Recent developments of Lattice QCD
（格子QCDの最近の進展）

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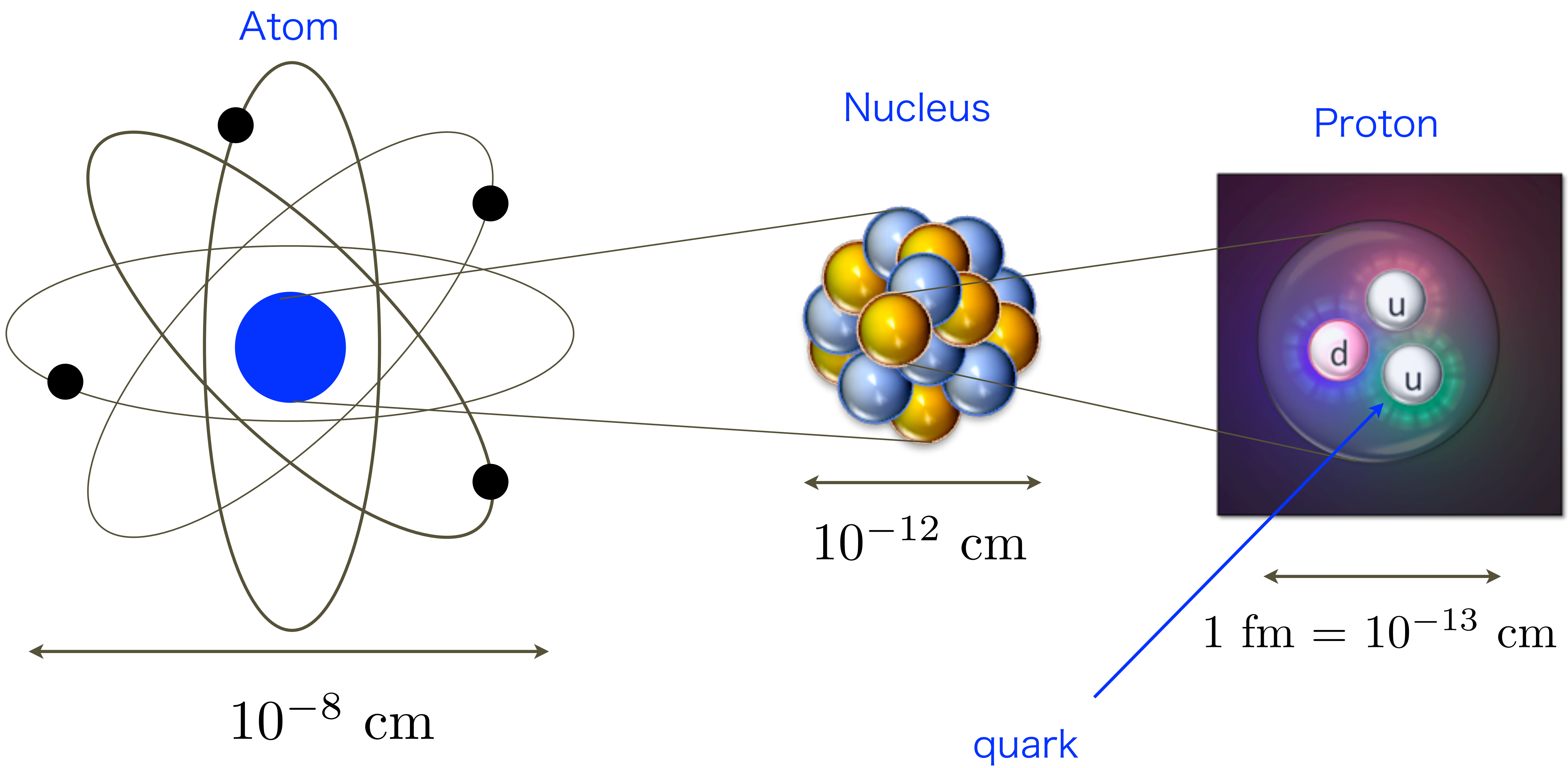
1. Introduction
格子QCDとは？

Quarks



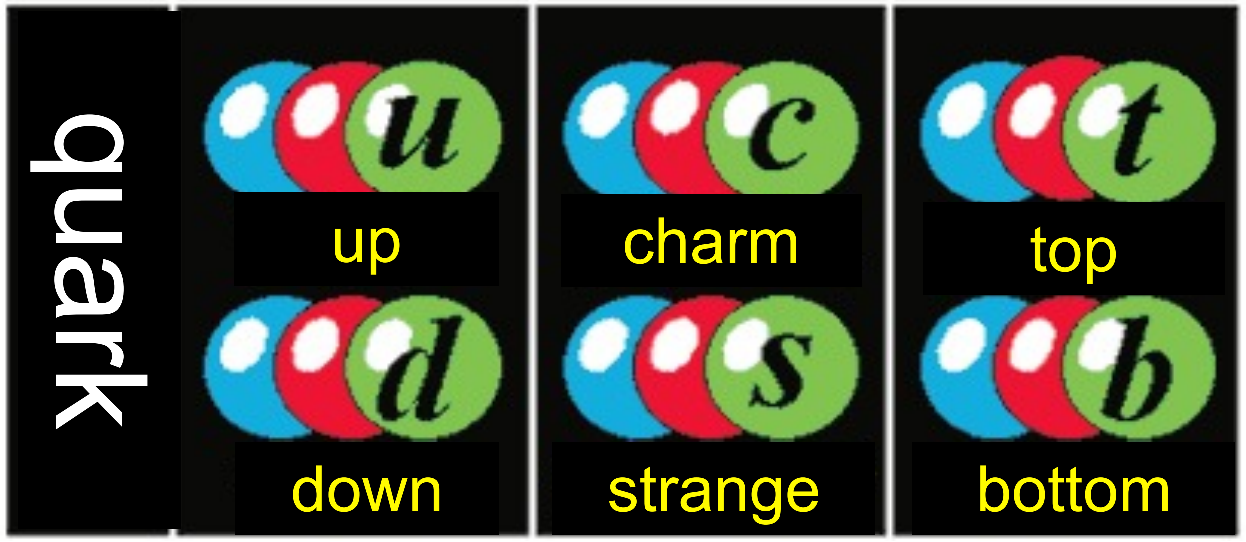
Hadrons are made of more fundamental objects, named “quarks”.

1973: Kobayashi and Maskawa predicted existences of 6 types(“flavor”) of quarks.

Kobayashi

Maskawa,
7th director of YITP

2008 Nobel prize



charge 2e/3

charge -e/3

QCD (Quantum ChromoDynamics)

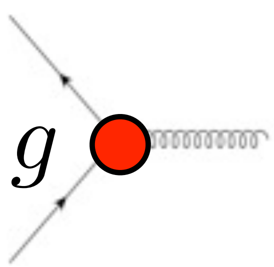
QCD: theory for dynamics of quarks   cf. QED (Quantum Electrodynamics)

$$\mathcal{L} = \bar{q}(x)\gamma^{\mu}\{\partial_{\mu} + igA_{\mu}(x)\}q(x) + \frac{1}{4}\{F^{a}_{\mu\nu}(x)\}^{2}$$

gluon   quark

quark

g



gluon

quark

$$a = 1 \sim 8$$

$$\bar{q}A_{\mu}q = \bar{q}^{A}\,T^{a}_{AB}A^{a}_{\mu}\,q^{B}$$

$$A, B = 1, 2, 3 \ (\text{color})$$

quarks-gluon interaction

(electrons-photon in QED)

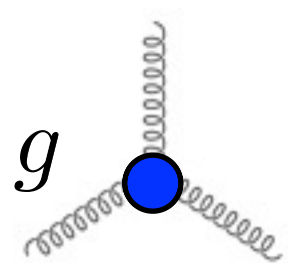
$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A_{\mu} - gf^{abc}A^{b}_{\mu}A^{c}_{\nu}$$

gluon field strength

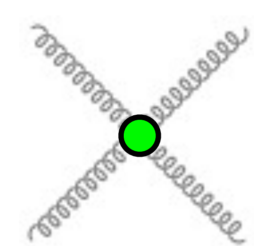
$(F^{a}_{\mu\nu})^{2}$ ⇒ self-interaction

(absent in QED)

g



g^{2}



g : unique coupling constant in QCD

universal for all flavors

Some Properties of QCD

Asymptotic freedom — forces becomes weaker at shorter distances

Gross Politzer Wilczek

2004 Nobel prize

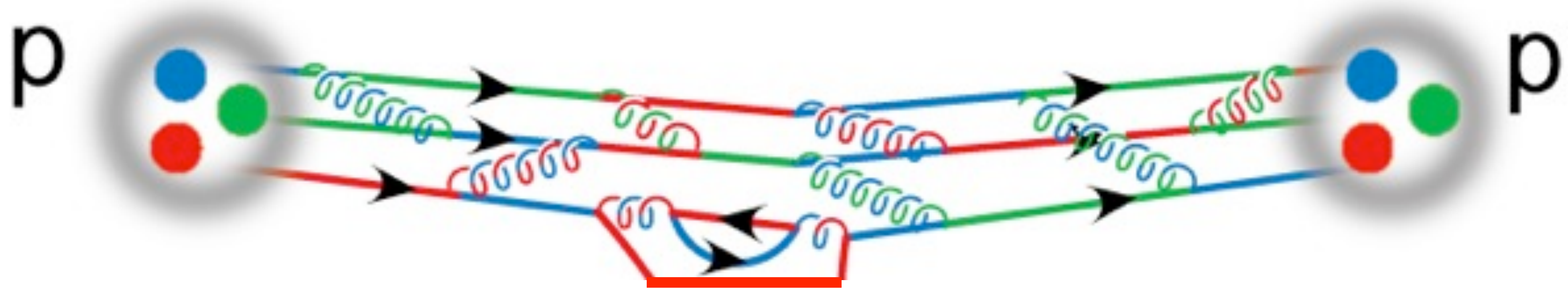
Quark confinement — forces becomes stronger at longer distances

no isolated quark can be observed

structure of nucleon

quark confinement

Difficulties of QCD

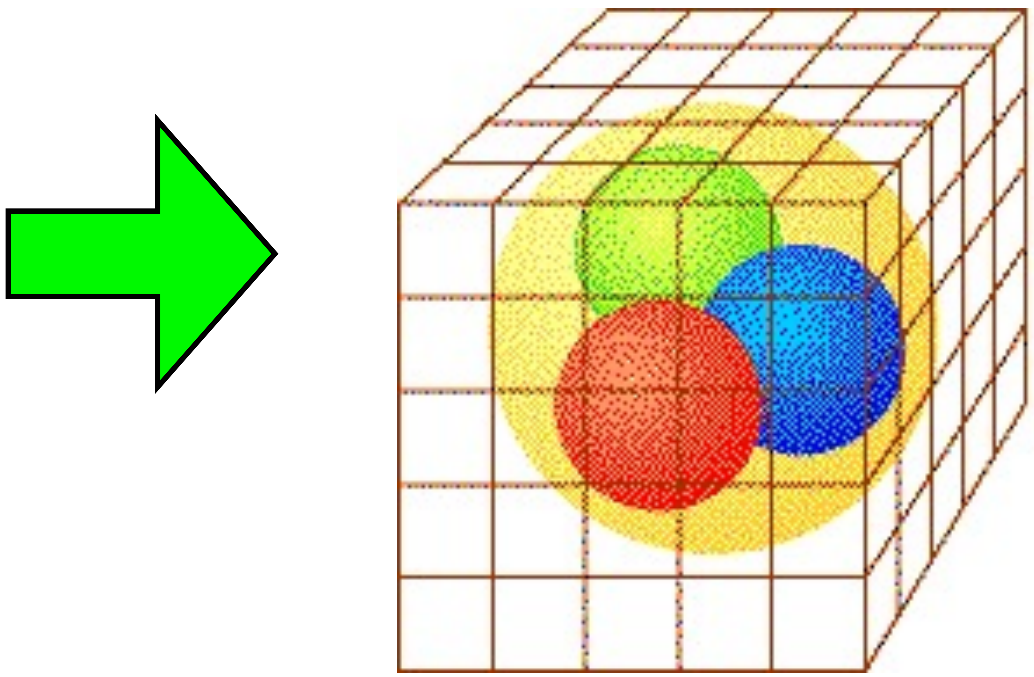


"Free" proton = 3 quarks interacting with each others by exchanging a lot of gluons, so that they move coherently.

Clearly, perturbation theory does not work !

Lattice QCD

We need a non-perturbative method.



Lattice Field Theories

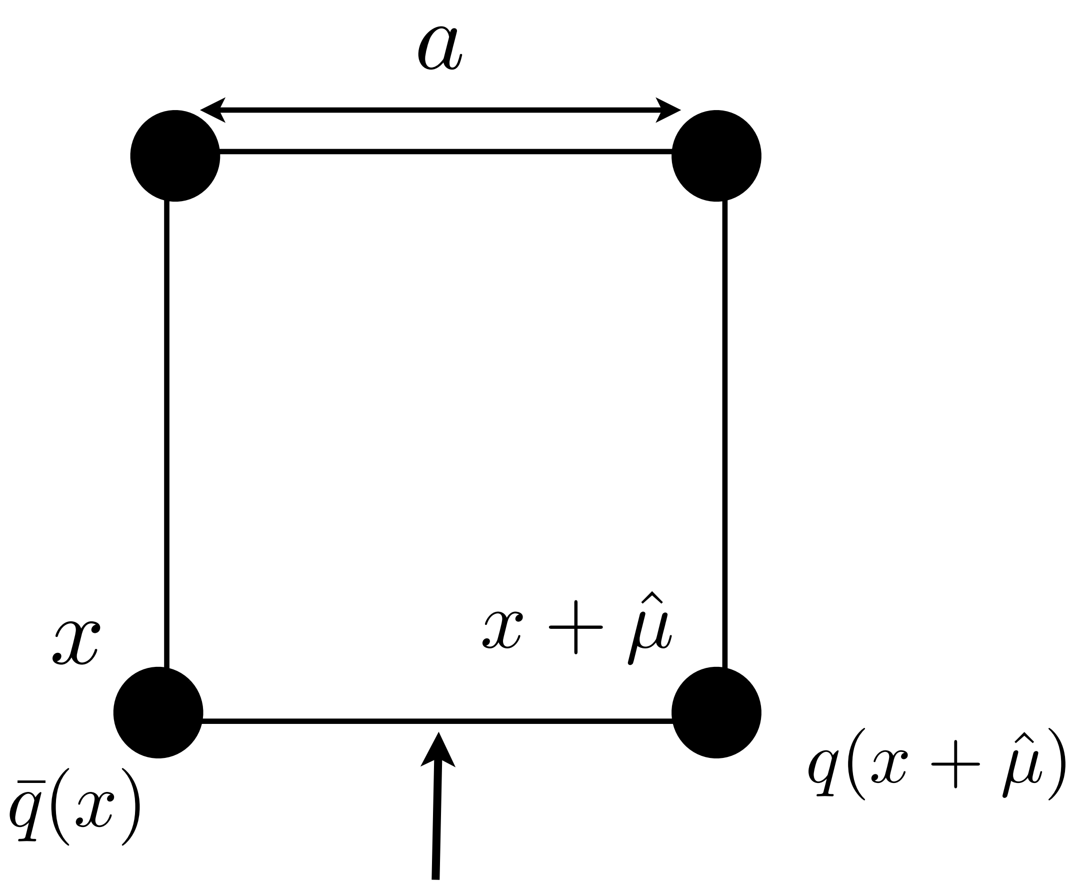
Definition of Quantum Field Theories

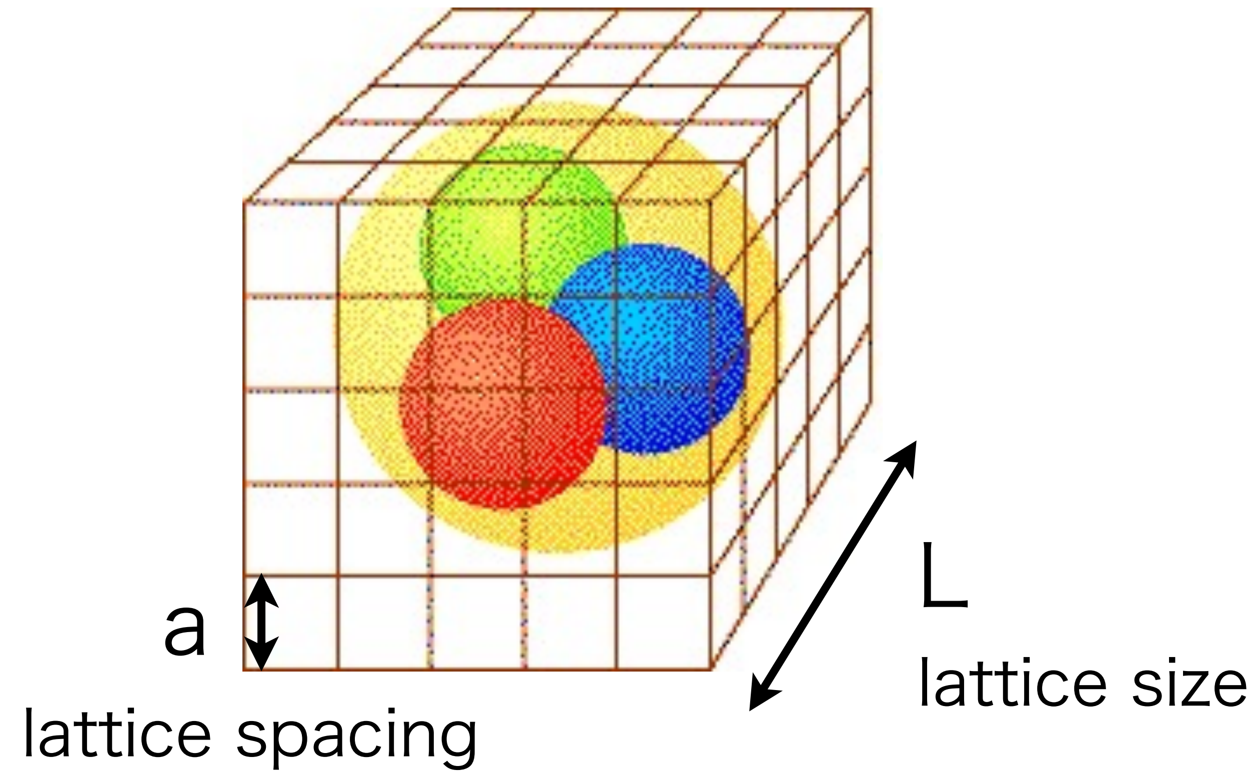
1. Continuum (quantum) Field theories
   - Perturbative expansion: needed to define the theory
   - Divergences ⇒ Regularization/Renormalization
   - Gauge volume ⇒ Gauge fixing
   - Path-integral quantization, canonical quantization

2. Lattice (quantum) field theories
   - does not rely on perturbation theory
   - lattice spacing a ⇒ regularization
   - continuum limit ($a \rightarrow 0$) has to be taken (renormalization)
   - Path-integral in Euclidean space
   - Strong or weak coupling expansions, Monte Carlo method

Lattice QCD

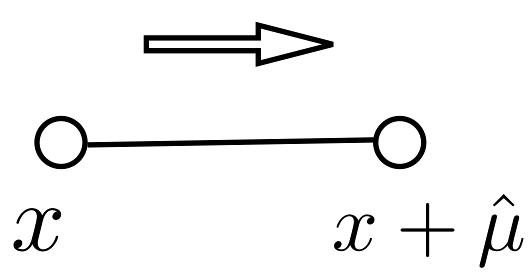
define QCD on a discrete space-time (lattice)



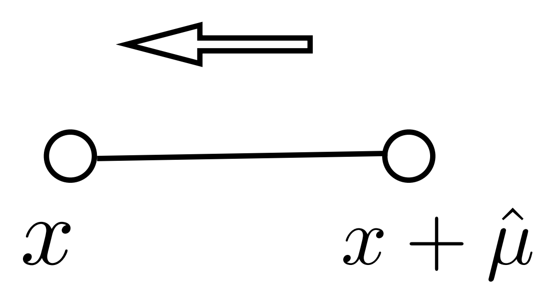


$$U_\mu(x) = e^{igaA_\mu(x)} = 1 + igaA_\mu(x) + \frac{\{igaA_\mu(x)\}^2}{2!} + \cdots \in \mathrm{SU}(3)$$ SU(3) matrix

gluon (lives on link) infinite numbers of gluons (non-perturbative) !



$U_\mu(x)$



$U_{-\mu}(x+\hat{\mu}) \equiv U_\mu(x)^\dagger$

continuum QCD

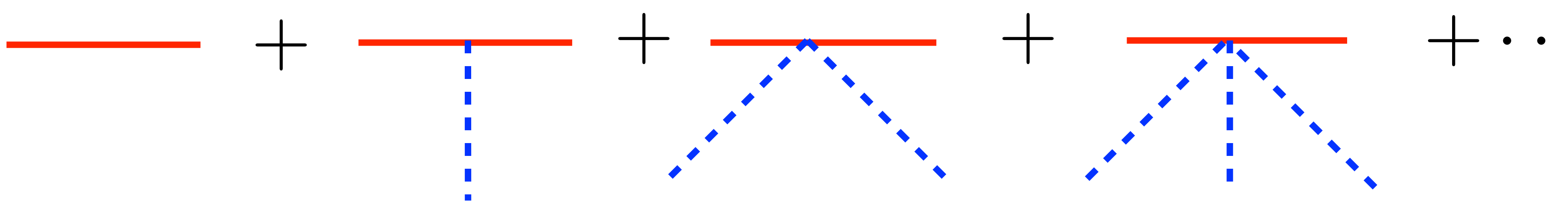
lattice QCD

$$\bar{q}(x)\gamma^{\mu}\{\partial_{\mu}+igA_{\mu}(x)\}q(x) \longleftarrow \bar{q}(x)\gamma^{\mu}\frac{U_{\mu}(x)q(x+\hat{\mu})-U_{-\mu}(x)q(x-\hat{\mu})}{2a}$$

$a \rightarrow 0$

quarks(covariant derivative)

$$\bar{q}(x)U_{\mu}(x)q(x+\hat{\mu}) =$$



quark interacts with many gluons in a very short distance !

quark action

$$S_F = \sum_{x,\mu}\bar{q}(x)\gamma^{\mu}\frac{U_{\mu}(x)q(x+\hat{\mu})-U_{-\mu}(x)q(x-\hat{\mu})}{2a}+m\sum_{x}\bar{q}(x)q(x)$$

gauge invariant

gauge transformation

$$q(x) \rightarrow \Omega(x)q(x)$$

$$\bar{q}(x) \rightarrow \bar{q}(x)\Omega(x)^{\dagger}$$

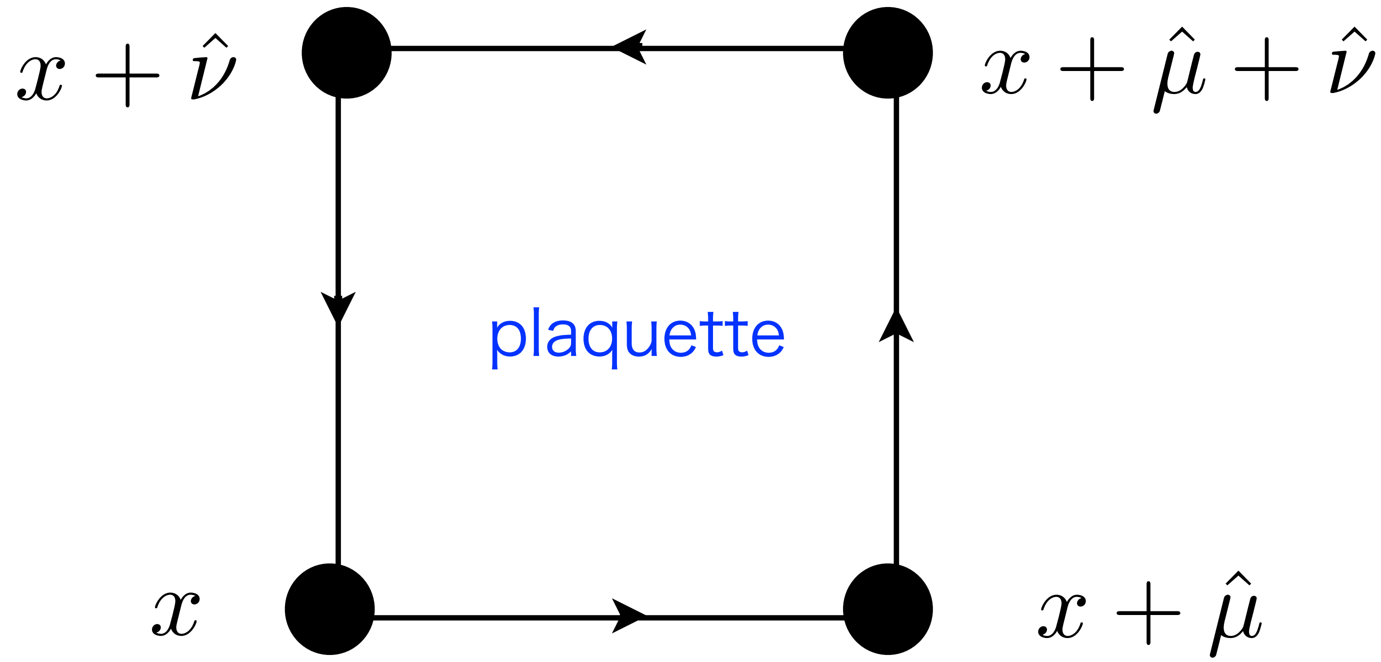
$$U_{\mu}(x) \rightarrow \Omega(x)U_{\mu}(x)\Omega(x+\hat{\mu})^{\dagger}$$

continuum QCD

lattice QCD

$$\frac{g^2}{2}\mathrm{tr}\, F_{\mu\nu}(x)^2 \xleftarrow[a \to 0]{} \mathrm{tr}\, U_\mu(x) U_\mu(x+\hat{\mu}) U_\mu(x+\hat{\nu})^\dagger U_\nu(x)^\dagger$$

gluons



gluon action

$$S_G = \frac{1}{g^2} \sum_x \sum_{\mu \neq \nu} \mathrm{tr}\, U_\mu(x) U_\mu(x+\hat{\mu}) U_\mu(x+\hat{\nu})^\dagger U_\nu(x)^\dagger$$

gauge invariant

Path integral

continuum QCD

$$\langle \mathcal{O}(A_\mu, q, \bar{q}) \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{D}q \mathcal{D}\bar{q}\, \mathcal{O}(A_\mu, q, \bar{q}) e^{-S_0 - S_{\rm int}}$$
$$= \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{D}q \mathcal{D}\bar{q}\, \mathcal{O}(A_\mu, q, \bar{q}) \sum_{n=0}^{\infty} \frac{(-S_{\rm int})^n}{n!} e^{-S_0}$$

perturbative expansion

lattice QCD

$$\langle \mathcal{O}(U_\mu, q, \bar{q}) \rangle = \frac{1}{Z} \int \mathcal{D}U_\mu \mathcal{D}q \mathcal{D}\bar{q}\, \mathcal{O}(U_\mu, q, \bar{q}) e^{-S_F - S_G}$$

calculate without perturbative expansion

important properties

$$\int \mathcal{D}U_\mu(x)\, U_\mu(x) = 0$$

gluon is random

$$\int \mathcal{D}U_\mu(x)\, U_\mu(x) U_\mu(x)^\dagger = \mathbf{1}_{3\times 3}$$

$$\int \mathcal{D}U_\mu(x)\ \det U_\mu(x) = 1$$

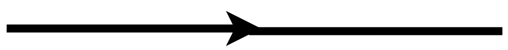
Strong coupling expansion

$$S_G = O\left(\frac{1}{g^2}\right) \to 0 \qquad g^2 \to \infty$$ strong coupling limit

quark path-integral

quark

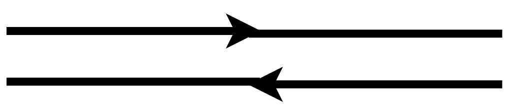
$U_\mu(x)$



$= 0$ by U integral quark confinement

meson

$U_\mu(x)$



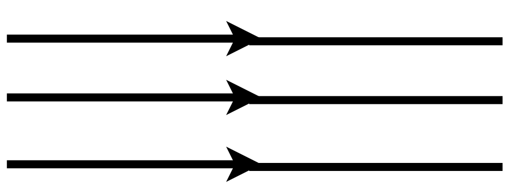
$U_\mu(x)^\dagger$

$\neq 0$

after U integral

baryon

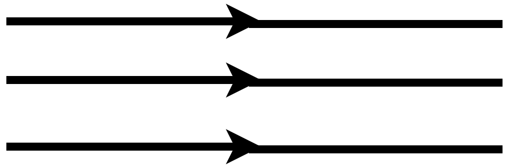
$U_\mu(x)^3$



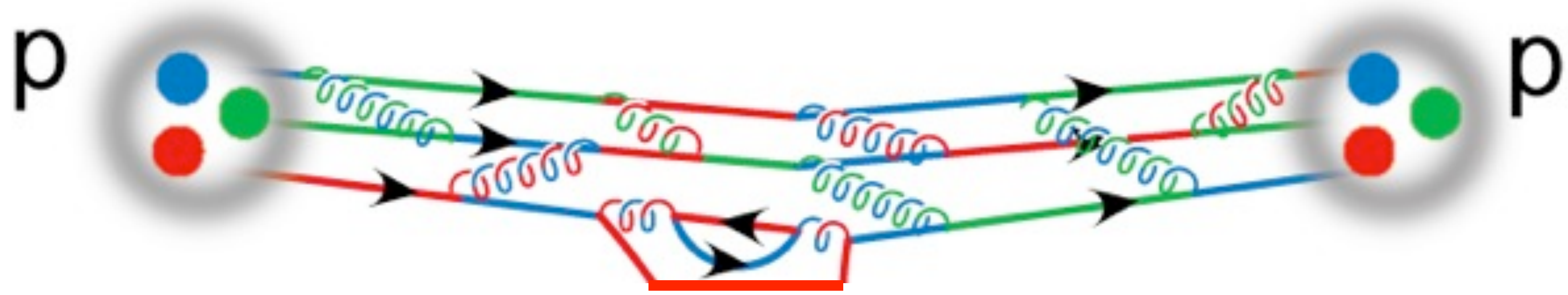
$\neq 0$

meson and baryon can propagate !

$U_\mu(x)^3$

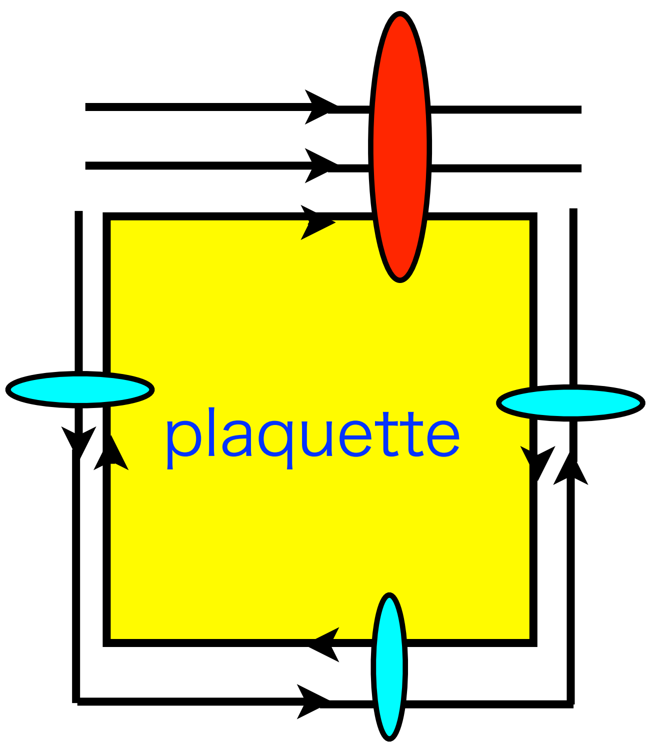


$\simeq$



in terms of perturbation theory

If $\frac{1}{g^2}$ is small but non-zero



$$= O\left(\frac{1}{g^2}\right)$$

strong coupling expansion

3 quarks can propagate separately but still coherently, as a free baryon.

Monte-Carlo simulations

After integral over quarks

$$\langle \mathcal{O}(q,\bar q,U)\rangle = \int \mathcal{D}\,q\mathcal{D}\bar q\,\mathcal{D}U\ \exp[\bar q\, D(U)\, q + S_G(U)]\mathcal{O}(q,\bar q,U)$$

$$= \int \mathcal{D}U\ \underline{\det D(U) e^{S_G(U)}}\hat{\mathcal{O}}(U)$$

probability of U $\equiv P(U)$

Importance sampling according to P(U)   "Monte-Carlo simulations"

calculate complicated QCD processes by computer simulations

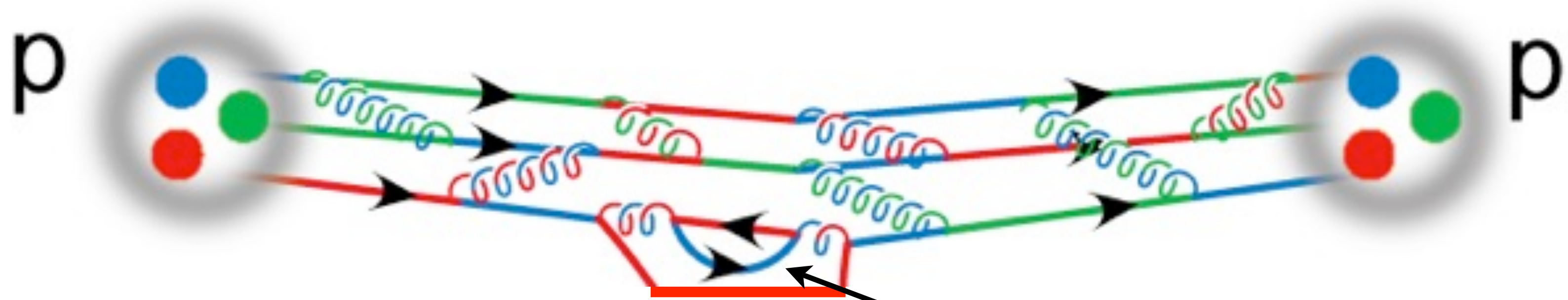
⇒ uses of super-computers are required.

Yet calculations are not so easy.

