場の理論によるブラックブレーンの解析と p-soup モデル

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Black brane thermodynamics

- Black branes are the solutions of supergravity:

$$ds^2 = \alpha' \left(\frac{U^{\frac{7-p}{2}}}{\sqrt{a_p \lambda}} (-f dt^2 + dx^i dx^i) + \sqrt{a_p \lambda} \left(U^{-\frac{7-p}{2}} \frac{dU^2}{f} + U^{\frac{p-3}{2}} d\Omega^2_{(8-p)} \right) \right)$$

$$f(U) = 1 - \left(\frac{U_0}{U} \right)^{7-p}, \qquad a_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2} \right)$$

- The dynamics of black branes can be studied by using the analogy of thermodynamics:

  area of horizon ~ entropy (Bekenstein-Hawking's formula)

  Hawking temperature ~ temperature

  other quantities: free energy, energy, … $F \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda^{-\frac{3-p}{5-p}} V_p$

- We can reproduce these quantities by studying the field theories on branes!

"p-soup model" (1)

- We consider the black Dp-branes:
  The field theory is the (p+1)-dim super Yang-Mills theory.
- First, we obtain the effective action of scalar moduli ϕ:
  The classical part is

$$S_{\mathrm{D}p}^{\text{classical}} = \frac{N}{\lambda} \int d\tau d^p x \sum_a \left(\frac{1}{2} \partial^\mu \phi_a^I \partial_\mu \phi_a^I \right)$$

  The 1-loop part is

$$S_{\mathrm{D}p,T=0}^{\text{one-loop}} = - \int d\tau d^p x \sum_{a<b} \frac{\Gamma\left(\frac{7-p}{2}\right)}{(4\pi)^{\frac{1+p}{2}}} \left(2 \frac{\{\partial_\mu(\phi_a^I - \phi_b^I)\partial_\nu(\phi_a^I - \phi_b^I)\}^2}{|\phi_a - \phi_b|^{7-p}} - \frac{\{\partial_\mu(\phi_a^I - \phi_b^I)\partial^\mu(\phi_a^I - \phi_b^I)\}^2}{|\phi_a - \phi_b|^{7-p}} \right) + \dots$$

  More than 1-loop parts can be obtained similarly.
- Notes: we don't care rational factors of coefficients.

"p-soup model" (2)

[Wiseman '13]

- Next, we make some assumptions.
- Branes should be uniformly distributed: $\boxed{\phi_a^I \sim \phi_a^I - \phi_b^I \sim \phi}$
- Higher derivative terms should be negligible: $|(\partial\phi)^2/\phi^4| \ll 1$
- Temperature-dependent terms should be negligible:
- These two conditions coincide if we impose $\beta|\phi_a - \phi_b| \gg 1$

$$\boxed{\partial\phi_a^I \sim \partial(\phi_a^I - \phi_b^I) \sim \frac{\pi}{\beta}\phi}$$

- All loop terms should be of the same order ("virial theorem"):

$$\boxed{S_{\mathrm{D}p}^{\text{classical}} \sim S_{\mathrm{D}p,T=0}^{\text{one-loop}} \sim S_{\mathrm{D}p}}$$

Non-perturbative region!

- We call it "p-soup model":
  The p-branes are (uniformly) scattered but strongly coupled.

“p-soup model” (3)

[Wiseman ‘13]

- We can reproduce the energy, the radius of horizon, and so on.
- This is not only in D-brane cases, but also in M-brane cases: ABJM theory for M2, dimensional analysis for M5. [Morita-SS ’13]

[Morita-SS-Wiseman-Withers ’13]

- The scalar moduli action can be reproduced by the probe brane action in external background (in supergravity)

$$S_{probe} = -\mu_2 \int dt dx^p \frac{1}{H} \left(\sqrt{-\det\left(\eta_{\mu\nu} + H \partial_\mu \vec{Z} \cdot \partial_\nu \vec{Z}\right)} - 1 \right)$$

