A hidden-variables version of Gisin’s theorem

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Based on

K. Fujikawa and K. Umetsu,
arXiv: 1410.1702 [quant-ph]

Introduction

ベルの不等式:

局所実在論と量子力学を明確に区別するcriterion

局所実在論 (local realism) :

文脈依存性のない局所的な隠れた変数理論によって記述

(Non-contextual and local hidden variables models)

局所実在論の予言（CHSH inequality）:

$$
\begin{aligned}
|\langle\psi|B|\psi\rangle_{CHSH}| &= |\langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle + \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}'\cdot\sigma|\psi\rangle \\
&\quad +\langle\psi|\mathbf{a}'\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle - \langle\psi|\mathbf{a}'\cdot\sigma\otimes\mathbf{b}'\cdot\sigma|\psi\rangle| \leq 2,
\end{aligned}
$$

量子力学はCHSH inequality の破れを予言する:

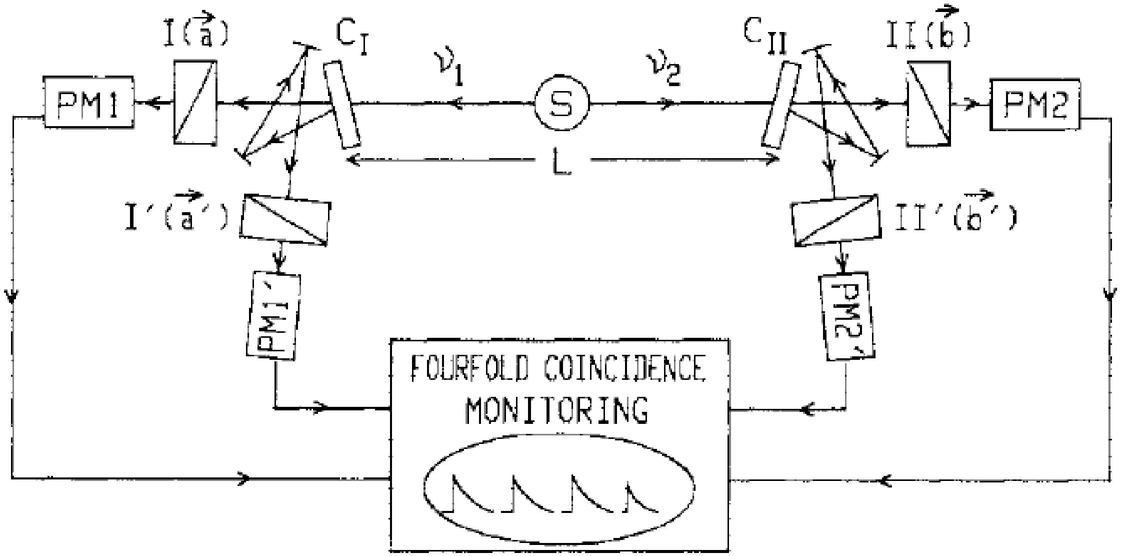
$$
\begin{aligned}
\langle\psi|B|\psi\rangle_{QM} &= \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle + \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}'\cdot\sigma|\psi\rangle \\
&\quad +\langle\psi|\mathbf{a}'\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle - \langle\psi|\mathbf{a}'\cdot\sigma\otimes\mathbf{b}'\cdot\sigma|\psi\rangle
\end{aligned}
$$

$$
\Longrightarrow \quad |\langle\psi|B|\psi\rangle_{QM}| \leq 2\sqrt{2}.
$$

CHSH inequality and Aspect's experiment

Aspect et.al., PRL (1982)

$$\langle B \rangle = \langle \mathbf{a}\cdot\sigma \otimes \mathbf{b}\cdot\sigma \rangle + \langle \mathbf{a}\cdot\sigma \otimes \mathbf{b}'\cdot\sigma \rangle + \langle \mathbf{a}'\cdot\sigma \otimes \mathbf{b}\cdot\sigma \rangle - \langle \mathbf{a}'\cdot\sigma \otimes \mathbf{b}'\cdot\sigma \rangle.$$