Recently hadron masses have been accurately calculated. (free hadrons)

2. Hadron spectra
ハドロン質量の計算

Hadron mass calculations



creation/annihilation of quark-antiquark pair

"effect of det D(U)"

2-pt correlation function

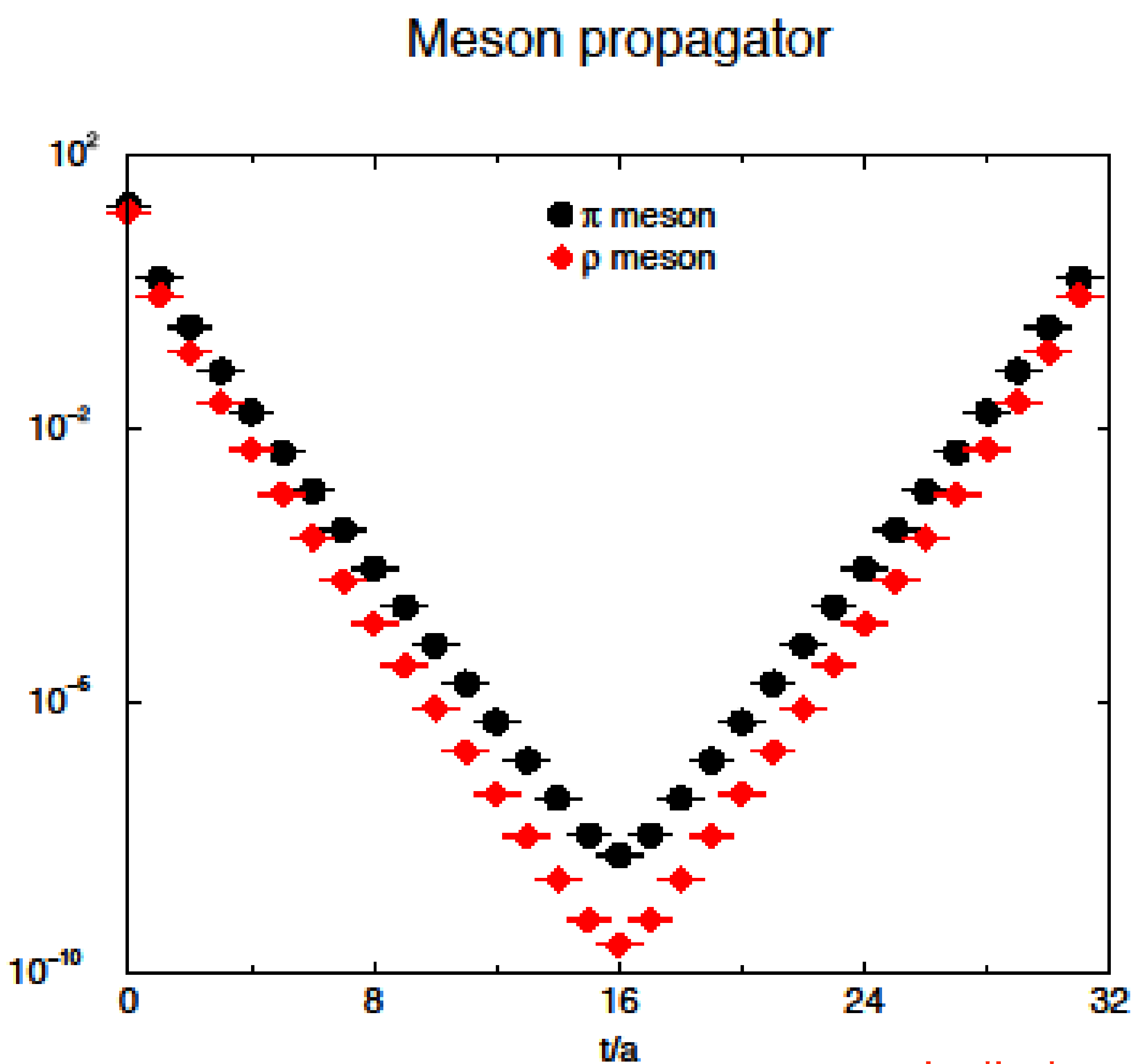
set det $D(U) = 1$: quenched approximation

$$\langle 0|p(0,\vec{0})\frac{1}{V}\sum_{\vec{x}}\bar{p}(t,\vec{x})|0\rangle = \langle 0|p(0,\vec{0})\sum_n |n\rangle\langle n|\frac{1}{V}\sum_{\vec{x}}\bar{p}(t,\vec{x})|0\rangle$$

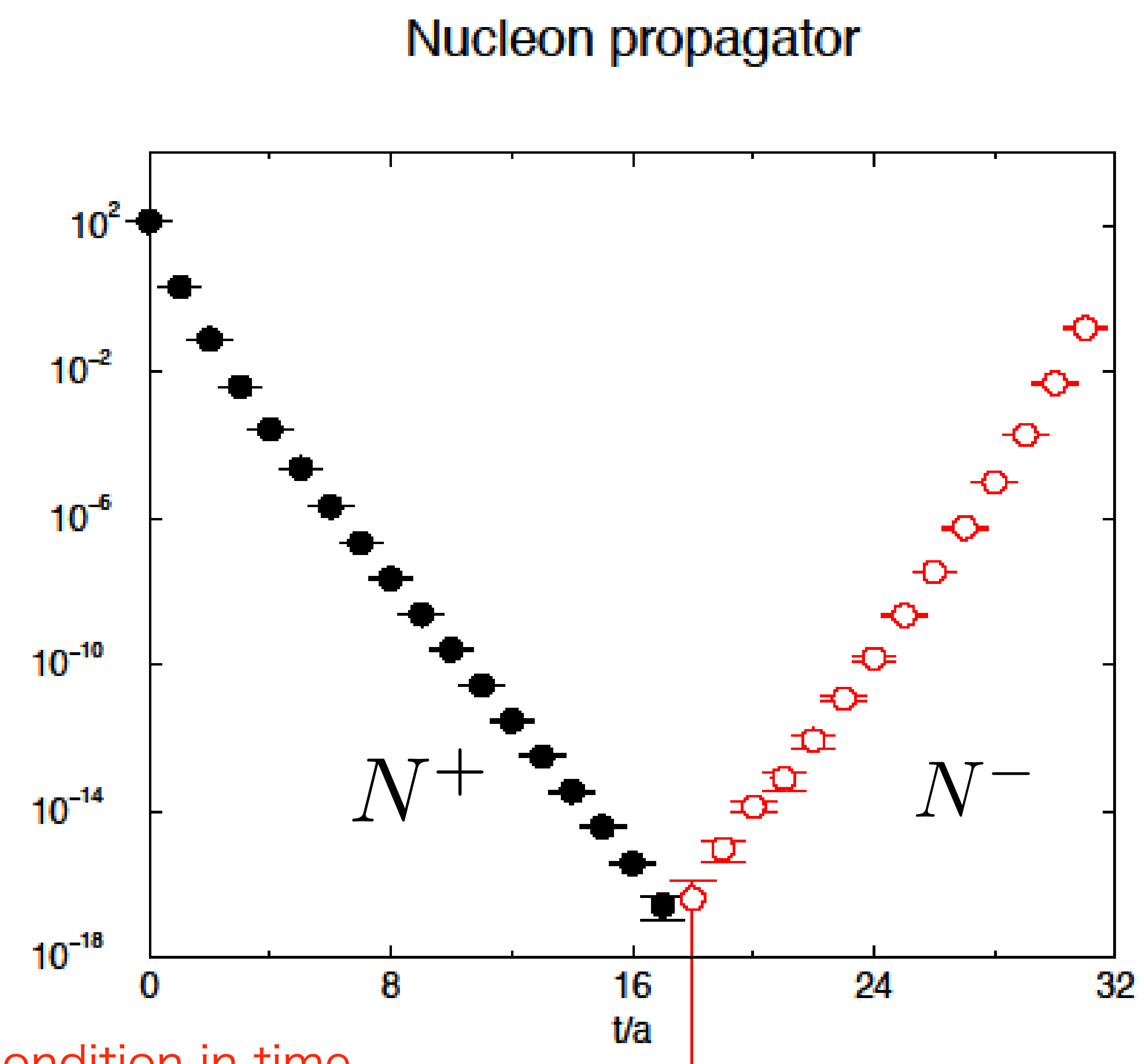
$$= \sum_n |\langle 0|p(0,\vec{0})|n\rangle|^2 e^{-m_n t} = C_0 e^{-m_0 t} + C_1 e^{-m_1 t} + \cdots$$

extract the ground-state hadron mass m_0 from the large t behavior

Meson propagator



Nucleon propagator



periodic boundary condition in time

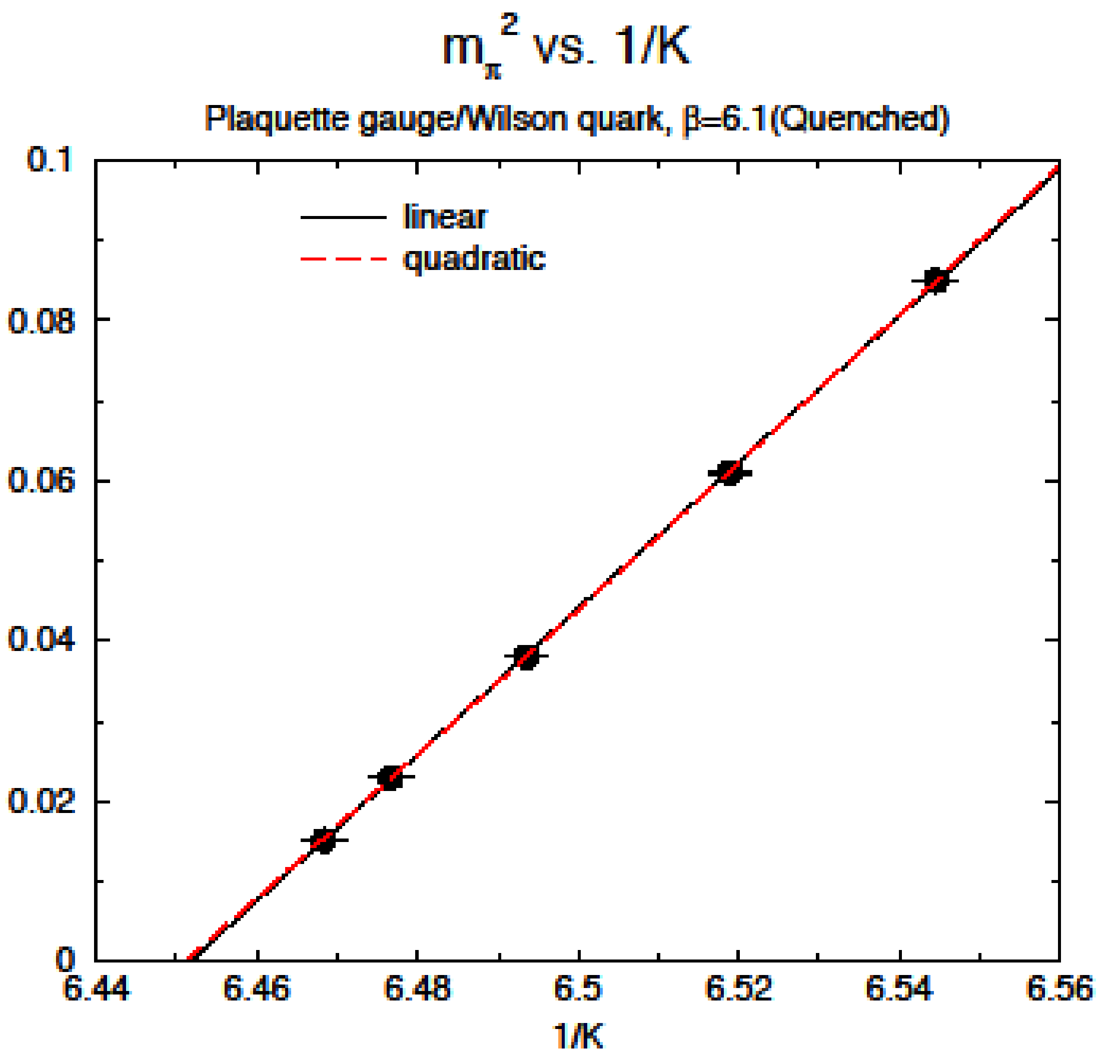
pion is lighter than rho.

Nucleon is lighter than its negative-parity state.

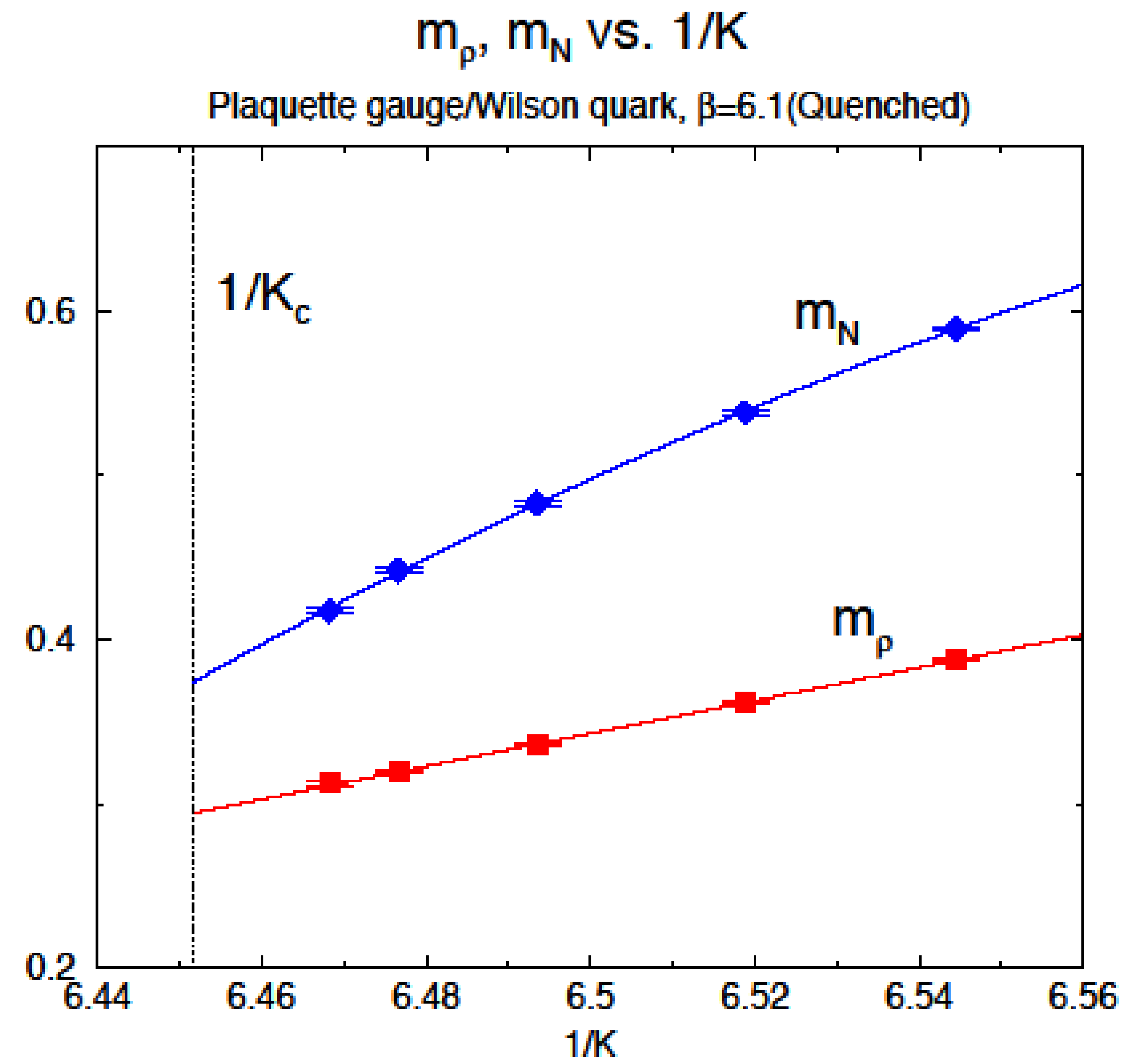
Chiral extrapolation

It is difficult to make quark mass as small as the “experimental” value in numerical simulations. Extrapolations from heavier quark masses are usually made.

Pion mass



Other hadrons

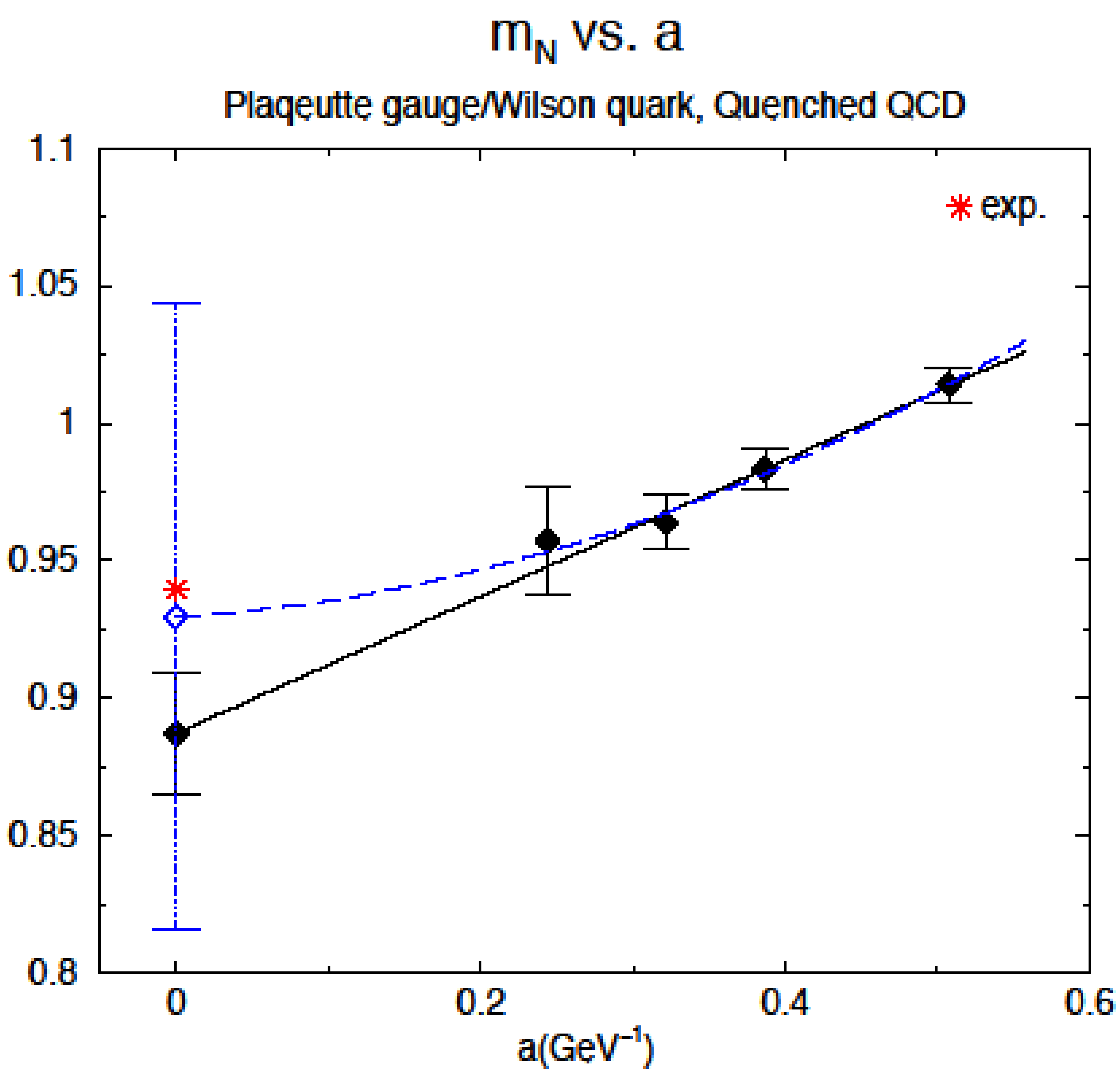


quark mass $$2m_q a = \frac{1}{K} - \frac{1}{K_c}$$

Continuum extrapolation

$a \to 0$ limit should be taken.

Nucleon mass

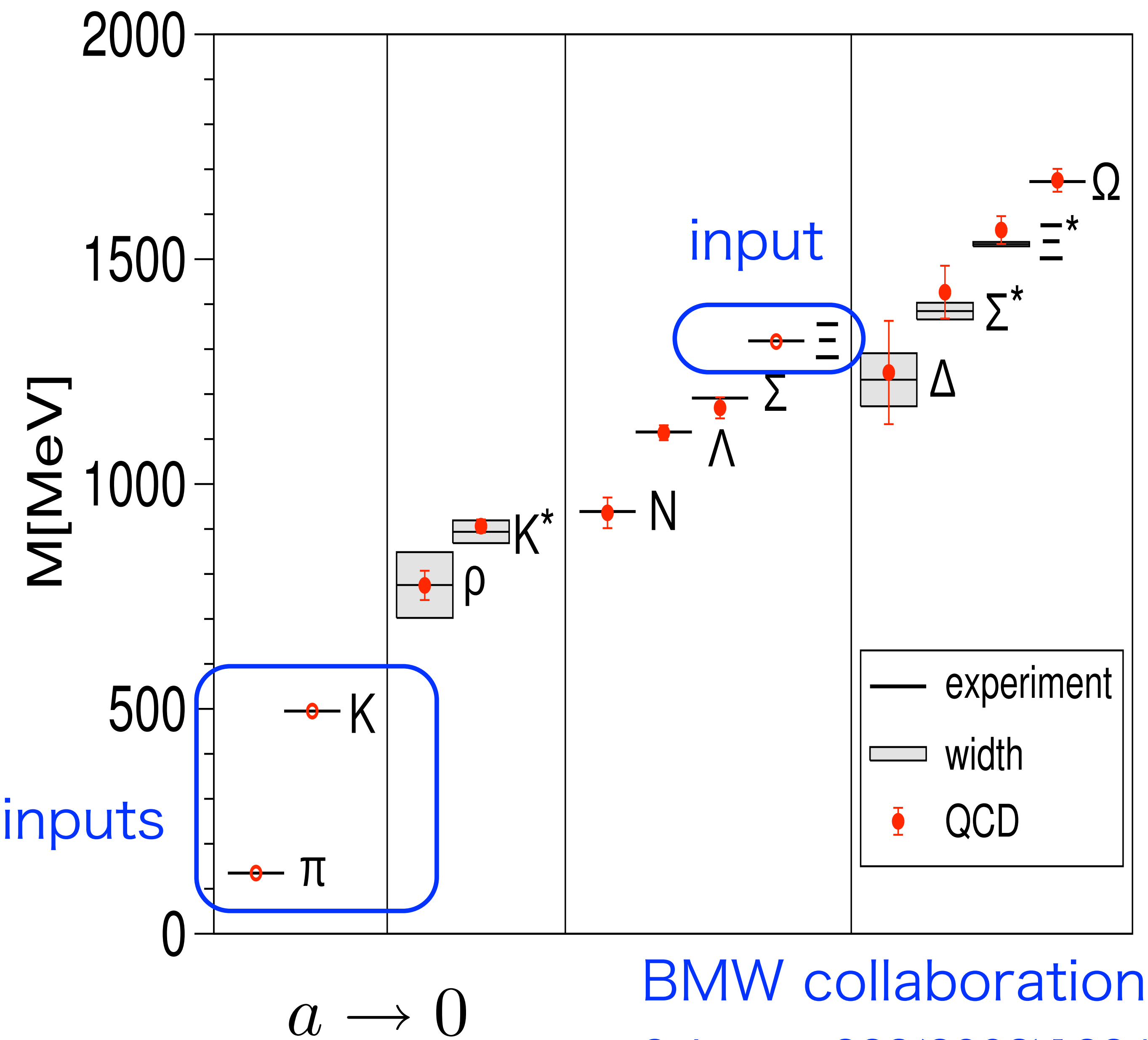


lattice spacing

continuum extrapolation by fit

$$
\begin{aligned}
m_N(a) &= m_N(0) + C_1 a \\
m_N(a) &= m_N + C_1 a^2 + C_2 a^2
\end{aligned}
$$

The state of arts for hadron masses



BMW collaboration
Sciences 322(2008)1224

Baryon

Quarks	Octet($\frac{1}{2}$)		Decouplet($\frac{3}{2}$)	
uuu				Δ^{++}
uud	N	p	Δ	Δ^{+}
udd		n		Δ^{-}
ddd				Δ^{0}
uus	Σ	Σ^{+}		$(\Sigma^{*})^{+}$
uds	Λ	Σ^{0}, Λ^{0}	Σ^{*}	$(\Sigma^{*})^{0}$
dds		Σ^{-}		$(\Sigma^{*})^{-}$
uss	Ξ	Ξ^{0}	Ξ^{*}	$(\Xi^{*})^{0}$
dss		Ξ^{-}		$(\Xi^{*})^{-}$
sss			Ω	Ω

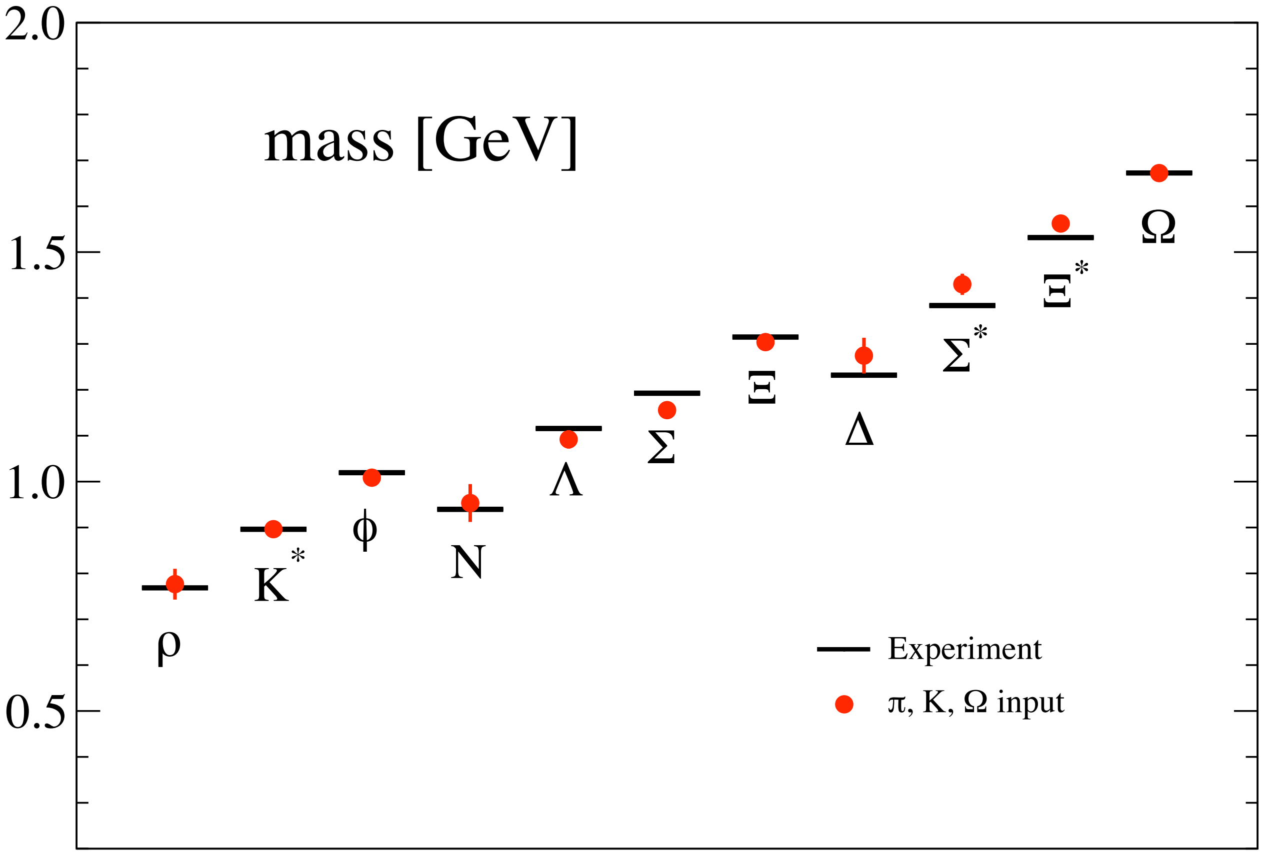
Meson

Quarks	PesudoScala(0)		Vector(1)	
$\bar{u}u - \bar{d}d$	π	π^{0}	ρ	ρ^{0}
$\bar{d}u$, $\bar{u}d$		$\pi^{\pm}$		$\rho^{\pm}$
$\bar{u}u + \bar{d}d$		η		ω
$\bar{s}d$, $\bar{d}s$	K	K^{0}, $\bar{K}^{0}$	K^{*}	$(K^{*})^{0}$, $(\bar{K}^{*})^{0}$
$\bar{s}u$, $\bar{u}s$		$K^{\pm}$		$(K^{*})^{\pm}$
$\bar{s}s$		η_s		ϕ

an agreement between lattice QCD
and experiments is good.

PACS-CS Collaboration

Phys. Rev. D79 (2009) 034503

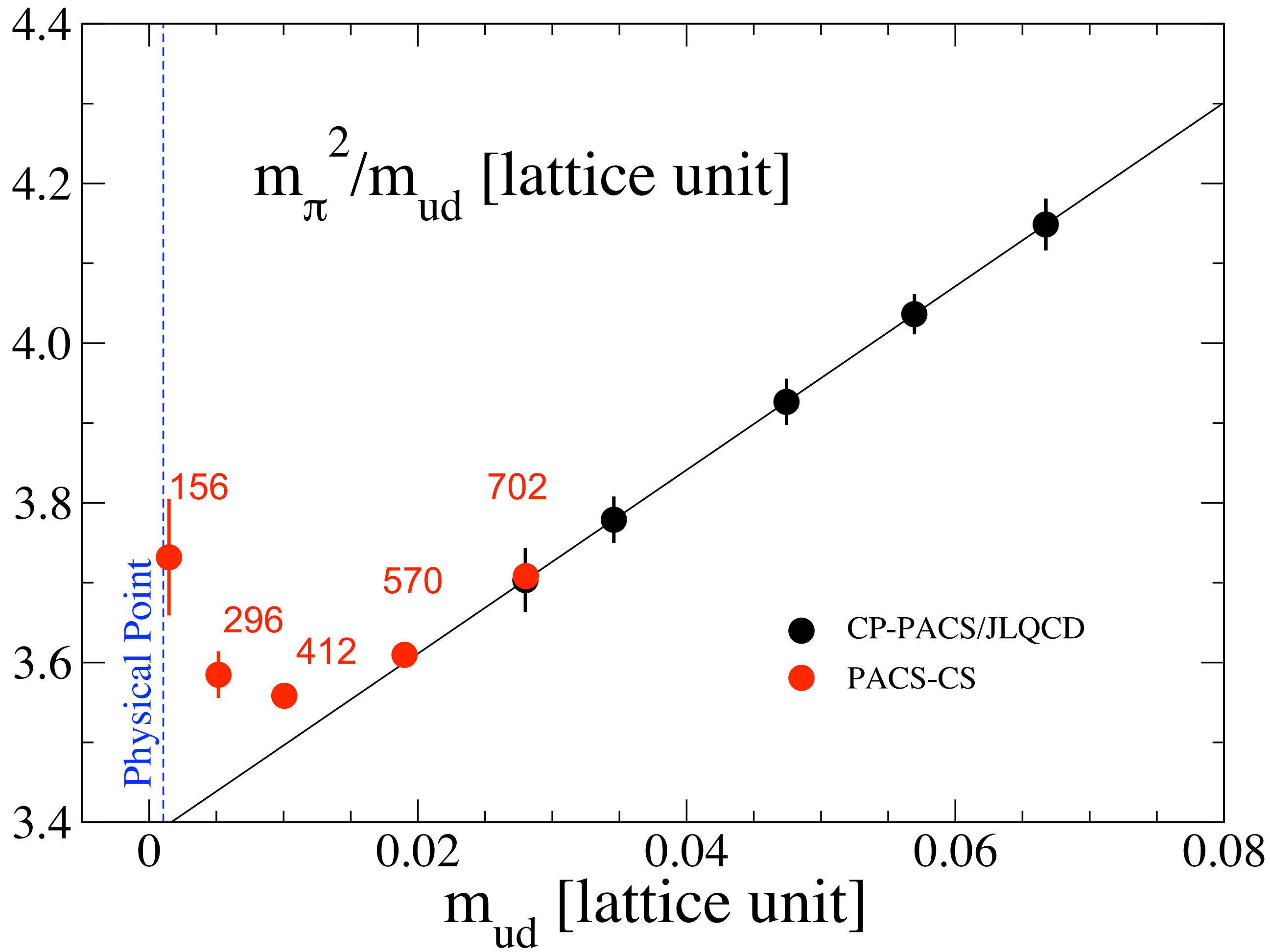


$$a = 0.09 \text{ fm}$$

$$L = 2.9 \text{ fm} \qquad m_\pi L = 2.3$$

$$m_\pi^{\text{min.}} = 156 \text{ MeV}$$

Almost on physical quark mass
(no chiral extrapolation)



chiral extrapolation vs. physical point

Chiral extrapolation sometimes becomes non-trivial due to the chiral-log, as shown in the figure.

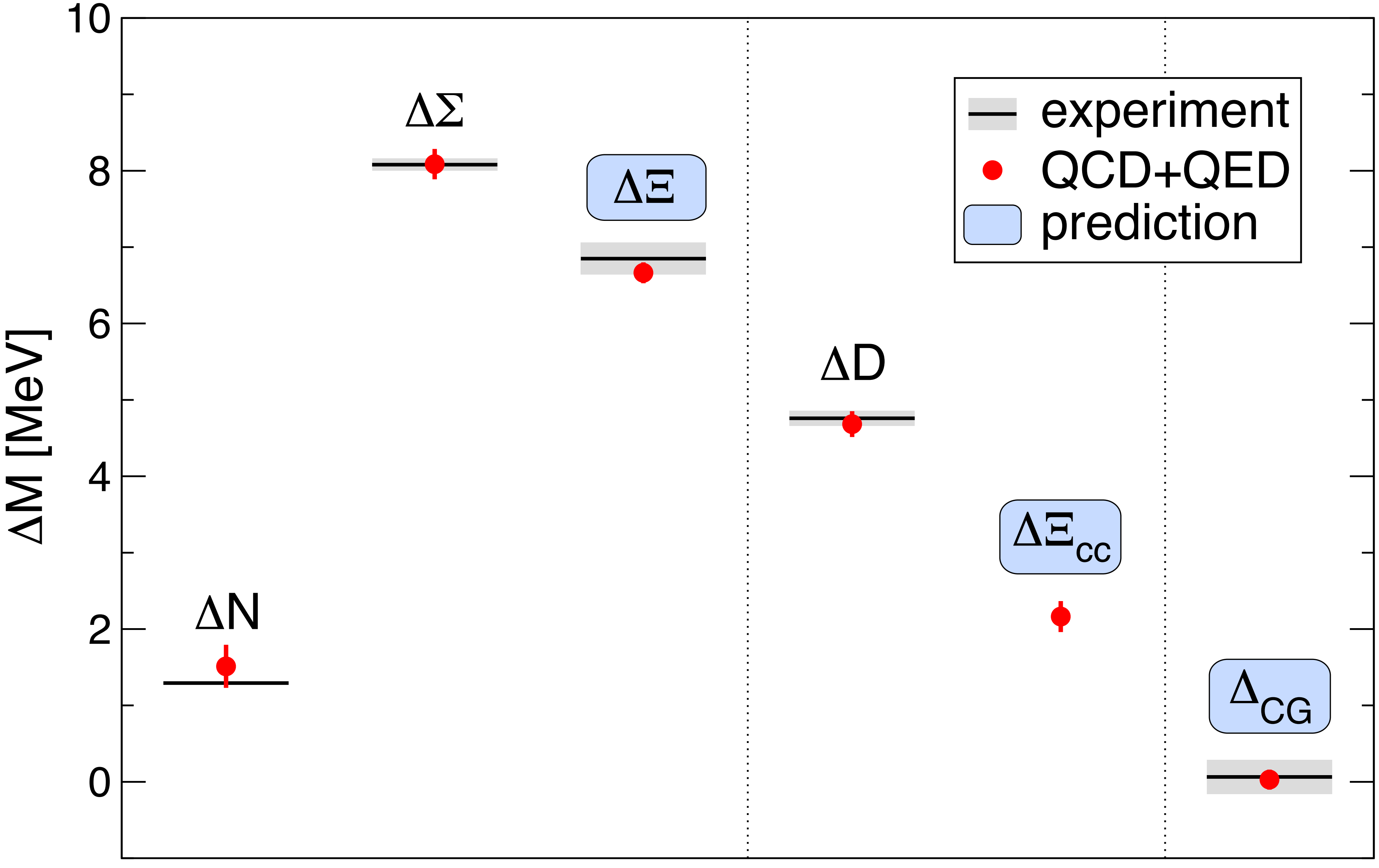
Further improvement

Borsanyi et al.
arXiv:1406.4088[hep-lat]
1+1+1+1 flavor QCD (u,d,s,c)
+ non-compact QED

Iso-spin breaking effects

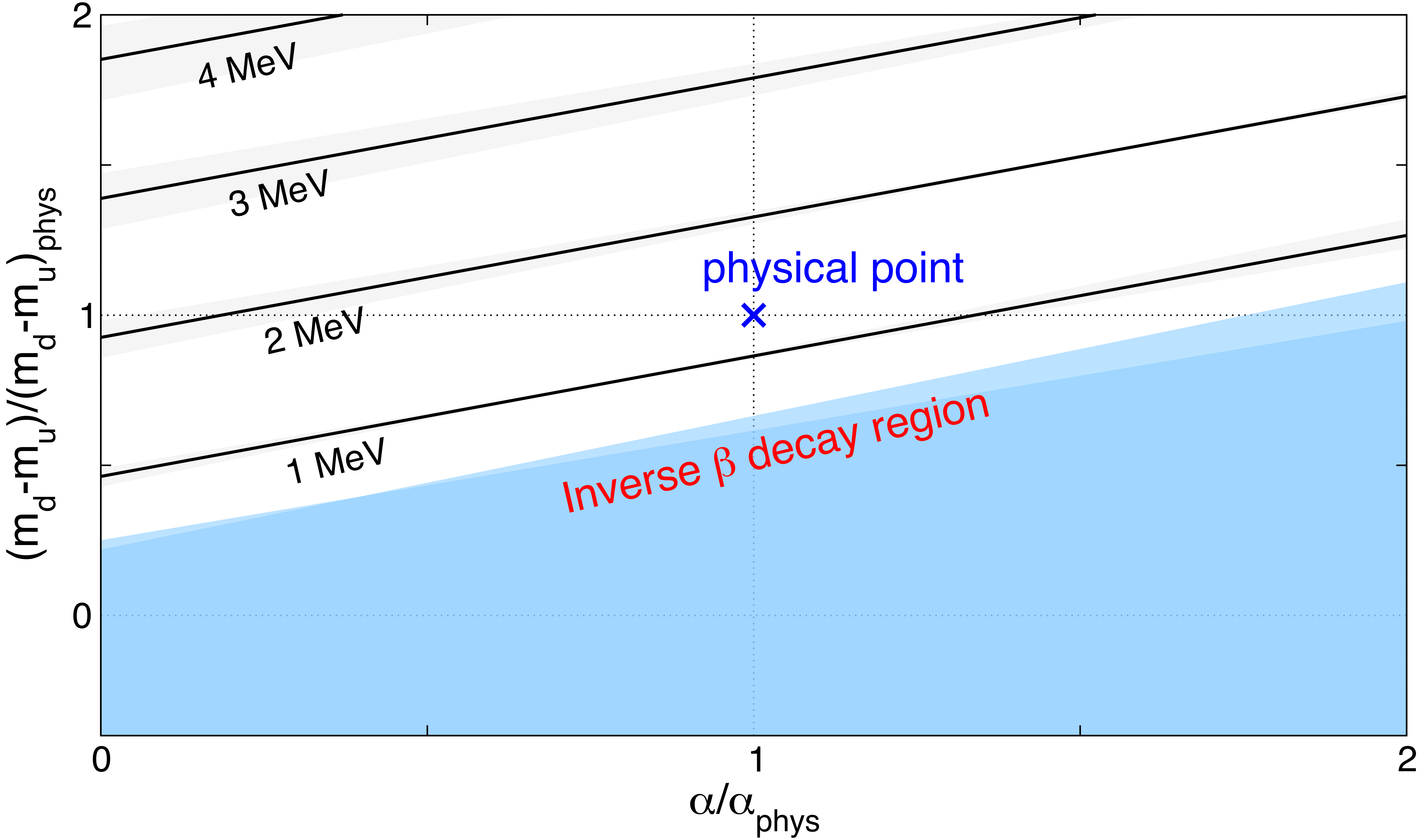
$m_u \neq m_d$ effect

QED effect ($\alpha_u = -2\alpha_d$)



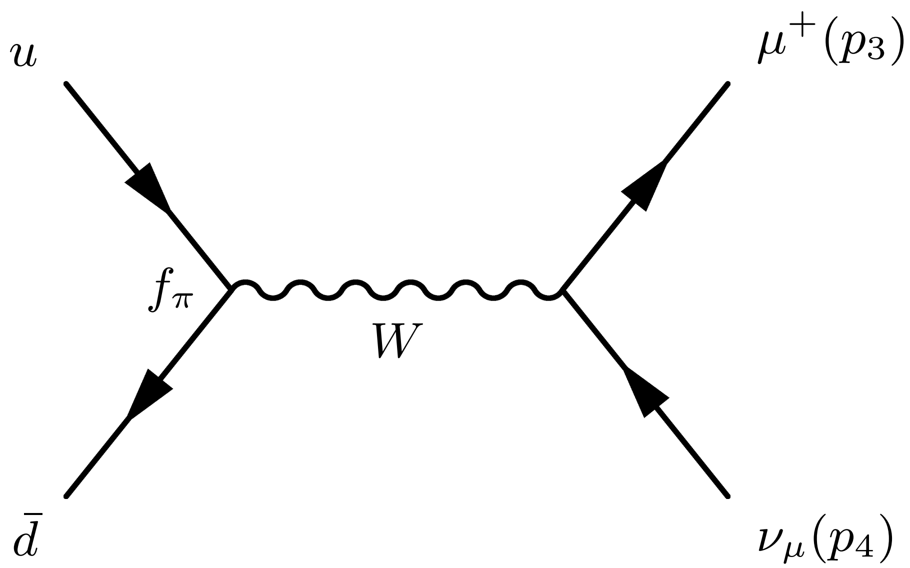
	mass splitting [MeV]	QCD [MeV]	QED [MeV]
$\Delta N = n - p$	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)
$\Delta\Sigma = \Sigma^- - \Sigma^+$	8.09(16)(11)	8.09(16)(11)	0
$\Delta\Xi = \Xi^- - \Xi^0$	6.66(11)(09)	5.53(17)(17)	1.14(16)(09)
$\Delta D = D^\pm - D^0$	4.68(10)(13)	2.54(08)(10)	2.14(11)(07)
$\Delta\Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^{+}$	2.16(11)(17)	-2.53(11)(06)	4.69(10)(17)
$\Delta_{\mathrm{CG}} = \Delta N - \Delta\Sigma + \Delta\Xi$	0.00(11)(06)	-0.00(13)(05)	0.00(06)(02)

Fine tuning in Nature ?



