衝撃波型背景時空におけるbi-local場

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2014.11.9 CST-MISC 連携シンポジュウム

-素粒子と時空・現象から探る素粒子-

2011/10/30-31（日大）

2012/11/2-3　（益川塾）

2014/3/15-16　（日大）

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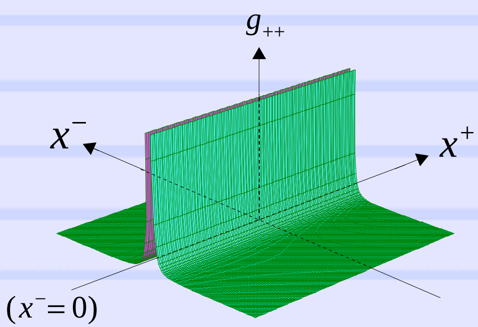
arXive:1410.4014 Bi-Local Field in Gravitational Shock Wave Background. (with N.Kanda)

§1 Introduction

Gravitational Shock Wave Background

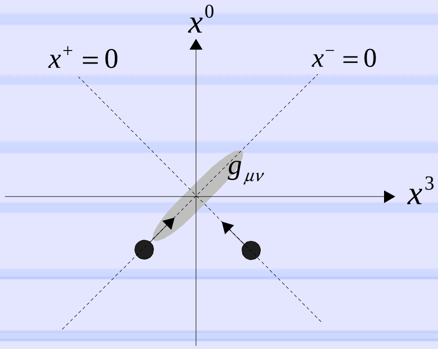
- P.C.Aichelburg and Sexl (1971)
  On the Gravitational Field of a Massless Particle
- T.Dray and G.'t Hoot (1984)
  The Graviational Shock Wave of a Massless Particle
- G. 't Hoot (1987)
  Graviton Dominance in Ultra-Heigh-Energy Scattering



$$x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$$

T_{--} : Energy-Momentum Tensor

- ▷ Electromagnetic field
- ▷ Fluids with the light velocity
- ▷ Particles with the light velocity



Bi-Local Fields

- H.Yukawa (1949) Quntum Theory of Non-Local Fields. Part I

$$\left.\begin{array}{l}[p_\mu,[p^\mu,U]]+m^2c^2U=0\\ [x_\mu,[x^\mu,U]]-\lambda^2U=0\\ [x^\mu,[p_\mu,U]]=0\end{array}\right\} \Rrightarrow \begin{array}{c} U(X,\bar{x})=\langle x'|U|x''\rangle \\ \left(X=\dfrac{x'+x''}{2},\ \bar{x}=x'-x''\right)\end{array}$$

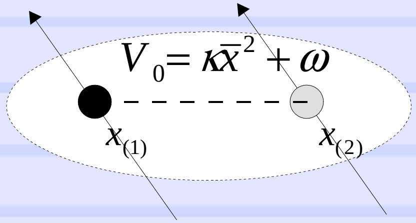
$\cdots$ T.Takabayasi, I.Sogami, S.Ishida, T.Gotō

$$S=\int d\tau\frac{1}{2}\sum_{i=1}^{2}\left\{e_{(i)}^{-1}\eta_{\mu\nu}\dot{x}_{(i)}^{\mu}\dot{x}_{(i)}^{\nu}-V_0e_{(i)}\right\}$$

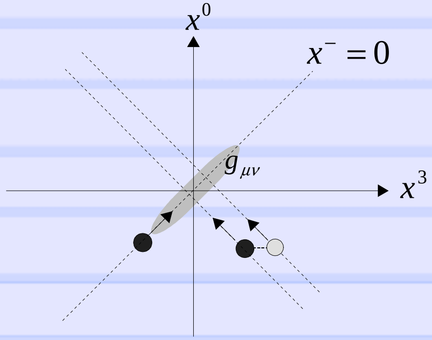
$\Downarrow \quad \delta e_{(i)}$

$$H=P^2+\frac{1}{\alpha'}a^\dagger\cdot a+m_0^2\approx 0,$$
$$T=P\cdot\bar{p}\approx 0 \quad (\rightarrow P\cdot a\approx 0)$$

$$\left\{\begin{array}{c}\bar{x}=\sqrt{\frac{1}{2\kappa}}(a^\dagger+a)\ \text{etc.}\\ \alpha'=\frac{1}{8\kappa},\ m_0^2=4\omega+16\kappa\end{array}\right\}$$



- ▷ relativistic bound state
- ▷ prototype of string
- ▷ purpose of this work



α' : effective ?

