History of Oriental Astronomy

Proceedings of the Joint Discussion-17 at the 23rd General Assembly of the International Astronomical Union, organised by the Commission 41 (History of Astronomy), held in Kyoto, August 25–26, 1997

Edited by

S. M. Razaullah Ansari

President of IUHPS Commission for History of Ancient and Medieval Astronomy
Former President of IAU Commission for History of Astronomy
Former Professor of Physics, Aligarh Muslim University, Aligarh, India

KLUWER ACADEMIC PUBLISHERS

DORDRECHT / BOSTON / LONDON

2002

1.3. The First Equation Table for Mercury in the *Huihui li*

Michio Yano*

1. Introduction

The structure of the *Huihui li*, a Chinese Islamic calendar originally compiled in A.D. 1383,[1] and the basic theory behind it were investigated by Kiyosi Yabuuti in his pioneering works.[2] However, we are not yet sure on which Islamic sources the *Huihui li* was actually based.[3]

The main body of the Arabic literature called *zīj* is astronomical tables together with the instructions how to use these tables. Very few *zīj*es give theoretical explanation of how these tables were prepared. Proofs are rarely given. In this sense we can call the *Huihui li* a typical *zīj*. What is recorded in the *Huihui li* are only astronomical tables and very brief explanations of how to use them. Nothing theoretical is found throughout the text. In order to discover the theory or the algorithm by which the tables were constructed, we can first hypothesize the theory or algorithm from the tables and, in turn, reconstruct the tables from the hypotheses. Only when the reconstructed tables agree with the actual tables to a sufficient degree we can say that the hypotheses were correct. It is only after such preliminary studies that we can guess the sources of the *Huihui li* and investigate its relation to Arabic and Persian astronomical texts.

The present author has been dealing with this problem with Benno van Dalen.[4] We read a joint paper concerning the tables of the planetary latitude at the 8th International Conference on the History of Science in East Asia, August 26–31, 1996, Seoul, Korea.[5] As van Dalen already suggested on that occasion, the *Huihui li* was very closely related to the *Sanjufīnī Zīj*[6] which was prepared by a certain Abū Muḥammad ʿAṭā ibn Ahmad ibn Muḥammad Khwāja Ghāzī al-Samarqandī al-Sanjufīnī in 1366 for the Mongol Viceroy of Tibet.[7]

The aim of the present paper is to point out the peculiar nature of the first equation table for Mercury in the *Huihui li* and to show that this peculiarity was shared not only by the *Sanjufīnī Zīj* but also by al-Bīrūnī in his *al-Qānūn al-Masʿūdī*.[8]

The reader of this paper is assumed to be familiar with the planetary theory of the Ptolemaic system.

2. The First Equation of Planets

In the case of planets except Mercury, the geometrical model by which the equation tables of the *Huihui li* was computed seems to have been similar to that of the *Almagest*.[9] Concerning the structure of the equation tables in the *Huihui li*, the following two points are worth mentioning.

---

* This is a slightly revised version of my paper published in the *Memoirs of the International Institute for Linguistic Sciences*, Kyoto Sangyo University, No. 1 (March 1999).

S.M.R. Ansari (ed.), History of Oriental Astronomy, 33–43.

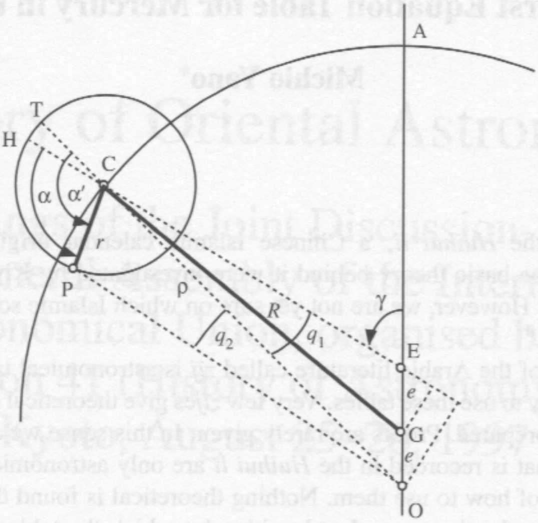

Figure 1 Planets except Mercury: q_1 and q_2.

1. The first equation (q_1 in Fig. 1), or the equation for the centre of the epicycle, was separated into two components in the *Almagest*. In the planetary equation tables the two components are arranged as column 3 and column 4. Following Neugebauer[10] I express the two sets of tabular values as c_3 and c_4, respectively.