3. Weak matrix elements
ハドロンの弱電時行列要素の計算

3-1. Decay constants for PS mesons



$$\langle 0|\,\bar{d}\gamma_\mu\gamma_5 u|\pi^+(p)\rangle = ip_\mu f_{\pi^+}, \quad \langle 0|\,\bar{s}\gamma_\mu\gamma_5 u|K^+(p)\rangle = ip_\mu f_{K^+}.$$

axial-vector current

$$A_\mu^{\pi^+}$$

A-P correlation function

$$\langle 0|A_0^{\pi^+}(0,\vec{0})\frac{1}{V}\sum_{\vec{x}} P^{\pi^-}(t,\vec{x})|0\rangle = \langle 0|A_0^{\pi^+}(0,\vec{0})|\pi^+(\vec{0})\rangle\langle\pi^+(\vec{0})|\frac{1}{V}\sum_{\vec{x}} P^{\pi^-}(t,\vec{x})|0\rangle + \cdots$$

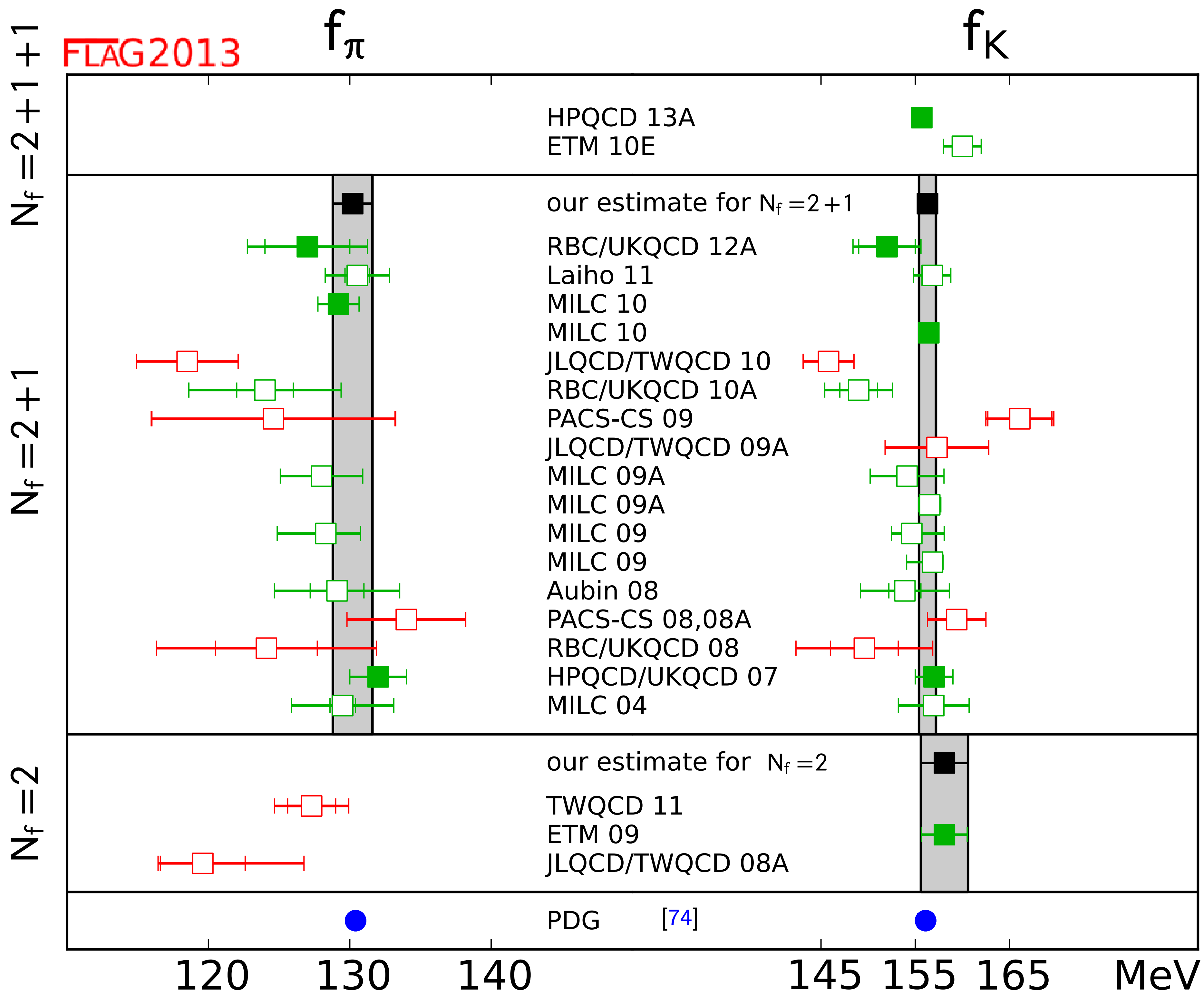
$$= \langle 0|A_0^{\pi^+}|\pi^+(\vec{0})\rangle\langle\pi^+(\vec{0})|P^{\pi^-}|0\rangle e^{-m_\pi t} + \cdots$$

P-P correlation function

$$\langle 0|P(0,\vec{0})\frac{1}{V}\sum_{\vec{x}} P^{\pi^-}(t,\vec{x})|0\rangle = \langle 0|P^{\pi^+}|\pi^+(\vec{0})\rangle\langle\pi^+(\vec{0})|P^{\pi^-}|0\rangle e^{-m_\pi t} + \cdots$$

$$\langle 0|A_0^{\pi^+}|\pi^+(\vec{0})\rangle = m_\pi f_{\pi^+}$$

The latest lattice results



Particle Data Group

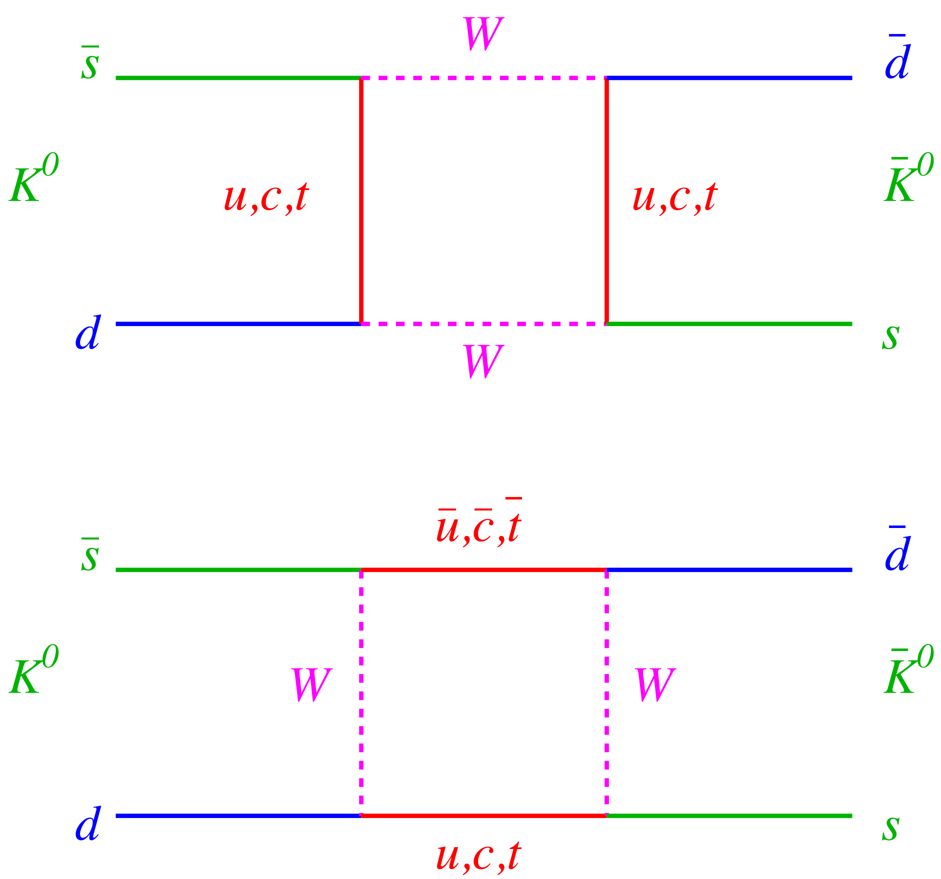
Lattice

$$f_\pi = 130.2\,(1.4)\ \text{MeV} \qquad (N_f = 2+1),$$
$$f_K = 156.3\,(0.9)\ \text{MeV} \qquad (N_f = 2+1),$$
$$f_K = 158.1\,(2.5)\ \text{MeV} \qquad (N_f = 2).$$

$$f_\pi^{(\text{PDG})} = 130.41\,(0.20)\ \text{MeV},$$
$$f_K^{(PDG)} = 156.1\,(0.8)\ \text{MeV},$$

3-2. Kaon B parameter B_K

K_0 - $\overline{K}_0$ mixing parameter (indirect CP violation)



effective 4-fermi oeprator

$$Q^{\Delta S=2} = \left[\bar{s}\gamma_\mu(1-\gamma_5)d\right]\left[\bar{s}\gamma_\mu(1-\gamma_5)d\right]$$

Weak Matrix Element

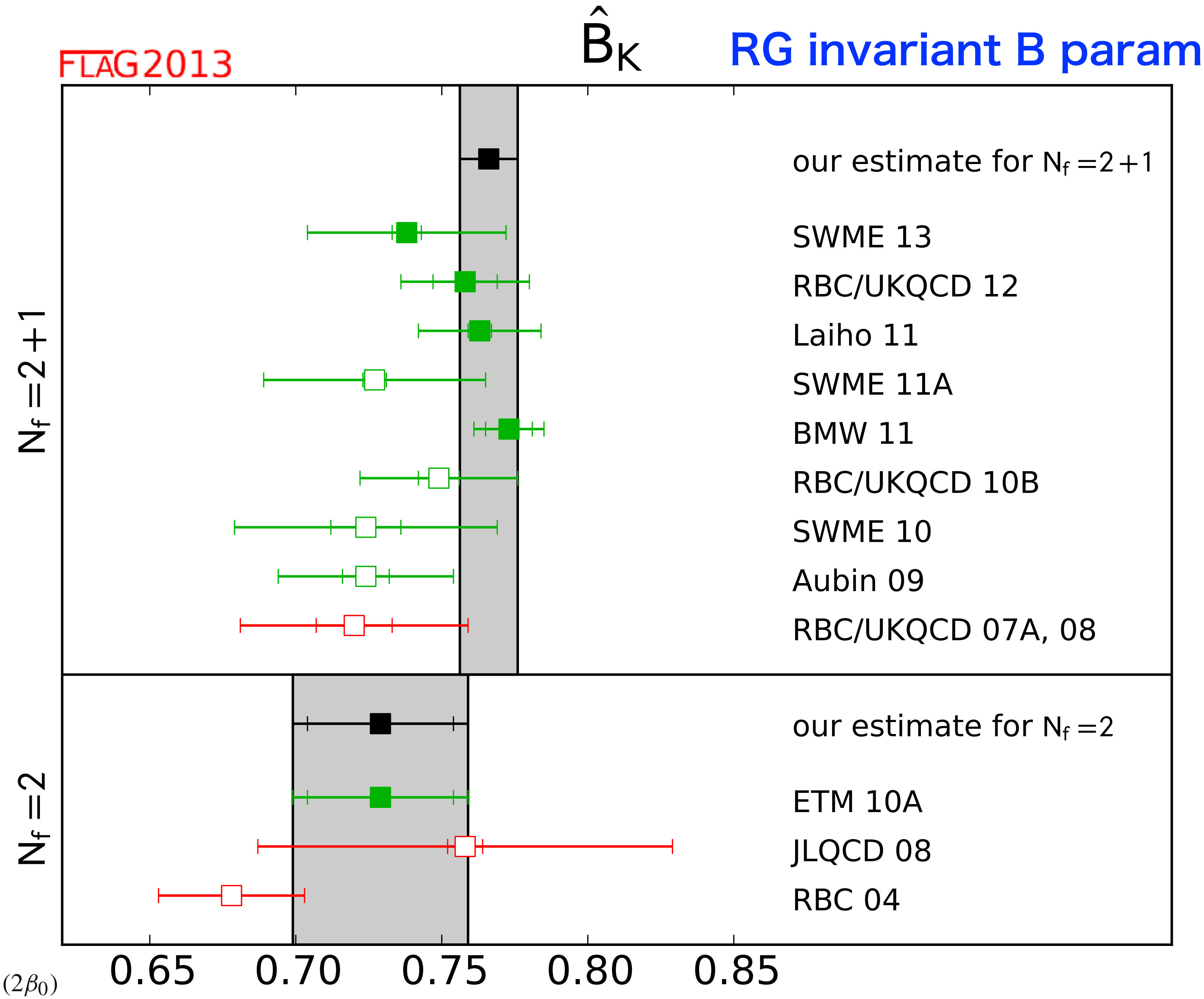
$$B_K(\mu) = \frac{\langle \bar{K}^0 \,|\, Q_R^{\Delta S=2}(\mu) \,|\, K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2} \,.$$

3-pt correlation function

$$\langle 0|K_0(t_1)Q_R^{\Delta S=2}(t_O)K_0(t_2)|0\rangle = \langle 0|K_0|\bar{K}_0\rangle\langle \bar{K}_0|Q_R^{\Delta S=2}|K_0\rangle\langle K_0|K_0|0\rangle e^{-m_{K_0}(t_2-t_1)} + \cdots$$

The latest lattice results

RG invariant B parameter



$$\hat{B}_K = \left(\frac{\bar{g}(\mu)^2}{4\pi}\right)^{-\gamma_0/(2\beta_0)} \times \left\{1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2}\left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2}\right]\right\} B_K(\mu).$$

$$N_f = 2+1: \quad \hat{B}_K = 0.7661(99),$$

$$N_f = 2+1: \quad B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.5596(72).$$

3-3. Kaon decays

$K \to \pi\pi$ decays

$$A(K^+ \to \pi^+\pi^0) = \sqrt{\frac{3}{2}} A_2 e^{i\delta_2}$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2}$$

$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}} A_2 e^{i\delta_2} .$$

$\delta_{0,2}$ strong phases

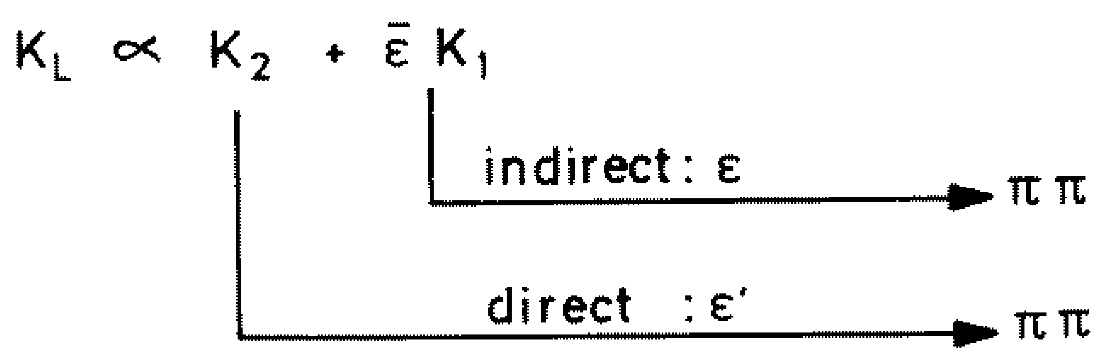
A_I: $K \to \pi\pi$ $(I = 0, 2)$ weak decay amplitude

$\Delta I = 1/2$ rule

$$\frac{\text{Re } A_0}{\text{Re } A_2} = 22.45(6)$$

Experiment

CP violation



direct CP violation

$$\varepsilon' = \frac{1}{\sqrt{2}} \text{Im}\left(\frac{A_2}{A_0}\right) e^{i\Phi}, \qquad \Phi = \pi/2 + \delta_2 - \delta_0,$$

Some lattice results

$K \to (\pi\pi)_{I=2}$ decay amplitude

T. Blum et al., PRL108(2012)141061
T. B.um et al., PRD86(2012)074513

Lattice

$$\mathrm{Re}A_2 = 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}} 10^{-8}\ \mathrm{GeV},$$
$$\mathrm{Im}A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}} 10^{-13}\ \mathrm{GeV}.$$

Experiment

$$\mathrm{Re}A_2 = 1.479(4) \times 10^{-8}\mathrm{GeV}$$

K^+ decays

$$a^{-1} = 1.364\ \mathrm{GeV},\ m_\pi = 142\ \mathrm{MeV},\ m_K = 506\ \mathrm{MeV}$$

$$W_{2\pi} = 486\ \mathrm{MeV}$$

$\Delta I = 1/2$ rule

P. Boyle et al., PRL110(2013)152001

Lattice

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = \begin{cases} 9.1(2.1) & \text{for } m_K = 878\ \mathrm{MeV},\quad m_\pi = 422\ \mathrm{MeV} \\ 12.0(1.7) & \text{for } m_K = 662\ \mathrm{MeV},\quad m_\pi = 329\ \mathrm{MeV}. \end{cases}$$

Experiment

$$\frac{\mathrm{Re}\ A_0}{\mathrm{Re}\ A_2} = 22.45(6)$$

$$a^{-1} = 1.73\ \mathrm{GeV}$$

4. EoS at Finite Temperature QCD
有限温度QCDの状態方程式

Finite temperature QCD

Phase transition at finite T

hadrons (quark confinement) → quark-gluon plasma (deconfinement)

$T \to \text{large}$

Lattice QCD at finite temperature

$N_s^3 \times N_T, N_T \ll N_s$ → $T = \frac{1}{N_t a}$

Equation of State (EoS)

$p(T), s(T), \varepsilon(T)$

free energy

$F = -T \log Z$

→

pressure $\frac{p(T)}{T} = \frac{\partial \log Z}{\partial V} \simeq \frac{\ln Z}{V}$

energy density $\varepsilon(T) = -\frac{1}{V}\frac{\partial \log Z}{\partial 1/T}$

entropy density $s(T) = \frac{1}{V}\frac{\partial (T \log Z)}{\partial T} = \frac{p(T)}{T} + \frac{\varepsilon(T)}{T}$

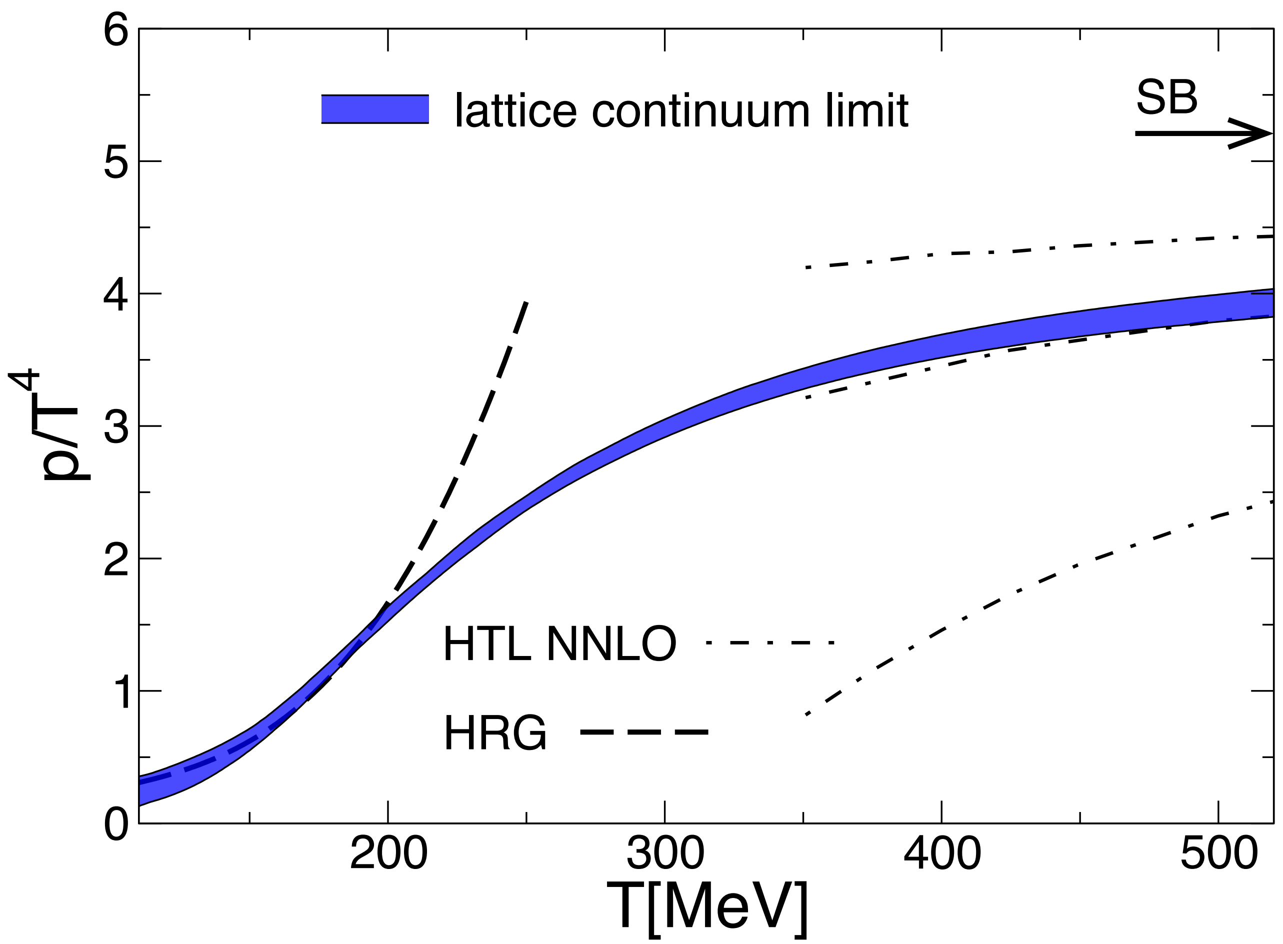
The latest lattice results

Equation of states from lattice QCD

Borsanyi et al.

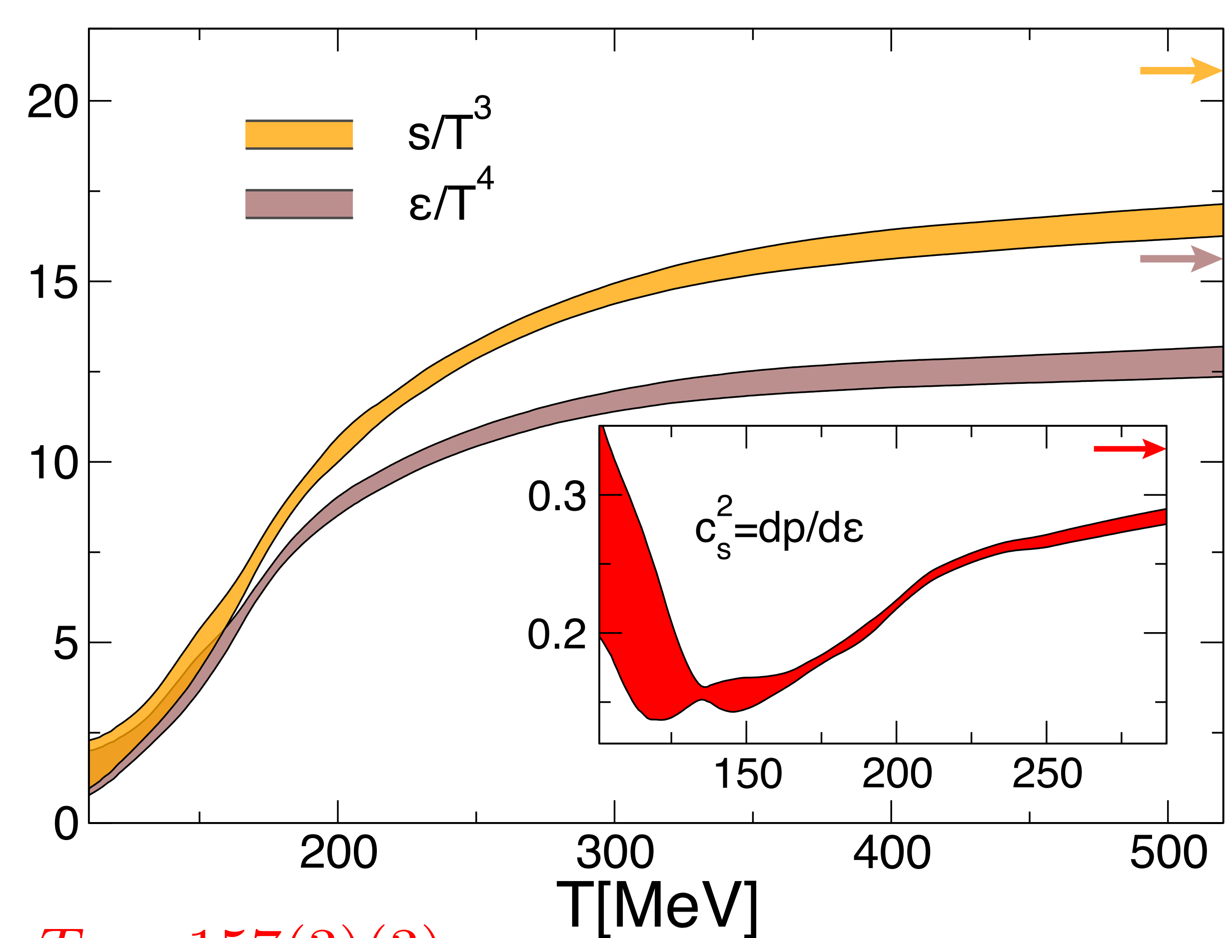
arXiv:1312.2193[hep-lat], 2+1 flavor QCD

Pressure



$T_c = 157(3)(3)$

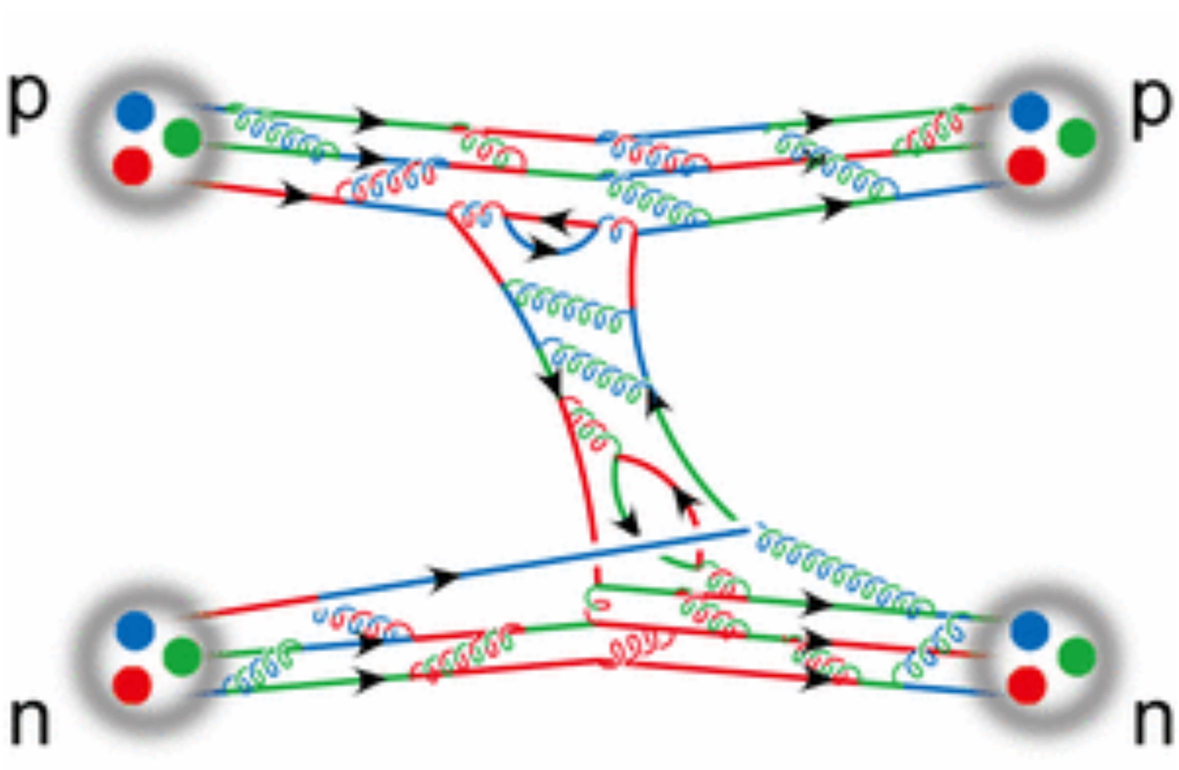
Entropy & energy density



$T_c = 157(3)(3)$

speed of sound

5. Hadron interactions
格子QCDによる核力の計算



--approaches to nuclear physics from lattice QCD--

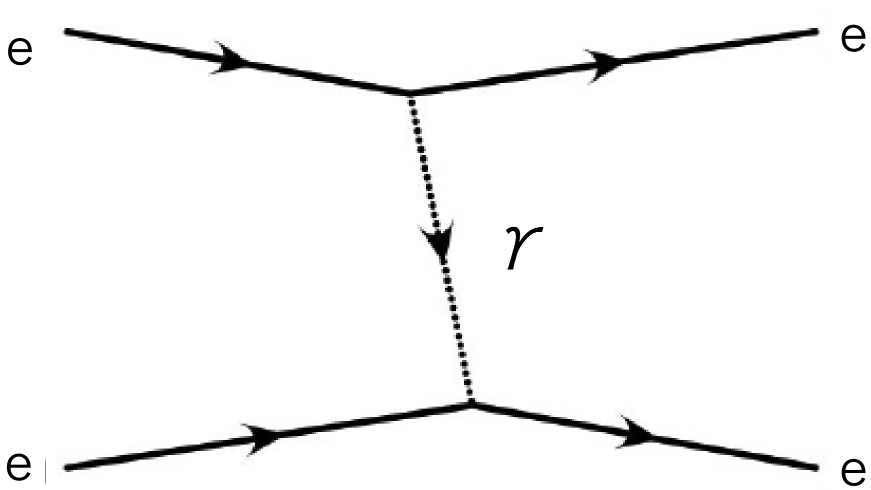
5-1. Hadron Interactions

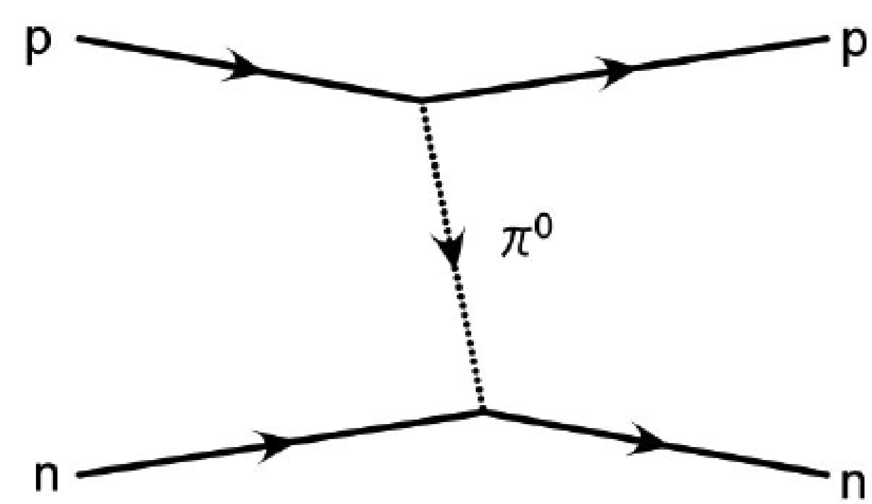
Ex. Nuclear Force

1949 Nobel prize
(1st in Japan)

Meson Theory (before quarks) 1935 Hideki Yukawa (1st director of YITP)

- Nucleons interact with each other by exchanging virtual particles.
- the interaction range is proportional to the inverse of the virtual particle's mass
    - -> the virtual particles are heavier than electrons but lighter than nucleons
        - ->(π)"meson"





Coulomb potential

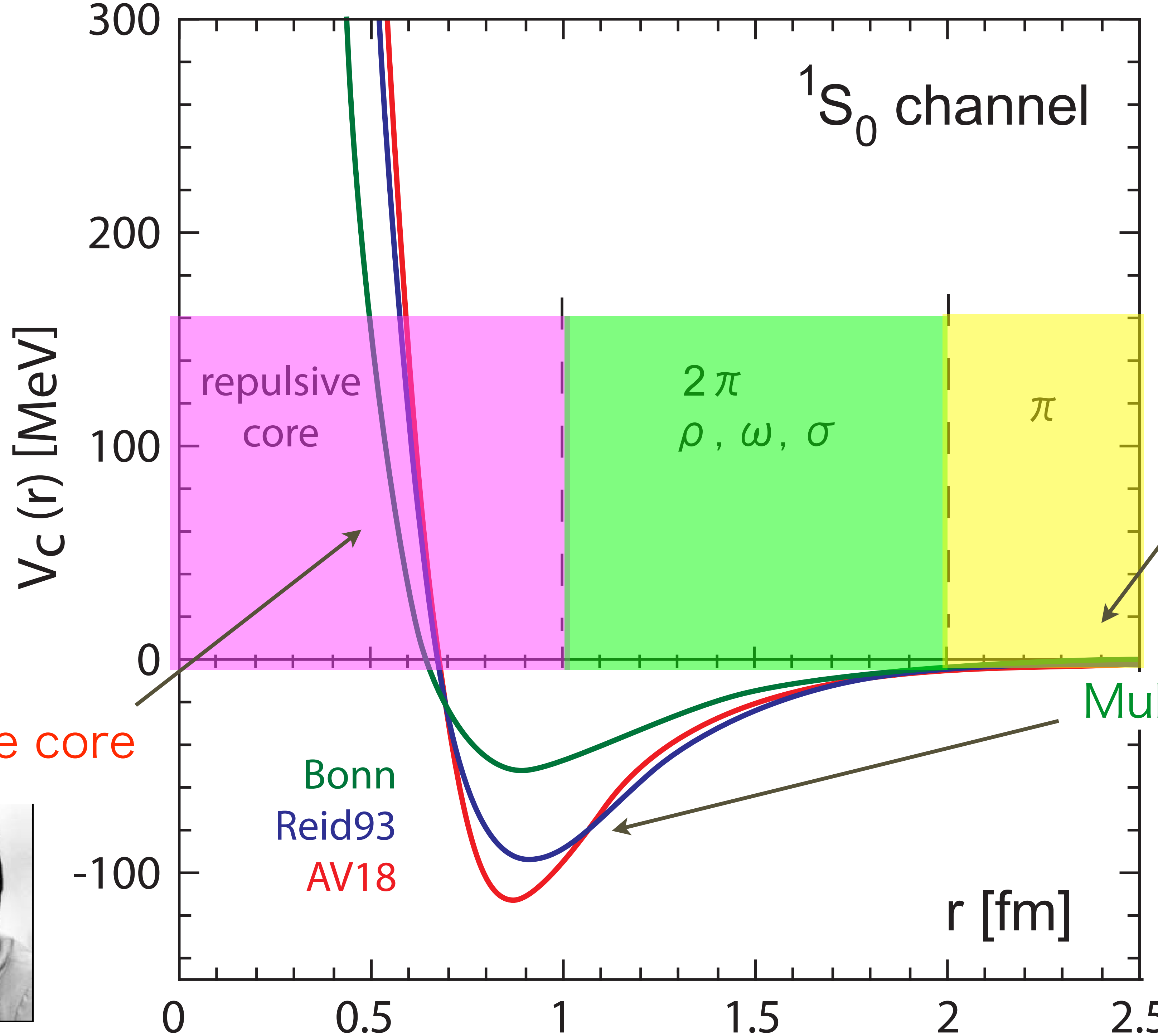
$$V(r) = \frac{e^2}{4\pi}\frac{1}{r}$$

Yukawa potential

$$V(r) = \frac{g^2}{4\pi}\frac{e^{-m_\pi r}}{r}$$

Modern nuclear forces after Yukawa

Nuclear Potential



Repulsive core

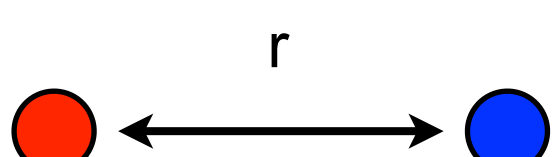
Jastrow(1951)

Yukawa(1935)

One-pion exchange

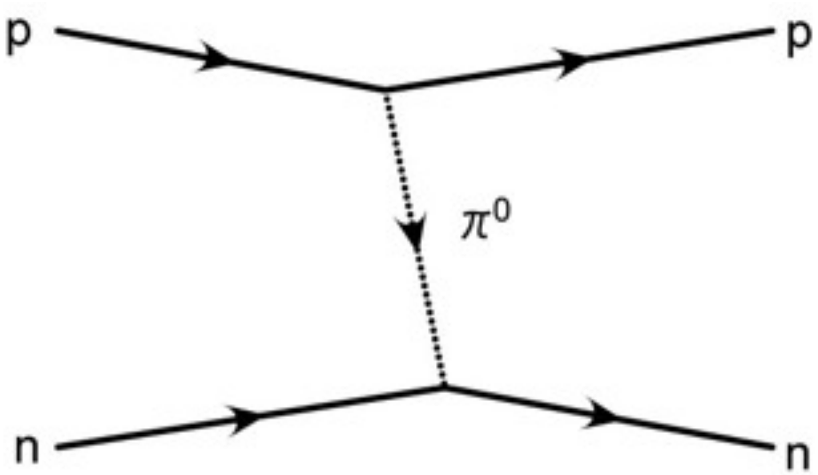
Multi-pions/heavy mesons

Taketani et al.(1951)

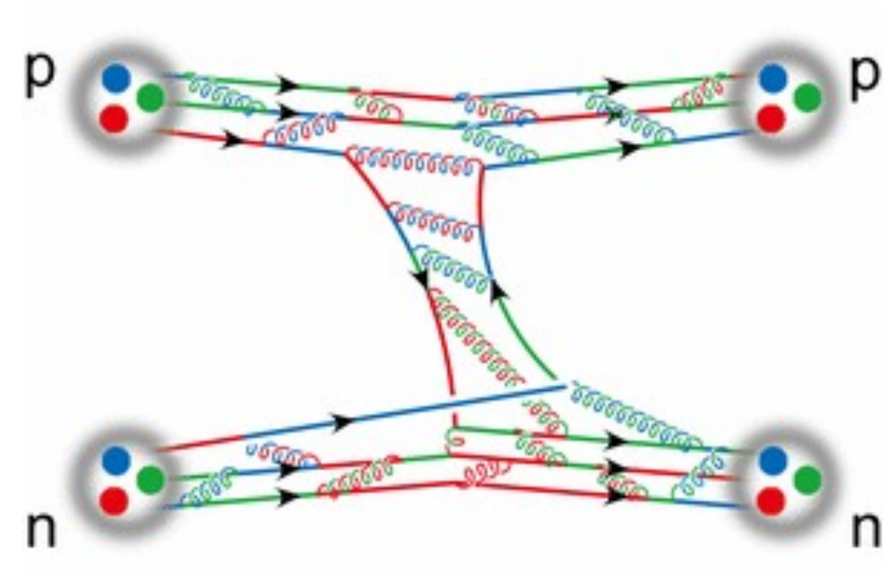


distance between nucleons

Nuclear forces in terms of quarks ?