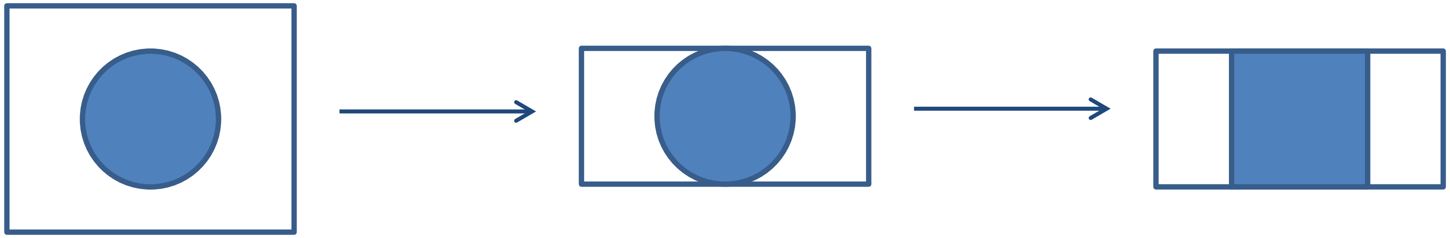
for small gravity coupling and weakly curved branes. $\left|\partial_\mu \vec{Z}\right| \ll 1$

- I’d like to show you further evidence: ex.) Gregory-Laflamme transition

$$H = 1 + \frac{2\kappa^2 \mu_1}{(n-2)\,\Omega_{n-1}} \frac{1}{|Z^m|^{n-2}}$$

Gregory-Laflamme transition

- It occurs when the black branes are compactified on a circle:



- If the circle becomes smaller than the radius of horizon, the horizon overlaps on the circle, and then the system becomes unstable. Then a phase transition occurs. As a result, the branes becomes smeared on the circle.
- This is the Gregory-Laflamme transition: for example, the derivatives of free energy are discontinuous.
- We can explain this transition by using "p-soup model".

Examples (1)

[Morita-SS-Wiseman-Withers, in progress]

- Ex.) GL transition between D2 and M2 (on M-theory circle)

  D2: winding on circle / M2: localized on circle

- D2 moduli action from 3d SYM with monopole corrections

$$S^{\text{3dSYM}}_{\text{one-loop}} \sim \int d\tau d^2x \sum_{a<b} \frac{2\pi}{\Omega_6} \frac{\left|\partial_\mu \left(\vec{\phi}_a - \vec{\phi}_b\right)\right|^4}{|\vec{\phi}_a - \vec{\phi}_b|^5} \left(1 + \underline{e^{-\frac{\phi_a - i\phi_b^8}{g_{YM}^2}}} + \dots\right)$$

  where ϕ^8 is the dual scalar of YM gauge field. $\phi_a^I = X_a^I / 2\pi\alpha'$

- M2 moduli action from ABJM with infinite mirror images

$$S^{\text{3dSYM}}_{\text{one-loop}} \sim \int d\tau d^2x \underline{\sum_{n\in\mathbb{Z}}} \sum_{a<b} \frac{(2\pi)^8 l_{\text{pl}}^9}{\Omega_7} \frac{\left|\frac{1}{(2\pi)^2 l_{\text{pl}}^3} \left\{\left(\partial\vec{X}_a - \partial\vec{X}_b\right)^2 + \left(\partial X_a^8 - \partial X_b^8\right)^2\right\}\right|^2}{\left|(\vec{X}_a - \vec{X}_b)^2 + (X_a^8 - X_b^8 + \underline{2\pi n R})^2\right|^3}$$

- These two actions are equivalent under Poisson resummation.

Examples (1)

- GL transition between D2 and M2 occurs when the monopole correction becomes O(1): $\phi \sim g_{YM}^2 \;\; (X \sim R)$
- According to (1-loop part of) moduli action, when $\boxed{\phi \ll g_{YM}^2}$

$$S_{\text{one-loop}}^{\text{3dSYM}} \sim \int d\tau d^2x \sum_{a<b} \frac{2\pi}{\Omega_6} \frac{\left|\partial_\mu\left(\vec{\phi}_a - \vec{\phi}_b\right)\right|^4}{|\vec{\phi}_a - \vec{\phi}_b|^5} \left(1 + e^{-\frac{\phi_a - i\phi_b^8}{g_{YM}^2}} + \dots\right)$$

potential for ϕ^8 is weak: uniform distribution along M-circle

- This moduli action is equivalent to $\qquad \phi_a^I = X_a^I / 2\pi\alpha'$

$$S_{\text{one-loop}}^{\text{3dSYM}} \sim \int d\tau d^2x \sum_{n\in\mathbb{Z}} \sum_{a<b} \frac{(2\pi)^8 l_{\text{pl}}^9}{\Omega_7} \frac{\left|\frac{1}{(2\pi)^2 l_{\text{pl}}^3}\left\{\left(\partial\vec{X}_a - \partial\vec{X}_b\right)^2 + \left(\partial X_a^8 - \partial X_b^8\right)^2\right\}\right|^2}{\left|(\vec{X}_a - \vec{X}_b)^2 + (\underline{X_a^8 - X_b^8} + 2\pi n R)^2\right|^3}$$

When $\boxed{\phi \sim g_{YM}^2}$, the attractive force between X^8's is provided: Branes are localized on the circle, and the transition occurs.

