Massive Supergravity

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1 Introduction

Cosmological Constant Problem:

$$\begin{aligned} \text{Higgs Condensation} \sim & \quad (\ 100\,\text{GeV}\ )^4 \\ \text{QCD Chiral Condensation} \sim & \quad (\ 100\,\text{MeV}\ )^4 \end{aligned} \tag{1}$$

These seem not contributing to the Cosmological Constant!

$\Longrightarrow$ Massive Gravity: an idea toward resolving it

However, Massive Gravity has its own problems:

- van Dam-Veltman-Zakharov (vDVZ) discontinuity

  Its $m \to 0$ limit does not coincides with the Einstein gravity.

- Boulware-Deser ghost

$$\underbrace{10}_{h_{\mu\nu}} - \underbrace{(1+3)}_{N=h_{00},\ N^i=h_{0i}} \quad = \quad 6 \quad = \quad \underbrace{5}_{\text{massive spin2}} + \underbrace{1}_{\text{BD ghost}} \tag{2}$$

We focus on the BD ghost problem here.

In addition, we believe that any theory should eventually be made supersymmetric, that is, Supergravity (SUGRA).

This may be of help also for the problem that the dRGT massive gravity allows no stable homogeneous isotropic universe solution.

In this talk, we

1. explain the dRGT theory
2. massive supergravity

2 Fierz-Pauli massive gravity (linearized)

Einstein-Hilbert action

$$\mathcal{L}_{\mathrm{EH}} = \sqrt{-g}R \tag{3}$$

$$\mathcal{L} = \Big[\mathcal{L}_{\mathrm{EH}}\Big]_{\text{quadratic part in } h_{\mu\nu}} + \underbrace{\left[-\frac{m^2}{4}(h_{\mu\nu}^2 - ah^2)\right]}_{=\ \mathcal{L}_{\mathrm{FP}}^{\mathrm{mass}}(a=1)} \tag{4}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{5}$$

In Fierz-Pauli theory with $a = 1$, there are only 5 modes describing properly massive spin 2 particle.

$\because$) No time derivative appears for h_{00}, h_{0i} in $\mathcal{L}_{\rm EH} \to \mathcal{L}_{\rm EH}$ is linear in $N,\ N_i$.

If $a = 1$, the mass term $\mathcal{L}_{\rm FP}^{\rm mass}$ is also clearly linear in $N \sim h_{00}$!

$\Longrightarrow$

- N_i can be solved algebraically and be eliminated.
- N e.o.m. $\frac{\delta S}{\delta N} = 0$ gives 1 constraint on other fields since S is linear in N so that

$$\underbrace{10}_{h_{\mu\nu}} - \underbrace{3}_{N_i} - (\underbrace{1}_{N} + \underbrace{1}_{\text{constraint}}) = 5 \tag{6}$$

Nonlinear completion of this theory was proposed by

dRGT: de Rham-Gabadadze-Tolley, Phys. Rev. Lett. 106 (2011)

which is claimed to be free of BD ghost on arbitrary background and to connect smoothly to Einstein gravity as $m \to 0$ by Vainshtein mechanism.

They use the Stückelberg field formalism by AHGS.

3 Arkani-Hamed-Georgi-Schwartz : Stückelberg formalism

Ann. Phys. 305 (2003) 96; the work preceding to dRGT.

AHGS have rewritten the Fierz-Pauli theory into GC invariant form: GC invariance is realized as a Fake Symmetry, or Hidden Local Symmetry.