Meson Theory



Quark Theory

Much more difficult than masses.

5-2. Three strategies to nuclear physics

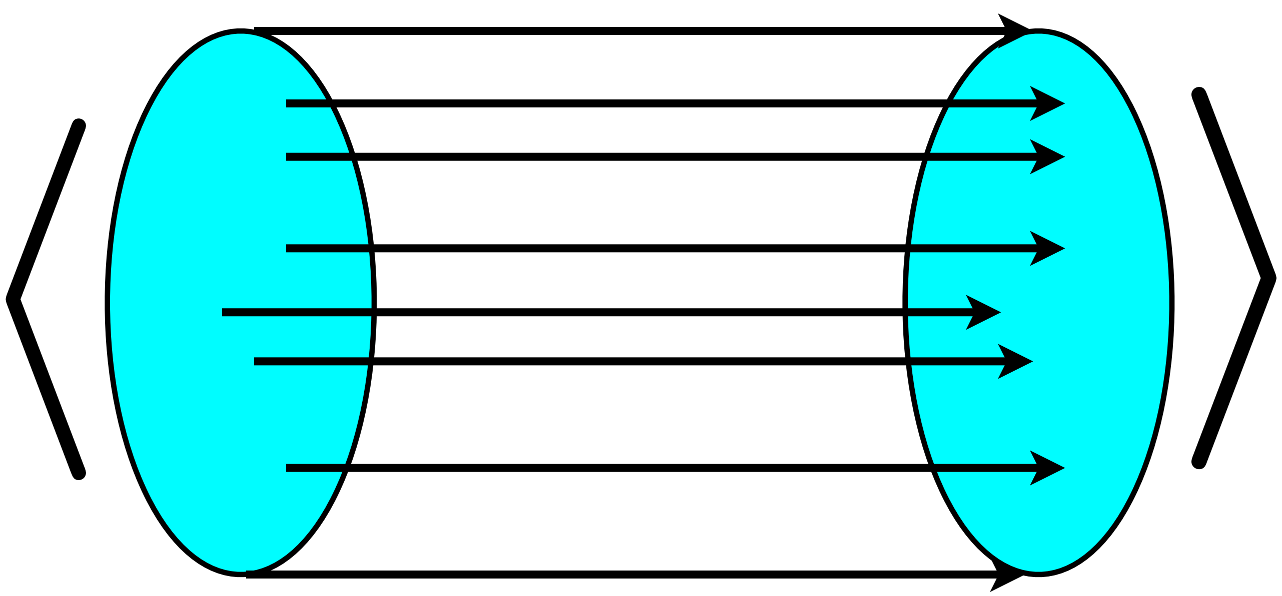
Extreme calculate nuclei directly from lattice QCD

Ab-Initio but (almost) impossible.

difficult to extract "physics" from results

difficult to apply results to other systems

nuclei propagator



$$\simeq e^{-m_A t} + \cdots$$

3A quark lines A: atomic number

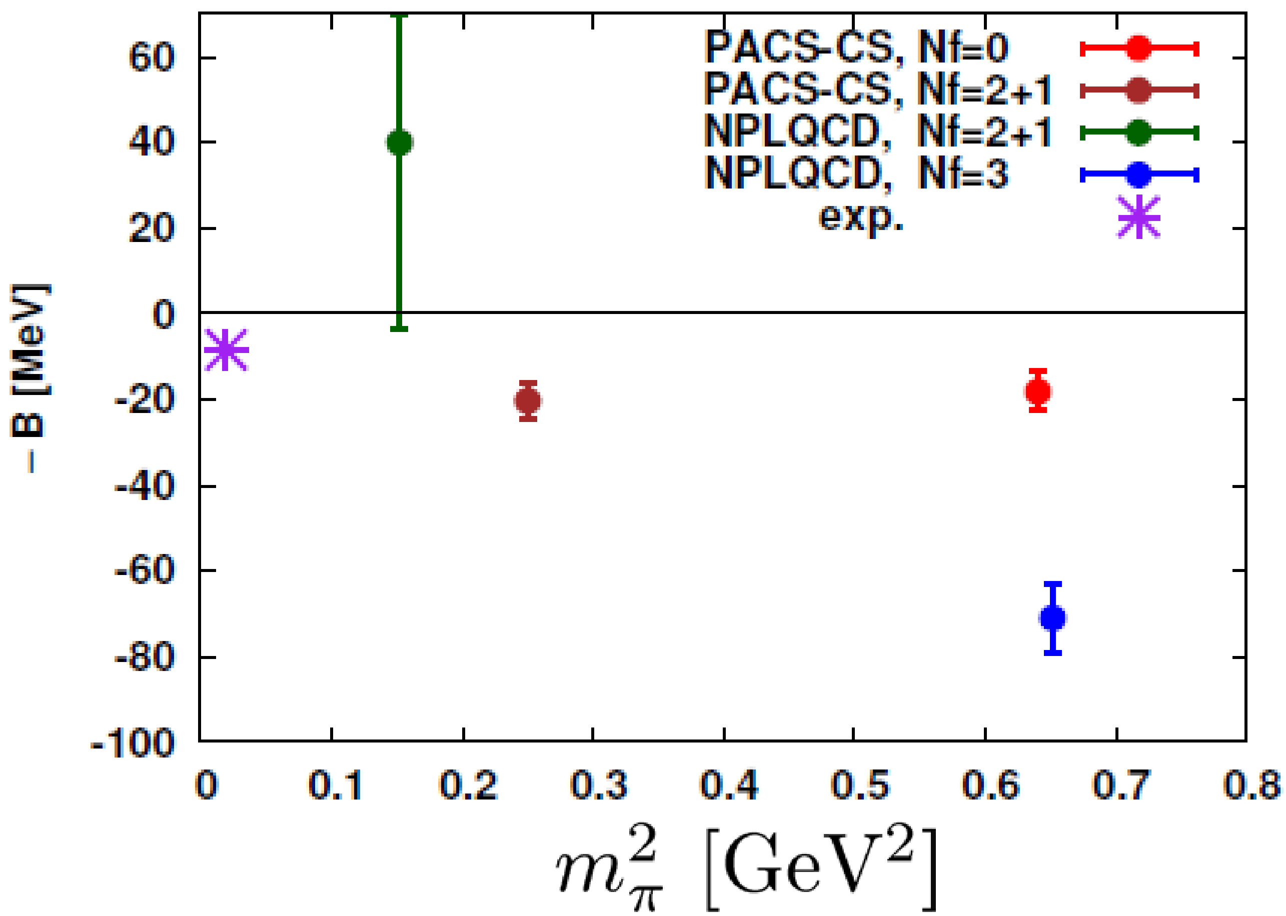
large number of contractions/very noisy

↑

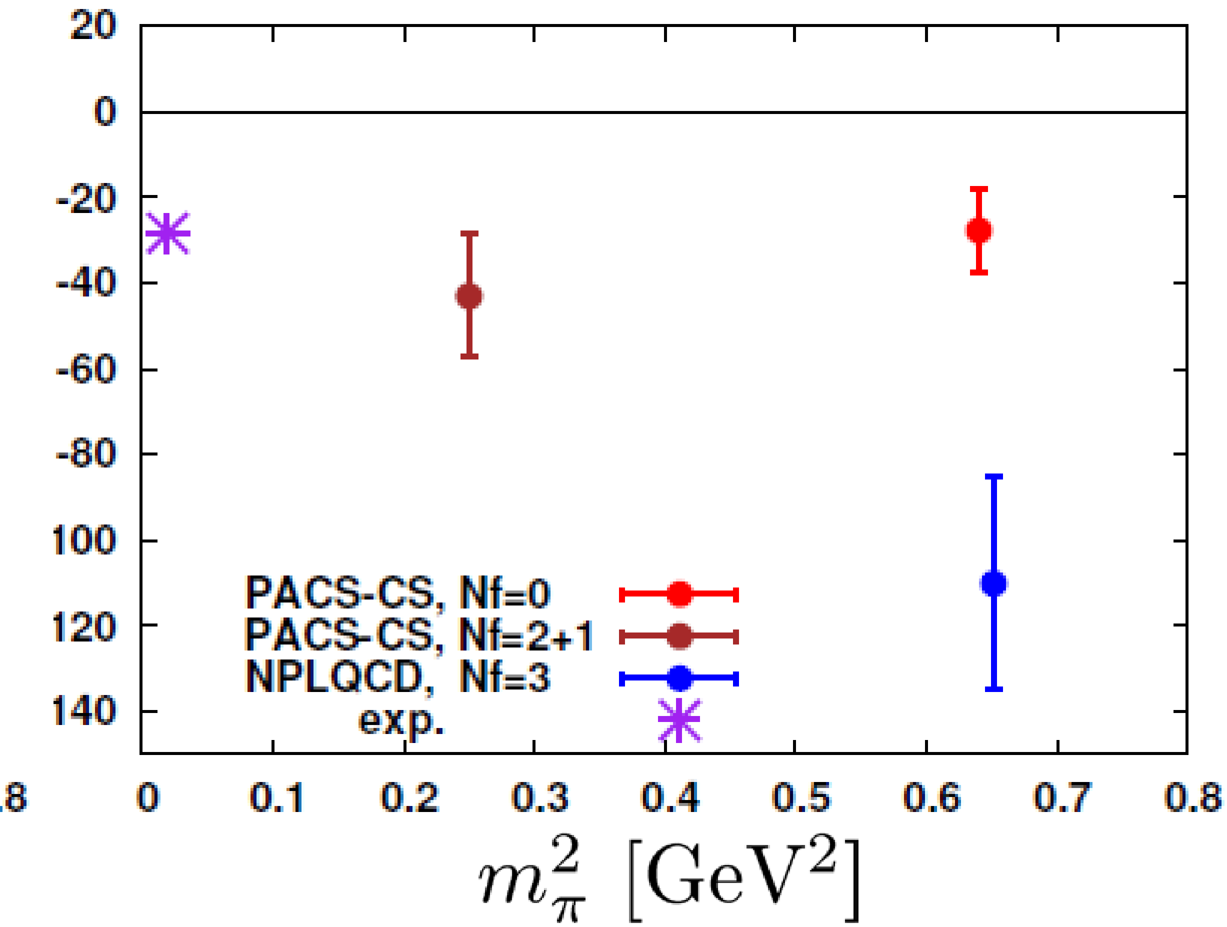
some reduction (Doi-Endres, CPC 184(2013)117)

binding energy of A=3,4 nuclei

$^{3}\mathrm{H}\ (={}^{3}\mathrm{He})$



$^{4}\mathrm{He}$



PACS-CS, PRD81(2010)111504, PRD86 (2012) 074514.
NPLQCD, PPNP66(2011)1, arXiv:1004.2935.

signals can be obtained, though results scatter.

Standard calculate NN phase shift from lattice QCD

Ab-Initio for phase shift. Results can not be directly applied to nuclear physics.

phase shift → nuclear potential → nuclear structure

Lüsher's finite volume method for the phase shift

two particles in the finite box $(V = L^3)$

energy $E = 2\sqrt{\mathbf{k}^2 + m^2}$ → $\mathbf{k} \neq \frac{2\pi}{L}\mathbf{n} \ (\mathbf{n} \in Z^3)$

due to the interaction between two particles

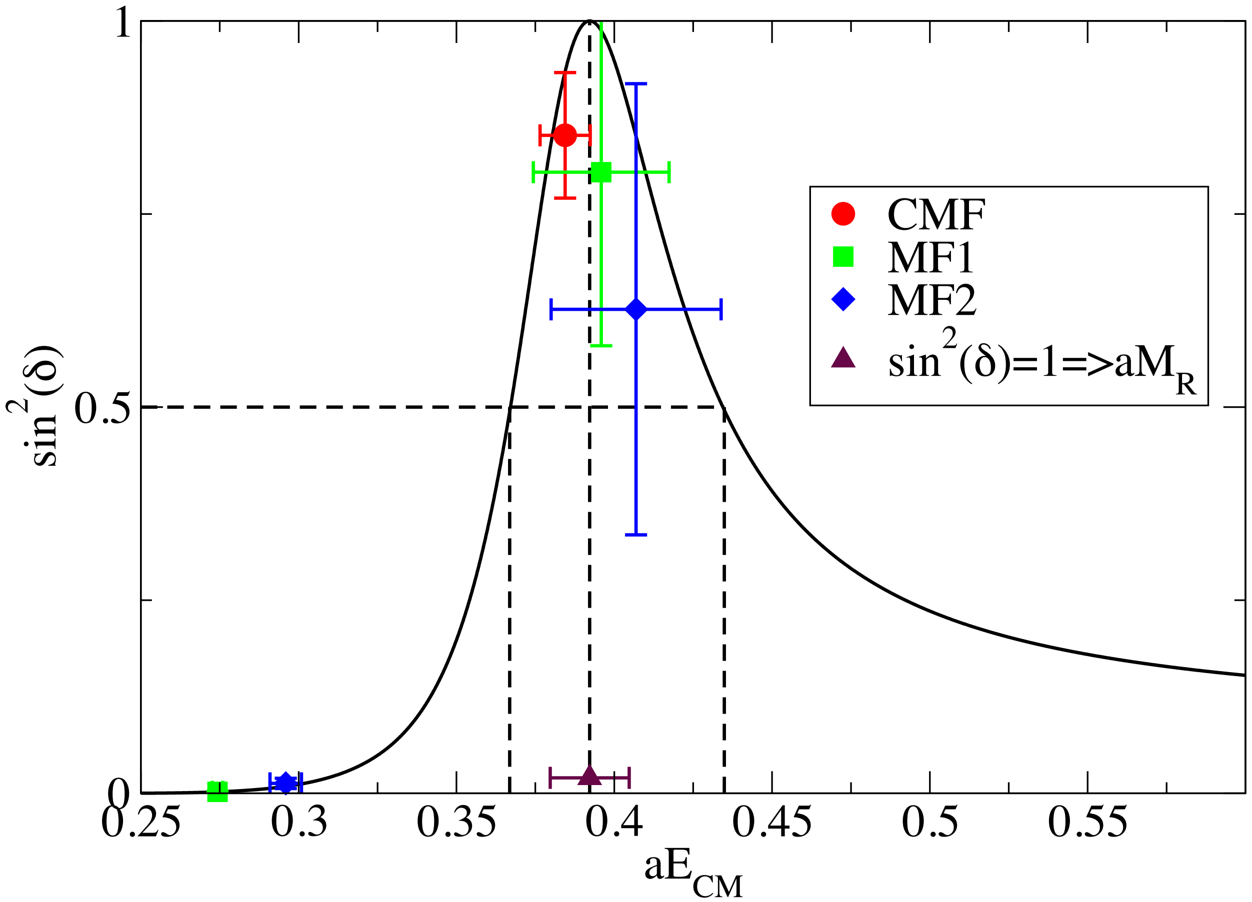
→ phase shift $\delta_l(k_n)$

Formula (Ex.) $k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} \underline{Z_{00}(1; q^2)}$ $k = |\mathbf{k}|$ $q = \frac{kL}{2\pi} \neq \mathbf{Z}$

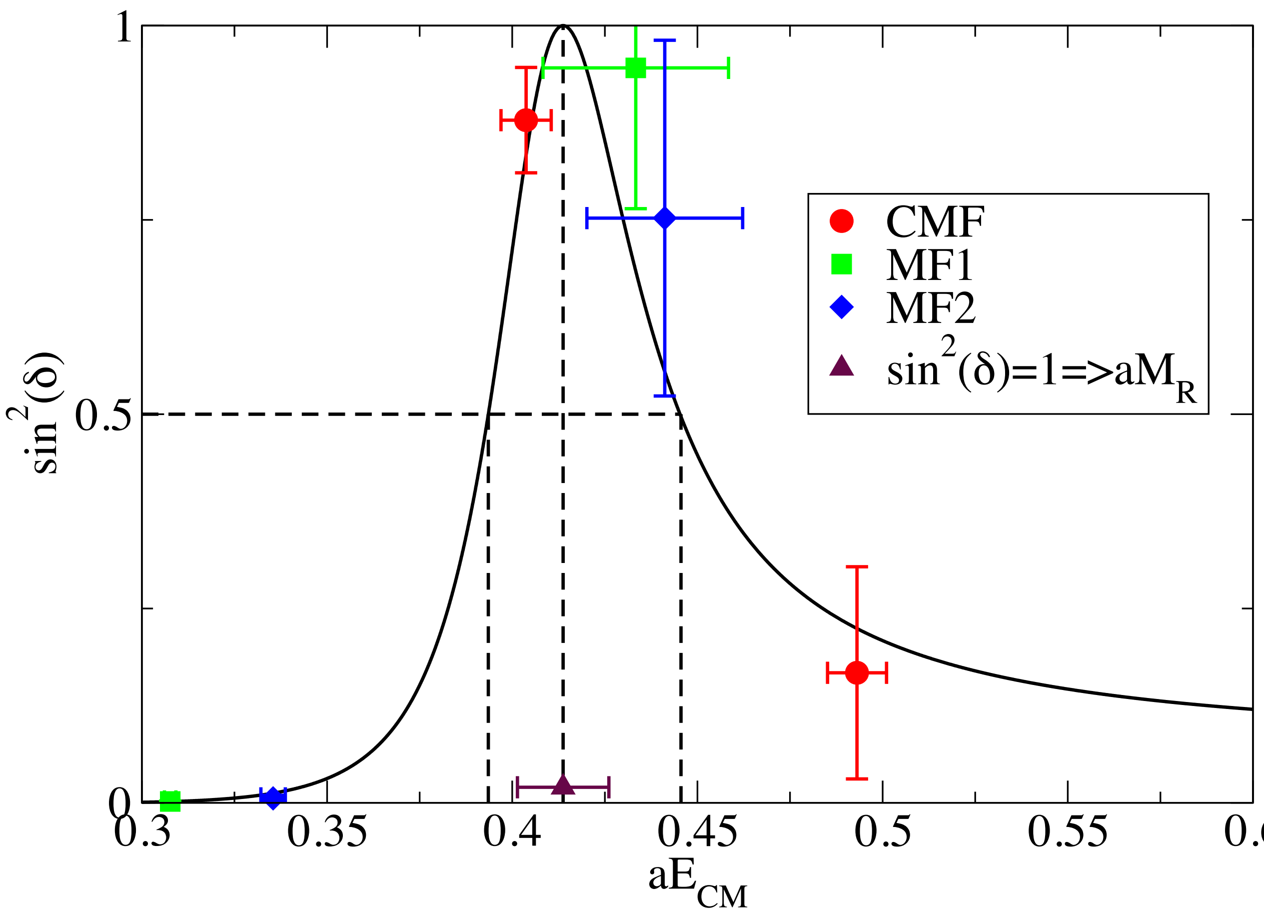
generalize zeta-function $Z_{00}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{n} \in \mathbf{Z}^3} (\mathbf{n}^2 - q^2)^{-s}$

$\pi^+\pi^-$ scattering (ρ meson width)

ETMC: Feng-Jansen-Renner, PLB684(2010)



$m_\pi = 290$ MeV



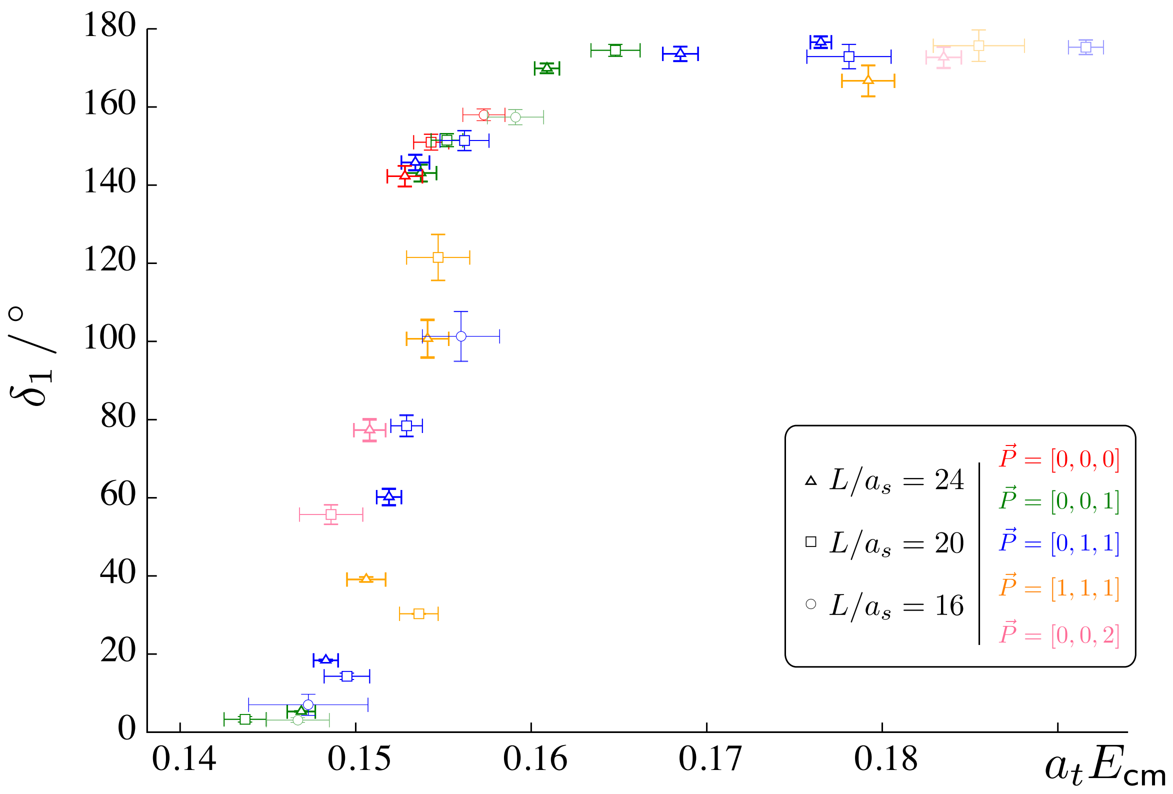
$m_\pi = 330$ MeV

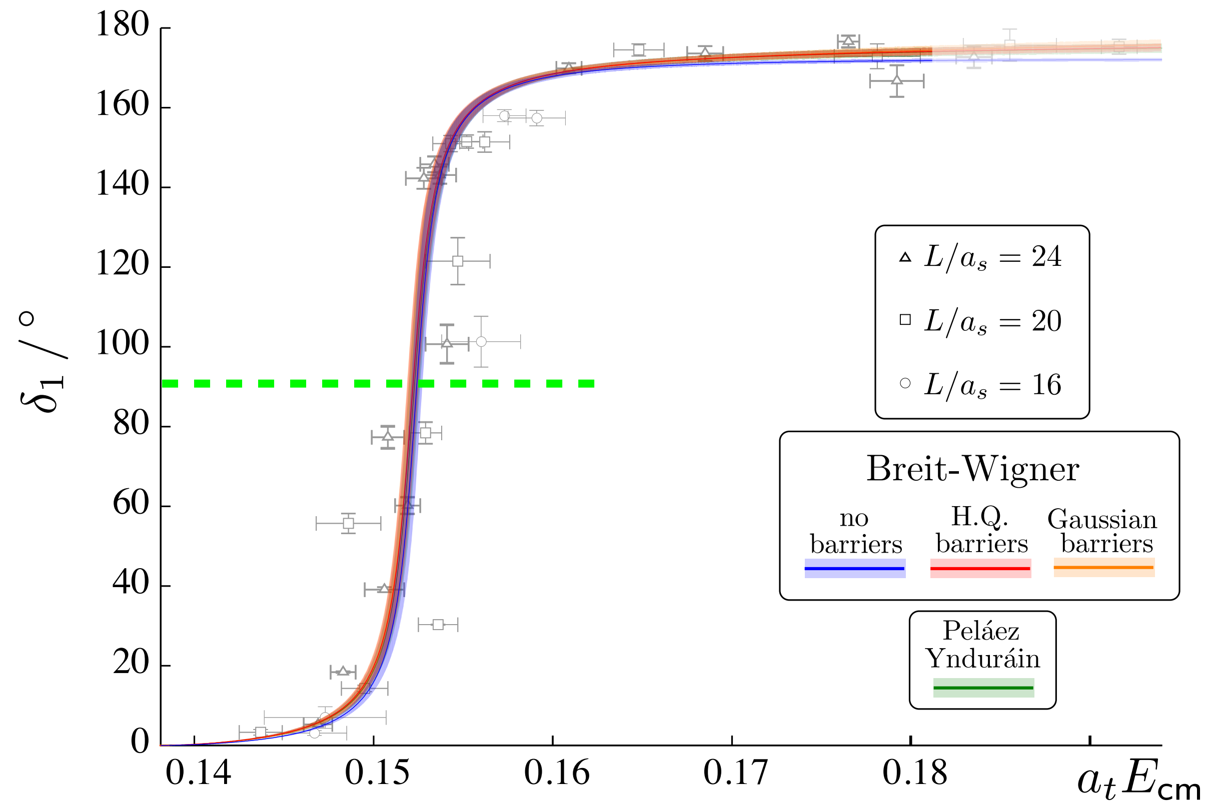
Resonance can be treated in this way.

$I = 1\ \pi\pi$ scattering (ρ resonance)

Dudek-Edwards-Thomas, PRD87(2013)034505

$\delta_1(E_{\rm cm})$





2-flavor anisotropic clover fermion

$a_s \sim 0.12\,\mathrm{fm}$

$m_\pi \sim 400\,\mathrm{MeV}$

Alternative calculate nuclear potential from lattice QCD strategy in this lecture

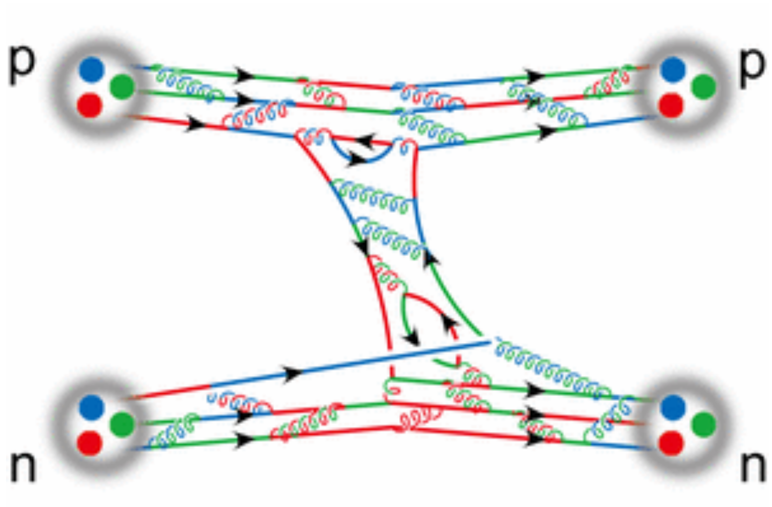
Ab-Initio for potential.

"Physics" is clear

nuclear potential ⇨ nuclear structure

Difficulties

A. Interactions are much more difficult than masses.



more complicated diagrams,
larger volume,
more Monte-Carlo sampling, etc.

B. Definition of potential in quantum theories ?

classical $V(x)$ ⇨ quantum $V(x)$ potential is an input

no classical NN potentials QCD $V_{NN}(x)$? output from QCD

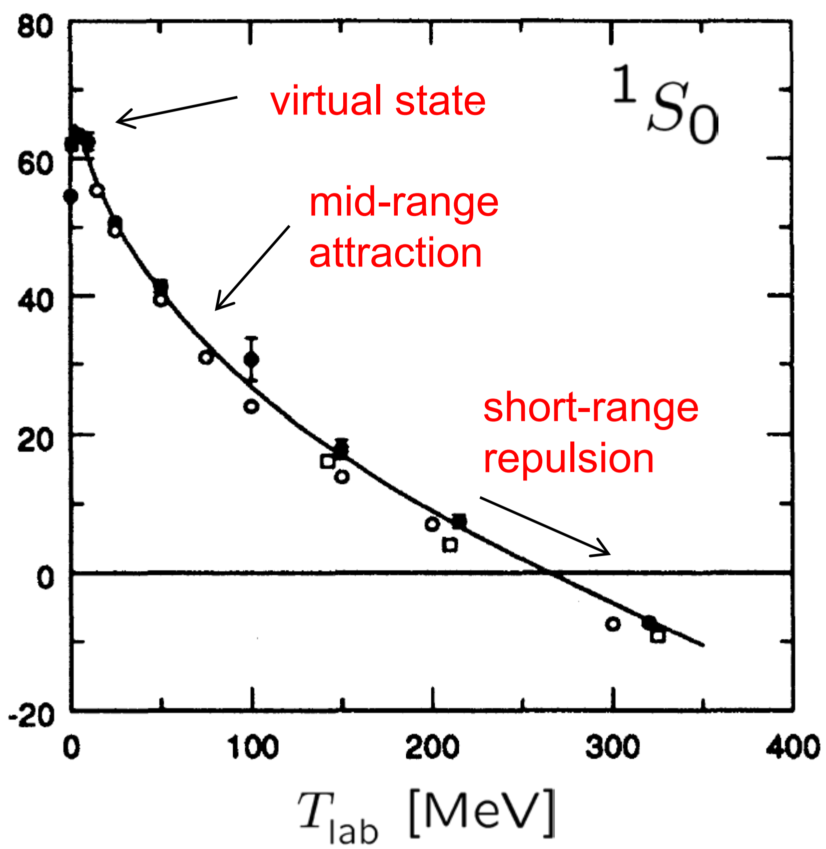
Potentials in QCD ?

What are “potentials” in quantum field theories such as QCD ?

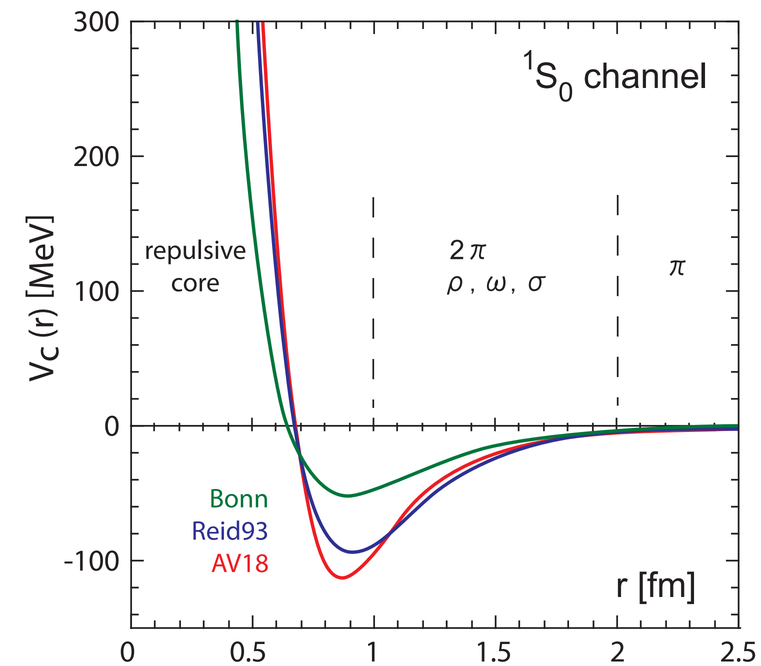
“Potentials” themselves can NOT be directly measured. cf. running coupling in QCD

scheme dependent, Unitary transformation

experimental data of scattering phase shifts



potentials, but not unique



useful to “understand” physics

cf. asymptotic freedom

“Potentials” are useful tools to extract observables such as scattering phase shift.

One may adopt a convenient definition of potentials as long as they reproduce correct physics of QCD.

5-3. Our strategy in lattice QCD

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89

Consider "elastic scattering"

$$NN \to NN \qquad \cancel{NN \to NN + \text{others}} \quad (\cancel{NN \to NN + \pi, NN + \bar{N}N, \cdots})$$

energy $\quad W_k = 2\sqrt{\mathbf{k}^2 + m_N^2} < W_{\rm th} = 2m_N + m_\pi \quad$ Elastic threshold

Quantum Field Theoretical consideration

Unitarity constrains S-matrix below inelastic threshold as

$$S = e^{2i\delta}$$

Ex. Scalar particles

$$\delta(k) = \begin{pmatrix} \delta_0(k) & & & \\ & \delta_1(k) & & \\ & & \delta_2(k) & \\ & & & \dots \end{pmatrix}$$

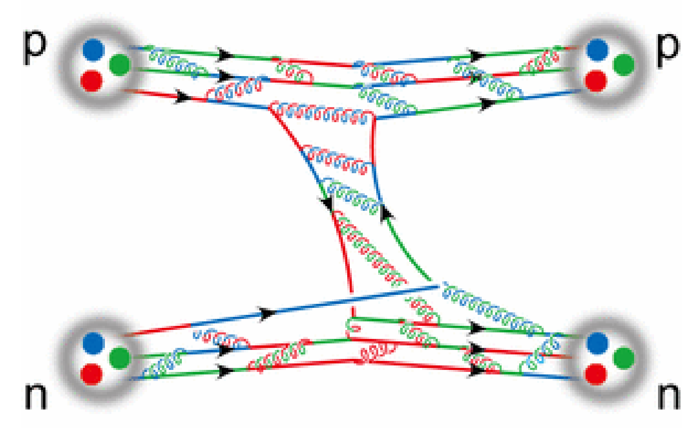
Step 1 define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

Spin model: Balog et al., 1999/2001

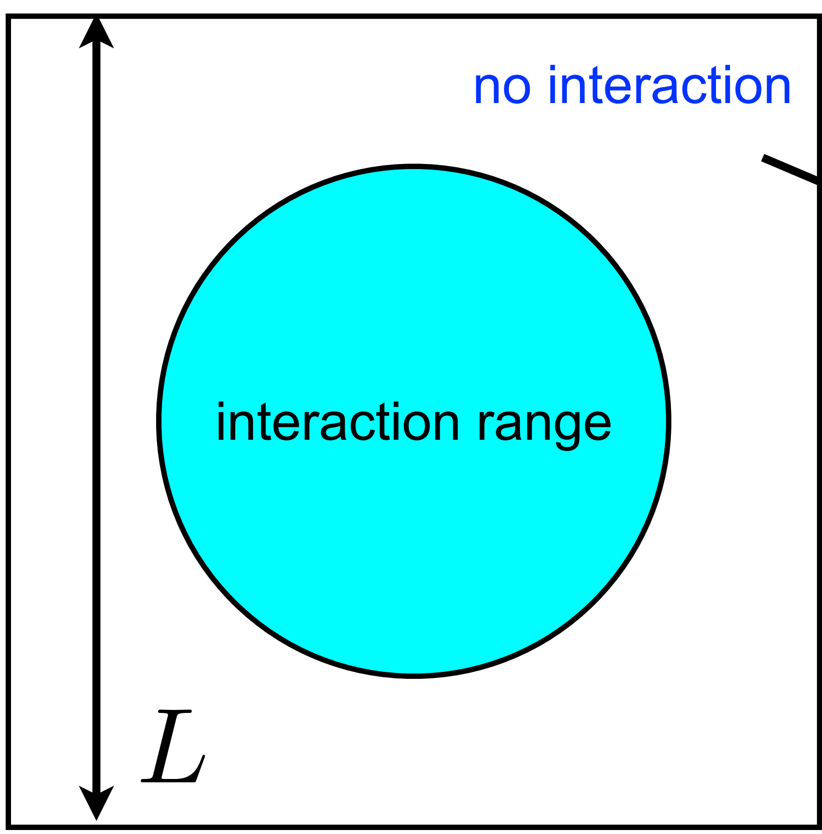
$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

QCD eigen-state

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator "scheme"



Asymptotic behavior of NBS wave function



$r = |\mathbf{r}| \to \infty$

partial wave

$$\varphi^l_{\mathbf{k}} \to A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}$$

NBS wave function ⟺ scattering wave function in quantum mechanics

cf. Luescher's finite volume method allowed k at L ⇒ $\delta_l(k_n)$

Step 2 define non-local but energy-independent "potential" as

$\mu = m_N/2$

reduced mass

$$\left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, \underline{U(\mathbf{x},\mathbf{y})} \varphi_{\mathbf{k}}(\mathbf{y})$$

non-local potential

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

(Trivial) proof of "existence"

We can construct a non-local but energy-independent potential easily as

inner product

$$U(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}^{W_k, W_{k'} \le W_{\rm th}} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \varphi^\dagger_{\mathbf{k}'}(\mathbf{y}) \qquad \eta^{-1}_{\mathbf{k},\mathbf{k}'}: \text{ inverse of } \eta_{\mathbf{k},\mathbf{k}'} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k}'})$$

For $^\forall W_{\mathbf{p}} < W_{\rm th}$

$$\int d^3y\, U(\mathbf{x},\mathbf{y}) \phi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(x) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \eta_{\mathbf{k}',\mathbf{p}} = \left[\epsilon_p - H_0\right] \varphi_{\mathbf{p}}(x)$$

Remark

Non-relativistic approximation is NOT used. We just take the specific (equal-time) frame.