Examples (2)

- Ex.) GL transition between D2 and D3 (on T-duality circle)
  D2: localized on circle / D3: winding on circle

- It is similar to the D2/M2 case, but the interpretation changes.

$$S^{E,4}_{compact} = -\int d\tau d^p x \sum_{a<b} \sum_{\underline{n\in\mathbb{Z}}} \frac{\Gamma\left(\frac{8-p}{2}\right)}{(4\pi)^{\frac{p}{2}} L} \frac{2\left(\partial_\mu \vec{\phi}_{ab}\cdot\partial_\nu \vec{\phi}_{ab}\right)\left(\partial^\mu \vec{\phi}_{ab}\cdot\partial^\nu \vec{\phi}_{ab}\right) - \left(\partial_\mu \vec{\phi}_{ab}\cdot\partial^\mu \vec{\phi}_{ab}\right)^2}{\left(\underline{\left(\frac{2\pi n}{L} - a^p_{ab}\right)}^2 + \left|\vec{\phi}_{ab}\right|^2\right)^{\frac{8-p}{2}}}$$

localized Dp

$$\sim -\int d\tau d^p x \sum_{a<b} \frac{\Gamma\left(\frac{7-p}{2}\right)}{(4\pi)^{\frac{1+p}{2}}} \frac{2\left(\partial_\mu \vec{\phi}_{ab}\cdot\partial_\nu \vec{\phi}_{ab}\right)\left(\partial^\mu \vec{\phi}_{ab}\cdot\partial^\nu \vec{\phi}_{ab}\right) - \left(\partial_\mu \vec{\phi}_{ab}\cdot\partial^\mu \vec{\phi}_{ab}\right)^2}{\left|\vec{\phi}_{ab}\right|^{7-p}}$$

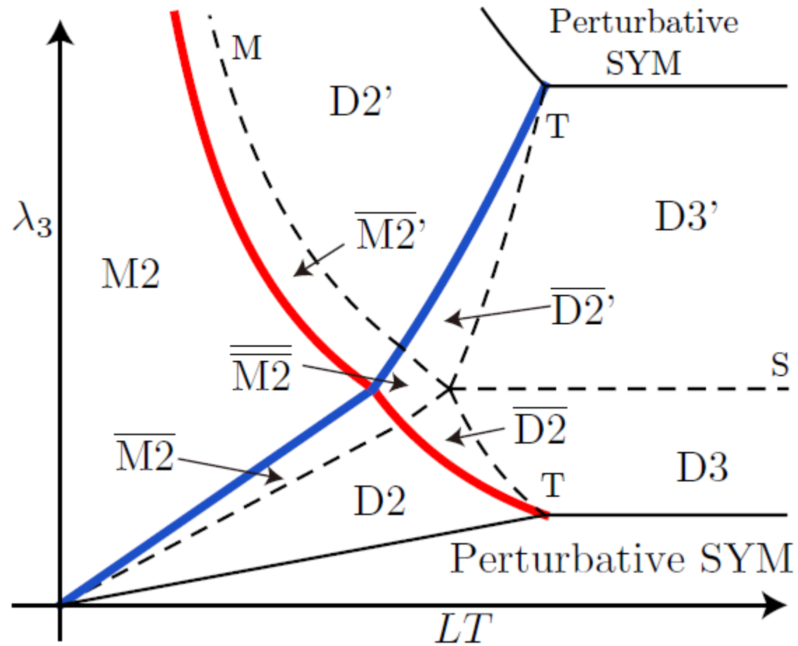
$$\times\left(1 + \underline{e^{-L|\phi_{ab}|}}\left(L\left|\phi_{ab}\right|\right)^{\frac{6-p}{2}} \cos\left(L a^p_{ab}\right) + \ldots\right)$$

winding D(p+1)

- The transition occurs at $L\phi \sim 1$: Polyakov loop becomes O(1).
  The expectation of value of Polyakov loop is the order parameter of this confinement/deconfinement transition.