As is shown in Fig. 2 the component c_3 ($= \angle$ODE) is the equation which would be produced if the equant (E) were the centre of the deferent, and thus the centre of the epicycle were carried on it (as the dotted circle in Fig. 2).

Actually, however, the centre of the deferent (with the radius R) carrying the epicycle C is G. The correction due to the shift from E to G is tabulated as c_4 ($= \angle$COD). Thus the first equation (q_1) is obtained by combining the two columns:

$$q_1 = c_3 + c_4.$$

Even though the first equation can be easily obtained by direct computation,[11] Ptolemy kept the two components separate 'for didactic reasons because he wanted the reader to see how c_3' (q_1 in our notation) had been obtained . . . ', as Neugebauer puts it.[12]

For the later astronomers whose aim was simply to prepare practical tables, there was no reason to provide the two separate columns, and it was natural that they tabulated only q_1. This was the case with Theon's *Handy Tables*,[13] which is originally ascribed to Ptolemy, as well as the Arabic and Persian *zīj*es and the *Huihui li*.

2. The sixth column of the equation tables in the *Almagest* gives the second equation (q_2 in Fig. 1 and Fig. 3), or the equation due to anomalistic motion of the planets. This column was prepared for the special case when the centre of the epicycle is at the mean distance on the deferent, and the differences which might be produced at the farthest distance and the nearerest distance are listed in the fifth column (negative) and the seventh column (positive), respectively. For the cases when the planet is in between the mean distance and

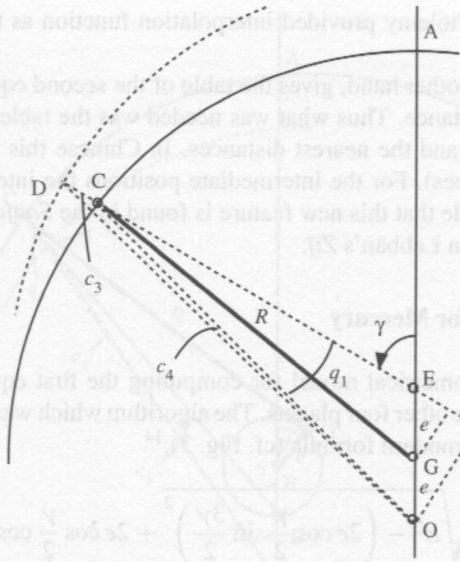

Figure 2 Planets except Mercury: c_3 and c_4.

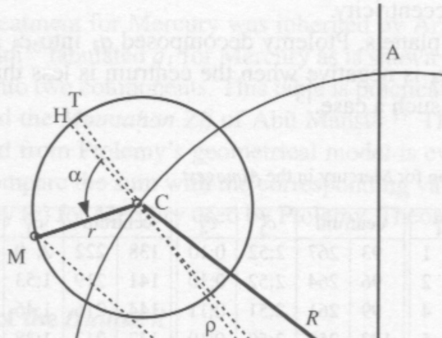

Figure 3 Mercury: q_1 and q_2.

these two extremities, Ptolemy provided interpolation function as the eighth column of the tables.

The *Huihui li*, on the other hand, gives the table of the second equation at the farthest, instead of the mean, distance. Thus what was needed was the table for the difference of equation at the farthest and the nearest distances. In Chinese this is called *yuan-jin du* (遠近度, distance degrees). For the intermediate positions the interpolation function is provided. It is remarkable that this new feature is found in the *Sanjufīnī Zīj*, but nowhere else except in Kūšyār ibn Labbān's *Zīj*.

3. The Special Case for Mercury

In the *Almagest* the geometrical model for computing the first equation of Mercury is different from that for the other four planets. The algorithm which was used in the *Almagest* can be expressed in the modern formula (cf. Fig. 3):[14]

$$\mathrm{EC} = \sqrt{R^2 - \left(2e\cos\frac{\gamma}{2}\sin\frac{3\gamma}{2}\right)^2} + 2e\cos\frac{\gamma}{2}\cos\frac{3\gamma}{2}$$

$$\rho\ (= \mathrm{OC}) = \sqrt{e^2 + \mathrm{EC}^2 + 2e\cdot\mathrm{EC}\cos\gamma}$$

$$q_1 = \arcsin\left(\frac{e\sin\gamma}{\rho}\right)$$

where γ is centrum and e is eccentricity.