Step 3 expand the non-local potential in terms of derivative as

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}) + V_T(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

spins

tensor operator $S_{12} = \frac{3}{r^2}(\boldsymbol{\sigma_1}\cdot\mathbf{x})(\boldsymbol{\sigma_2}\cdot\mathbf{x}) - (\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2})$

This expansion is a part of our "scheme" for potentials.

Step 4 extract the local potential at LO as

$$V_{\mathrm{LO}}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

Step 5 solve the Schroedinger Eq. in the infinite volume with this potential.

phase shifts and binding energy below inelastic threshold

We can check a size of errors of the LO in the expansion. (See later).

5.4 Results

Standard method to extract NBS wave function

NBS wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0|N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)|NN,W_k\rangle$$

Potential

$$[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

4-pt Correlation function

source for NN

$$F(\mathbf{r},t-t_0) = \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\overline{\mathcal{J}}(t_0)|0\rangle$$

complete set for NN

$$\begin{aligned} F(\mathbf{r},t-t_0) &= \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\sum_{n,s_1,s_2}|2N,W_n,s_1,s_2\rangle\langle 2N,W_n,s_1,s_2|\overline{\mathcal{J}}(t_0)|0\rangle + \cdots \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2}\varphi^{W_n}(\mathbf{r})e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N,W_n,s_1,s_2|\overline{\mathcal{J}}(0)|0\rangle. \end{aligned}$$

ground state saturation at large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r},t-t_0) = A_0\varphi^{W_0}(\mathbf{r})e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

NBS wave function

This is a standard method in lattice QCD and was employed for our first calculation.

Improved method

Ishii et al. (HALQCD), PLB712(2012) 437

normalized 4-pt function

$$R(\mathbf{r},t) \equiv F(\mathbf{r},t)/(e^{-m_N t})^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$-\frac{\partial}{\partial t} R(\mathbf{r},t) = \left\{ H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t)$$

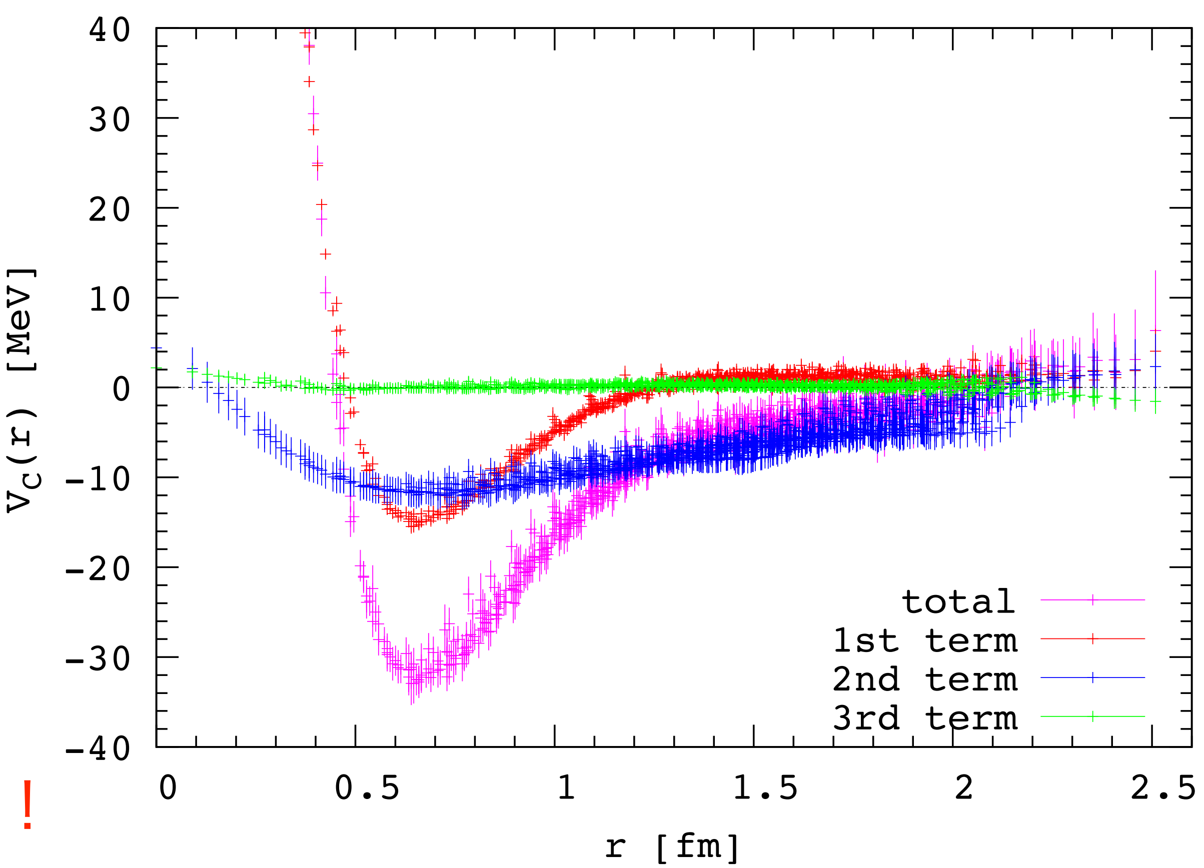
potential

Leading Order

$$\left\{ -H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t) = \int d^3r' \, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) = V_C(\mathbf{r}) R(\mathbf{r},t) + \cdots$$

1st 2nd 3rd total

3rd term(relativistic correction) is negligible.

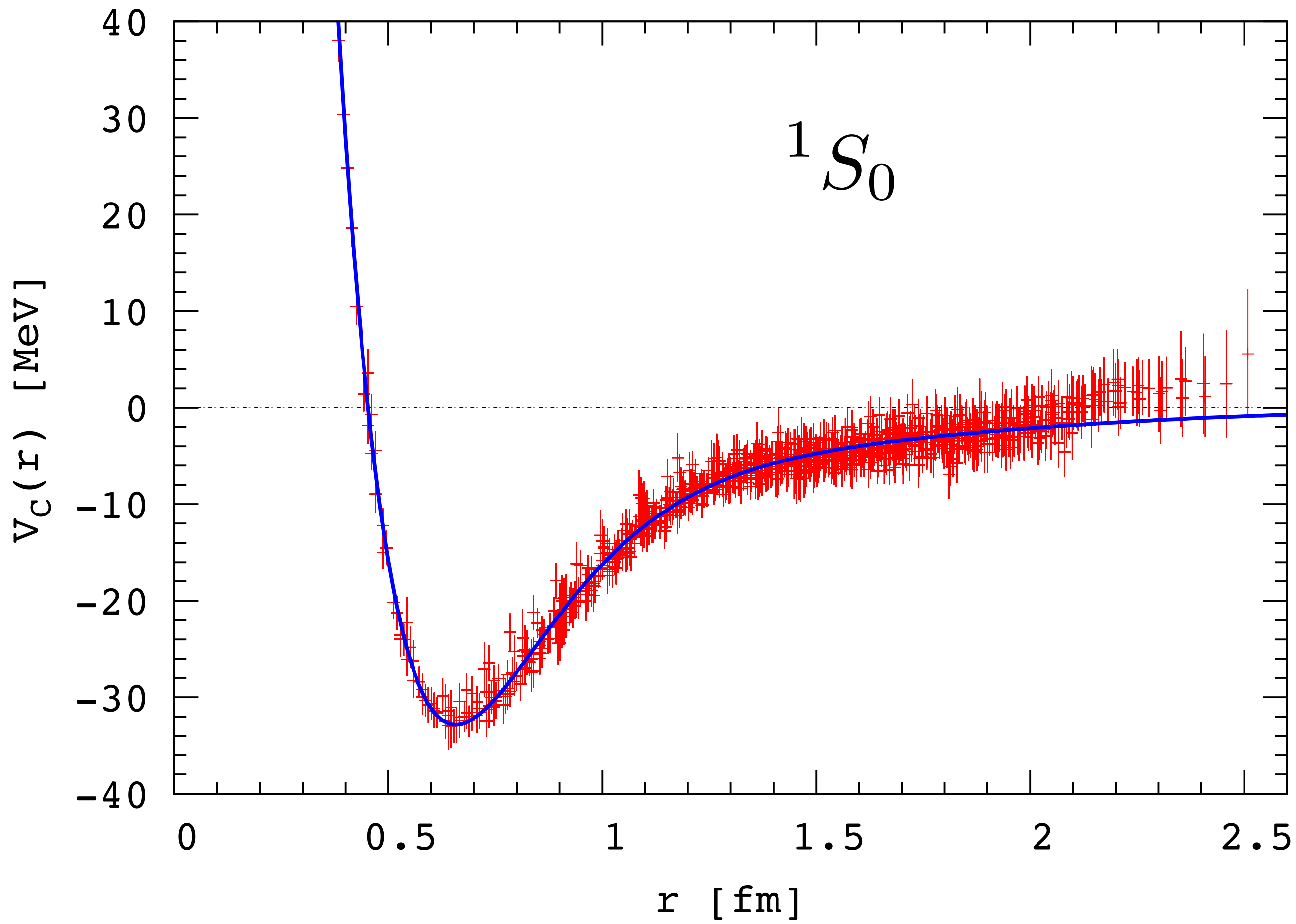


Ground state saturation is no more required !
(advantage over finite volume method.)

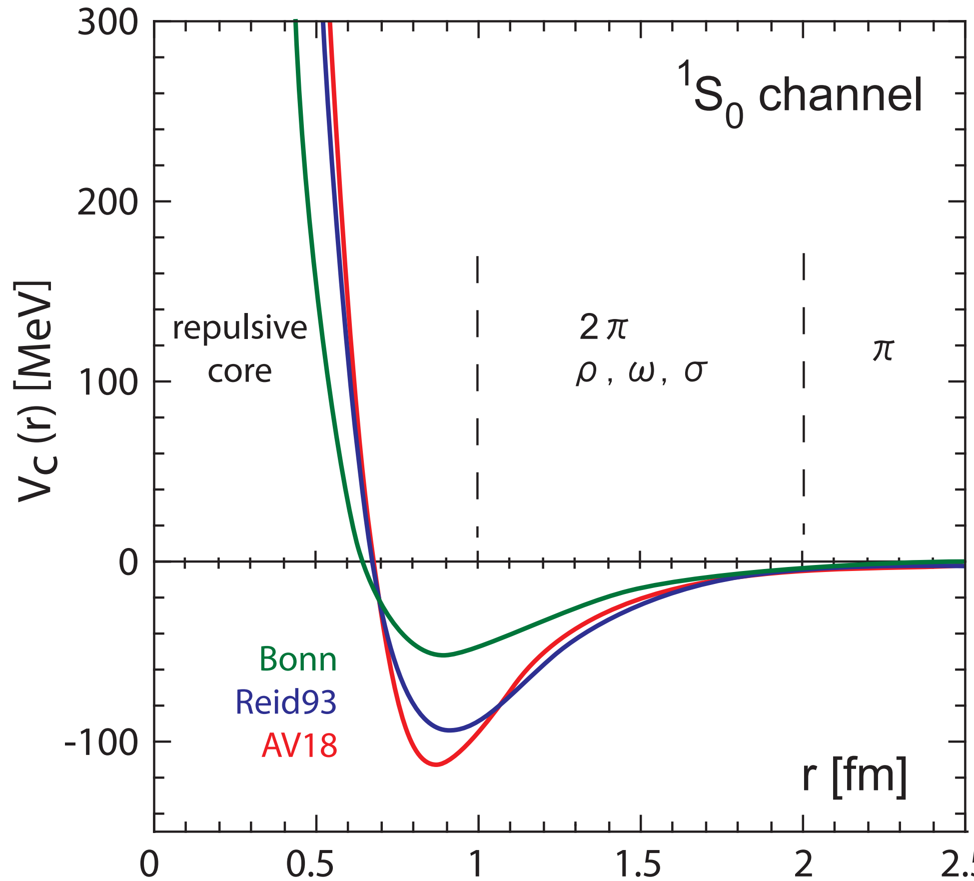
NN potential

2+1 flavor QCD, spin-singlet potential (PLB712(2012)437)

a=0.09fm, L=2.9fm $m_\pi \simeq 700$ MeV



phenomenological potential



Qualitative features of NN potential are reproduced !

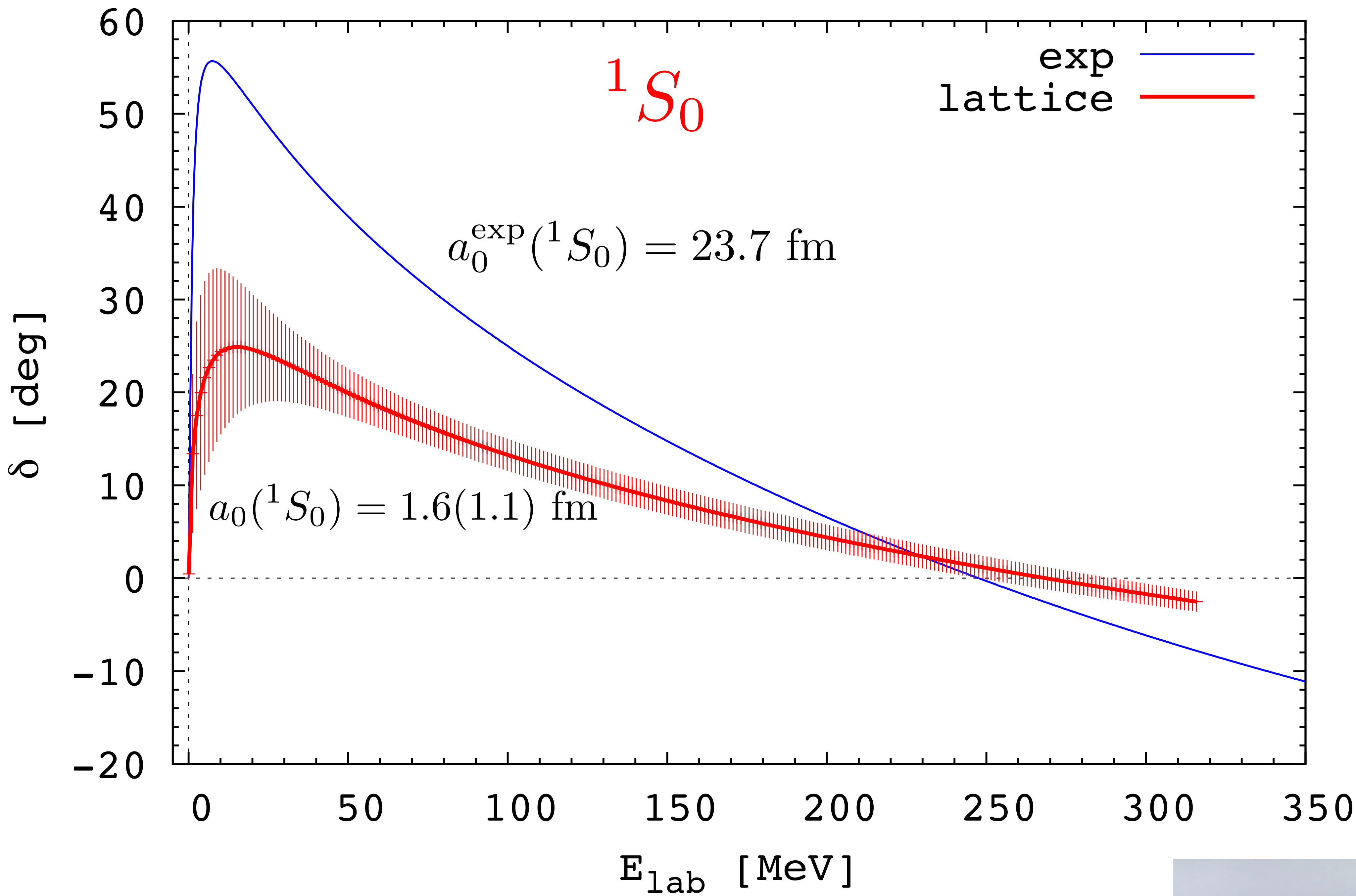
(1)attractions at medium and long distances

(2)repulsion at short distance(repulsive core)

1st paper(quenched QCD): Ishii-Aoki-Hatsuda, PRL90(2007)0022001

selected as one of 21 papers in Nature Research Highlights 2007. (One from Physics, Two from Japan, the other is on “iPS” by Sinya Yamanaka et al.)

NN potential → phase shift



It has a reasonable shape. The strength is weaker due to the heavier quark mass.

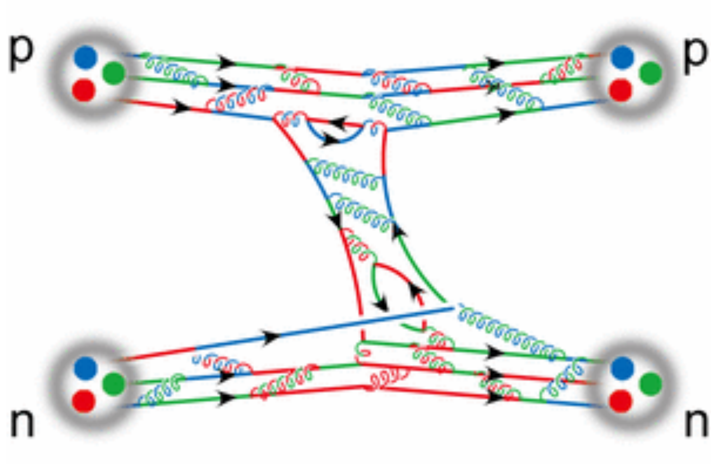
Need calculations at physical quark mass on K-computer.

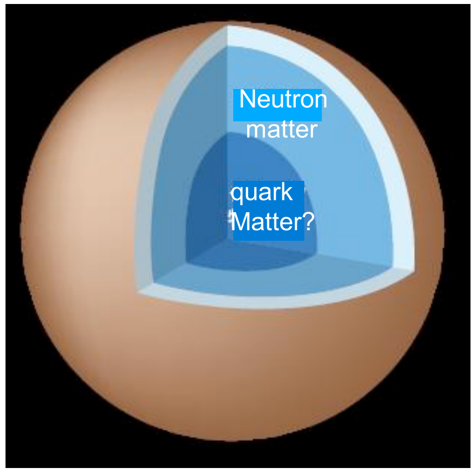
6. Summary

- Lattice QCD is a very powerful method to investigate dynamics of quarks
- not only hadron masses but also hadron interactions can be investigated from the 1st principle
- the potential (HALQCD) method is new but very useful to investigate not only the nuclear force but also general baryonic interactions in (lattice) QCD.
- the method can be easily applied also to meson-baryon and meson-meson interactions.

Our strategy

Potentials from lattice QCD → Nuclear Physics with these potentials → Neutron stars Supernova explosion





Back-up

Convergence of velocity expansion: estimate 1

If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different.(cf. LOC of ChPT).

Numerical check in quenched QCD

$m_\pi \simeq 0.53$ GeV

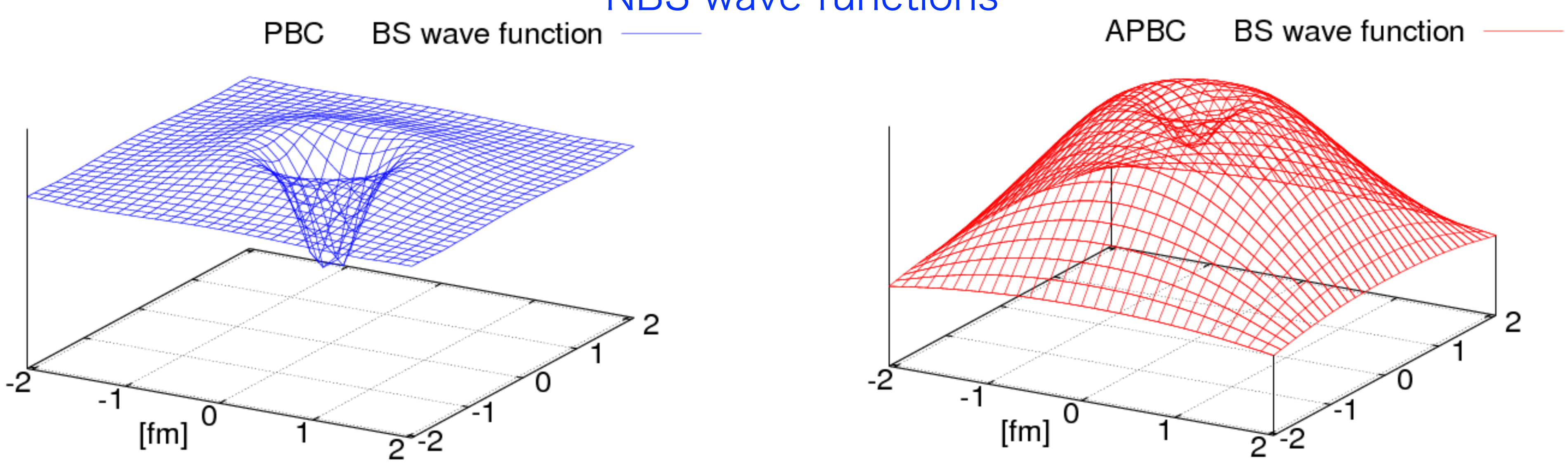
a=0.137fm, L=4.0 fm

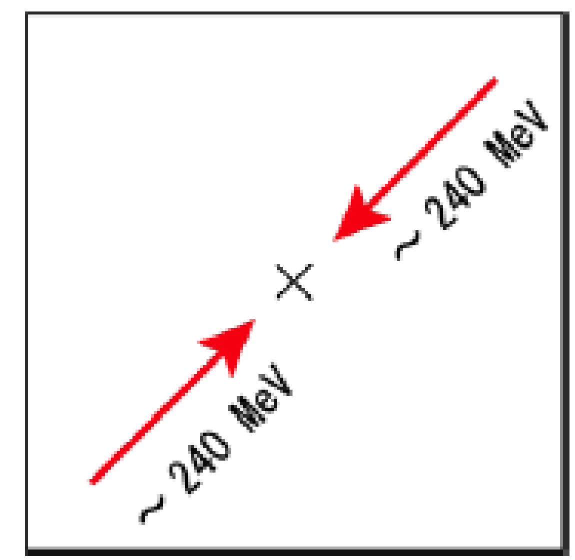
K. Murano, N. Ishii, S. Aoki, T. Hatsuda

PTP 125 (2011)1225.

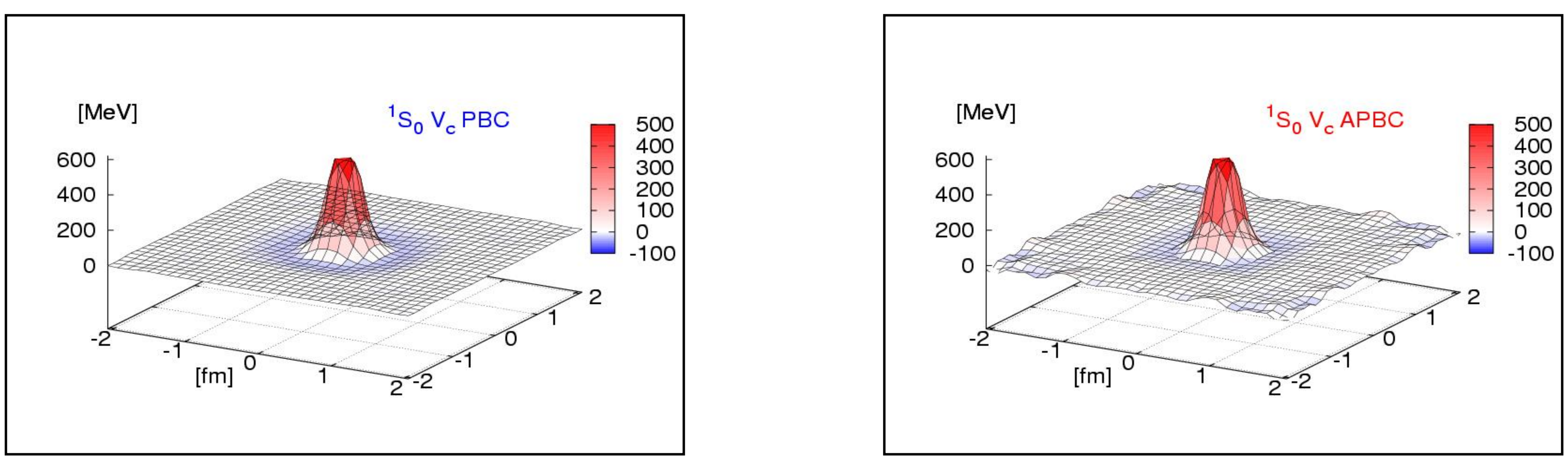
- PBC (E∼0 MeV)
- APBC (E∼46 MeV)

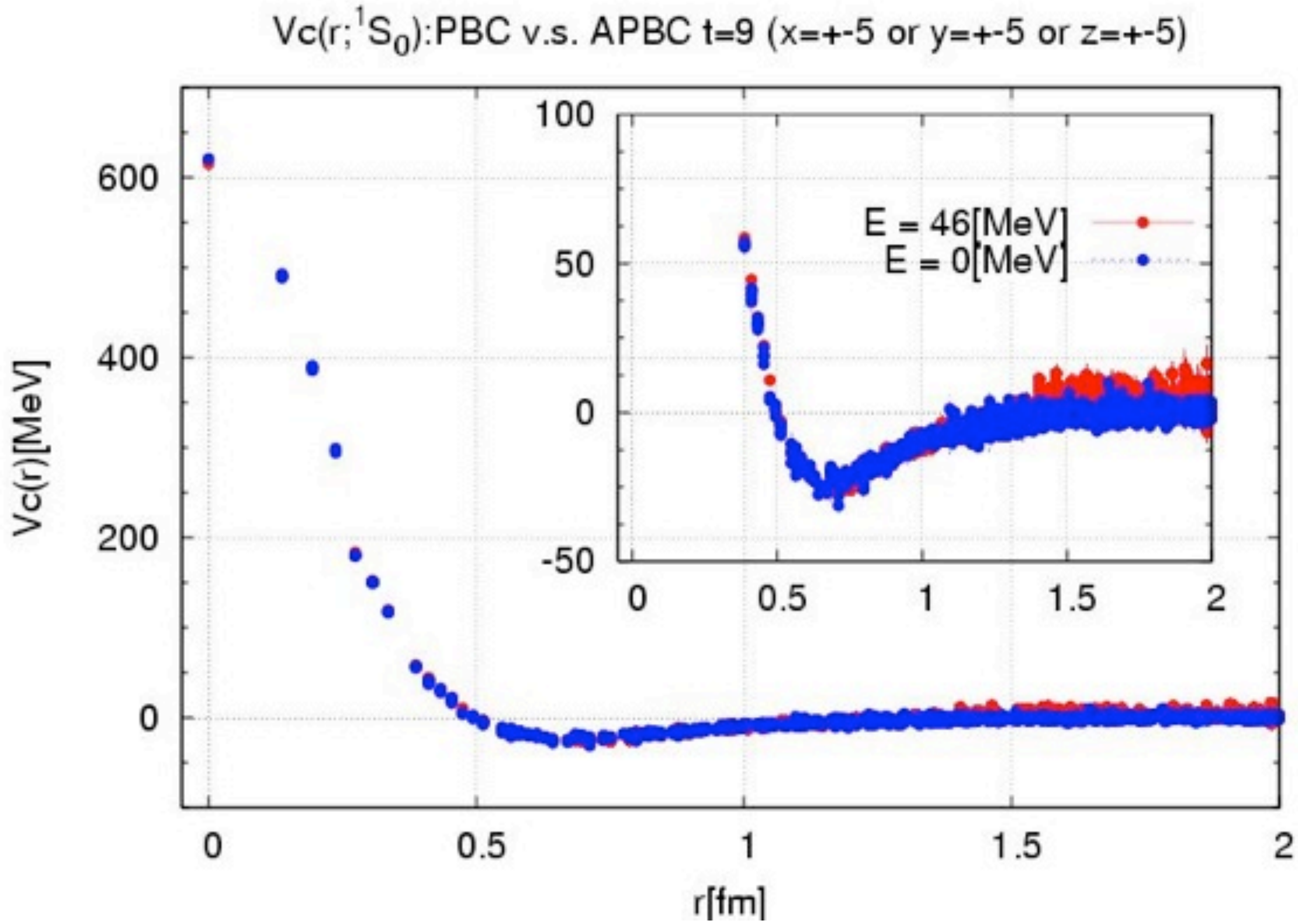
NBS wave functions

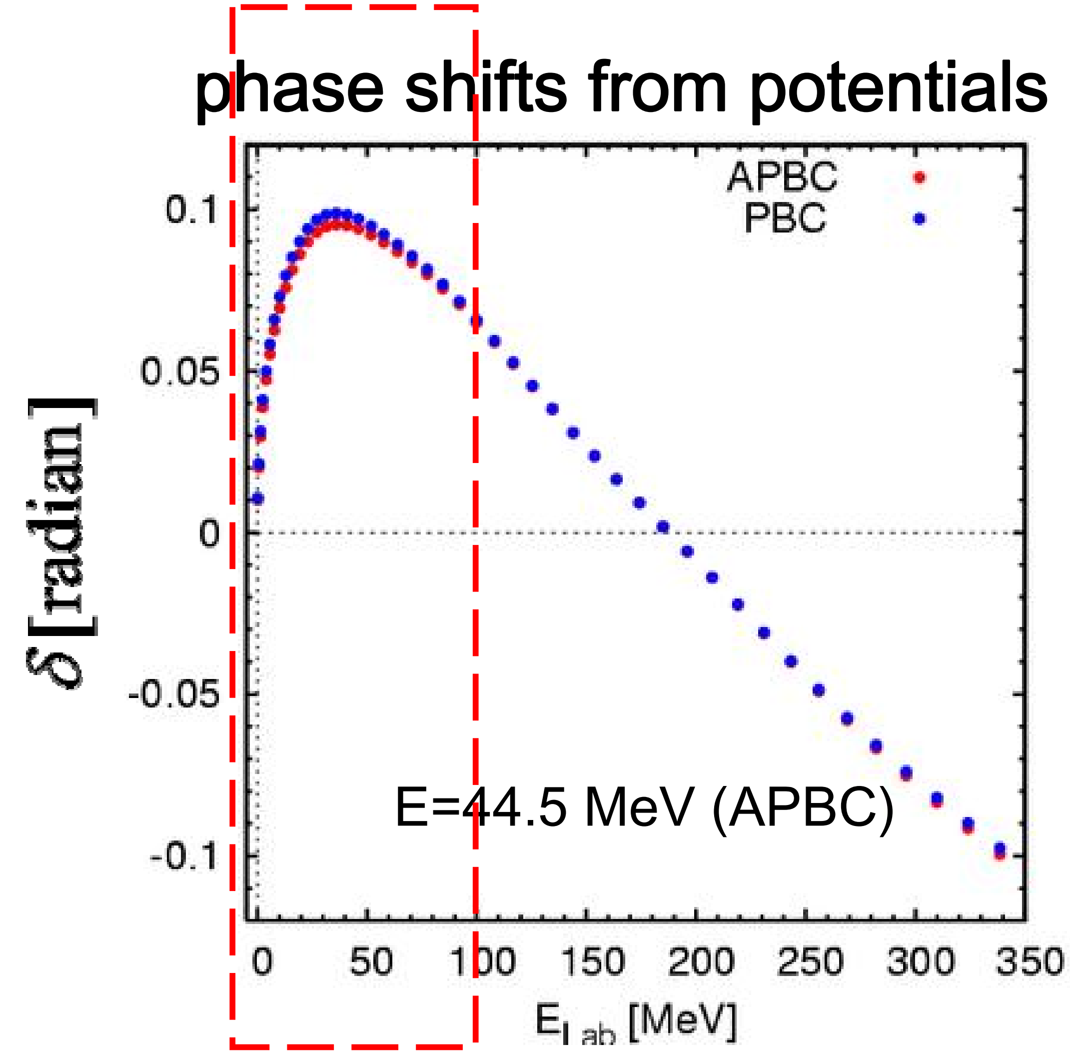




potentials







Higher order terms turn out to be very small at low energy in our scheme.

Need to be checked at lighter pion mass in 2+1 flavor QCD.

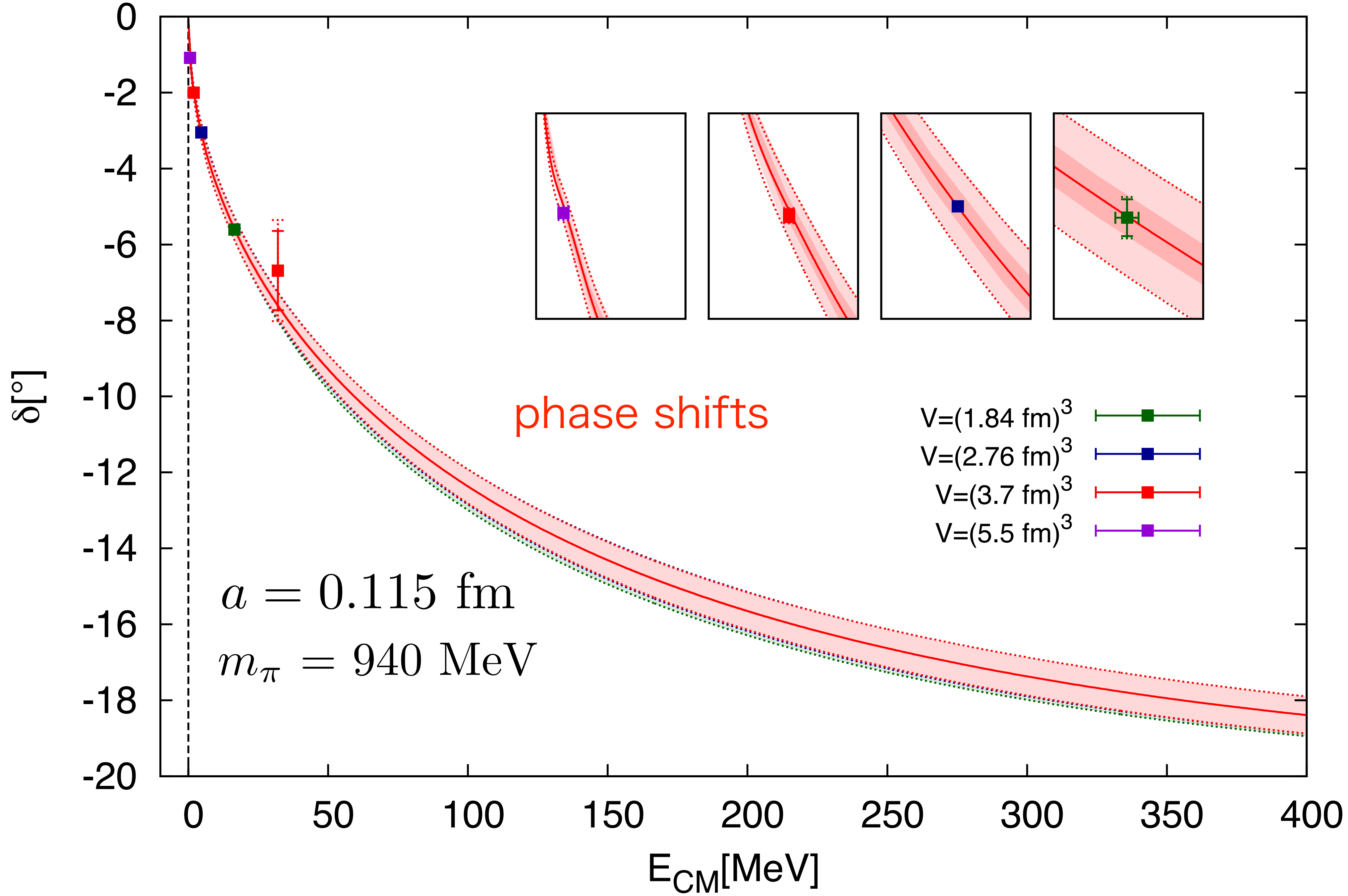
Note: convergence of the velocity expansion can be checked within this method.

(in contrast to convergence of ChPT, convergence of perturbative QCD)

Convergence of velocity expansion: estimate 2

Kurth, Ishii, Doi, Aoki & Hatsuda, JHEP 1312(2013)015

Potential vs Luescher (I=2 pi-pi scattering. Quenched QCD)



both methods agree very well.

This establishes a validity of the potential method and shows a good convergence of the velocity expansion.