この対決の最終的な決着は1982年のAspect et.al. の実験において,
この量が2を超えることがあることが実証された.

$$\boxed{\langle B \rangle_{\rm exp} = 2.697 \pm 0.015}$$

Gisin's Theorem

Gisin, PLA (1991)

Gisin's Theorem の対偶:

全ての $\mathbf{a},\ \mathbf{a}',\ \mathbf{b},\ \mathbf{b}'$ に対して常にCHSH 不等式が成立するならば，その状態は separable state である.

$$\boxed{\begin{aligned}&|\langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle+\langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}'\cdot\sigma|\psi\rangle\\&+\langle\psi|\mathbf{a}'\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle-\langle\psi|\mathbf{a}'\cdot\sigma\otimes\mathbf{b}'\cdot\sigma|\psi\rangle|\leq 2,\end{aligned}}$$

$$\Downarrow$$

$$|\psi\rangle=|+\rangle\otimes|-\rangle$$

※ CHSH 不等式と量子状態の関係を理解する上で，非常に意義深い.

A hidden-variables version of Gisin's Theorem

A hidden-variables version of Gisin's Theorem

$$\langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle = \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{1}|\psi\rangle\langle\psi|\mathbf{1}\otimes\mathbf{b}\cdot\sigma|\psi\rangle$$

Non-contextual and local hidden variables models の範囲内で満たされる等式.

この等式の証明は次の要請に基づく(詳細は時間の制限により省略):

The known concrete hidden-variables models in d=2 satisfy the relation:

Bell, Rev. Mod. Phys. (1966)

$$\langle\mathbf{1}\otimes(\mathbf{b}+\mathbf{b}')\cdot\sigma\rangle = \langle\mathbf{1}\otimes\mathbf{b}\cdot\sigma\rangle + \langle\mathbf{1}\otimes\mathbf{b}'\cdot\sigma\rangle$$

$\Downarrow$ *d*=4 でも成立することを要請

$$\langle\mathbf{a}\cdot\sigma\otimes(\mathbf{b}+\mathbf{b}')\cdot\sigma\rangle = \langle\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma\rangle + \langle\mathbf{a}\cdot\sigma\otimes\mathbf{b}'\cdot\sigma\rangle$$

$$\Rightarrow \quad \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle = \int\rho_1(\lambda_1)d\lambda_1 a(\psi,\lambda_1)\int\rho_2(\lambda_2)d\lambda_2 b(\psi,\lambda_2).$$

Numerical Test

Gisin’s example:

- 測定器の設定:

$$a_y = b_y = 0, \quad a_x = \sin\theta, \quad a_z = \cos\theta, \quad b_x = \sin\phi, \quad b_z = \cos\phi.$$

- 波動関数の設定:

$$|\psi\rangle = \frac{1}{\sqrt{|\alpha|^2 + |\beta|^2}} \left[\alpha|+\rangle_1|-\rangle_2 - \beta|-\rangle_1|+\rangle_2\right] \qquad \left(0 \le \alpha \le 1, \quad \beta = \sqrt{1-\alpha}. \right)$$

局所実在論の予言:

- CHSH inequality:

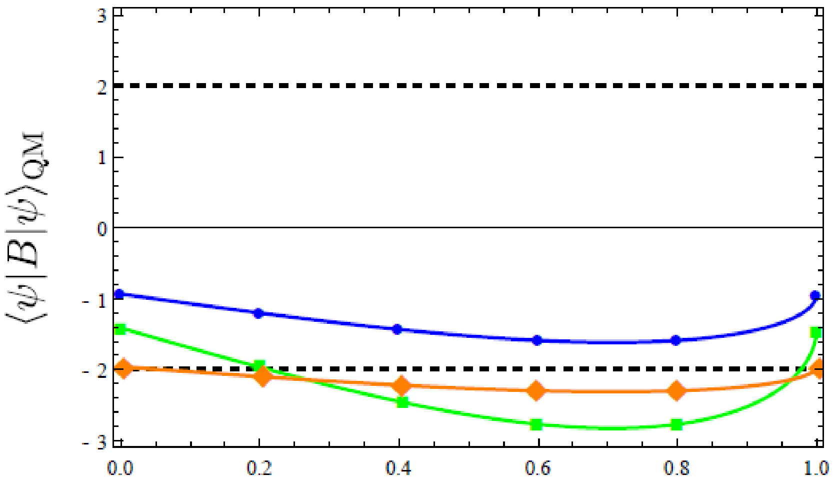
$$|\langle\psi|B|\psi\rangle_{CHSH}| \ \le \ 2,$$