As in the case of the other planets, Ptolemy decomposed q_1 into c_3 and c_4 which are shown in Table 1. Note that c_4 is negative when the centrum is less than 60° or greater than 300°. Fig. 4 is drawn for such a case.[15]

Table 1 First equation for Mercury in the *Almagest*

centrum		c_3°	c_4°	centrum		c_3°	c_4°	centrum		c_3°	c_4°
6	354	0;18	-0; 1	93	267	2;52	0;10	138	222	2; 0	0; 4
12	348	0;34	-0; 2	96	264	2;52	0;10	141	219	1;53	0; 4
18	342	0;51	-0; 4	99	261	2;51	0;11	144	216	1;46	0; 3
24	336	1; 7	-0; 5	102	258	2;50	0;10	147	213	1;38	0; 3
30	330	1;22	-0; 5	105	255	2;48	0;10	150	210	1;30	0; 2
36	324	1;37	-0; 4	108	252	2;46	0;10	153	207	1;22	0; 2
42	318	1;51	-0; 4	111	249	2;44	0; 9	156	204	1;13	0; 2
48	312	2; 4	-0; 3	114	246	2;41	0; 9	159	201	1; 5	0; 1
54	306	2;15	-0; 1	117	243	2;37	0; 9	162	198	0;56	0; 1
60	300	2;25	0; 0	120	240	2;33	0; 8	165	195	0;46	0; 1
66	294	2;34	0; 2	123	237	2;28	0; 7	168	192	0;38	0; 0
72	288	2;41	0; 4	126	234	2;23	0; 7	171	189	0;28	0; 0
78	282	2;46	0; 6	129	231	2;18	0; 6	174	186	0;19	0; 0
84	276	2;50	0; 7	132	228	2;12	0; 6	177	183	0; 9	0; 0
90	270	2;52	0; 9	135	225	2; 6	0; 5	180	180	0; 0	0; 0

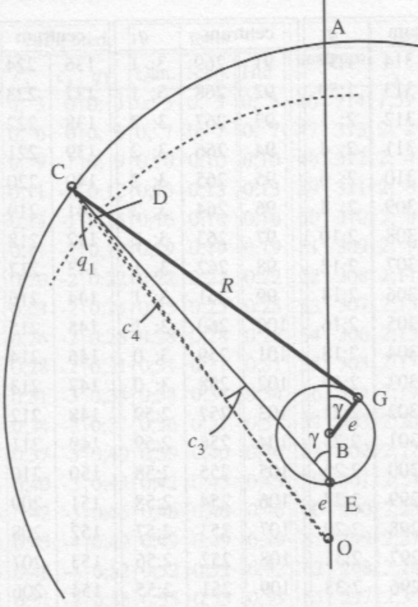

Figure 4 Mercury: c_3 and c_4.

This special treatment for Mercury was inherited by Arabic/Islamic astronomers. For instance, al-Battānī[16] tabulated q_1 for Mercury as is shown in Table 2, without, of course, decomposing it into two components. This table is practically the same as that in Theon's *Handy Tables* and the *Mumtaḥan Zīj* of Abū Manṣūr.[17] That these tabular values might have been derived from Ptolemy's geometrical model is evident when we add c_3 and c_4 of Table 1 and compare the sum with the corresponding values q_1 in Table 2.

The eccentricity (e) for Mercury used by Ptolemy, Theon, Abū Manṣūr, al-Battānī, etc. is 3 for $R = 60$.

4. Peculiarity of the *Huihui li*

The *Huihui li* does not give any separate account for the case of Mercury, but there is no doubt that the table for Mercury was constructed in a different way from that for the other planets.

Strangely enough, however, I found that the table for the first equation (q_1) for Mercury in the *Huihui li* could not be produced by the same computer program that had produced al-Battānī's table for q_1 whichever value of e was input. What is actually given in the *Huihui li* is shown in the eighth column of Table 3. After several attempts of rewriting my program in order to get the tabular values of the *Huihui li*, I happened to find that these values can be produced by subtracting Ptolemy's c_4 from c_3, instead of adding them together! In Table 3 I have shown c_3 (in degrees), c_4 (in minutes), and q_1 which I got using my program with $e = 3$.[18]