§2 Bi-Local Field in G. S. W. B

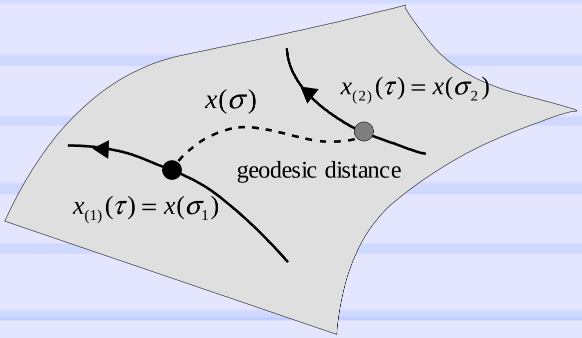
- G.S.W.B.

$$ds^2 = -2dx^+dx^- + f(x_\perp)\delta(x^-)d^2x^- + d^2x_\perp,$$

$$f(x_\perp) = f_0 - 2Q\log\left(\frac{r}{r_0}\right), \quad \left(r = \sqrt{x_\perp^2},\ Q = \frac{4\sqrt{2}P_S}{E_P^2} \sim E_P^{-1}\right)$$

$\Downarrow$ embedding of a bi-local model

$$S = \int d\tau \frac{1}{2}\sum_{i=1}^{2}\left\{ g_{\mu\nu}\frac{\dot{x}^\mu_{(i)}\dot{x}^\nu_{(i)}}{e_{(i)}} - \underbrace{V\left(x_{(1)}, x_{(2)}\right)}_{\text{bi-scalar function}} e_{(i)} \right\}$$



$$V = 2\kappa^2\sigma(x_{(1)}, x_{(2)}) + \omega$$

$$\sigma(x_{(1)}, x_{(2)}) = \frac{\sigma_2 - \sigma_1}{2}\int_{\sigma_1}^{\sigma_2} d\sigma g_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma}$$

$$\to \frac{1}{2}\bar{x}^2, \quad (g_{\mu\nu} \to \eta_{\mu\nu})$$

❋ Constraints

$$\delta S/\delta e_{(i)} = 0 \ \rightarrow \ g^{\mu\nu} p_{(i)\mu} p_{(i)\nu} + V 0, \ (i = 1, 2)$$

$$\approx \qquad \Downarrow$$

$$\left.\begin{array}{ll} \text{sum.} & : \ H \approx 0 \cdots \text{wave equation} \\ \text{sub.} & : \ T \approx 0 \cdots \text{supplementary condiition} \end{array}\right\} \ni \delta(x^-_{(i)})$$

❋ Canonical transformation

$$U = \exp\left\{ -\frac{i}{2} \sum_{i=1}^{2} f(x_{(i)\perp}) \theta(x^-_{(i)}) p^-_{(i)} \right\}$$

$$\Downarrow$$

$$U \left\{ \begin{array}{c} H \\ T \end{array} \right\} U^\dagger = \left\{ \begin{array}{c} \tilde{H} \\ \tilde{T} \end{array} \right\} \ni \theta(x^-_{(i)}), \quad \not\ni \delta(x^-_{(i)})$$

$\cdots$ $\tilde{H} \approx 0$ & $\tilde{T} \approx 0$ are <u>complex / not compatible</u>

$$\sim O(Q^2), O(\alpha' Q)$$

- **Approximation:** $(Q^2), O(\alpha' Q) \sim 0$

$$\tilde{H} \simeq \left(P^2 + \alpha'^{-1} a^\dagger \cdot a + m^2 + \Delta M^2\right) \approx 0,$$

$$(\tilde{T} \simeq P \cdot \bar{p} \approx 0) \to P \cdot a \approx 0,$$

$$\Delta M^2 = -2 \sum_{i=1}^{2} \left(\{p_\perp, A\}\theta p^-\right)_{(i)},$$

$$A(x_\perp) = -\frac{1}{2}\left(\partial_\perp f(x_\perp)\right) = Q\frac{x_\perp}{r^2}$$

$$(\ \to [P \cdot a, \tilde{H}] \not\approx 0 \ )$$

- **Projection:**

$$\Delta M^2 \to [\Delta M^2] \quad \left(= \frac{1}{2\pi}\int_0^{2\pi} d\theta e^{i\theta N} \Delta M^2 e^{-i\theta N}, \ N = \frac{P\cdot a^\dagger P\cdot a}{P^2}\right)$$