The simplest case is the "two site model", in which case easiest way to understand is to regard it as "space-time filling d-brane" in $D = d + 1$ dimensional target space-time.

$$\begin{aligned} &\text{Target Space :} \quad X^M \quad \text{with metric} \quad G_{MN}(X) = \eta_{MN} \\ &\text{brane (world sheet) :} \quad x^\mu \quad \text{with metric} \quad g_{\mu\nu}(x) \end{aligned} \tag{7}$$

Embedding function

$$X^M = Y^M(x) \tag{8}$$

Induced metric on the brane

$$\boxed{f_{\mu\nu}(x) = \partial_\mu Y^M(x) \cdot \eta_{MN} \cdot \partial_\nu Y^N(x) \qquad (9)}$$

From world sheet viewpoint,

$$Y^M(x) \quad : \quad D \text{ scalar functions}$$
$$\text{then, } \Rightarrow f_{\mu\nu}(x) : \text{GC tensor} \tag{10}$$

GC-invariant form of FP theory by AHGS:

$$\begin{aligned} \mathcal{L}_{\text{AHGS}} &= \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{AHGS}}^{\text{mass}} \\ \mathcal{L}_{\text{AHGS}}^{\text{mass}} &= -\frac{m^2}{4}\sqrt{-g}g^{\mu\nu}g^{\alpha\beta}\left(H_{\mu\alpha}H_{\nu\beta} - aH_{\mu\nu}H_{\alpha\beta}\right) \end{aligned} \tag{11}$$

where

$$\begin{aligned} H_{\mu\nu} &= g_{\mu\nu} - f_{\mu\nu} \\ &= g_{\mu\nu} - \partial_\mu Y^M \cdot \eta_{MN} \cdot \partial_\nu Y^N \end{aligned} \tag{12}$$

is a GC tensor and the AHGS lagrangian $\mathcal{L}_{\text{AHGS}}$ is clearly GC invariant.

This is achieved by the introduction of the mapping function $Y^M(x)$ which is analogous to $g^{5M}(x)$ from deconstruction point of view.

$$Y^M(x) = x^\mu\, \delta_\mu^M + \phi^M(x) \tag{13}$$

$\phi^M = 0$: "Unitary Gauge" (or, "static gauge" from brane viewpoint)

$$\Longrightarrow \quad \partial_\mu Y^M(x) = \delta_\mu^M \quad \Longrightarrow \quad f_{\mu\nu}(x) = \eta_{\mu\nu} \tag{14}$$

This mass term reduces to $\mathcal{L}_{\mathrm{FP}}^{\mathrm{mass}}$ at linearized level.

We can see more explicitly the absence of BD-ghost in this AHGS formulation of massive gravity on flat background $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$.

Generally, before fixing gauge,

$$\boxed{f_{\mu\nu} = \partial_\mu Y^M \eta_{MN} \partial_\nu Y^N = \eta_{\mu\nu} + \partial_\mu \phi_\nu(x) + \partial_\nu \phi_\mu(x) + \partial_\mu \phi^M \cdot \partial_\nu \phi_M(x)}$$

Introduce a Stückelberg Vector A_μ and scalar field π by writing

$$\boxed{\phi_\mu(x) \equiv \frac{1}{m} A_\mu(x) - \frac{1}{m^2} \partial_\mu \pi(x)}$$

Then, clearly this system is invariant under the GC and additional U(1) gauge transformation independently of a value:

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad \delta A_\mu = m\xi_\mu + \partial_\mu \Lambda, \quad \delta \pi = m\Lambda \tag{15}$$

For the Fierz-Pauli value $a = 1$, we can quantize the system as usual based on the BRS symmetry.

Counting of physical degrees of freedom:

$$\underbrace{10 \;+\; 4 \;+\; 1}_{g_{\mu\nu}+A_\mu+\pi} - (\underbrace{4 \;+\; 4}_{\text{GCghosts:}c_\mu+\bar{c}_\mu}) - (\underbrace{1 \;+\; 1}_{\text{U(1)ghosts:}c+\bar{c}}) = 5 \qquad ! \tag{16}$$

Note that U(1) gauge invariance was a fake gauge symmetry which was brought into the system by introducing the Stückelberg scalar π.

But it gave subtracting 2 modes $c + \bar{c}$.

Isn't this STRANGE ?