Examples (2)

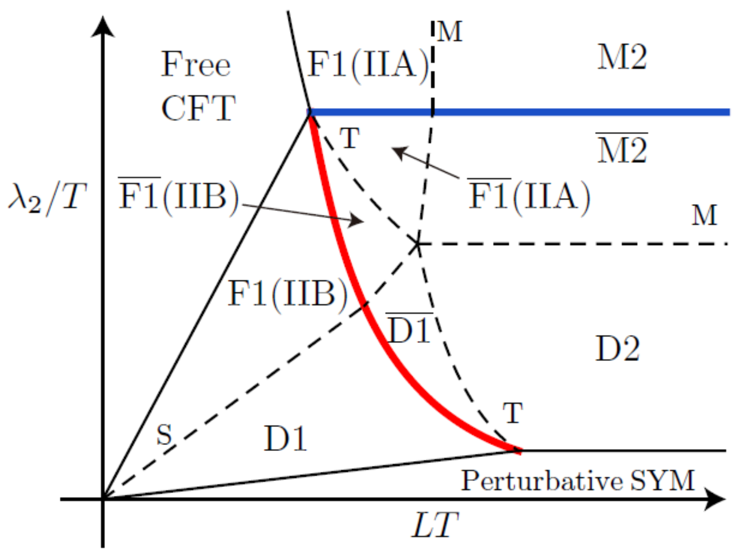
- Discussion of the previous two examples enable us to make the phase diagram for black D3-brane winding on the circle.

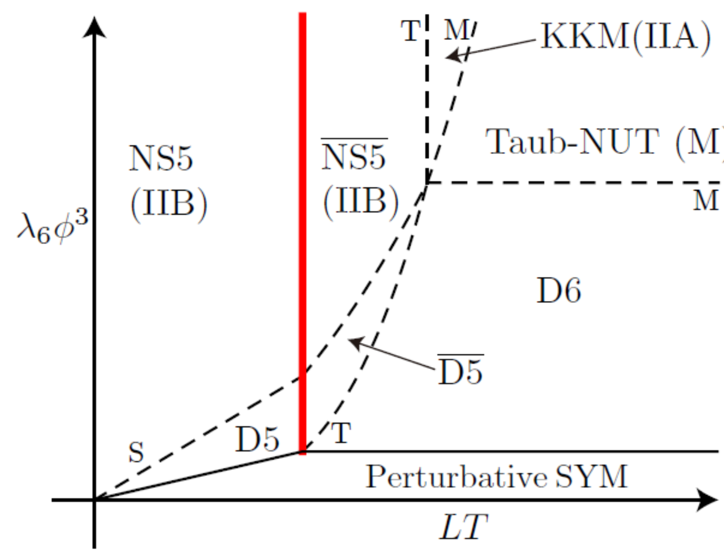


- This is consistent with the results from supergravity. Note that… we use only the knowledge on field theory!

Examples (3)

- Similarly we can draw other phase diagrams for other branes.
- They contain the phases of F1-strings, NS5-branes, Kaluza-Klein monopoles…, whose moduli actions are unknown.
- In our discussion, we speculate those moduli actions by simply rewriting parameters under S, T-duality relations.
- As a result, we can reproduce the correct phase diagrams.





Systems with less SUSYs?

[Morita-SS, '14]
[Morita-SS, in progress]

- Ex.) Intersecting branes:
  especially, D1/D5 and D1/D5/P systems
- In these cases, we need to speculate the moduli action from a probe brane action of each brane.
  (This includes nontrivial speculations about multi-brane interactions.)
- After writing down the moduli action in this way, we can use the "p-soup" discussion!
- We can reproduce the correct energy, size of horizon, entropy and so on. (… at the same point B=0 in the moduli space!)
- The Gregory-Laflamme transitions and the duality relations to other intersecting D/M-brane systems can be also discussed.

Conclusion

- We propose a new picture of black branes, which is based on the field theory on p-branes. We call it “p-soup model”.
- This model can explain the dynamics of black brane systems where all the D/M-branes are the same kind and parallel.
- We can also understand the Gregory-Laflamme transition on a compact circle in terms of the field theory.
- Moreover, this model can explain the dynamics of intersecting black brane systems, which have less supersymmetries.
- Other possible applications…?