- Hidden-variables version of Gisin’s theorem:

$$G(\mathbf{a},\mathbf{b}) \equiv \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle - \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{1}|\psi\rangle\langle\psi|\mathbf{1}\otimes\mathbf{b}\cdot\sigma|\psi\rangle = 0,$$

Results

$$\langle B\rangle = \langle \mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma\rangle + \langle \mathbf{a}\cdot\sigma\otimes\mathbf{b}'\cdot\sigma\rangle + \langle \mathbf{a}'\cdot\sigma\otimes\mathbf{b}\cdot\sigma\rangle - \langle \mathbf{a}'\cdot\sigma\otimes\mathbf{b}'\cdot\sigma\rangle.$$



$$\mathbf{a} = (\sin\theta, 0, \cos\theta),\ \mathbf{b} = (\sin\phi, 0, \cos\phi)$$

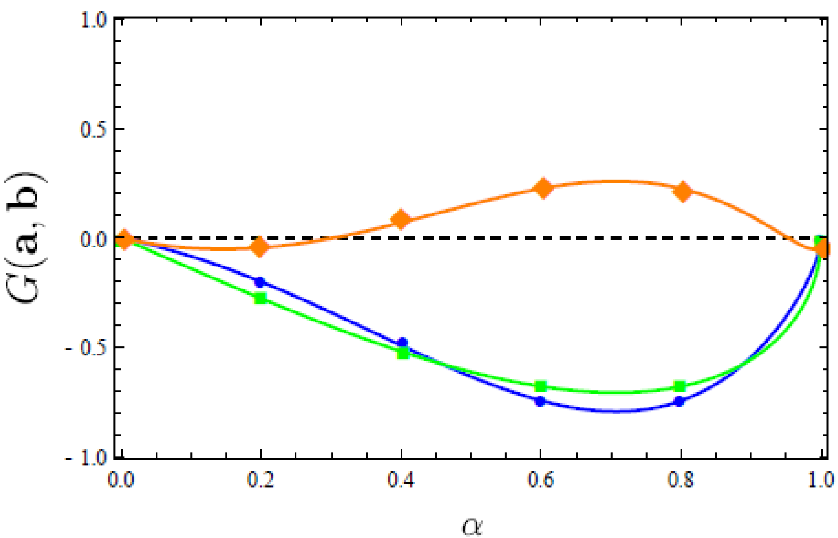
$$(\theta, \phi, \theta', \phi')$$

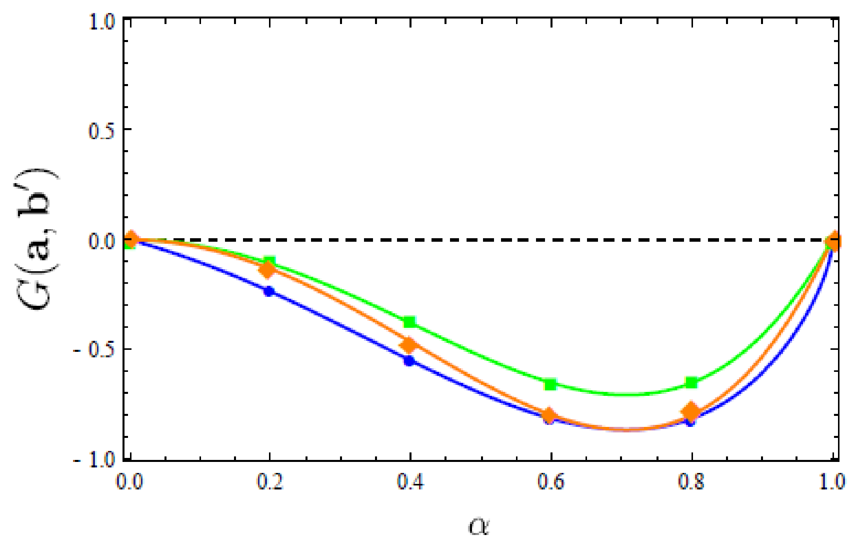
Case A: $\left(\frac{\pi}{3}, \frac{\pi}{8}, \frac{\pi}{4}, \frac{\pi}{6}\right)$