The point is that usually

$$H_{\mu\nu} \supset \partial_\mu \phi_\nu \supset \partial_\mu \partial_\nu \pi H_{\mu\nu}^2 - a\, H^2 \supset (1-a)\Box\pi \cdot \Box\pi$$

That is, When $a \neq 1$ there appears Higher Derivative Term so that the single field π actually contains (1+1)-modes! (one of them is of negative metric = BD ghost).

So the problem is boiled down to confirm that the absence of higher derivative term for π.

AHGS model (Fierz-Pauli at linearlized level on flat background) con tains BD ghost apart from the flat background!

4 "Ghost-free" massive gravity of de Rham-Gabadadze-Tolley

PRL 106 (2011)

$$\boxed{\mathcal{L} = \mathcal{L}_{\rm EH} - \frac{m^2}{4}\sqrt{-g}\,U(g_{\mu\nu}, H_{\mu\nu})}$$

dRGT have given their mass term U as follows:

Noting

$$(g^{-1}H)^{\mu}{}_{\nu} = g^{\mu\lambda}(g_{\lambda\nu} - f_{\lambda\nu}) = \delta^{\mu}{}_{\nu} - (g^{-1}f)^{\mu}{}_{\nu} \tag{17}$$

they define a tensor $K^{\mu}{}_{\nu}$ by

$$K^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \sqrt{(g^{-1}f)^{\mu}{}_{\nu}} \tag{18}$$

Define a generating function

$$\begin{aligned}\det(\delta^{\mu}{}_{\nu} + \lambda K^{\mu}{}_{\nu}) &= (1/4!)\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}(\delta+K)_{\mu}{}^{\alpha}(\delta+K)_{\nu}{}^{\beta}(\delta+K)_{\rho}{}^{\gamma}(\delta+K)_{\sigma}{}^{\delta} \\ &= 1 + \lambda U^{(1)}(K) + \lambda^2 U^{(2)}(K) + \lambda^3 U^{(3)}(K) + \lambda^4 U^{(4)}(K)\end{aligned}$$

giving $U^{(n)}(K)$ $(n = 1, 2, 3, 4)$

$$\begin{aligned}
U^{(1)}(K) &= \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\alpha\nu\rho\sigma} K^{\mu}{}_{\alpha} = 3!\,[K] \\
U^{(2)}(K) &= \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\rho\sigma} K^{\mu}{}_{\alpha} K^{\nu}{}_{\beta} \\
&= 2\left([K^2] - [K]^2\right) \quad \rightarrow \ \text{Fierz-Pauli} \\
U^{(3)}(K) &= \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma\sigma} K^{\mu}{}_{\alpha} K^{\nu}{}_{\beta} K^{\rho}{}_{\gamma} \\
U^{(4)}(K) &= \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma\delta} K^{\mu}{}_{\alpha} K^{\nu}{}_{\beta} K^{\rho}{}_{\gamma} K^{\sigma}{}_{\delta}
\end{aligned} \tag{19}$$

Then the dRGT mass term is given:

$$\boxed{\sqrt{-g}\,U(g,\,H) = \sqrt{-g}\left(2U^{(2)}(K) + \alpha_3 U^{(3)}(K) + \alpha_4 U^{(4)}(K)\right)}$$

minimal model $\alpha_3 = \alpha_4 = 0$.

Focusing only the Stückelberg scalar π on the flat background $g_{\mu\nu} = \eta_{\mu\nu}$, decoupling limit,

$$\begin{aligned}\phi_\mu(x) = A_\mu(x) + \partial_\mu \pi(x) &\Rightarrow \phi_\mu(x) = -\partial_\mu \pi(x) \\ &\Rightarrow \partial^\mu \phi_\nu(x) = \partial^\mu \partial_\nu \pi(x) \equiv \Pi^\mu{}_\nu \end{aligned} \tag{20}$$

$$\begin{aligned} g^{-1} f \to \eta^{-1} f &= 1 + \eta^{-1}(\partial\phi) + \eta^{-1}(\partial\phi)^T + \eta^{-1}(\partial\phi)\eta^{-1}(\partial\phi)^T \\ &= 1 + 2\Pi + \Pi^2 = (1+\Pi)^2 \end{aligned} \tag{21}$$

so that, in decoupling limit,

$$K = \sqrt{g^{-1} f} - 1 \to \Pi^\mu{}_\nu = \partial^\mu \partial_\nu \pi(x) \tag{22}$$

these $U^{(n)}(\Pi)$ $(n = 1, 2, 3, 4)$ give total derivative. No higher derivatives appear.