More structure at LO

Tensor potential

$$(H_0 + V_C(r) + V_T(r)S_{12})\psi(\mathbf{r};1^+) = E\psi(\mathbf{r};1^+)$$

J=1, S=1

mixing between $^3S_1$ and $^3D_1$ through the tensor force

$$\underbrace{\psi(\mathbf{r};1^+) = \mathcal{P}\psi(\mathbf{r};1^+)}_{^3S_1} + \underbrace{\mathcal{Q}\psi(\mathbf{r};1^+)}_{^3D_1}$$

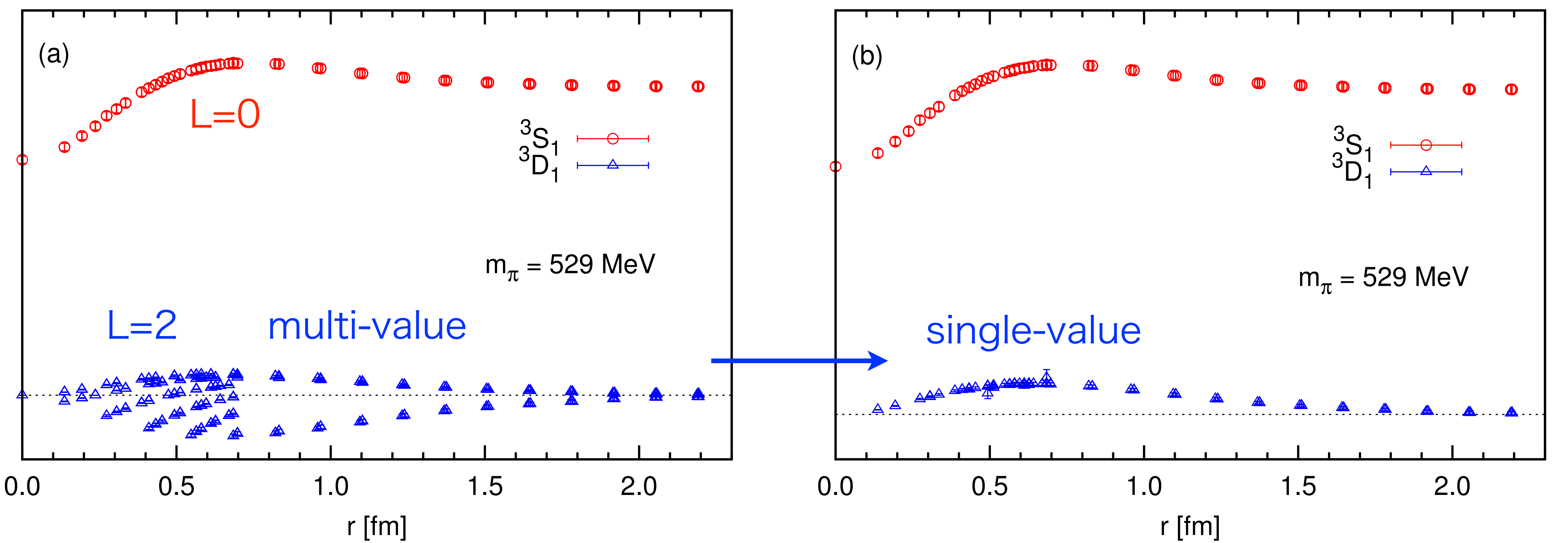
"projection" to L=0   "projection" to L=2

$$\Rightarrow \begin{aligned} H_0[\mathcal{P}\psi](\mathbf{r}) + V_C(r)[\mathcal{P}\psi](\mathbf{r}) + V_T(r)[\mathcal{P}S_{12}\psi](\mathbf{r}) &= E[\mathcal{P}\psi](\mathbf{r}) \\ H_0[\mathcal{Q}\psi](\mathbf{r}) + V_C(r)[\mathcal{Q}\psi](\mathbf{r}) + V_T(r)[\mathcal{Q}S_{12}\psi](\mathbf{r}) &= E[\mathcal{Q}\psi](\mathbf{r}) \end{aligned}$$

Wave functions

Quenched

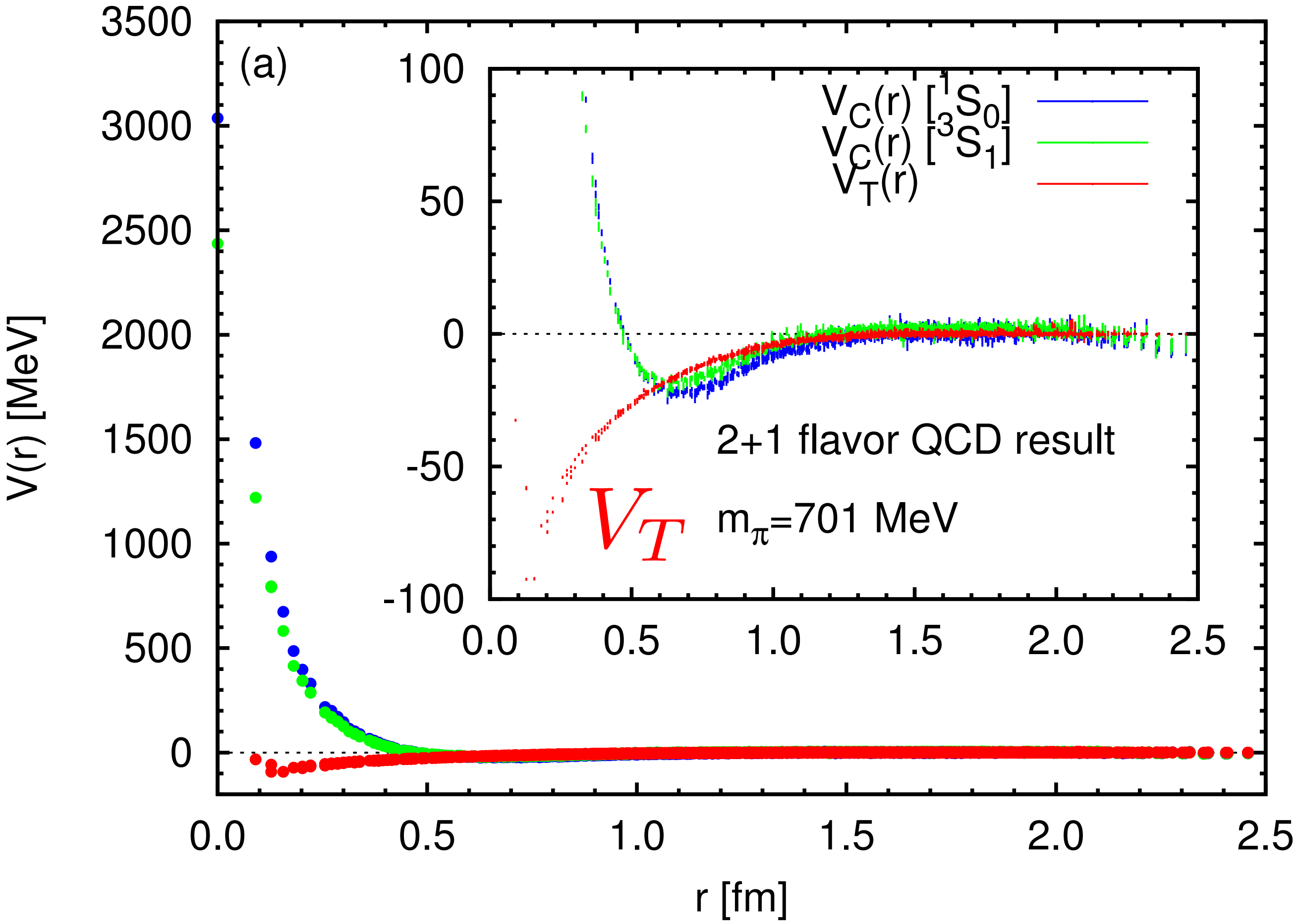
Aoki, Hatsuda, Ishii, PTP 123 (2010)89



divided by $Y_{20}(\theta, \phi)$

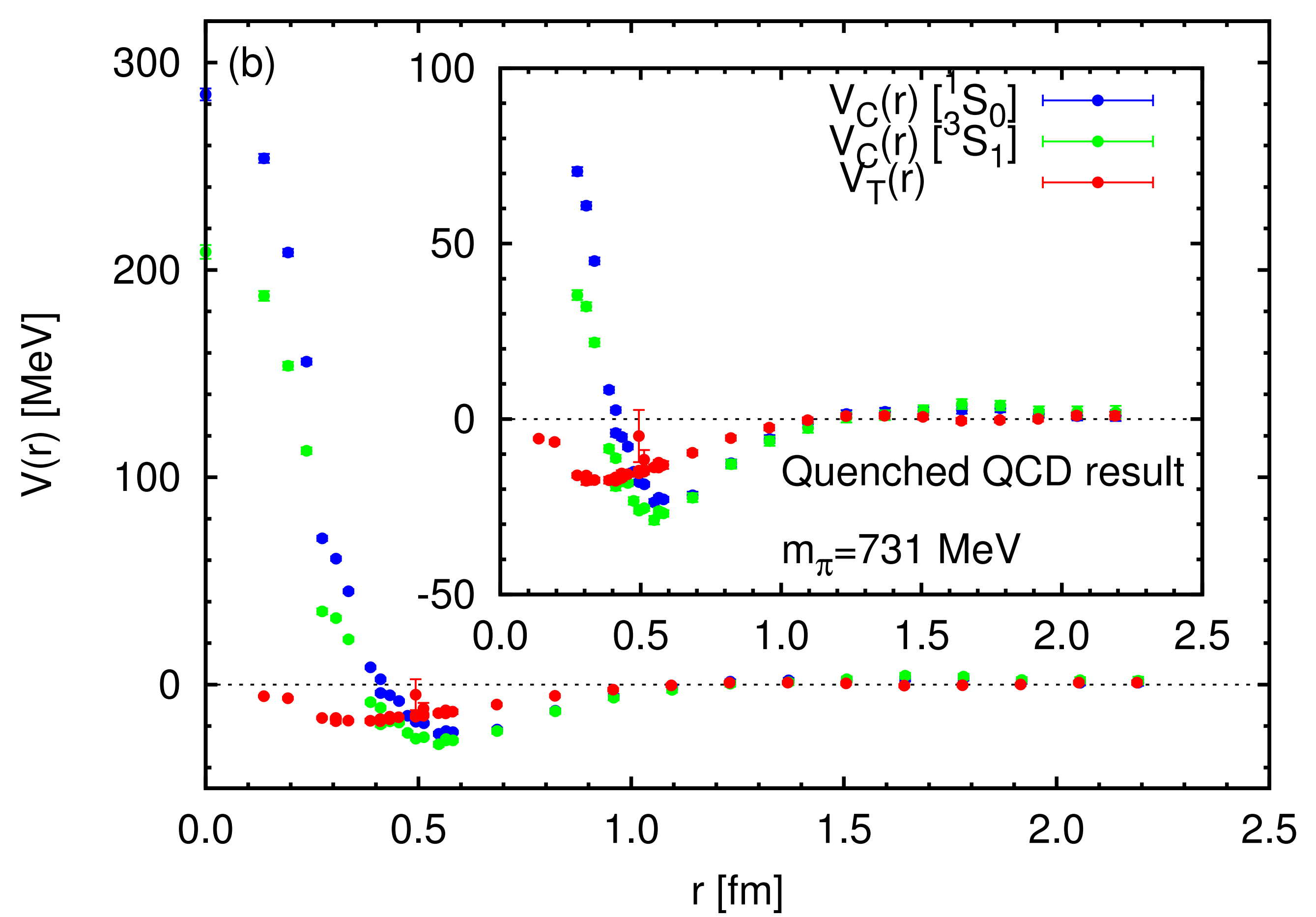
Potentials

full QCD



$a \simeq 0.091$ fm $\quad L \simeq 2.9$ fm

quenched QCD



$a \simeq 0.137$ fm $\quad L \simeq 4.4$ fm

- no repulsive core in the tensor potential.
- the tensor potential is enhanced in full QCD

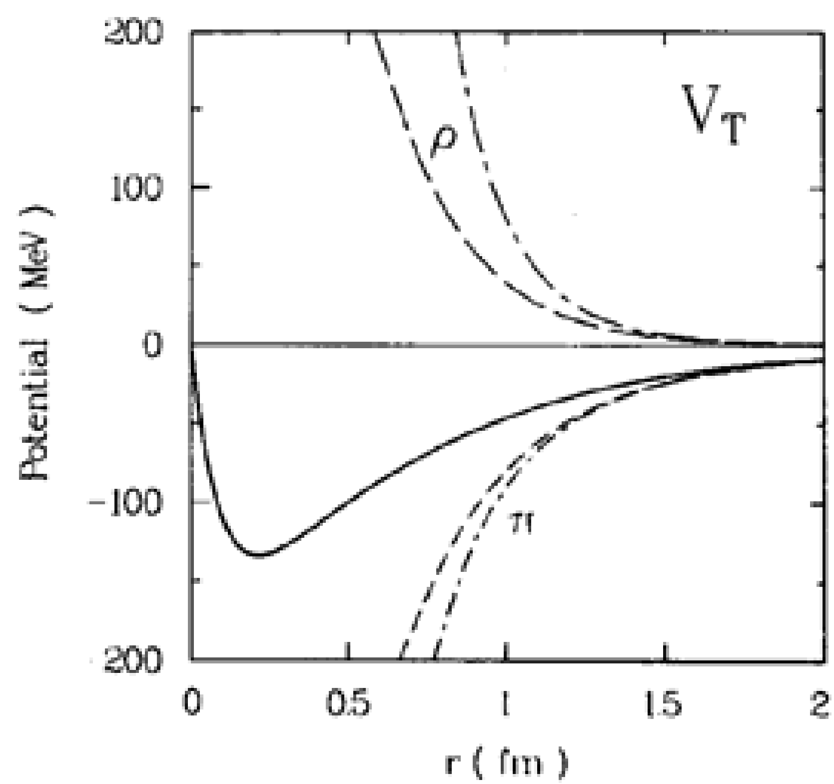
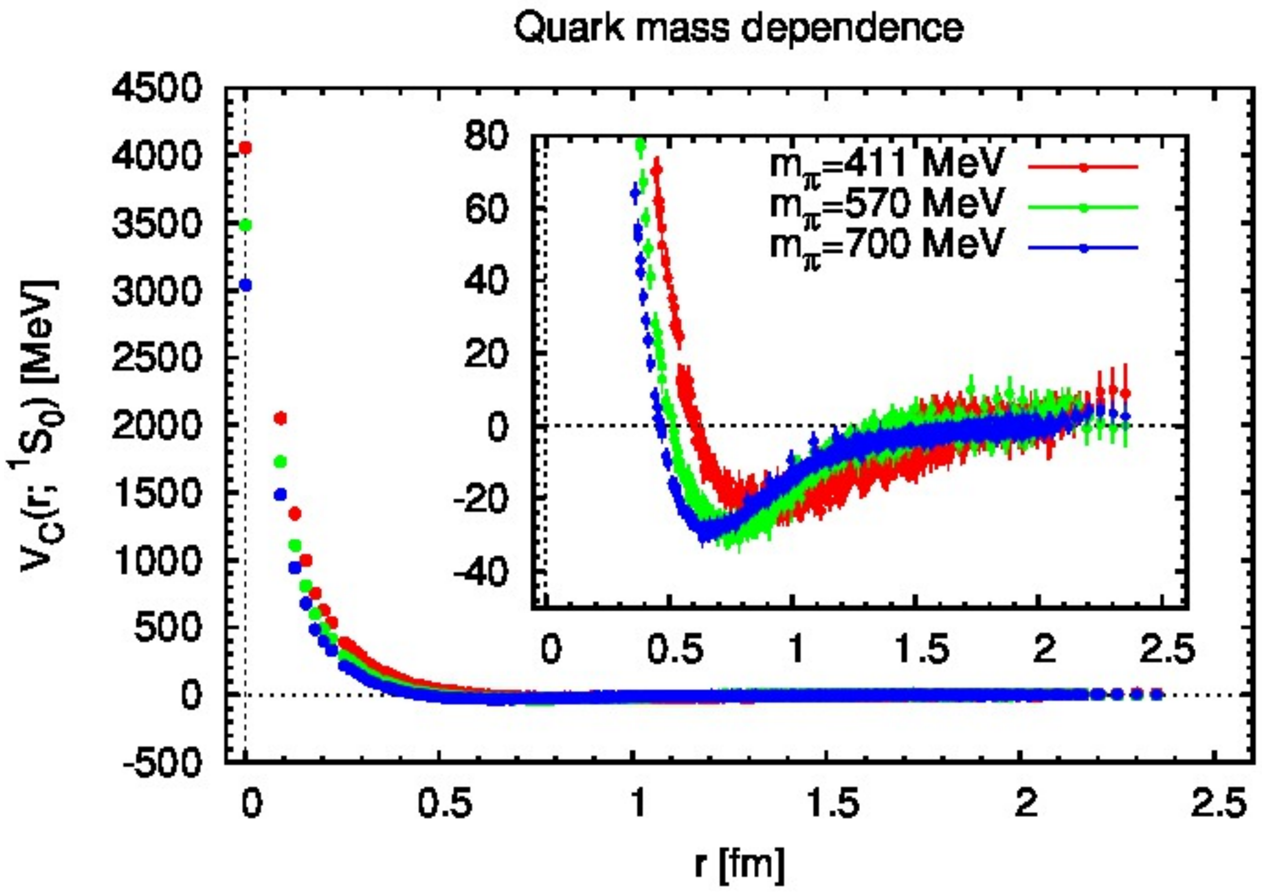
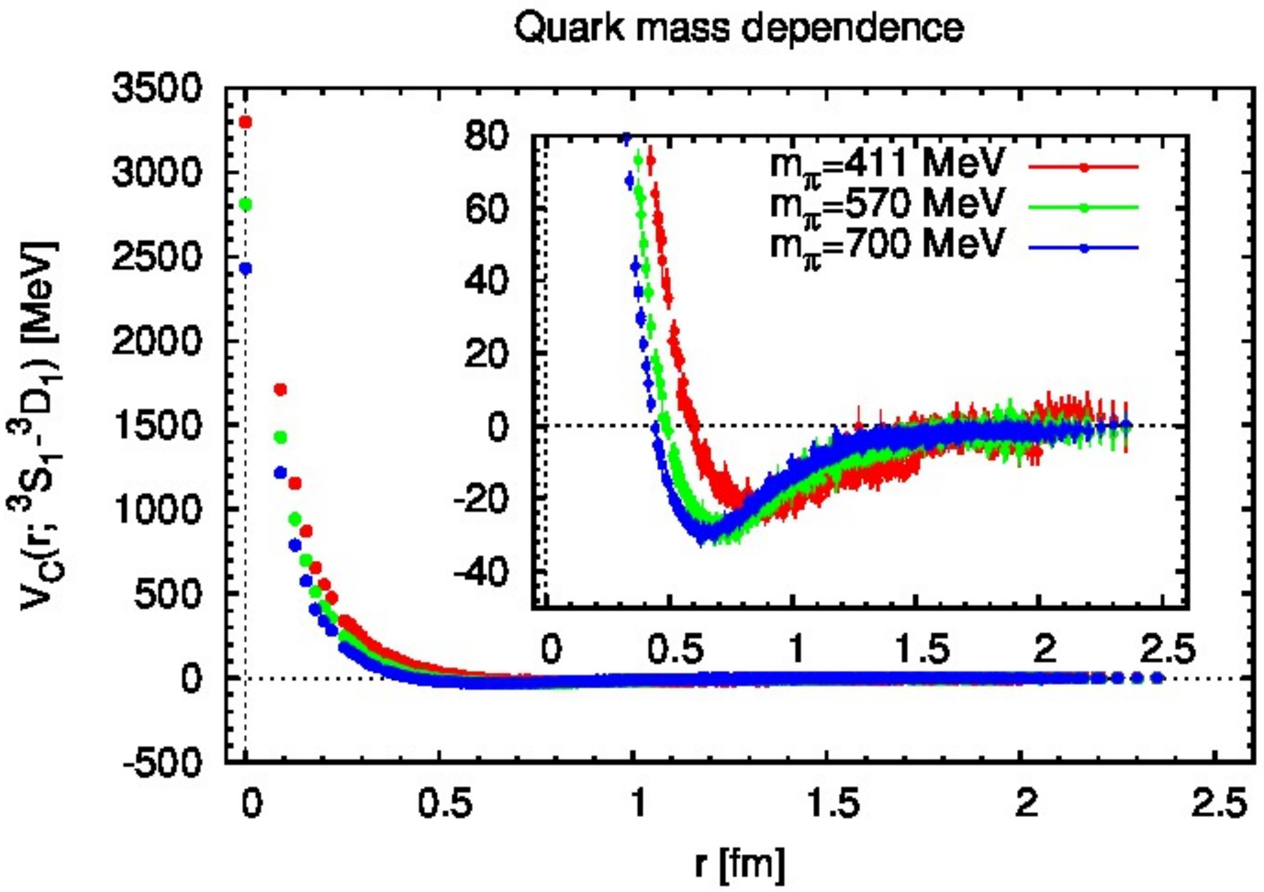


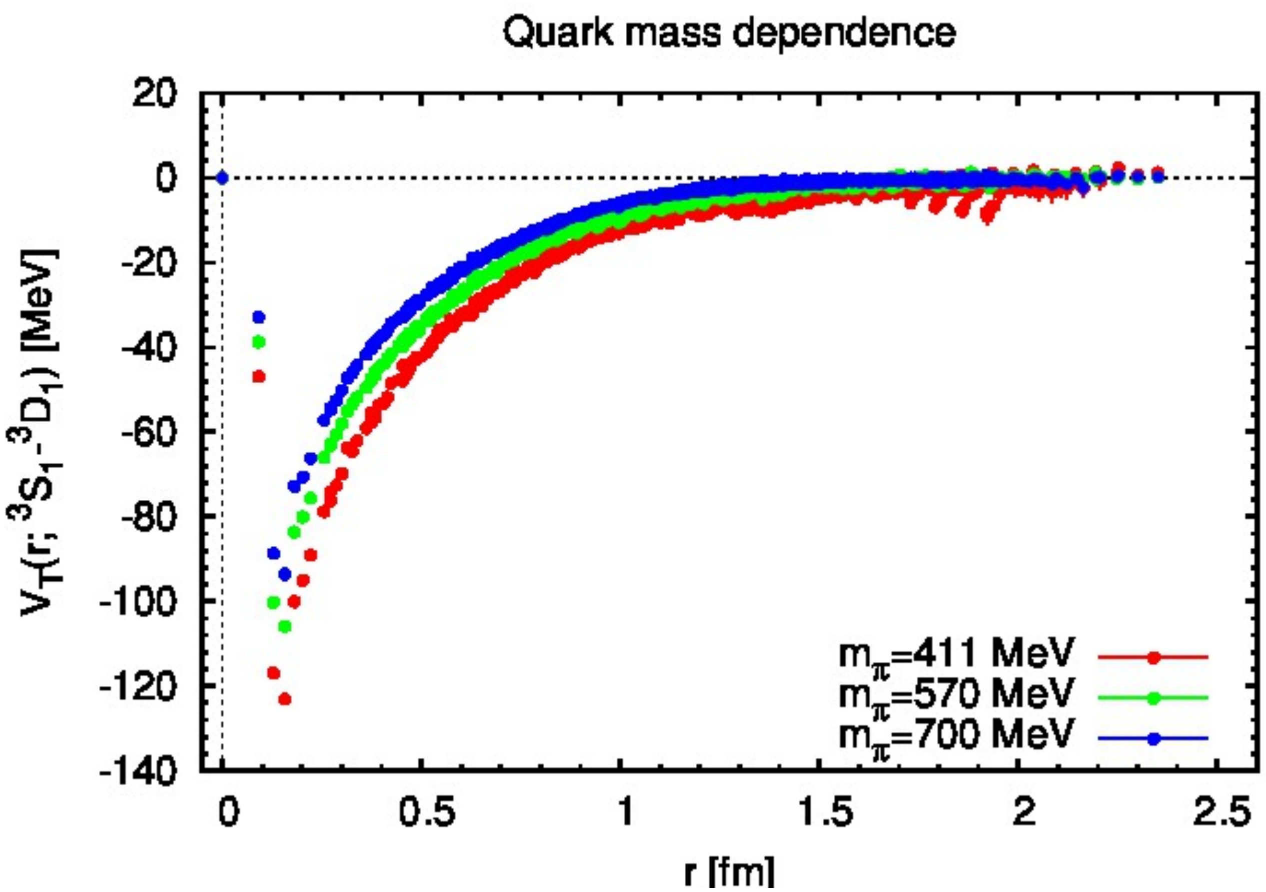
Fig. 3.7. The contributions from π and ρ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

from R.Machleidt, Adv.Nucl.Phys.**19**

Quark mass dependence (full QCD)

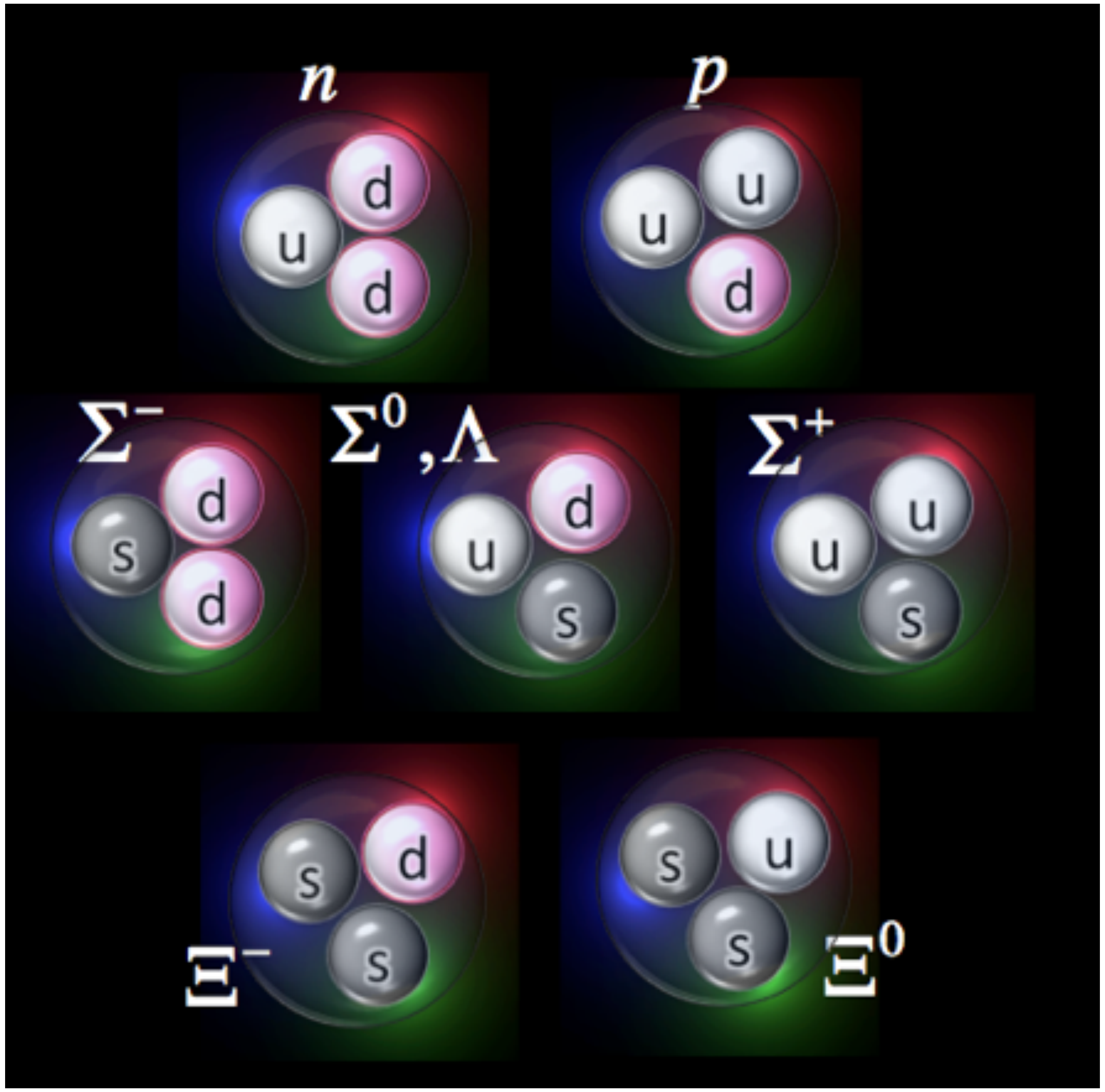






- the tensor potential increases as the pion mass decreases.
  - manifestation of one-pion-exchange ?
- both repulsive core and attractive pocket are also grow as the pion mass decreases.

2. Hyperon Interactions

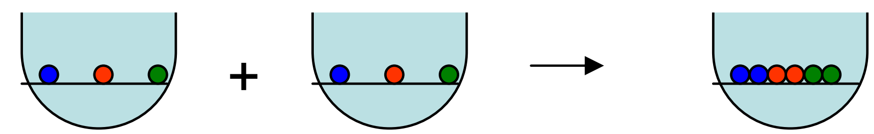


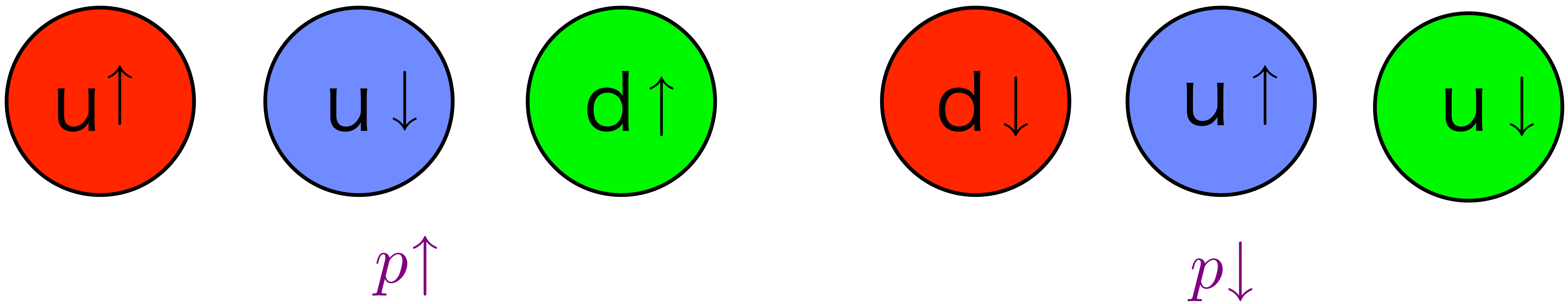
Origin of the repulsive core ?

quarks are "fermion" → two can not occupy the same position. ("Pauli principle")

they have 3 colors(red,blue,green), 2 spin(↑ ↓), 2 flavors(up,down)

→ 6 quark can occupy the same position





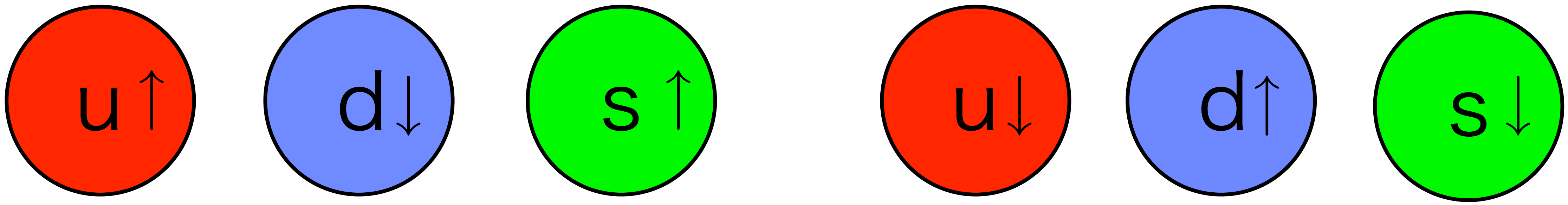
but allowed color combinations are limited + interaction among quarks

?

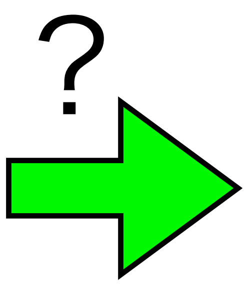
→ repulsive core ?

What happen if strange quarks are added ?

$\Lambda(uds)$ - $\Lambda(uds)$ interaction

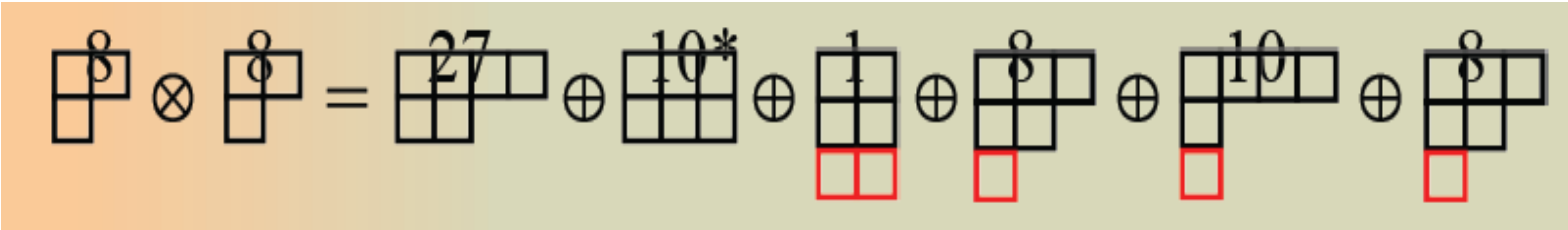


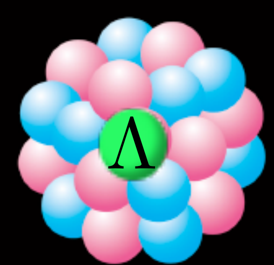
all color combinations are allowed

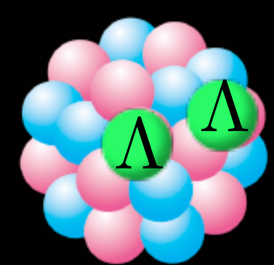


no repulsive core ?

Octet Baryon interactions







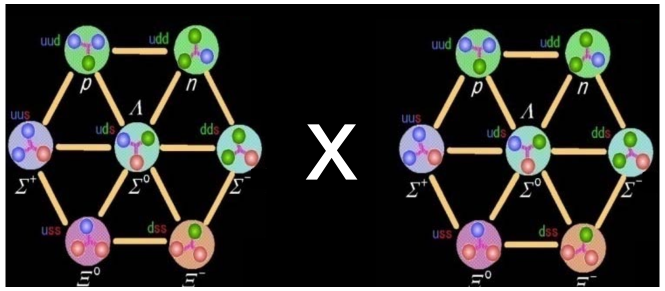
- phase shift available for YN and YY scattering are limited
- plenty of hyper-nucleus data will be soon available at J-PARC

- prediction from lattice QCD
- difference between NN and YN ?