Case B: $\left(\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, 0\right)$

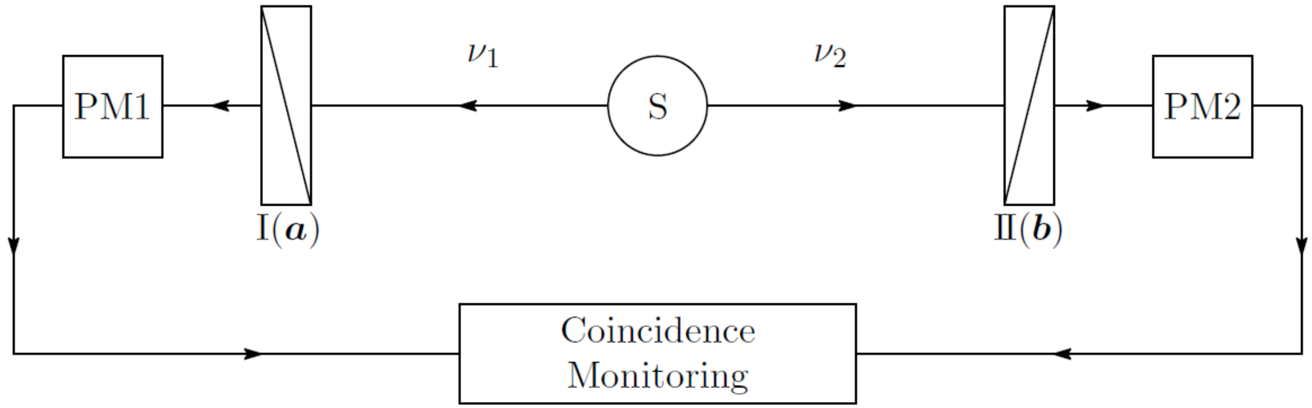
Case C: $\left(\frac{\pi}{6}, \frac{3\pi}{4}, \pi, 0\right)$

$$G(\mathbf{a},\mathbf{b}) \equiv \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle - \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{1}|\psi\rangle\langle\psi|\mathbf{1}\otimes\mathbf{b}\cdot\sigma|\psi\rangle$$





Aspect's Experiment



Notations 1:

