^*
$F_{\eta\eta}^2 \cdots$ potential terms for ϕ, ϕ^*

- Caonical structure

$$[\hat{x}^i, \hat{x}^j] = i\theta^{ij}$$

$\cdots$quantum mechanics, D-brane, etc.

- Lie structure

$$[\hat{x}^i, \hat{x}^j] = i\theta^{ij}_k \hat{x}^k$$

$\cdots$ continuous group, doubly relativity, etc.

- Quantum space structure

$$[\hat{x}^i, \hat{x}^j] = i\theta^{ij}_{kl} \hat{x}^k \hat{x}^l$$

$\cdots$ quantum group

ex. quantum phase space

$$x p_x = q p_x x$$

$$\{x, p_x\}_q = 1 \to \partial_x x - q x \partial_x = 1$$

$$\hat{p}_x = -\frac{i\hbar}{2}(\partial_x + \partial_x^\dagger) = -i\hbar \frac{q+1}{2q} D_x$$

- $D_x \psi(x) = \frac{\psi(qx) - \psi(q^{-1}x)}{(q - q^{-1})x}$

- $\underbrace{i\hbar D_t \psi(t,x)}_{\text{time diff.}} = E\psi(t,x)$

$\Downarrow$

domain equation ?

q-deformed bi-local field

q-deformation of h.o.

$[a, a^\dagger] = 1, \quad (a^\dagger a = N)$

$\Downarrow$

$a_q a_q^\dagger - q a_q^\dagger a_q = q^{-(N+\beta)}$

mapping: $a_q = a\sqrt{\frac{[N]_q}{N}}$

with $[N]_q = \dfrac{q^{N+\beta} - q^{-N-\beta}}{q - q^{-1}}$

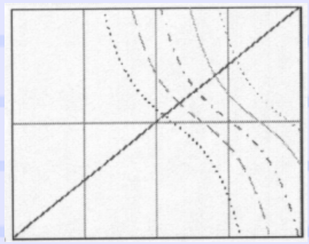
$\Downarrow$

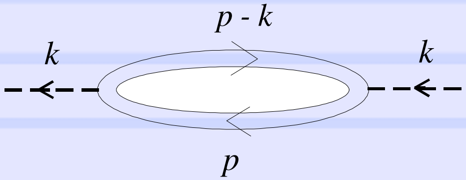
$a_q^\dagger a_q = [N]_q = \frac{\sinh[(N+\beta)\log q]}{\sinh[\log q]}$

deformed bi-local field

$$\left(\alpha' P^2 + \frac{1}{2}\{a_{q\mu}^\dagger, a_q^\mu\} + const.\right)\psi = 0$$

$$\frac{1}{2}\{a_{q\mu}^\dagger, a_q^\mu\} = \frac{\sinh[(-a_\mu^\dagger a^\mu - \frac{3}{4}\alpha' P^2 + c)\log q]}{\sinh[\frac{1}{2}\log q]}$$





on-shell eq.
$(P^2 - m_n^2)|\Psi\rangle = 0$

$\delta m^2 \sim \frac{1}{\alpha'}\left(\frac{\sinh(\frac{1}{2}\log q)}{\log q}\right)^2$

finite for $q \neq 1$

$\cdots$ Naka et. al (2005)

Synder's quantized spacetime

Snyder's Qntized Spacetime

$$-\eta_0^2+\eta_1^2+\eta_2^2+\eta_3^2+\eta_4^2=a^2$$

$$M_{ij}=i\hbar\left(\eta_i\frac{\partial}{\partial\eta_j}-\eta_j\frac{\partial}{\partial\eta_i}\right)$$

$$M_{0j}=i\hbar\left(\eta_0\frac{\partial}{\partial\eta_j}+\eta_j\frac{\partial}{\partial\eta_0}\right)$$

$$x_i=ia\left(\eta_4\frac{\partial}{\partial\eta^i}-\eta_i\frac{\partial}{\partial\eta_4}\right)$$

$$x_0=ia\left(\eta_4\frac{\partial}{\partial\eta_0}+\eta_0\frac{\partial}{\partial\eta_4}\right)$$

$$[x_\mu,x_\nu]=\frac{ia^2}{\hbar}M_{\mu\nu}$$

$$p_i=\frac{\hbar}{a}\frac{\eta_i}{\eta_4},\ p_0=\frac{\hbar c}{a}\frac{\eta_0}{\eta_4}\rightarrow M_{ij}=x_{[i}p_{j]}$$