Baryon Potentials in the flavor SU(3) limit

$m_u = m_d = m_s$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric Anti-symmetric

6 independent potentials in flavor-basis

$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r)$ ← $^1S_0$

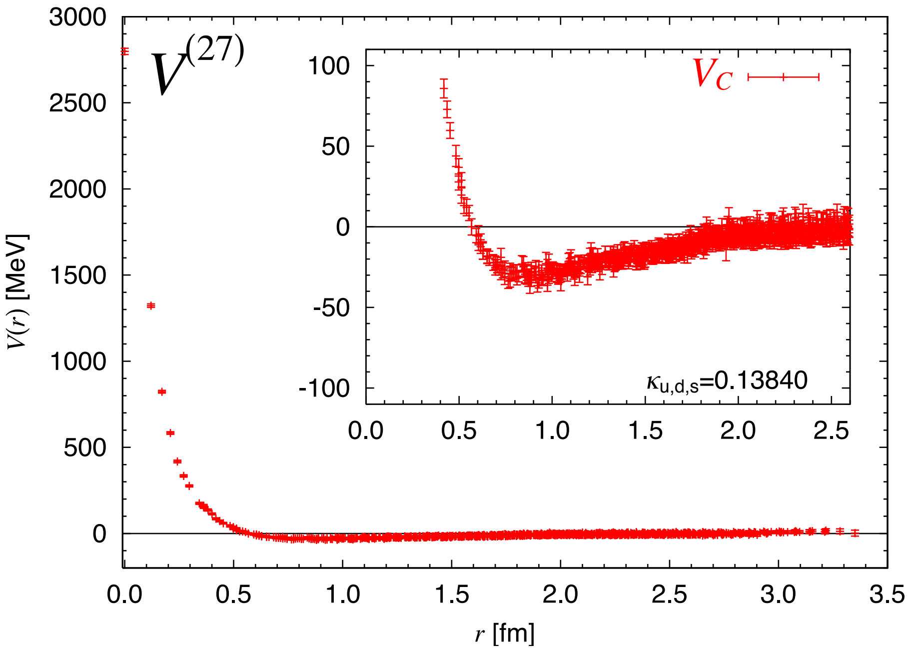
$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r)$ ← $^3S_1$

3-flavor QCD a=0.12 fm

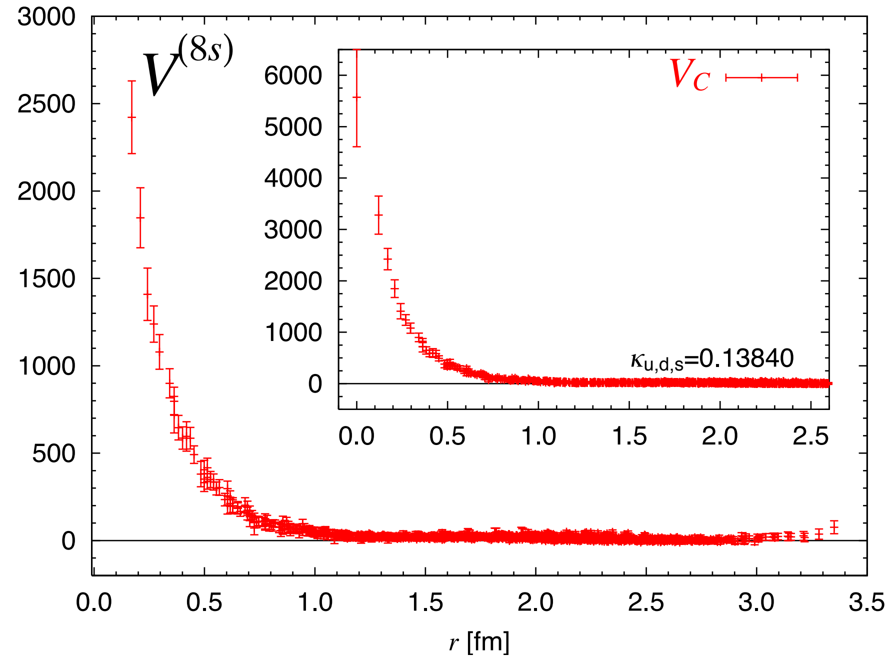
Inoue et al. (HAL QCD Coll.), PTP124(2010)591 L=2 fm

Inoue et al. (HAL QCD Coll.), NPA881(2012)28 L=2-4 fm

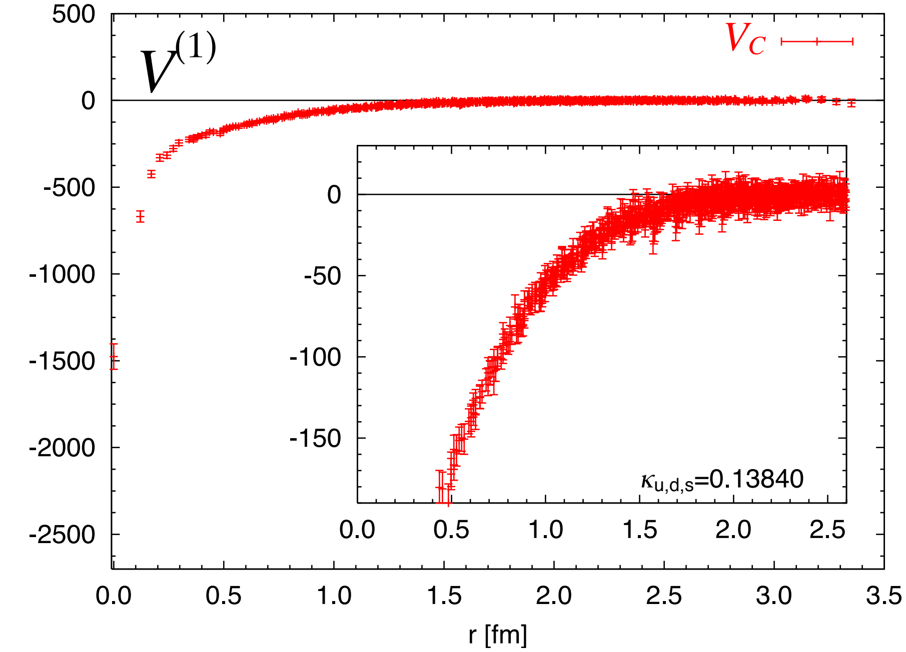
$L \simeq 4$ fm, $\quad m_\pi \simeq 470$ MeV



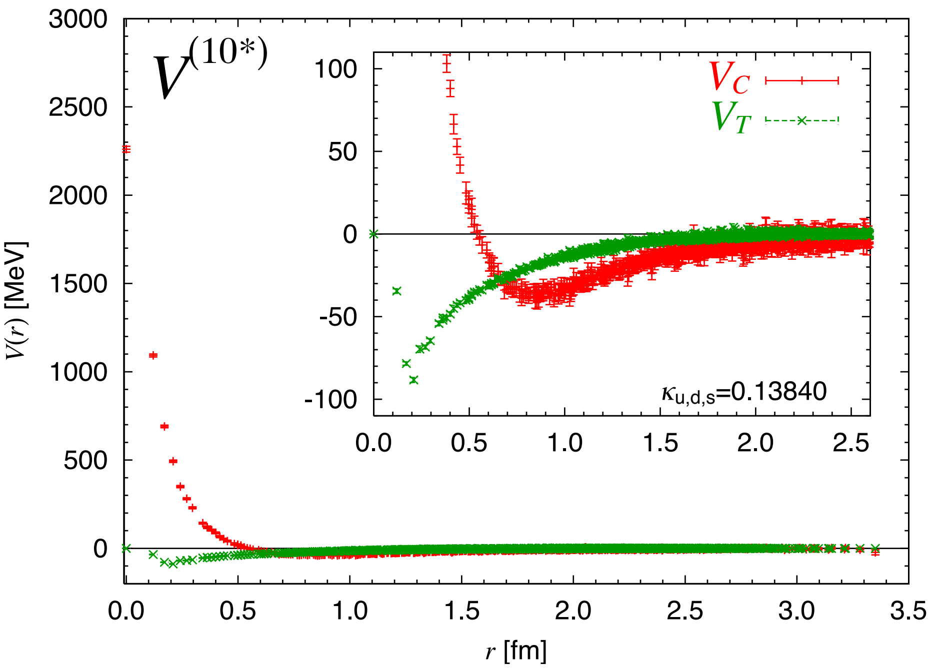
same as NN



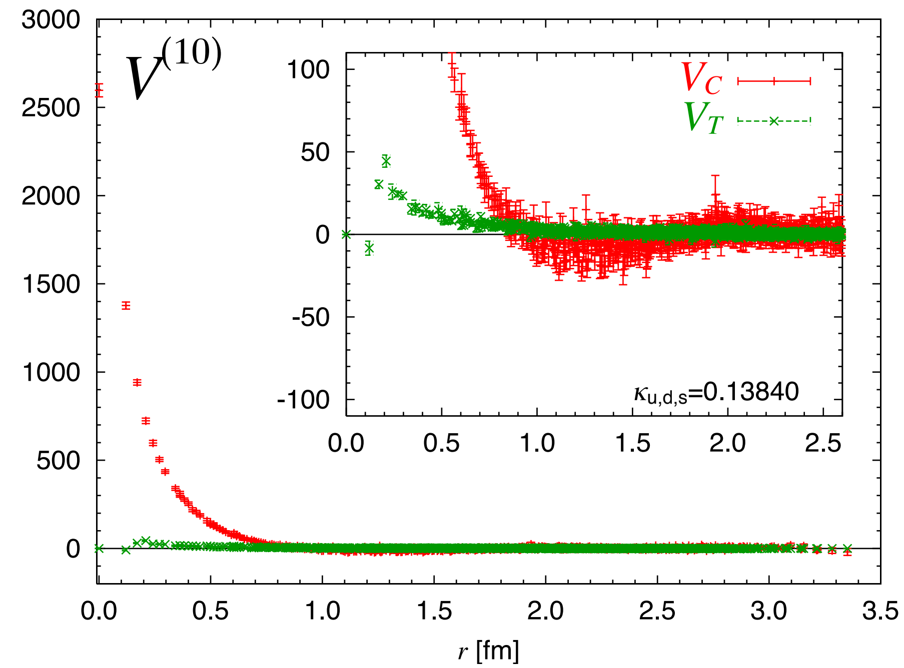
8s: strong repulsive core. repulsion only.



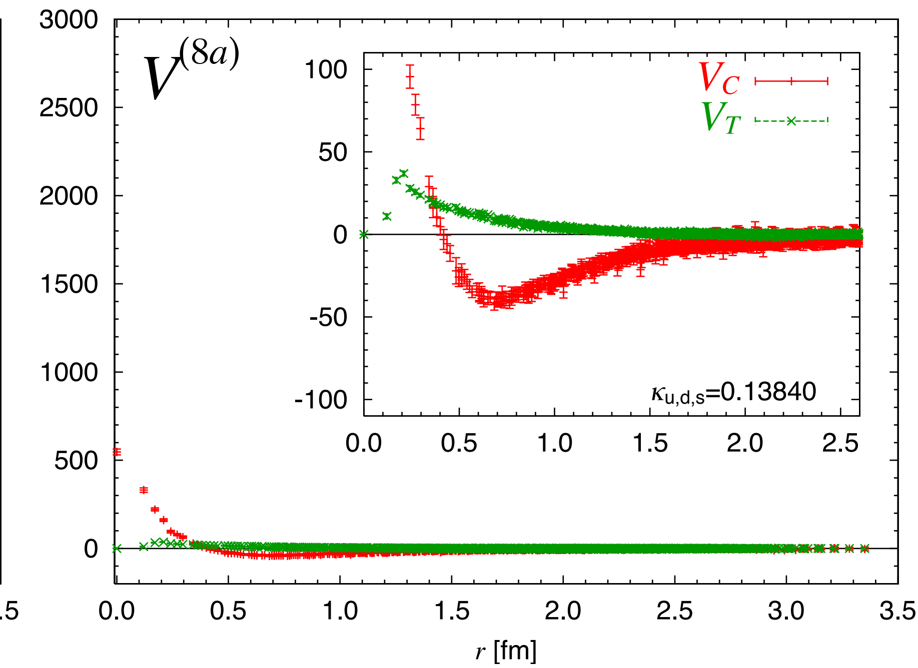
1: attractive instead of repulsive core ! attraction only . H-dibaryon.



same as NN



10: strong repulsive core. weak attraction.



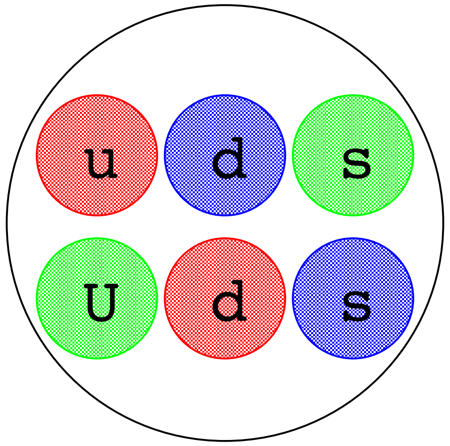
8a: weak repulsive core. strong attraction.

Flavor dependences of BB interactions become manifest in SU(3) limit !

H-dibaryon:

a possible six quark state(uuddss)

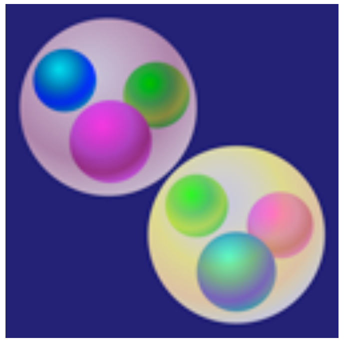
predicted by the model but not observed yet.



http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.106.162001

Binding baryons on the lattice

April 26, 2011



H-dibaryon in the flavor SU(3) limit

Inoue et al. (HAL QCD Coll.), PRL106(2011)162002

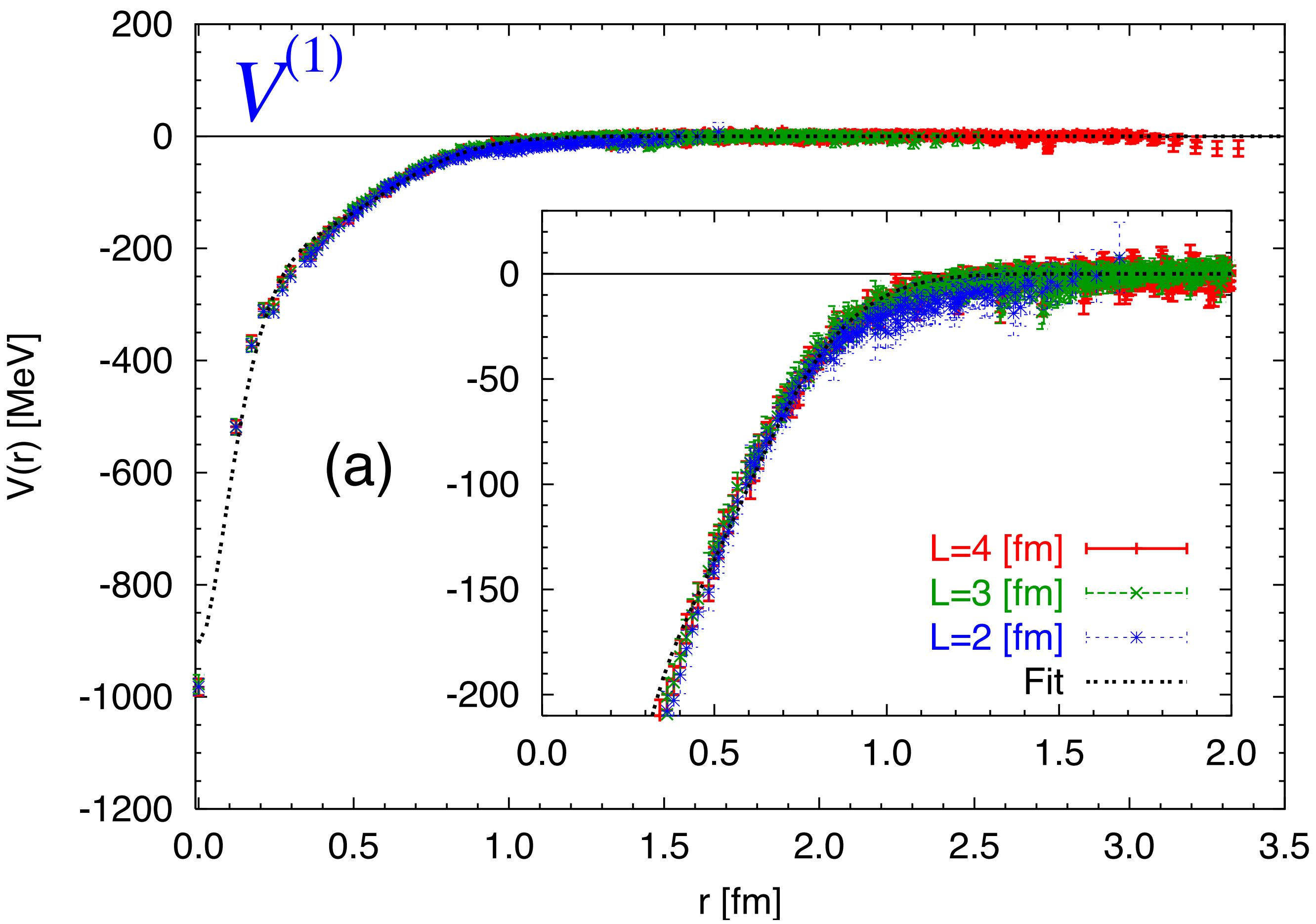
a=0.12 fm

Attractive potential
in the flavor singlet channel

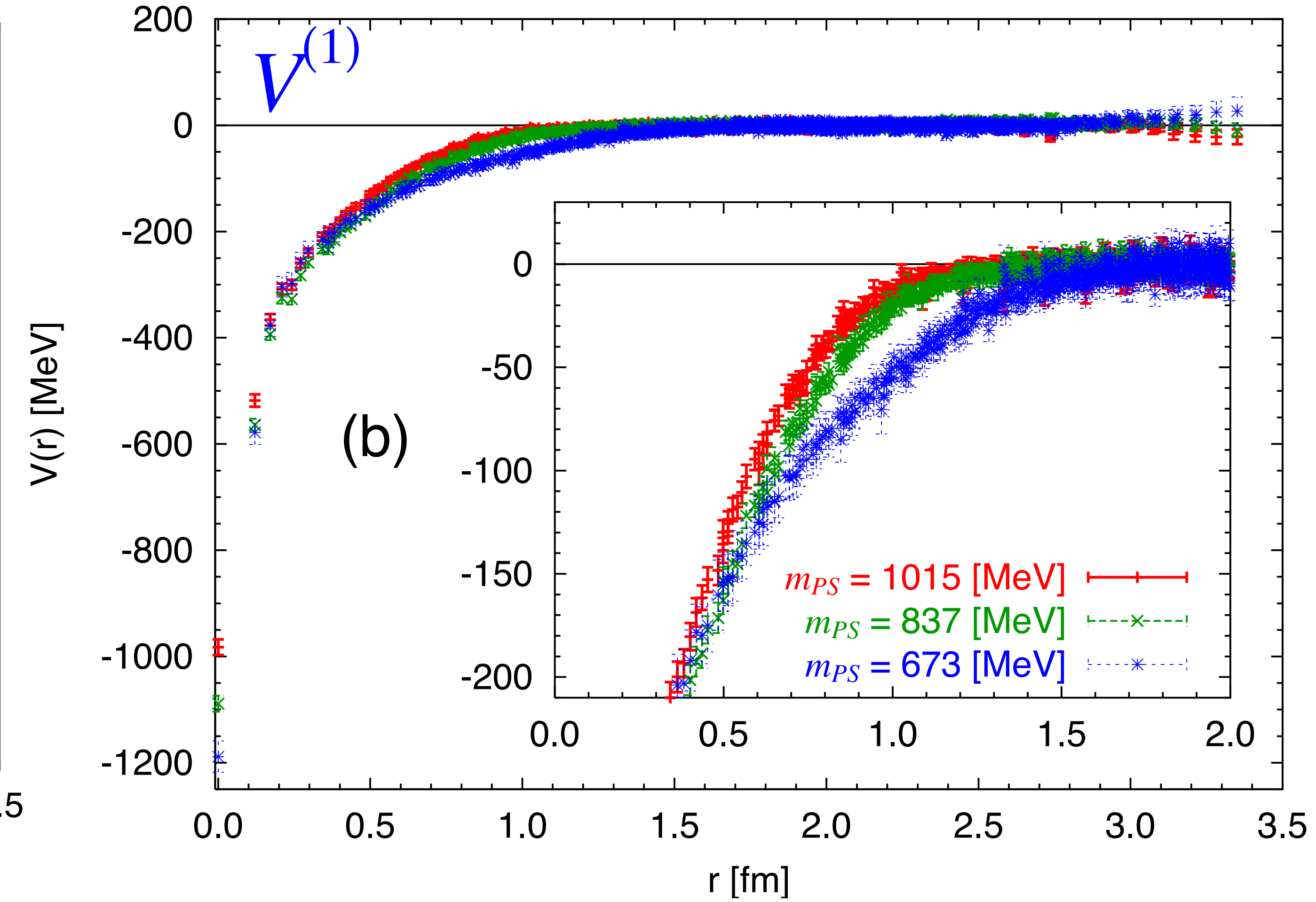
→ possibility of a bound state (H-dibaryon)

$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$

volume dependence



pion mass dependence



L=3 fm is enough for the potential.

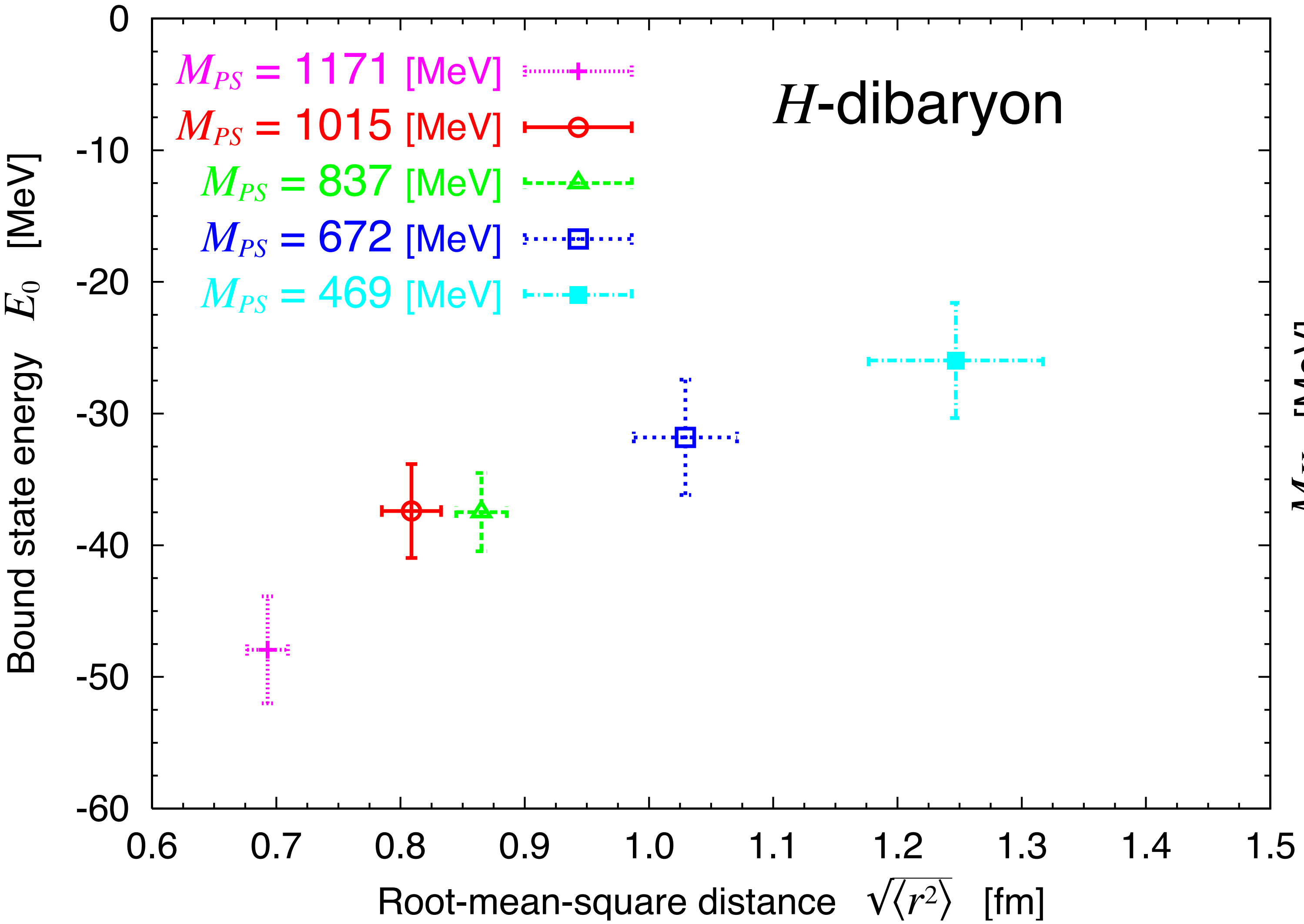
lighter the pion mass, stronger the attraction

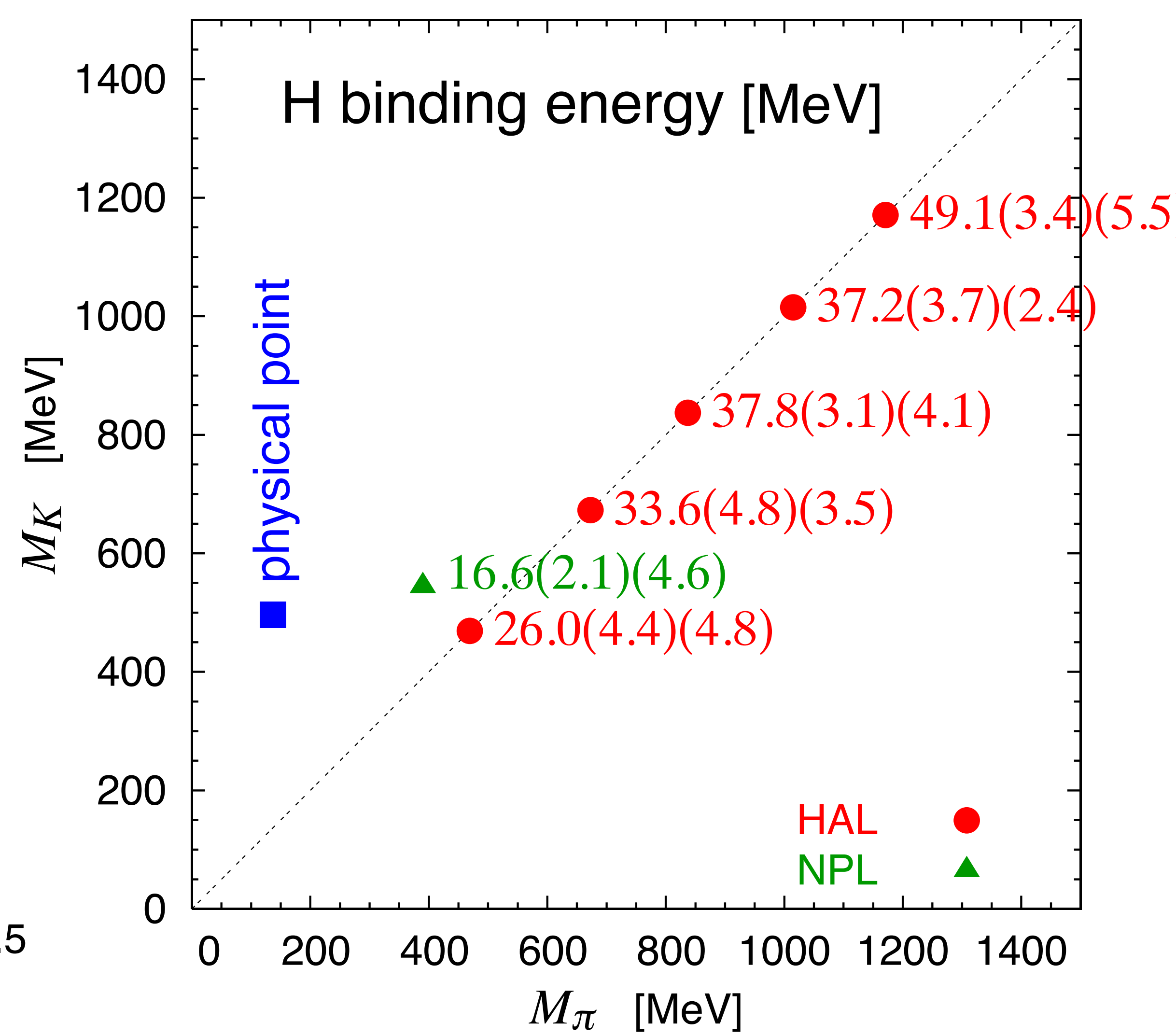
fit potentials at L=4 fm by

$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

Solve Schroedinger equation
in the infinite volume

One bound state (H-dibaryon) exists.

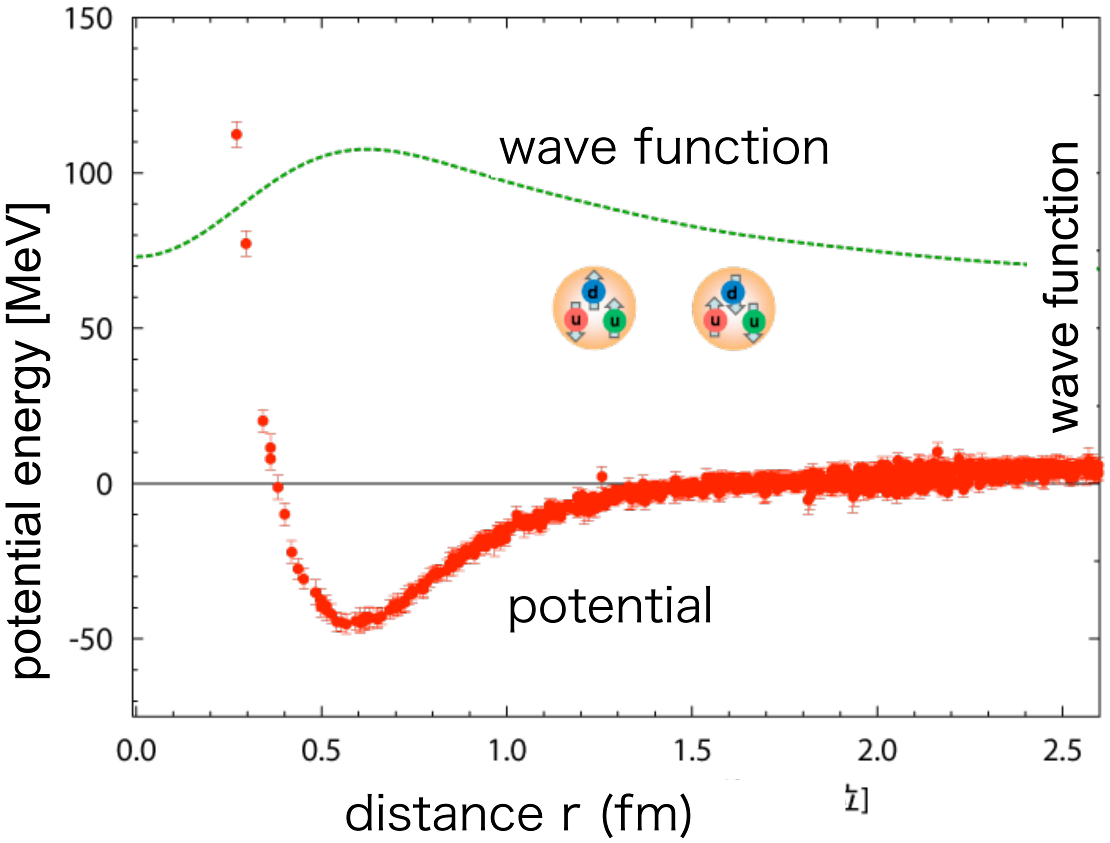




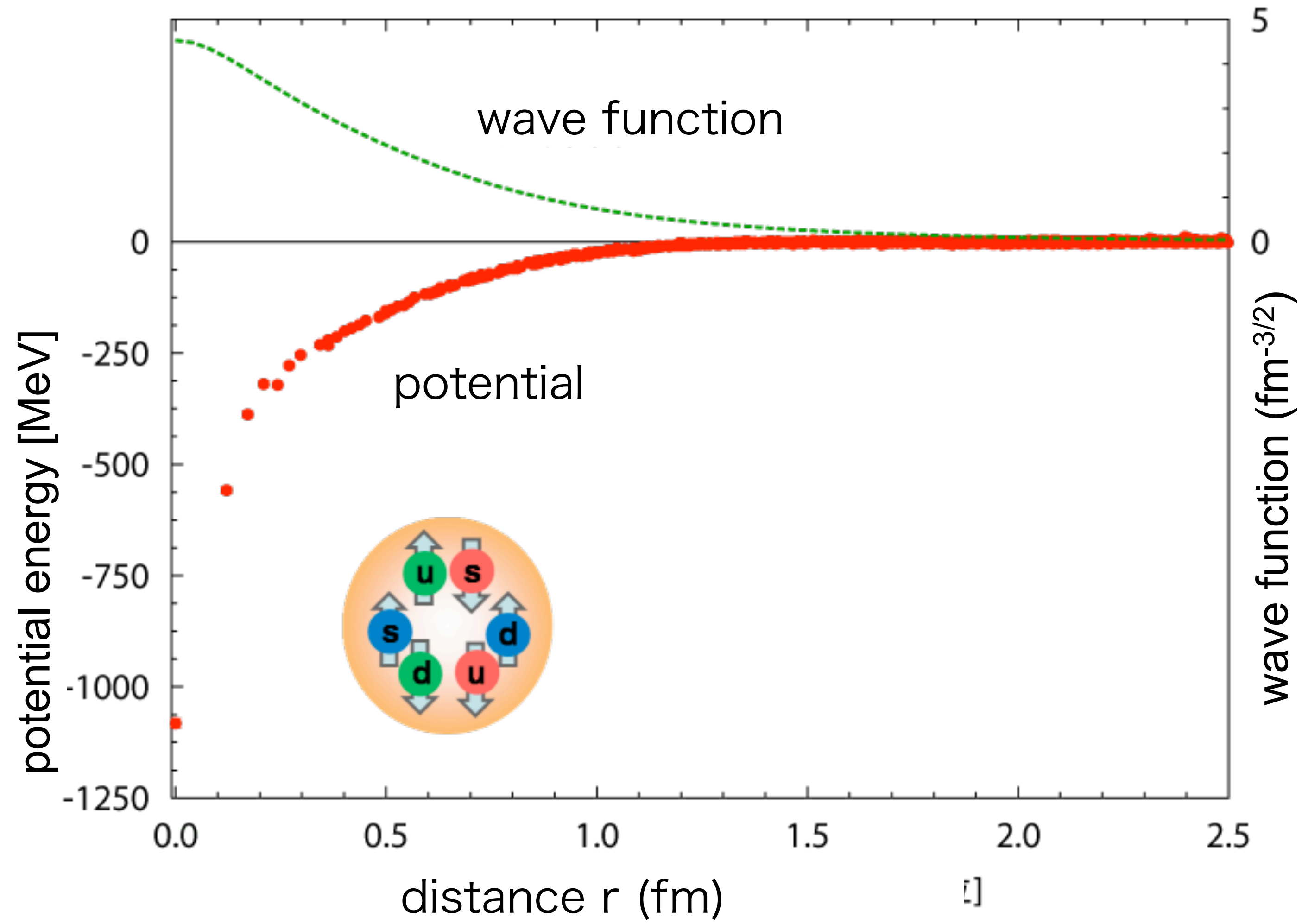
An H-dibaryon exists in the flavor SU(3) limit.
Binding energy = 25-50 MeV at this range of quark mass.
A mild quark mass dependence.

Real world ?

Deuteron

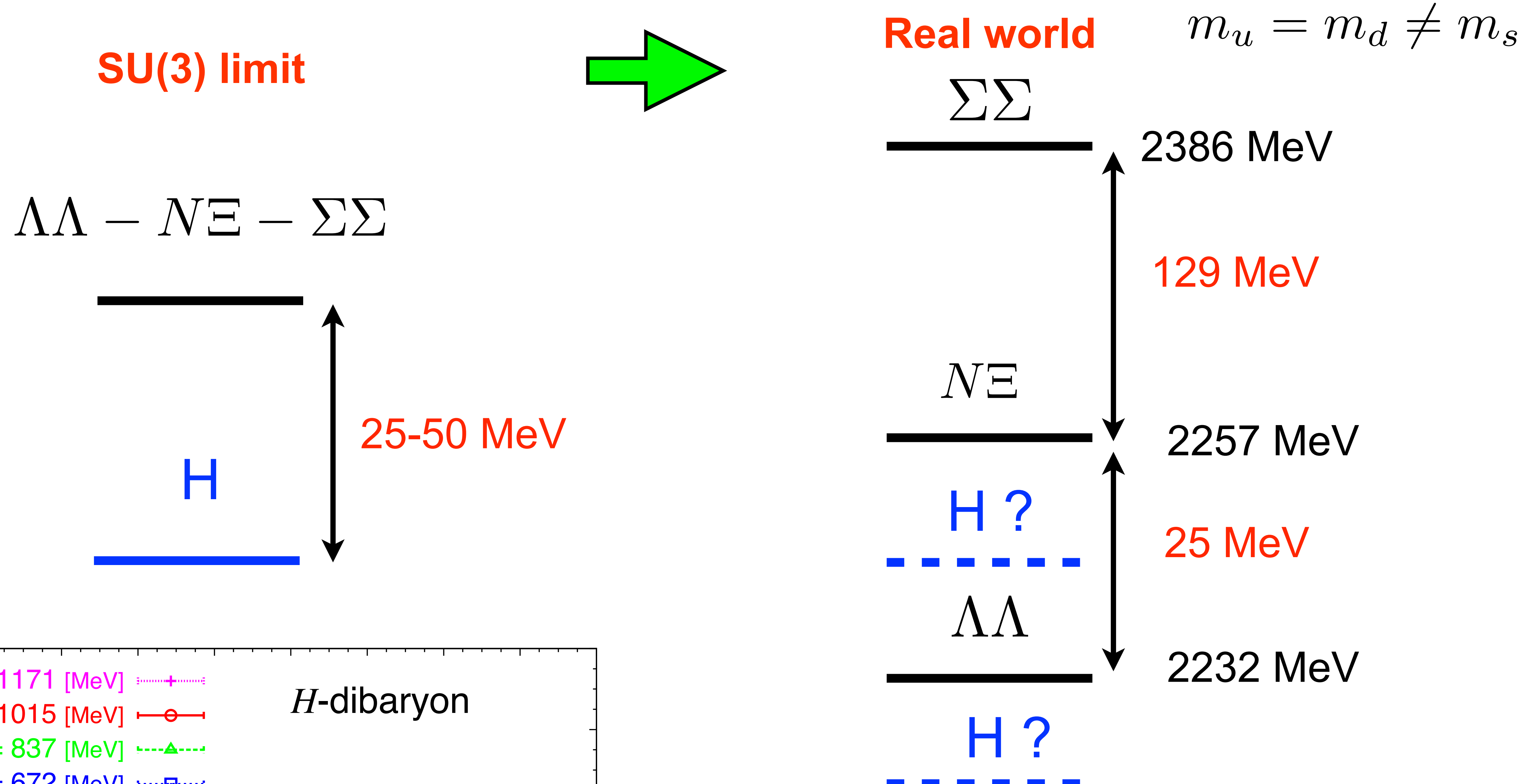


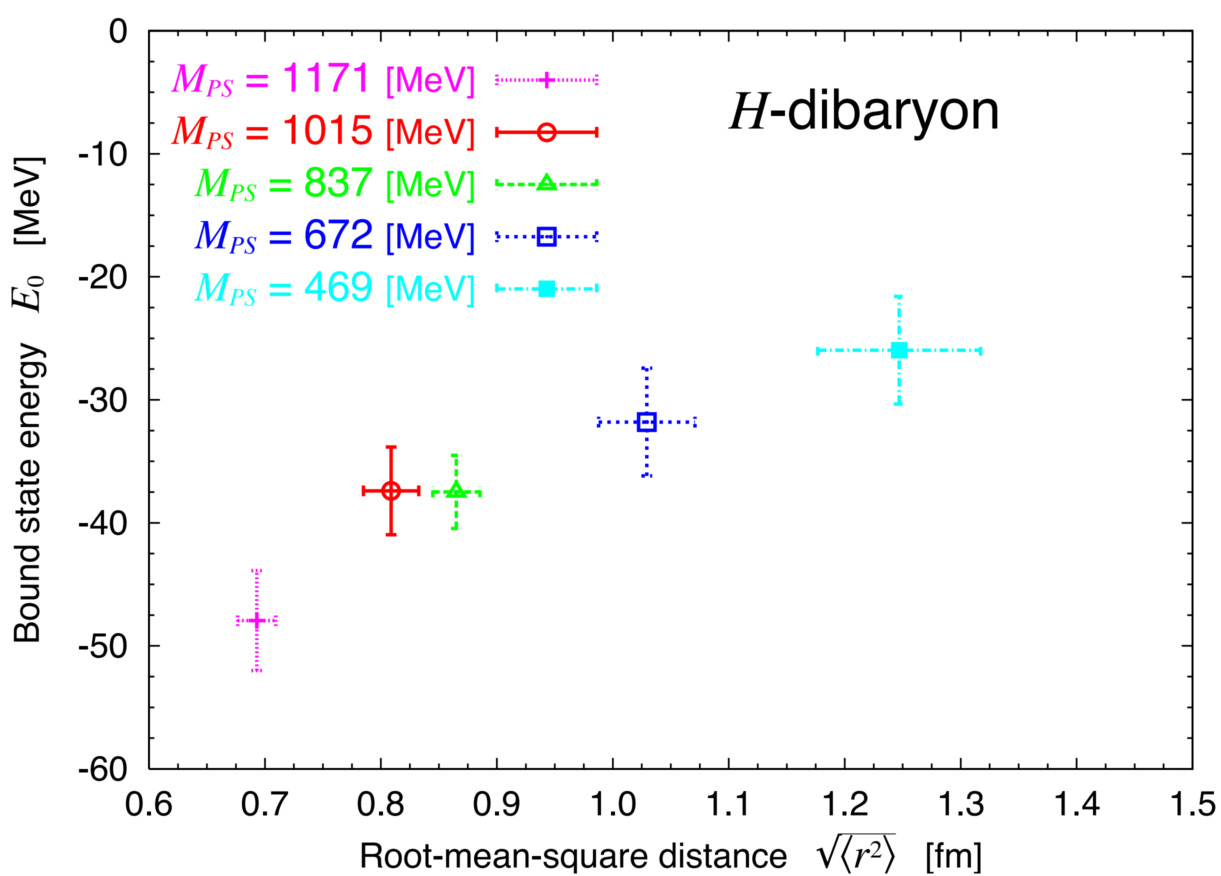
H-dibayon



3. Extensions

H-dibaryon with the flavor SU(3) breaking





S=-2 "Inelastic" scattering

$$m_N = 939 \text{ MeV}, \; m_\Lambda = 1116 \text{ MeV}, \; m_\Sigma = 1193 \text{ MeV}, \; m_\Xi = 1318 \text{ MeV}$$

S=-2 System(I=0)

$$M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV}$$

The eigen-state of QCD in the finite box is a mixture of them:

$$|S=-2, I=0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

$$E = 2\sqrt{m_\Lambda^2 + \mathbf{p}_1^2} = \sqrt{m_\Xi^2 + \mathbf{p}_2^2} + \sqrt{m_N^2 + \mathbf{p}_2^2} = 2\sqrt{m_\Sigma^2 + \mathbf{p}_3^2}$$

In this situation, we can not directly extract the scattering phase shift in lattice QCD.