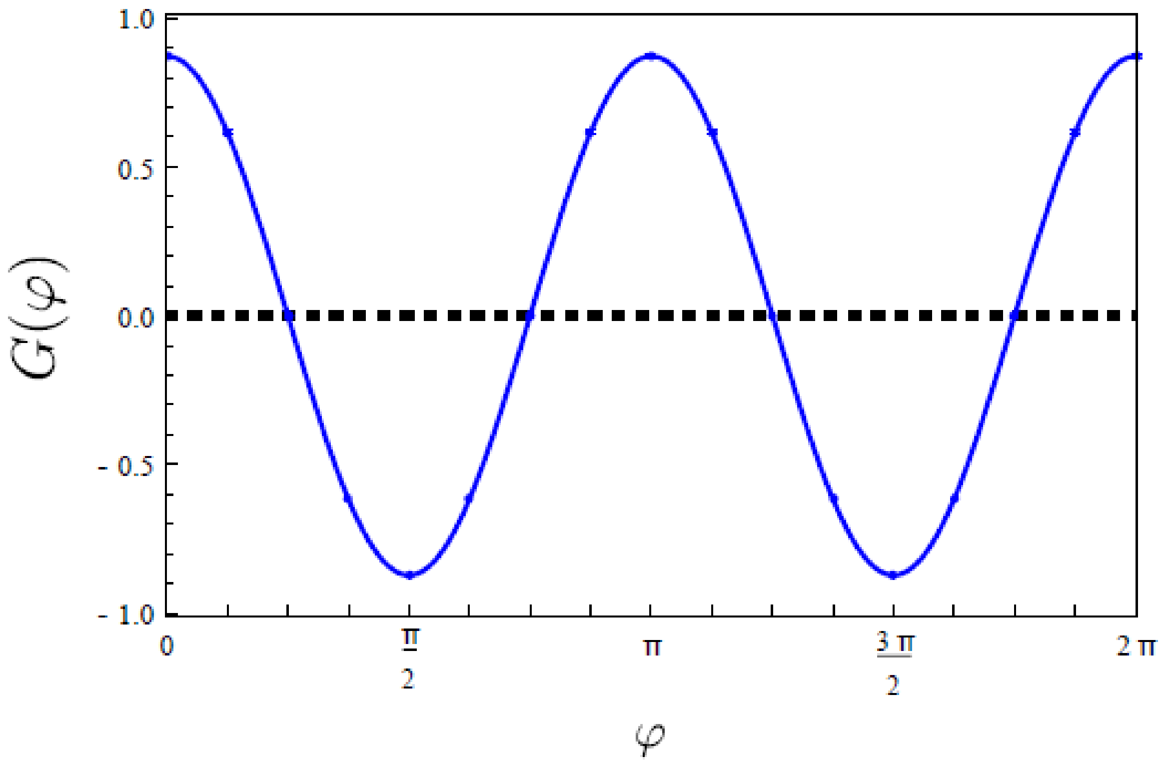
Freedman et.al. PRL (1972), Aspect et.al., PRL (1981)

- S: 光源　（光子対を放出）
- $\mathrm{I}_{\boldsymbol{a}}$, $\mathrm{II}_{\boldsymbol{b}}$: 偏光板　（ $\boldsymbol{a}$, $\boldsymbol{b}$ は調節可能なパラメータ）
- PM: Photomultiplier
  （光子が飛んで来れば+1, 飛んで来なければ -1 の値を取る. ）
- 同時検出が観測される.

Hidden-variable version of Gisin’s theorem

Aspect’s experimental values

$$G(\varphi) = 4[\frac{R(\varphi)}{R_0} - \frac{R_1 R_2}{R_0^2}] = (0.971 - 0.029)(0.968 - 0.028)0.984\cos 2\varphi$$



この実験結果は局所実在論の予言を明確に破る.

CHSH inequality

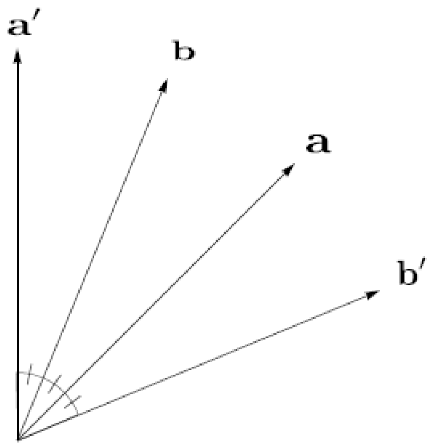
CHSH inequality

A. Aspect, et.al., PRL (1981)

$$\boxed{-1 \leq S = \frac{1}{R_0}\left[R(\boldsymbol{a},\boldsymbol{b}) + R(\boldsymbol{a},\boldsymbol{b}') + R(\boldsymbol{a}',\boldsymbol{b}) - R(\boldsymbol{a}',\boldsymbol{b}') - R_1(\boldsymbol{a}) - R_2(\boldsymbol{b})\right] \leq 0.}$$