§3 Scattering amplitudes

❋ light-like time : $T^- = X^-$

$$i\frac{\partial}{\partial T^-}|\tilde{\Phi}\rangle = \frac{1}{2\alpha' P^-}\left\{\alpha'(P_\perp^2 + m_0^2) + a^\dagger \cdot a + \alpha'[\Delta M^2]\right\}|\tilde{\Phi}\rangle$$

$$= (\tilde{H}_0 + \Delta\tilde{H})|\Phi\rangle$$

$$\left(\begin{array}{c} \tilde{H}_0 = \frac{1}{2\alpha' P^-}\left\{\alpha'(P_\perp^2 + m_0^2) + a^\dagger \cdot a\right\} \\ \Delta\tilde{H} \simeq \frac{i}{2}[\tilde{H}_0, \sum_{i=1}^2 f_i] P^- \theta(X^-) \end{array} \right)$$

❋ S-matrix

$$S = \lim_{\substack{T_2^- \to \infty \\ T_1^- \to -\infty}} U_2^\dagger \left(T e^{-i\int_{T_1^-}^{T_2^-} dT^- (\Delta\tilde{H})_D} \right) U_1 \ , \quad ([P^-, S] = 0)$$

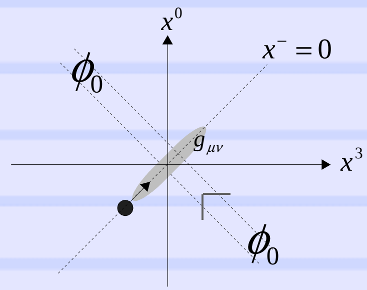
$$\to \quad \langle\Phi_b|S|\Phi_a\rangle = \langle\Phi_b|\Phi_a\rangle + i(2\pi)\delta(P_b^- - P_a^-)T_{ba}$$

examples: states/T-matrices

- $|\Phi\rangle = |\phi_{\text{rel.}}\rangle \otimes |P_\perp\rangle \otimes |P^-\rangle$

  $T_{ba} = T_{ba}^{(0)} + T_{ba}^{(1)} + \cdots\,, \quad (T_{ba}^{(n)} \sim O((\Delta H)^n)$

1 $\quad |\phi_i\rangle = |0_-\rangle \otimes |0_\perp\rangle, \ (i = a, b)$



$$T_{ba}^{(0)} = \frac{e^{\frac{i}{2}f_0 P_a^-}}{(2\pi)i}\int \frac{d^2X_\perp}{(2\pi)^2} e^{-i\Delta P_\perp \cdot X_\perp} \langle 0_\perp | e^{-\frac{i}{2}P_a^- Q \log\left(\frac{r_1 r_2}{r_0^2}\right)} |0_\perp\rangle\ , \ (\Delta P_\perp = P_{\perp b} - P_{\perp a})$$

$$= \underbrace{\frac{e^{\frac{i}{2}f_0 P_a^-}}{(2\pi)^3 i} \times r_0^2 \frac{\pi \Gamma(1 - \frac{i}{2}P_a^- Q)}{\Gamma(\frac{i}{2}P_a^- Q)} \left\{\frac{4}{r_0^2 (\Delta P_\perp)^2}\right\}^{1 - \frac{i}{2}P_a^- Q}}_{\text{'t Hooft amplitude}} \left[1 - O(P^- \alpha' Q)\right]$$

$$T_{ba}^{(1)} = i f_0 P_a^- T^{(0)} + 2iP_a^- e^{\frac{i}{2}f_0 P_a^-} \frac{\partial}{\partial P_a^-}\left(e^{-\frac{i}{2}f_0 P_a^-} T^{(0)}\right)$$