Generating function of general mass term:

$$\sqrt{-g} U(g,\, H) = \sqrt{-g}\,\det(1 + \lambda\sqrt{g^{-1} f}) \tag{23}$$

5 Massive Supergravity

5.1 dRGT massive gravity theory can be obtained by dimensional reduction $d = 5 \to 4$

dRGT have shown that the 2 site model of 5d Einstein theory, written in terms of vierbein discretized on 2 points y_1 and y_2 on 5-th dimension, yields after fixing all the 5d gauge symmetries

$$\int \varepsilon_{abcd} \left(R_1^{ab} \wedge e_1^c \wedge e_1^d + m^2 (e_1 - e_2)^a \wedge (e_1 - e_2)^b \wedge e_1^c \wedge e_1^d \right) + (1 \leftrightarrow 2) \tag{24}$$

If both fields $e_1 \equiv e$ and $e_2 \equiv f$ on y_1 and y_2 are taken as dynamical $\to$ bigravity.

If the field $e_{2,\mu}^{a} = f_\mu^a$ on y_2 are fixed to, e.g., $f_\mu^a = \delta_\mu^a$, then dRGT massive gravity for $e_{1,\mu}^a \equiv e_\mu^a$.

$$\begin{aligned} &\int \varepsilon_{abcd} m^2 (e - f)^a \wedge (e - f)^b \wedge e^c \wedge e^d) \\ &\to \int d^4x \sqrt{-g} m^2 U^{(2)}(K) = \int d^4x \sqrt{-g} m^2 2 \delta^\mu_{[a} \delta^\nu_{b]} K^\mu{}_a K^\nu{}_b \end{aligned} \tag{25}$$

Performing the similar thing on 2site model of 5d SUGRA is naively expected to yield a SUSY version of the massive gravity, massive SUGRA. Unfortunately, however, we were not yet able to get massive SUGRA, since the SUSY is fragil against discretization.

But in any case, it gives us strong clue for the possible massive SUGRA. so consider the KK reduction of $d = 5$ SUGRA, which consists of the fields

$$e_M^A(x,y),\ \psi_M^i(x,y),\ A_M(x,y), \qquad (i = 1, 2 : \mathrm{SU}(2)\ \text{label}) \tag{26}$$

5.2 Kaluza-Klein counting

Consider 5d SUGRA on $M_4 \times S^1$, and expansion into KK modes:

$$\varphi(x,\, y) = \sum_{n \in Z} \varphi_{(n)}(x)\, e^{inmy}\ , \qquad m \equiv \frac{1}{R} \tag{27}$$