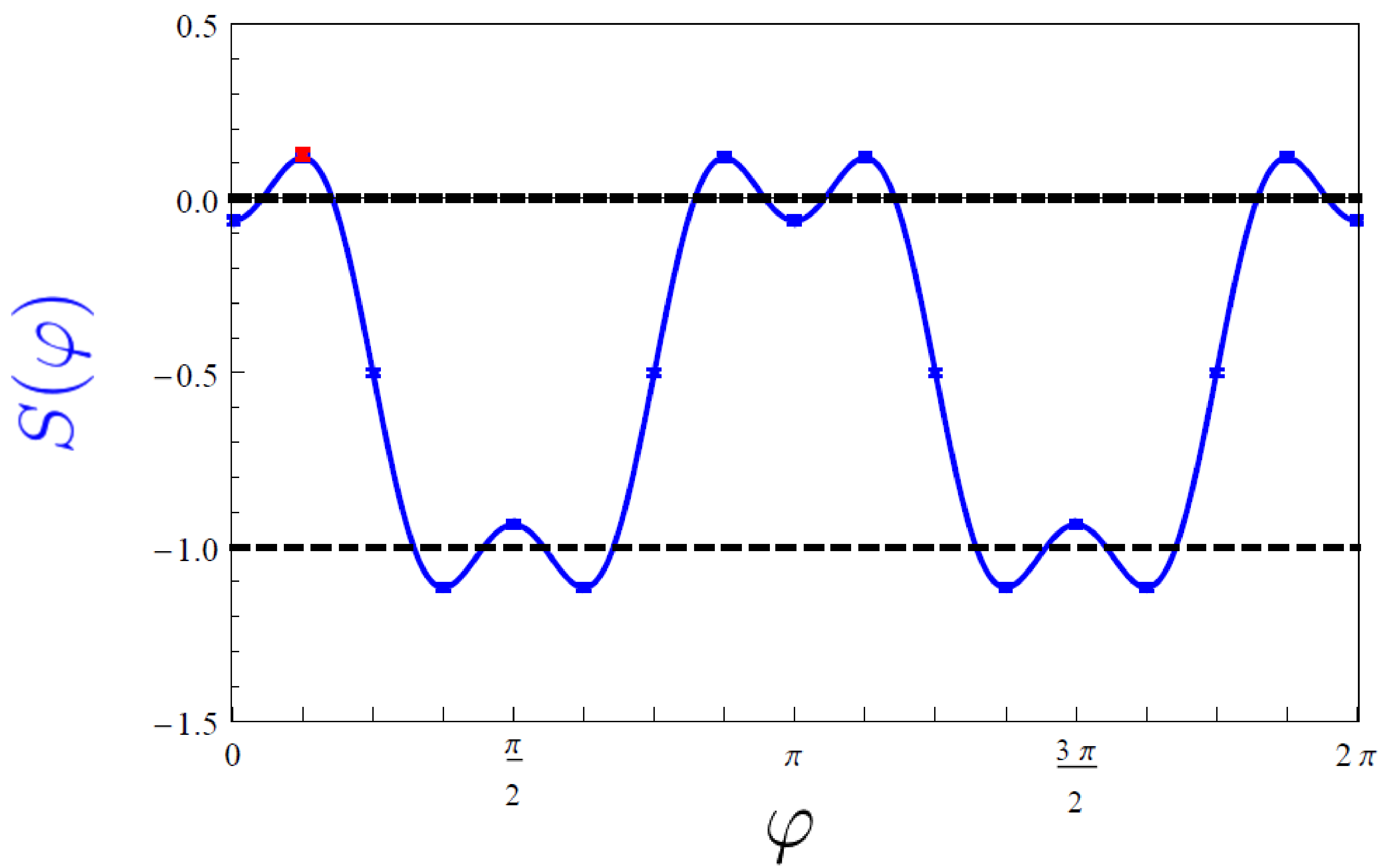
$$\varphi_{(\boldsymbol{a},\boldsymbol{b})} = \varphi_{(\boldsymbol{a},\boldsymbol{b}')} = \varphi_{(\boldsymbol{a}',\boldsymbol{b})} = \varphi, \quad \text{and} \quad \varphi_{(\boldsymbol{a}',\boldsymbol{b}')} = 3\varphi,$$

$$\boxed{S(\varphi) = \frac{3R(\varphi)}{R_0} - \frac{R(3\varphi)}{R_0} - \frac{R_1 + R_2}{R_0},}$$



CHSH inequality

$$S(\varphi) = \frac{3R(\varphi)}{R_0} - \frac{R(3\varphi)}{R_0} - \frac{R_1 + R_2}{R_0},$$