$\Delta x\Delta p\geq\frac{\hbar}{2}\left(1+\frac{a^2}{\hbar^2}\langle p^2\rangle\right)$ etc.

> $\langle\eta|U(x)|\eta'\rangle$ bi-local field ?
> $\cdots$ S. Tanaka (2000)

κ-Minkowski Spacetime

Lie structure: $[x_i,x_j]=0$

$$[x_0,x_i]=-\frac{i}{\kappa}x_i$$

surface in a 5-dim. Minkowski space (one-forms $\{dx_i,dx_4\}$)

$$[x_o,dx_0]=\frac{i}{\kappa}dx_4,[x_0,dx_4]=\frac{i}{\kappa}dx_0,$$

$\cdots$ $[x_i,dx_j]=\frac{i}{\kappa}\delta_{ij}(dx_0-dx_4)$,etc.

$$d:e^{ik\cdot x}:=-iP_A:e^{ik\cdot x}:dx_A$$

$$P_0=\kappa\sinh\frac{k_0}{\kappa}+\frac{\boldsymbol{k}^2}{2}e^{\frac{k_o}{\kappa}}$$

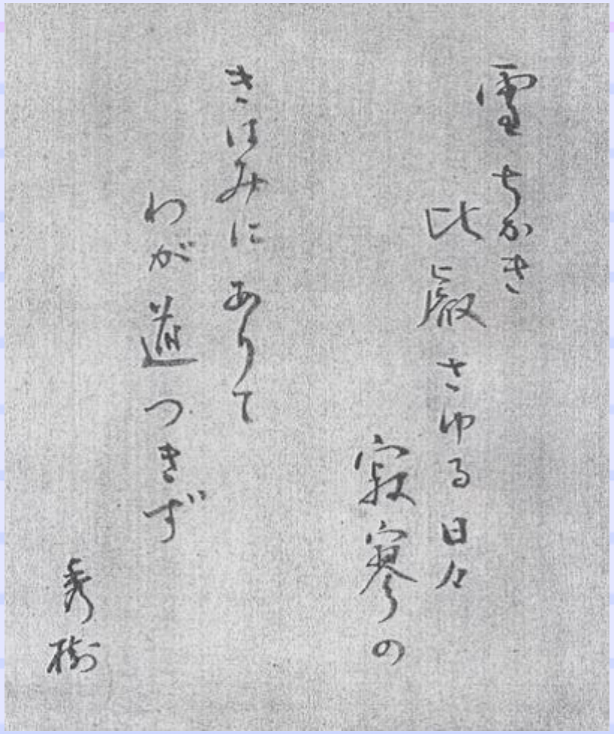
$$P_i=k_ie^{\frac{k_0}{\kappa}}$$

$$P_4=\kappa\cosh\frac{k_0}{\kappa}-\frac{\boldsymbol{k}^2}{2}e^{\frac{k_o}{\kappa}}$$

$$-P_0^2+P_1^2+P_2^2+P_3^2+P_4^2=\kappa^2$$

Casimir: $\left(2\kappa\sinh\frac{k_0}{2\kappa}\right)^2-\boldsymbol{k}^2e^{\frac{k_0}{\kappa}}$

Summary



In an article in 1979, Yukawa said "Thus, even if such a model turns out to be essentially a correct one, it should be only be the beginning of the story. The hope of the author is that it would play in future development of elementary particle theory a role somehow analogous to that of Rutherford model of the formulation of quantum theory."