$$
\begin{array}{rcccc}
\text{comp.} & \text{gauge trfs} & n=0 & n=\pm 1,\pm 2,\cdots \\
e^a_\mu : & \delta e^a_{\mu(n)} = \partial_\mu \xi^a_{(n)} & \text{massless } (j=2)\ 2 & \text{massive } (j=2)\ 5=2+2+1 \\
 & & & \uparrow \\
e^4_\mu : & \delta e^4_{\mu(n)} = \partial_\mu \xi^4_{(n)} & \text{massless } B_\mu\ (j=1)\ 2 & \text{NG } (j=1)\ 2 \\
e^a_y : & \delta e^a_{y(n)} = inm\xi^a_{(n)} & 0 \text{ by LL gauge} & 0 \text{ by LL gauge} \\
 & & & \uparrow \\
e^4_y : & \delta e^4_{y(n)} = inm\xi^4_{(n)} & \text{massless } \phi\ (j=0)\ 1 & \text{NG } (j=0)\ 1 \\
 & \text{---} & \text{---} & \text{---} \\
A_\mu : & \delta A_{\mu(n)} = \partial_\mu \theta_{(n)} & \text{massless GB } (j=1)\ 2 & \text{massive } (j=1)\ 3=2+1 \\
 & & & \uparrow \\
A_y : & \delta A_{y(n)} = inm\theta_{(n)} & \text{massless } (j=0)\ 1 & \text{NG } (j=0)\ 1 \\
 & \text{---} & \text{---} & \text{---} \\
\psi^i_\mu : & \delta \psi^i_{\mu(n)} = \partial_\mu \varepsilon^i_{(n)} & \text{massless GF } (j=3/2)\ 2 & \text{massive } (j=3/2)\ 4=2+2 \\
 & & & \uparrow \\
\psi^i_y : & \delta \psi^i_{y(n)} = inm\varepsilon^i_{(n)} & \text{massless } (j=1/2)\ 2 & \text{NG fermion } (j=1/2)\ 2 \\
 & \text{---} & \text{---} & \text{---}
\end{array}
\tag{28}
$$

$n = 0$ sector is identical with $N = 2$ $d = 4$ SUGRA, obtained by trivial reduction.

The massive towers with $n = \pm 1, \pm 2, \cdots$ for each n look like

$$N = 1 \text{ massive multiplet} \quad 1\, e^a_{\mu(n)} + 2\, \psi^i_{\mu(n)} + 1\, A_{\mu(n)} \ , \tag{29}$$

which actually gives $N = 2$ massive multiplets for each n with central charge $m = Z = p^4$. But note that they are complex; $\varphi^\dagger_{(n)} = \varphi_{(-n)}$.

Clearly, at the linearlized level, those KK modes $\varphi_{(n)}$ close within any pair $+|n|$ and $-|n|$ for any n under the $y \equiv x^4$-independent SUSY transformation $\varepsilon_{(n=0)}(x)$ as well as under the inhomogeneous shift trf $\partial_M \varepsilon_{(n)}(x)$ (because of p_4 momentum conservation).

So we can truncate to $n = \pm 1$ modes on flat background, or cosine and sine real modes:

$$\varphi(x)^{\cos} = \frac{1}{2}\big(\varphi_{(1)} + \varphi_{(-1)}\big), \qquad \varphi(x)^{\sin} = \frac{1}{2i}\big(\varphi_{(1)} - \varphi_{(-1)}\big), \tag{30}$$

We can further truncate to only y-parity even modes:

$$e_\mu^{a\,\cos}, \qquad e_\mu^{4\,\sin}, \qquad e_y^{a\,\cos}, \qquad e_y^{4\,\sin}, \qquad A_\mu^{\sin}, \qquad A_y^{\cos} \tag{31}$$

$$\psi_{\mu+}^{\cos}, \qquad \psi_{\mu-}^{\sin}, \qquad \psi_{y+}^{\sin}, \qquad \psi_{y-}^{\cos} \tag{32}$$

where, for 5D SU(2) Majorana spinors $\psi_{i=1,2}$,

$$\begin{aligned} \psi_+ &\equiv \psi_{1\mathrm{R}} + \psi_{2\mathrm{L}} \\ \psi_- &\equiv \psi_{2\mathrm{R}} - \psi_{1\mathrm{L}}, \end{aligned} \tag{33}$$

are both 4D Majorana $\overline{\psi_\pm} = \psi_\pm^{\mathrm{T}} C$.