Summary

- Hidden-variables version of Gisin's theorem を定式化:

$$\boxed{\langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle = \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{1}|\psi\rangle\langle\psi|\mathbf{1}\otimes\mathbf{b}\cdot\sigma|\psi\rangle}$$

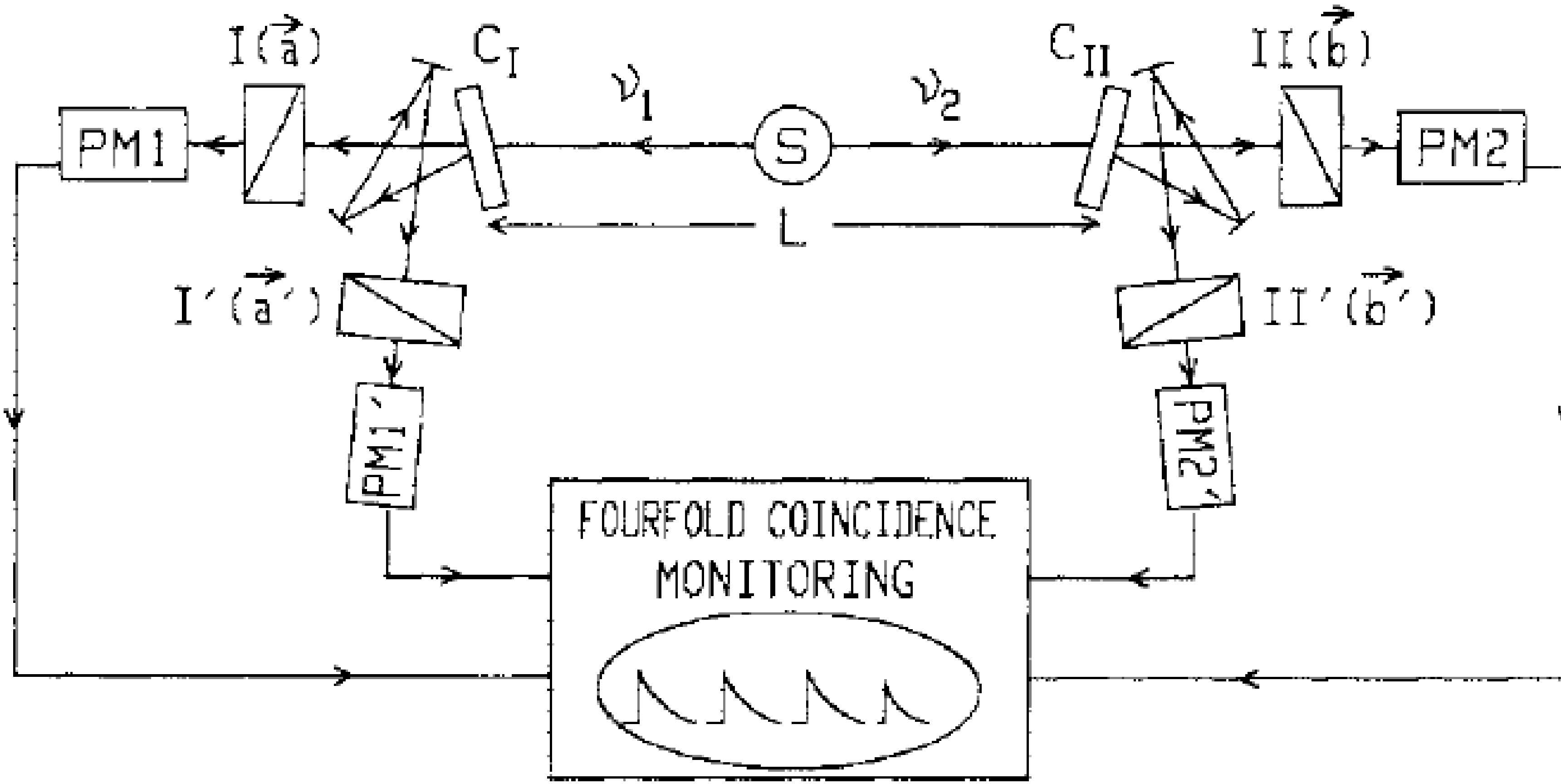
- CHSH inequality と比べ，Hidden-variables version of Gisin's theorem は量子力学と隠れた変数理論を区別できる領域が広い.