2 $|\phi_a\rangle = |z_a\rangle \otimes |0_\perp\rangle, \; |\phi_b\rangle = |0_-\rangle \otimes |0_\perp\rangle$

$$T_{ba}^{(0)} = \int d^2\mu_{\mathrm{tH}}(X_\perp)\left[1 - i\pi\alpha' P_a^- Q\delta^2(X_\perp) - \frac{\alpha'}{2}\frac{(Q\bar{p}_a^-)^2}{r^2} + \cdots\right],$$

$$\left(\int d^2\mu_{\mathrm{tH}}(X_\perp) \times 1 = (\text{'tHooft amplitude}) \right)$$

3 $|\phi_a\rangle = |0_-\rangle \otimes |0_\perp\rangle, \; |\phi_b\rangle = |0_-\rangle \otimes |\bar{x}_{\perp b}\rangle$

$$T_{*a}^{(0)} = \sqrt{\frac{\kappa}{\pi}} \int d^2\bar{x}_{\perp b} T_{ba}^{(0)} \simeq \int d\mu_{\mathrm{tH}}(X_\perp)\left[1 - \frac{2i\alpha' Q P_a^-}{r^2} + \cdots\right]$$

- Corrections to the 't Hooft amplitude appear with the factors

$$\alpha' Q P^-, \quad \alpha'(Q\bar{p}^-)^2 \quad (\gg Q^2)$$

($\times$ $\alpha'^{-1}Q \sim \kappa Q$ terms)

§4 Summary

- Bi-local model in G.S.W.B.

  $$ds^2 = -2dx^+ dx^- + f(x_\perp)\delta(x^-)d^2x^- + d^2x_\perp,$$

- Embedding of a bi-local model in a curved spacetime:

  two-body potential: $V(x_{(1)}, x_{(2)}) = \kappa^2 \underbrace{2\sigma(x_{(1)}, x_{(2)})}_{\text{geodesic distance}} + \omega$

- Primary constraints: $H_i = g^{\mu\nu} p_{(i)\mu} p_{(i)\nu} + V \approx 0$, $(i = 1, 2)$
  contain a δ-function singularity

- Canonical transformation: $\tilde{H}_i = U H_i U^\dagger \ni \theta(x^-), \not\ni \delta(x^-)$

- S-matrix

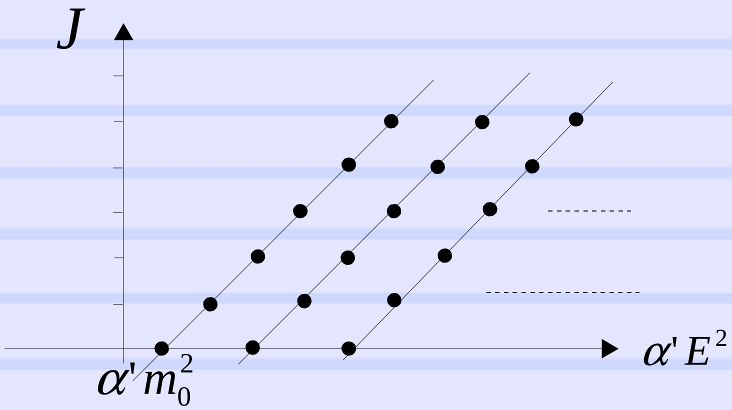
$$S = \lim_{\substack{T_2^- \to \infty \\ T_1^- \to -\infty}} U_2^\dagger \left(T e^{-i\int_{T_1^-}^{T_2^-} dT^- (\Delta \tilde{H})_D} \right) U_1$$

$$\to \lim_{\substack{T_2^- \to \infty \\ T_1^- \to -\infty}} U_2^\dagger U_1 \ , \ (\Delta H \to 0)$$

$\to$ 'tHooft amplitude , $(\alpha' \to 0)$

- Correction due to α': $\alpha' Q P^{-}$ ($\gg Q^2$) , ($\times$ $\alpha^{-1} Q$)

- ? infinite slope limit $\alpha' \to \infty$



heigher spin summetry?