Then the SUSY invariance also reduces to

$$\begin{aligned} \varepsilon_{(0)}^i(x) \;&\rightarrow\; \varepsilon_{(0)+} \equiv \varepsilon_+ : \text{const} \qquad \leftarrow \because \text{ fixed to flat backgrond} \\ \varepsilon_{(\pm 1)}^i(x) \;&\rightarrow\; \varepsilon_+^{\cos}(x) \equiv \eta_+(x), \quad \varepsilon_-^{\sin}(x) \equiv \eta_-(x) \end{aligned} \tag{34}$$

5.3 Linearized theory by Kaluza-Klein truncation of 5D SUGRA

Action:

With definitions

$$e_\mu{}^a \eta_{a\nu} \equiv e_{\mu\nu}, \qquad h_{\mu\nu} \equiv e_{\mu\nu} + e_{\nu\mu} \equiv 2e_{(\mu\nu)}, \quad e_\mu{}^4 \equiv B_\mu e^\phi, \quad e_y{}^4 = e^\phi$$
$$h \equiv h^\mu_\mu, \qquad h_\mu \equiv \partial^\nu h_{\mu\nu} = 2e^\mu_\mu, \tag{35}$$

the Action at the linearlized level is ($e_y{}^a = 0$ gauge):

$$\begin{aligned} S = & \frac{1}{4} h^{\mu\nu}\Big(\partial_\mu\partial_\nu h - 2\partial_{[\mu}h_{\nu]} + \Box h_{\mu\nu} + \eta_{\mu\nu}\big(\partial^\lambda h_\lambda - \Box h\big)\Big) \\ & - \frac{1}{4} F_{\mu\nu}{}^2(A) - 2i \sum_{i=\pm} \bar{\psi}_{\mu i}\gamma^{\mu\nu\rho}\partial_\nu \psi_{\rho i} \\ & + \Big(\frac{m}{2}h_{\mu\nu} + \partial_{(\mu}B_{\nu)} + \frac{1}{m}\partial_\mu\partial_\nu\phi\Big)^2 - \Big(\frac{m}{2}h + \partial\cdot B + \frac{1}{m}\Box\phi\Big)^2 \\ & + \frac{1}{2}m^2\big(A_\mu + \frac{1}{m}\partial_\mu A_y\big)^2 + (-2i)4m\big(\bar{\psi}_{\mu+} - \frac{1}{m}\partial_\mu\bar{\psi}_{y+}\big)\gamma^{\mu\nu}\big(\psi_{\nu-} + \frac{1}{m}\partial_\nu\psi_{y-}\big) \end{aligned}$$

Transformation law: (ε_+: const Weyl spinor)

$$
\begin{aligned}
\delta e_\mu{}^a &= -2i\bar{\varepsilon}_+\gamma^a\Big(\psi_{\mu+} - \frac{1}{m}\partial_\mu\psi_{y+}\Big) + \partial_\mu\xi^a \\
\delta e_\mu{}^4 &= \delta B_\mu = 2\bar{\varepsilon}_+\psi_{\mu-} - m\xi_\mu + \partial_\mu\xi^4 \\
\delta e_y{}^4 &= \delta\phi = 2\bar{\varepsilon}_+\psi_{y-} - m\xi^4 \\
\delta A_\mu &= \sqrt{6}\bar{\varepsilon}_+ i\gamma_5\psi_{\mu-} + \partial_\mu\theta \\
\delta A_y &= \sqrt{6}\bar{\varepsilon}_+ i\gamma_5\psi_{y-} - m\theta
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
\delta\psi_{\mu+} &= \partial_\mu\eta_+ - \frac{1}{4}\omega_{\mu,ab}\gamma^{ab}\varepsilon_+ + \frac{1}{2\sqrt{6}}\big(\gamma_{\mu\nu} - 2\eta_{\mu\nu}\big)\gamma_5\varepsilon_+(\partial_\nu A_y + mA^\nu) \\
&\qquad \text{with} \quad \omega_{\mu,ab} = \partial_\mu e_{[ab]} - \partial_{[a}h_{b]\mu} \\
\delta\psi_{y+} &= m\eta_+ - \frac{1}{4}\Big(\frac{1}{2}F_{\mu\nu}(B) + me_{[\mu\nu]}\Big)\gamma^{\mu\nu}\varepsilon_+ - \frac{i}{4\sqrt{6}}\gamma^{ab}\gamma_5\varepsilon_+F_{ab}(A) \\
\delta\psi_{\mu-} &= \partial_\mu\eta_- - \frac{i}{4}(F_{\mu\nu}(B) - mh_{\mu\nu})\gamma^\nu\varepsilon_+ - \frac{1}{4\sqrt{6}}\big(\gamma_{\mu ab} - 4\eta_{\mu a}\gamma_b\big)\gamma_5\varepsilon_+F^{ab}(A) \\
\delta\psi_{y-} &= -m\eta_- + \frac{i}{2}(\partial_\mu\phi + mB_\mu)\gamma^\mu\varepsilon_+ - \frac{1}{\sqrt{6}}(\partial_\mu A_y + mA^\mu)\gamma^\mu\gamma_5\varepsilon_+
\end{aligned}
\tag{37}
$$