- 実験を行う立場からも，扱うパラメータが少ないため実験の手間は軽減される.

- このHidden-variables version of Gisin's theorem はCHSH inequality に代わる量子力学と局所実在論を区別する新しいcriterionになるかもしれない.

CHSH inequality and Aspect's experiment

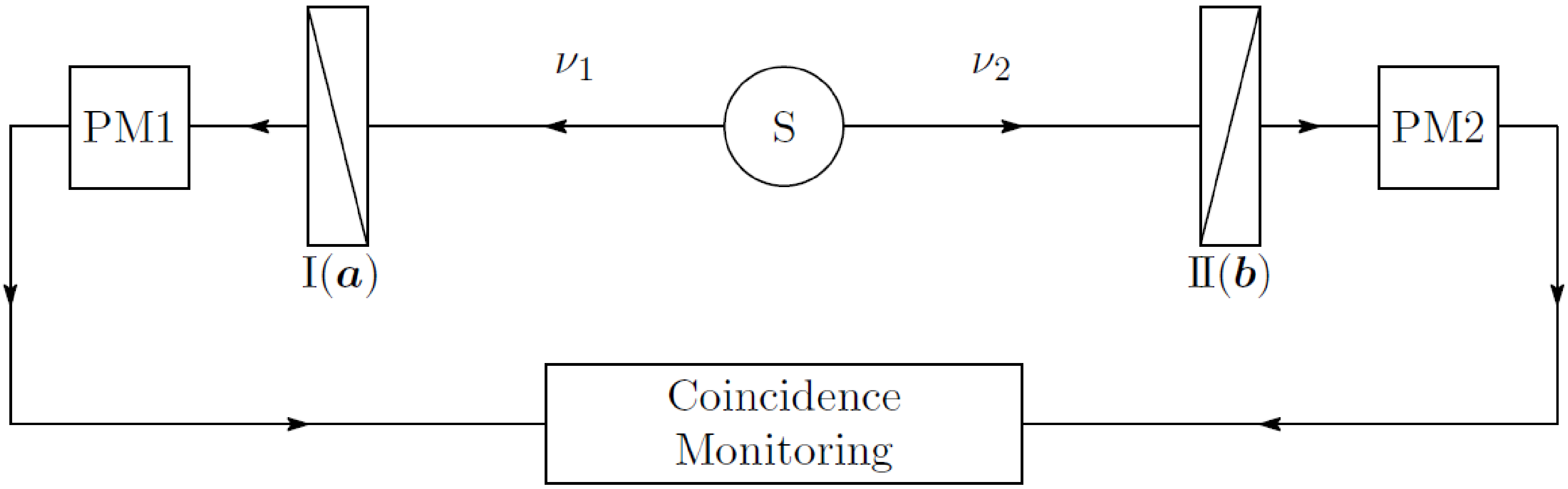
Aspect et.al., PRL (1982)



$$\langle B \rangle = \langle \mathbf{a} \cdot \sigma \otimes \mathbf{b} \cdot \sigma \rangle + \langle \mathbf{a} \cdot \sigma \otimes \mathbf{b}' \cdot \sigma \rangle + \langle \mathbf{a}' \cdot \sigma \otimes \mathbf{b} \cdot \sigma \rangle - \langle \mathbf{a}' \cdot \sigma \otimes \mathbf{b}' \cdot \sigma \rangle.$$

Hidden-variables version of Gisin's theorem and Aspect's experiment

Freedman et.al. PRL (1972), Aspect et.al., PRL (1981)



$$G(\mathbf{a},\mathbf{b}) \equiv \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{b}\cdot\sigma|\psi\rangle - \langle\psi|\mathbf{a}\cdot\sigma\otimes\mathbf{1}|\psi\rangle\langle\psi|\mathbf{1}\otimes\mathbf{b}\cdot\sigma|\psi\rangle$$