Extended method

Let us consider 2-channel problem for simplicity.

NBS wave functions for 2 channels at 2 values of energy:

$$\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = \langle 0|\Lambda(\mathbf{x})\Lambda(\mathbf{0})|E_\alpha\rangle$$
$$\Psi_\alpha^{\Xi N}(\mathbf{x}) = \langle 0|\Xi(\mathbf{x})N(\mathbf{0})|E_\alpha\rangle$$

$$\alpha = 1, 2$$

They satisfy

$$(\nabla^2 + \mathbf{p}_\alpha^2)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = 0$$
$$(\nabla^2 + \mathbf{q}_\alpha^2)\Psi_\alpha^{\Xi N}(\mathbf{x}) = 0$$

$$|\mathbf{x}| \to \infty$$

We define the "potential" from the coupled channel Schroedinger equation:

$$\left(\frac{\nabla^2}{2\mu_{\Lambda\Lambda}} + \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}\right)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

diagonal — off-diagonal

$$\left(\frac{\nabla^2}{2\mu_{\Xi N}} + \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}\right)\Psi_\alpha^{\Xi N}(\mathbf{x}) = V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Xi N\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

off-diagonal — diagonal

μ: reduced mass

$$\begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix} \begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix}$$

$$E_\alpha = \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}, \ \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}$$

$X \neq Y$ $\quad X, Y = \Lambda\Lambda$ or ΞN

$$\begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix}^{-1} \begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix}$$

Using the coupled channel potentials:

$$\begin{pmatrix} V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x}) & V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x}) \\ V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x}) & V^{\Xi N\leftarrow\Xi N}(\mathbf{x}) \end{pmatrix}$$

we solve the coupled channel Schroedinger equation in the infinite volume with an appropriate boundary condition.

For example, we take the incomming $\Lambda\Lambda$ state by hand.

In this way, we can avoid the mixture of several "in"-states.

$$|S=-2, I=0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

Lattice is a tool to extract the interaction kernel ("T-matrix" or "potential").

Preliminary results from HAL QCD Collaboration

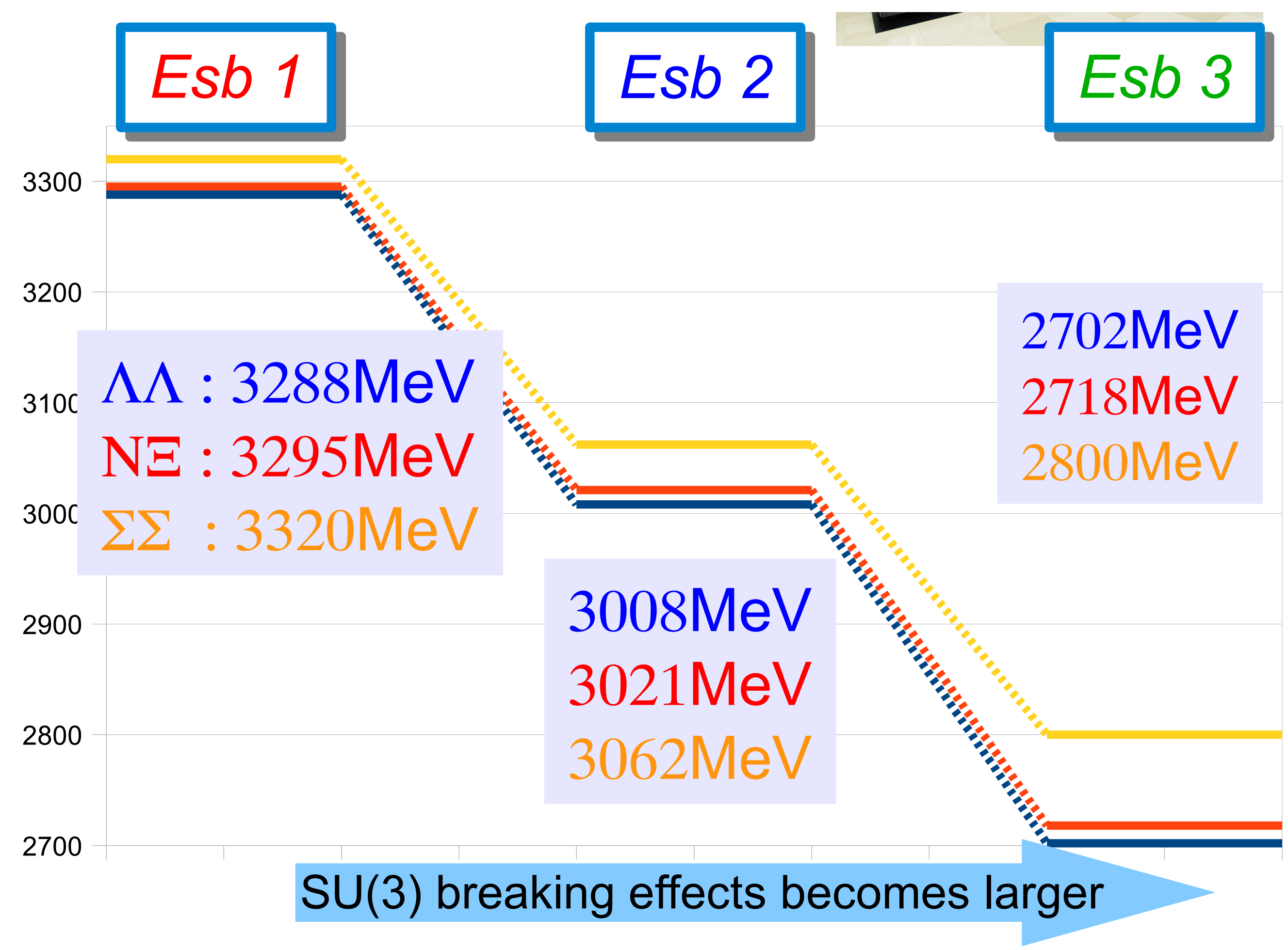
Sasaki for HAL QCD Collaboration

Gauge ensembles

In unit of MeV	*Esb 1*	*Esb 2*	*Esb 3*
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

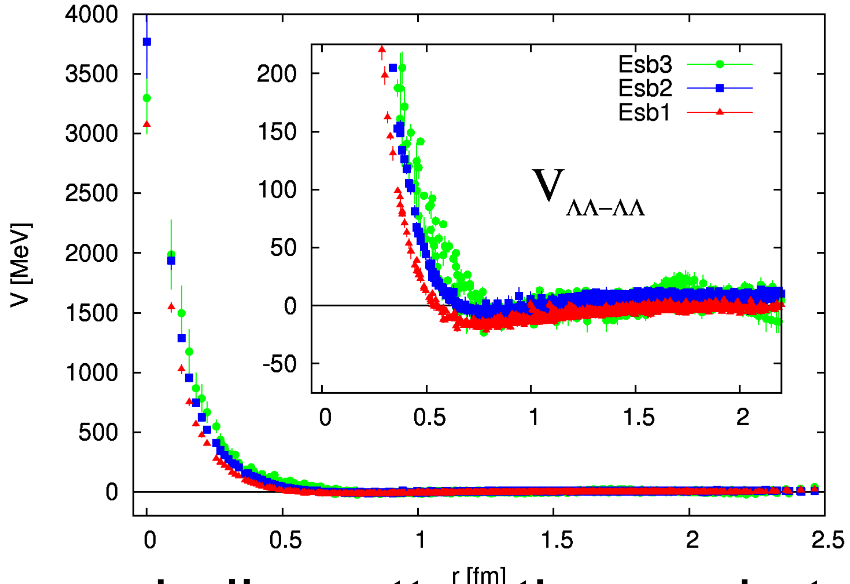
u,d quark masses lighter →

thresholds

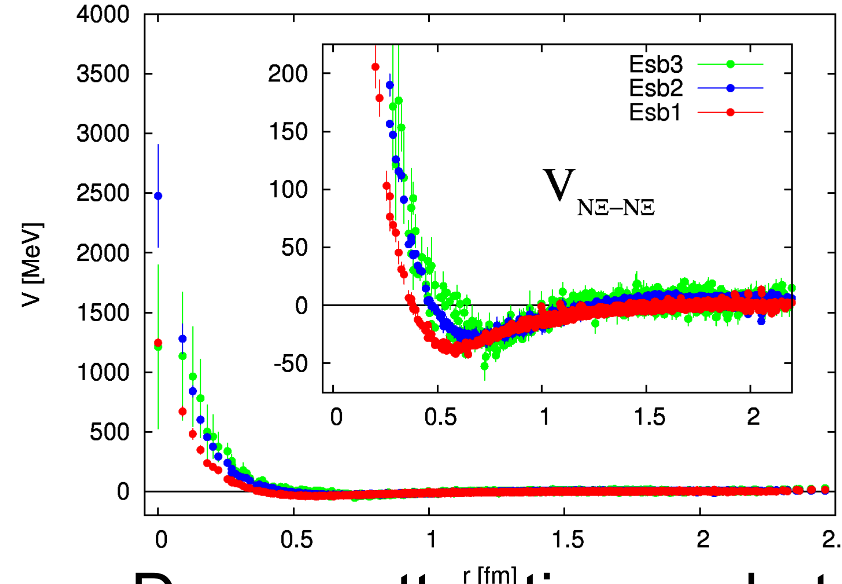


coupled channel 3x3 potentials

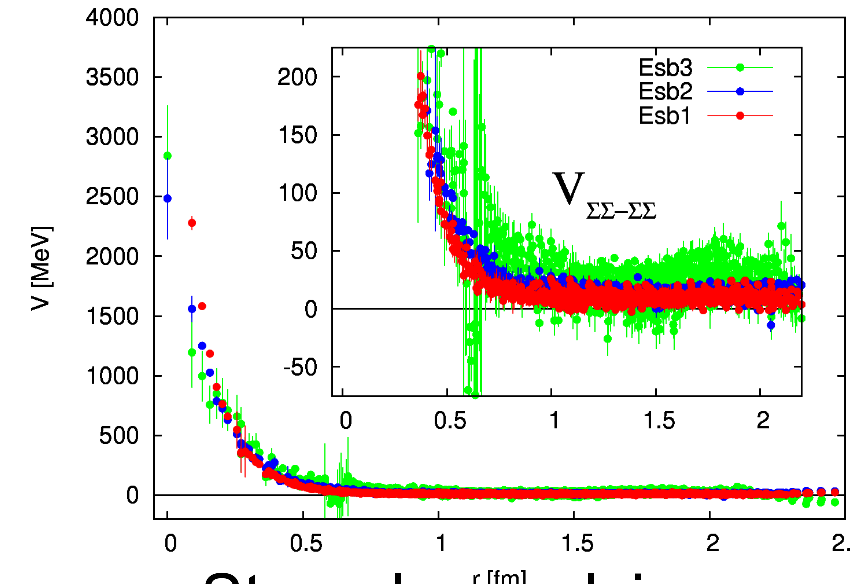
Diagonal elements



shallow attractive pocket



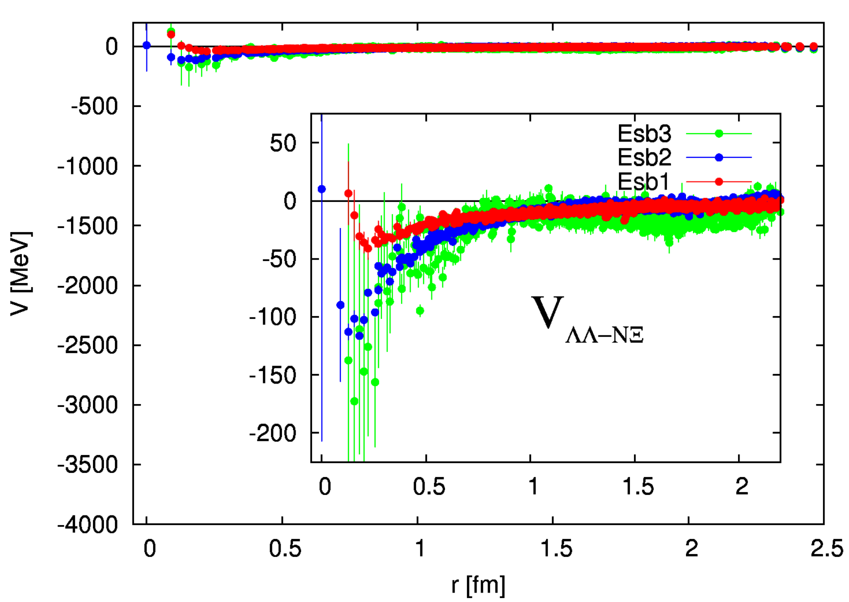
Deeper attractive pocket

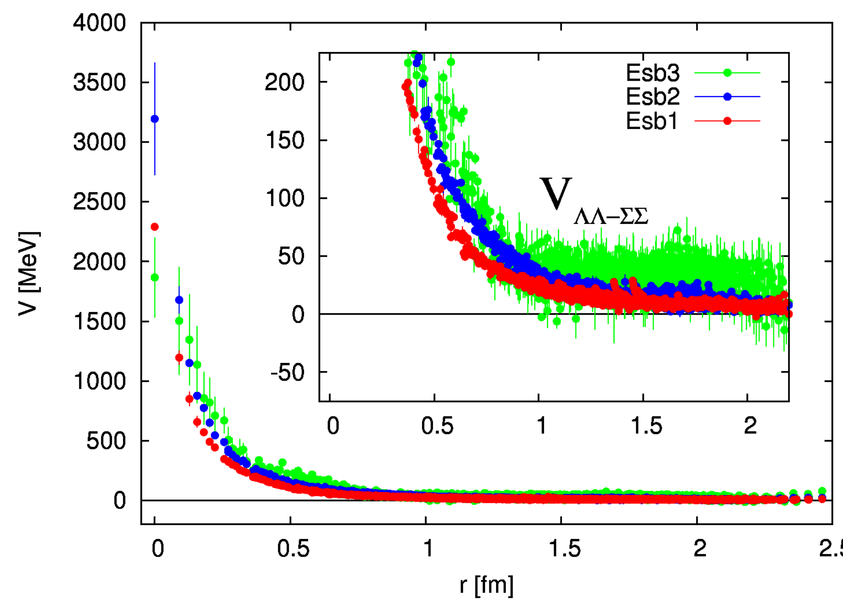


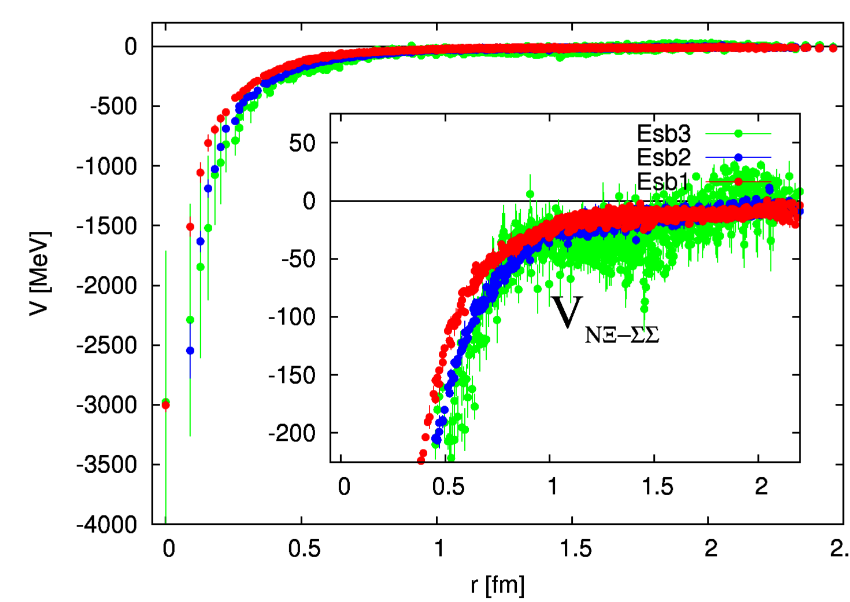
Strongly repulsive

All channels have repulsive core

Off-diagonal elements



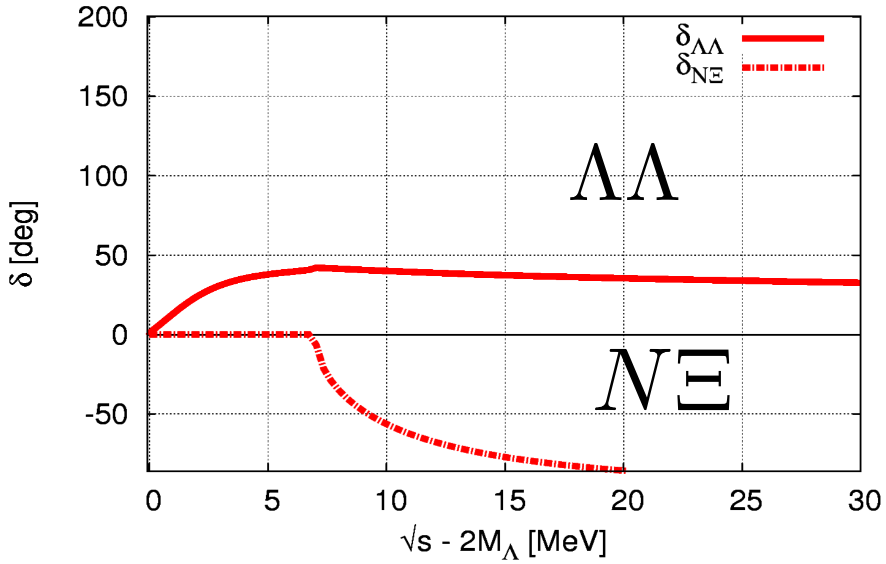




$\Lambda\Lambda$ and $N\Xi$ phase shift

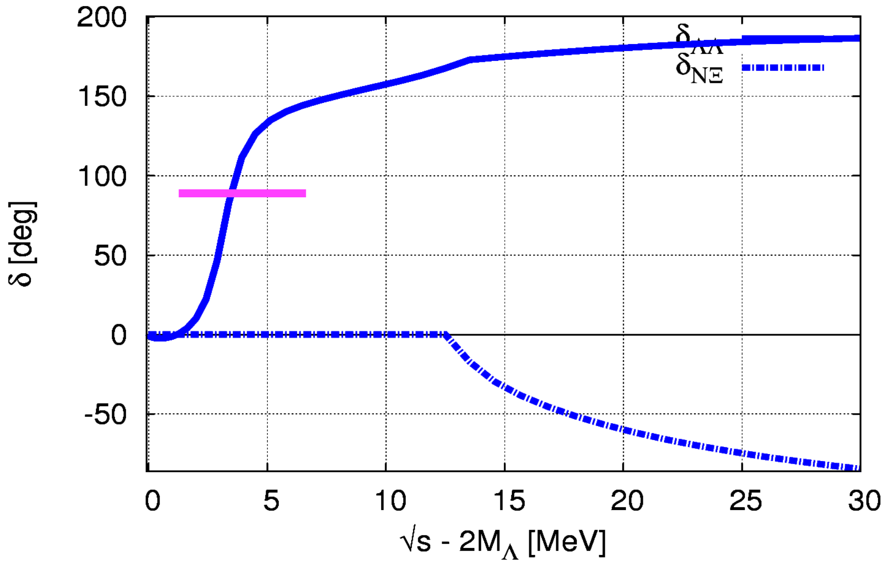
Preliminary !

Esb1 : mπ= 701 MeV



Bound H-dibaryon coupled to NΞ

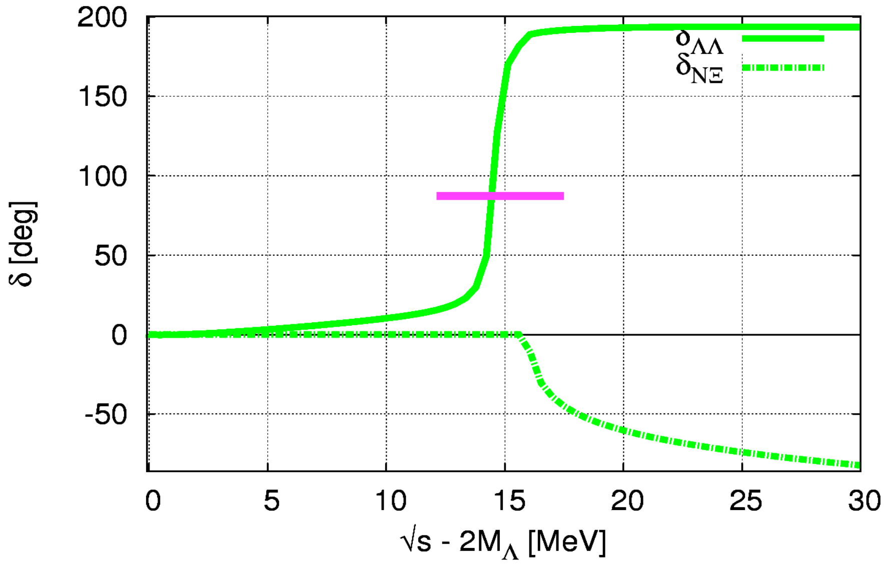
Esb2 : mπ= 570 MeV



H as ΛΛ resonance

H as bound NΞ

Esb3 : mπ= 411 MeV



H as ΛΛ resonance

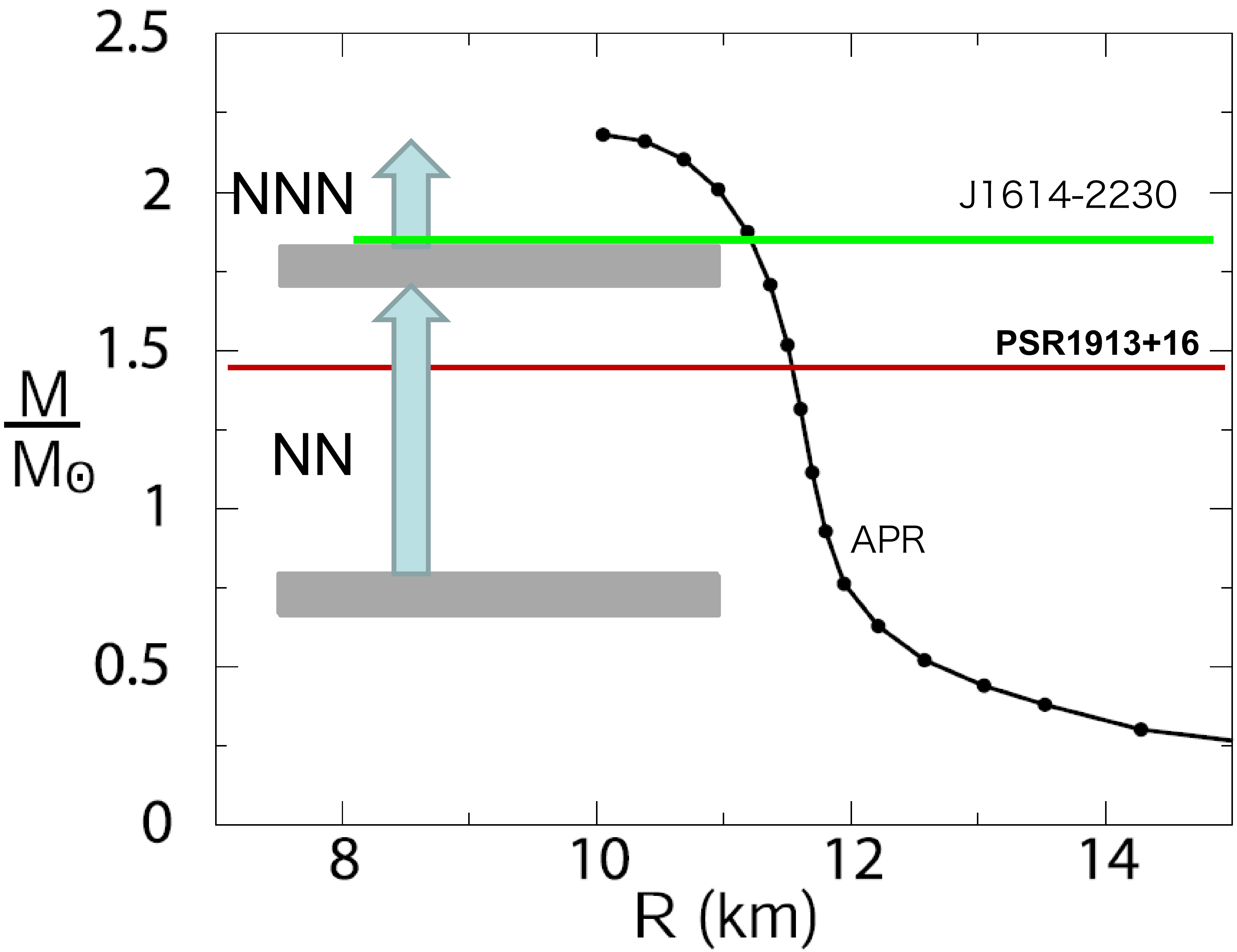
H as bound NΞ

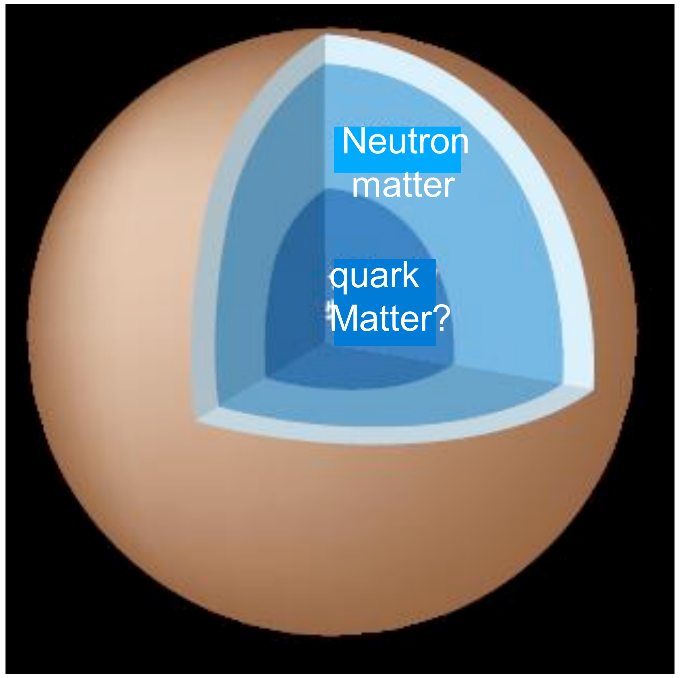
This suggests that H-dibaryon becomes resonance at physical point.
Below or above NΞ ? Need simulation at physical point.

Physically, it is essential that H-dibaryon is a bound state in the flavor SU(3) limit.

4. Challenge: Three nucleon force (TNF)

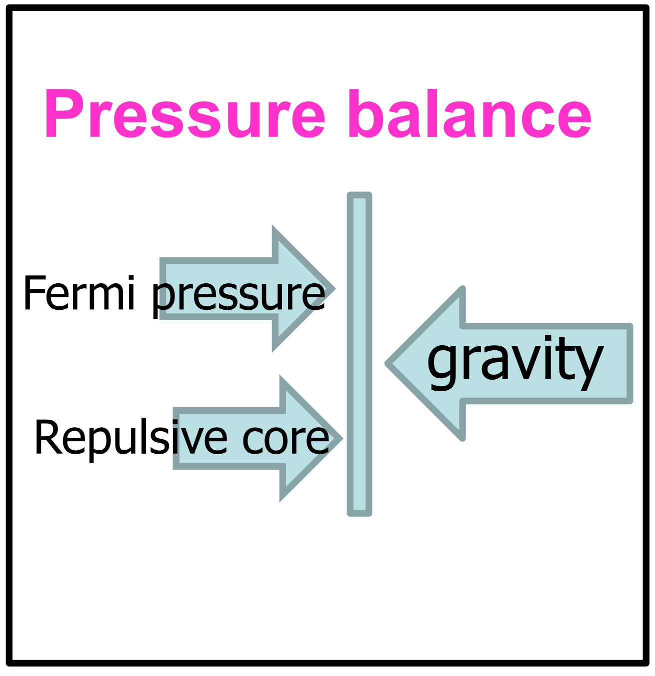
Maximum mass of neutron stars





($\rho_{max} \sim 6\rho_0$)

sustains neutron stars against gravitational collapse



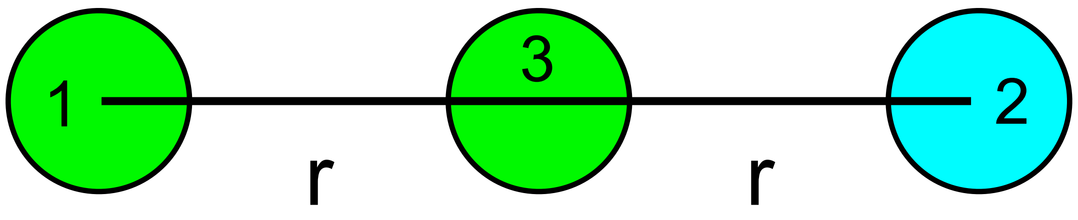
TNF from lattice QCD

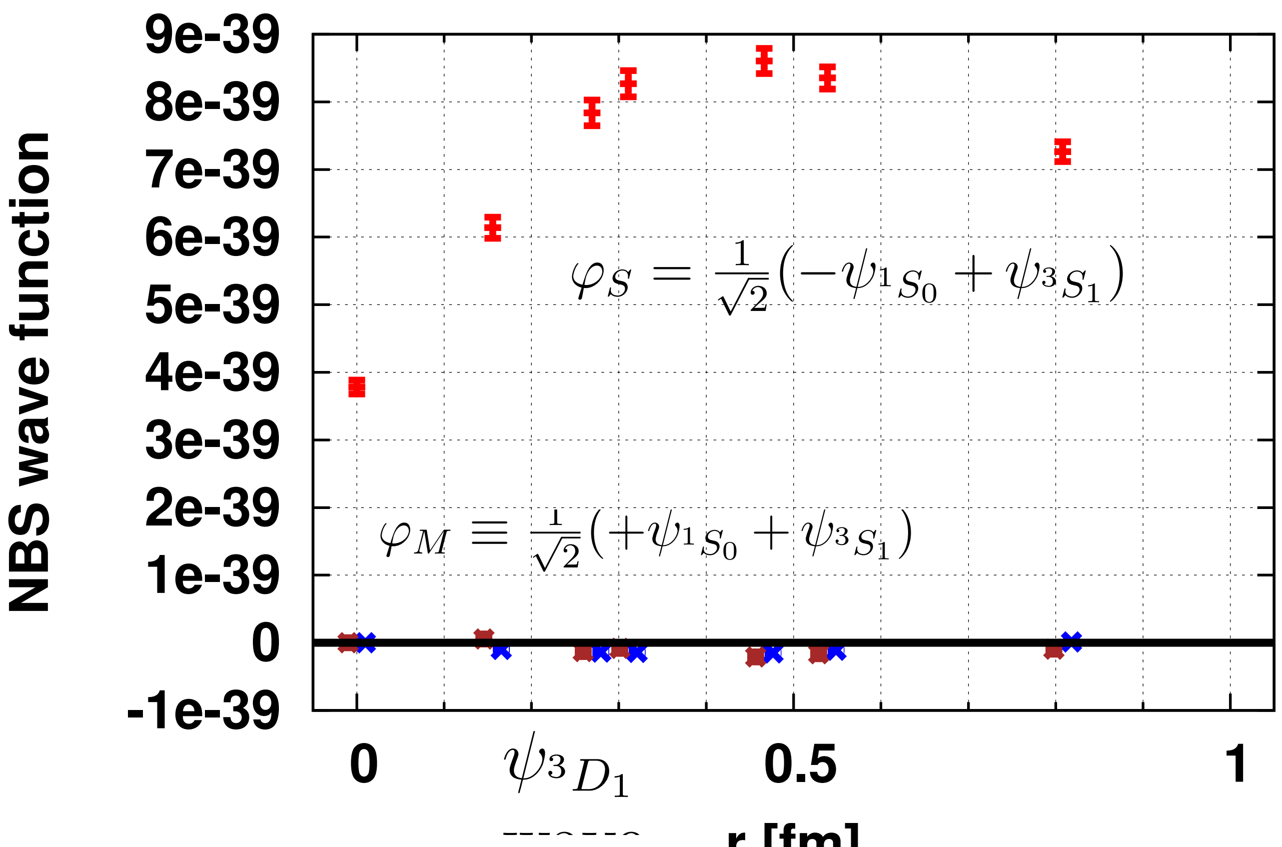
Doi et al. (HAL QCD), PTP 127 (2012) 723

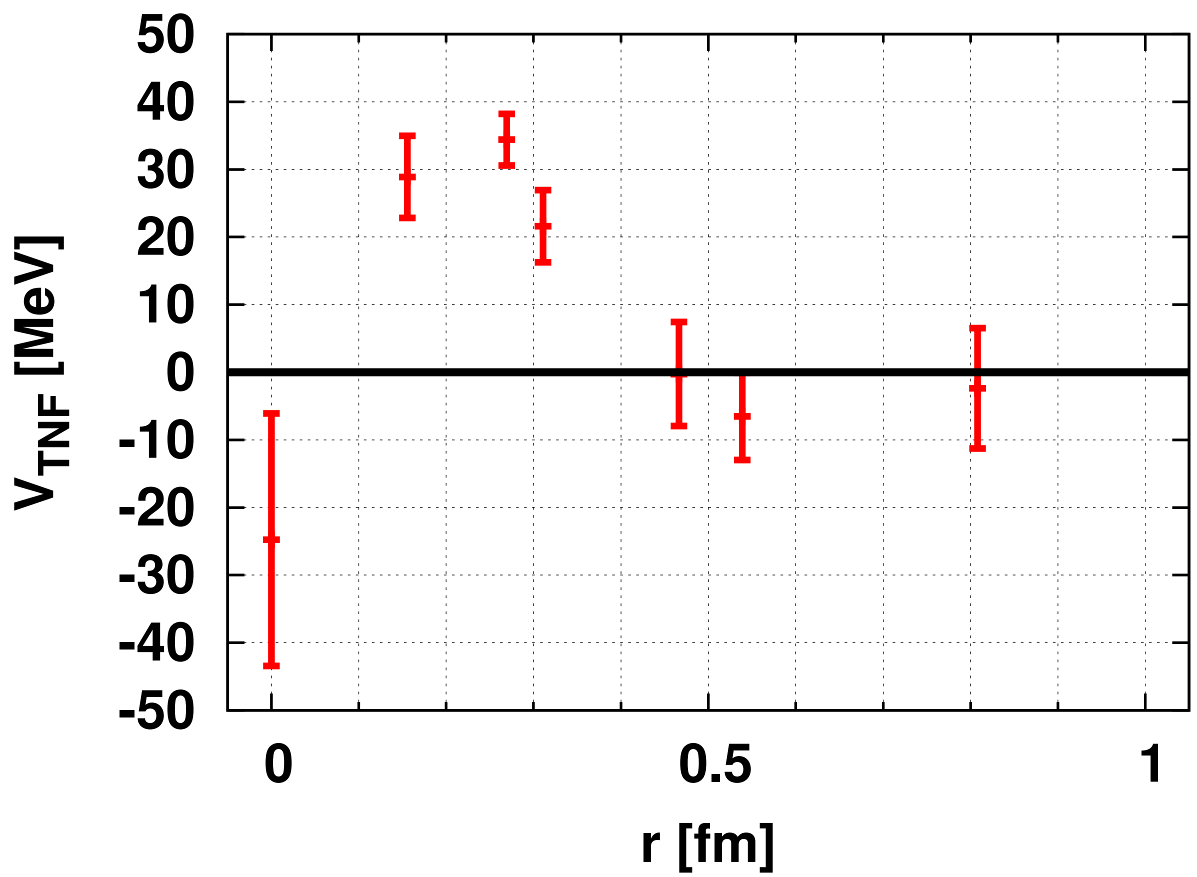
(1,2) pair $^1S_0,\ ^3S_1,\ ^3D_1$ S-wave only

$\mathrm{Triton}(I = 1/2,\ J^P = 1/2^+)$

Linear setup







scalar/isoscalar TNF is observed at short distance.

further study is needed to confirm this result.