5.4 Toward full super version of the dRGT theory

In the dRGT theory, the AHGS's ückeberg field was used to recover the GC invariance; the mapping function Y^M introduced by AHGS:

$$f^a_\mu(x) = \partial_\mu Y^M(x)\,\delta^A_M\,u^a_A(x) \tag{38}$$

$Y^M(x)$ is the coordinate in the 'target space', so that it must be 4 world scalar functions.

$$\begin{aligned} Y^M(x) = x^\mu\delta^M_\mu + \phi^M(x) &: \text{Stückelberg for GC} \\ \phi_\mu(x) = \phi^M\eta_{M\mu} = B_\mu + \partial_\mu\pi &: \pi(x)\ \text{Stückelberg for U(1)} \\ u^a_A(x) &: \text{Stückelberg for LL} \end{aligned} \tag{39}$$

If Y^M is treated as a world scalar, the induced vierbein $f^a_\mu(x)$ exactly transforms as a vierbein on the worldsheet, and so the mass term

$$\int m^2\varepsilon_{abcd}\left((e-f)^a\wedge(e-f)^b\wedge e^c\wedge e^d)\right) \tag{40}$$

is GC invariant.

It should be possible to extend this to the SUGRA case: introduce

$$Y^M(x) \to \text{a supermultiplet} \begin{cases} Y^M(x) & \text{target space vector} \\ \lambda_\pm(x) & \text{target space spinors} \\ \theta(x) & \text{target space scalar} \end{cases} \tag{41}$$

as Stückelberg fields so as to make

$$\begin{aligned} \text{induced viebein} : f^a_\mu(x) &= \partial_\mu Y^M(x)\, \delta^A_M\, u^a_A(x) \\ \text{induced Rarita-Schwinger} : \Psi_{\mu\pm,\alpha}(x) &= \partial_\mu \lambda_{\pm,\hat{\alpha}}(x)\, \mathcal{U}^{\hat{\alpha}}_\alpha(x) \\ \text{induced vector} : f_\mu(x) &= \partial_\mu \theta(x) \end{aligned} \tag{42}$$

Then the mass term, which should be $N = 2$ SUGRA invariant, will take the form

$$\begin{aligned} \mathcal{L}_{\text{boson mass}} &= \frac{M_P^2}{4} m^2 \varepsilon^{\mu\nu\rho\sigma} \varepsilon_{abcd} (e^a_\mu - f^a_\mu)(e^b_n - f^b_\nu) e^c_\rho e^d_\sigma - \frac{1}{2} m^2 (A_\mu - f_\mu)^2 \\ \mathcal{L}_{\text{fermion mass}} &= m(\bar{\psi}_{\mu+} - \bar{\Psi}_{\mu+}) \gamma^{\mu\nu} \gamma_5 (\psi_{\nu-} - \Psi_{\nu-}) \end{aligned} \tag{43}$$

The invariance under the GC and LL transformation is clear. Non-trivial is the SUSY.

We are at present trying to find the full SUSY transformation rule, from the linearlized result. But very non-trivial is the SUSY transformation rule for the Stückelberg fields Y^M, λ^i and u^a_A. We infer something like

$$
\begin{aligned}
\delta Y^M &= (\bar{\varepsilon}^i \gamma^\rho \lambda_i) \partial_\rho Y^M \\
\delta \lambda^i &= \varepsilon^i + (\bar{\varepsilon}^i \gamma^\rho \lambda_i) \partial_\rho \lambda^i \\
\delta u^a_A &= \cdots
\end{aligned}
\tag{44}
$$

which should, probably, reproduce the 'usual' N=2 SUGRA transformation for the induced SUGRA multiplet f^a_μ, Ψ^i_μ, f_μ:

$$
\begin{aligned}
\delta f^a_\mu &= -2i \bar{\varepsilon}^i \gamma^a \Psi_{\mu i} \\
\delta \Psi^i_\mu &= \left(\partial_\mu - \frac{1}{4} \Omega_\mu{}^{ab}(f, \Psi) \gamma_{ab} \right) \varepsilon^i - \frac{\sqrt{6}}{8} \left(\gamma_{\mu ab} - \frac{2}{3} \gamma_\mu \gamma_{ab} \right) \hat{F}^{ab}(f) \\
\delta f_\mu &= \sqrt{6} i \bar{\varepsilon}^i \Psi_{\mu i}
\end{aligned}
\tag{45}
$$

5.5 mode number counting

If we succeeded in constructing this mass term invariant under the $N = 2$ SUGRA transformation, the mode counting after quantization a la BRST will be the following:

Graviton sector:

$$
\begin{array}{llcl}
g_{\mu\nu} & & : & 10 \\
Y^M \ \rightarrow & \left\{ \begin{array}{l} \phi_\mu \\ \partial_\mu \pi \end{array} \right. & \begin{array}{c} : \\ : \end{array} & \begin{array}{l} 4 \\ 1 \end{array} \\
\text{GC ghost} & c_\mu, \ \bar{c}_\mu & : & -4 \times 2 \\
\text{U(1) ghost} & c, \ \bar{c} & : & -1 \times 2 \\
\multicolumn{4}{l}{10 + 4 + 1 - (4 \times 2 + 1 \times 2) = 5}
\end{array}
\tag{46}
$$

Gravitino sector

$$\begin{aligned} &\psi_{\mu\pm} : 4 \times 4(\mu) \times 2(\pm) \\ \text{Nakanishi-Lautrup spinor} \quad &\bar{B}_{\pm}\partial^{\mu}\psi_{\mu\pm} : 4 \times 2(\pm) \\ &\lambda_{\pm} : 4 \times 2 \\ \text{super ghosts} \quad &\gamma_{\pm},\ \bar{\gamma}_{\pm} : 4 \times 2(\pm) \times 2 \end{aligned} \tag{47}$$

The fields $\psi_{\mu\pm},\ B_{\pm},\ \lambda_{\pm}$ all have first order derivative kinetic terms so that the counting reads

$$\frac{4 \times 4 \times 2 + 4 \times 2 + 4 \times 2}{2} - 4 \times 2 \times 2 = 4 \times 2 \tag{48}$$

in coincide with the d.o.f. massive spin 3/2 particle.

Essential is the point that the Stückelberg spinor $\lambda_{\pm}$ has only first order derivative kinetic term despite it appears in the derivative form $\Psi_{\mu\pm} = \partial_{\mu}\lambda_{\pm}$ in the mass term:

$$\mathcal{L}_{\text{fermionmass}} = m(\bar{\psi}_{\mu+} - \bar{\Psi}_{\mu+})\gamma^{\mu\nu}(\psi_{\nu-} - \Psi_{\nu-}) \tag{49}$$

It is guaranteed by the antisymmetry property of $\gamma^{\mu\nu}$.

This might be thought to be automatic by the balance between boson and fermion mode numbers in SUSY theory. But it is not so because $N = 2$ SUSY are spontaneously broken (non-linearly realized) here, where λ^i are the Goldstinos.