Table 2 al-Battānī's first equation for Mercury

centrum		q_1	centrum		q_1	centrum		q_1	centrum		q_1
1	359	0; 3	46	314	1;57	91	269	3; 1	136	224	2; 9
2	358	0; 6	47	313	1;59	92	268	3; 1	137	223	2; 7
3	357	0; 9	48	312	2; 1	93	267	3; 2	138	222	2; 5
4	356	0;12	49	311	2; 4	94	266	3; 2	139	221	2; 2
5	355	0;15	50	310	2; 6	95	265	3; 2	140	220	2; 0
6	354	0;17	51	309	2; 8	96	264	3; 2	141	219	1;57
7	353	0;20	52	308	2;10	97	263	3; 2	142	218	1;55
8	352	0;23	53	307	2;12	98	262	3; 1	143	217	1;52
9	351	0;25	54	306	2;14	99	261	3; 1	144	216	1;49
10	350	0;28	55	305	2;16	100	260	3; 1	145	215	1;47
11	349	0;30	56	304	2;18	101	259	3; 0	146	214	1;44
12	348	0;32	57	303	2;19	102	258	3; 0	147	213	1;41
13	347	0;35	58	302	2;21	103	257	2;59	148	212	1;38
14	346	0;38	59	301	2;23	104	256	2;59	149	211	1;34
15	345	0;40	60	300	2;25	105	255	2;58	150	210	1;32
16	344	0;43	61	299	2;27	106	254	2;58	151	209	1;30
17	343	0;45	62	298	2;29	107	253	2;57	152	208	1;27
18	342	0;47	63	297	2;31	108	252	2;56	153	207	1;24
19	341	0;50	64	296	2;33	109	251	2;55	154	206	1;21
20	340	0;53	65	295	2;35	110	250	2;54	155	205	1;18
21	339	0;55	66	294	2;36	111	249	2;53	156	204	1;15
22	338	0;58	67	293	2;38	112	248	2;52	157	203	1;12
23	337	1; 0	68	292	2;40	113	247	2;51	158	202	1; 9
24	336	1; 2	69	291	2;41	114	246	2;50	159	201	1; 6
25	335	1; 5	70	290	2;43	115	245	2;49	160	200	1; 3
26	334	1; 8	71	289	2;44	116	244	2;48	161	199	1; 0
27	333	1;10	72	288	2;45	117	243	2;46	162	198	0;57
28	332	1;13	73	287	2;47	118	242	2;45	163	197	0;54
29	331	1;15	74	286	2;48	119	241	2;43	164	196	0;51
30	330	1;17	75	285	2;49	120	240	2;41	165	195	0;48
31	329	1;20	76	284	2;50	121	239	2;39	166	194	0;45
32	328	1;23	77	283	2;51	122	238	2;37	167	193	0;42
33	327	1;25	78	282	2;52	123	237	2;35	168	192	0;39
34	326	1;28	79	281	2;53	124	236	2;34	169	191	0;35
35	325	1;31	80	280	2;54	125	235	2;32	170	190	0;32
36	324	1;33	81	279	2;55	126	234	2;30	171	189	0;28
37	323	1;36	82	278	2;56	127	233	2;28	172	188	0;25
38	322	1;38	83	277	2;57	128	232	2;26	173	187	0;22
39	321	1;40	84	276	2;57	129	231	2;24	174	186	0;19
40	320	1;43	85	275	2;58	130	230	2;22	175	185	0;16
41	319	1;45	86	274	2;59	131	229	2;20	176	184	0;13
42	318	1;47	87	273	2;59	132	228	2;18	177	183	0; 9
43	317	1;50	88	272	3; 0	133	227	2;16	178	182	0; 6
44	316	1;52	89	271	3; 0	134	226	2;14	179	181	0; 3
45	315	1;54	90	270	3; 1	135	225	2;11	180	180	0; 0

Table 3 First equation for Mercury in *al-Qānūn*, *Sanjufīnī*, and *Huihui li*

centrum		computed			texts			centrum		computed			texts		
		c_3°	c'_4	q_1	Qān.	Sanj.	Hui.			c_3°	c'_4	q_1	Qān.	Sanj.	Hui.
1	359	0; 3	0	0; 3	0; 3	0; 3	0; 3	46	314	1;59	-3	2; 3	2; 3	2; 3	2; 3
2	358	0; 6	0	0; 6	0; 7	0; 7	0; 7	47	313	2; 2	-3	2; 5	2; 5	2; 5	2; 5
3	357	0; 9	-1	0; 9	0;10	0;10	0;10	48	312	2; 4	-3	2; 6	2; 7	2; 7	2; 7
4	356	0;11	-1	0;12	0;13	0;13	0;13	49	311	2; 6	-3	2; 8	2; 9	2; 9	2; 9
5	355	0;14	-1	0;15	0;16	0;16	0;16	50	310	2; 8	-2	2;10	2;10	2;10	2;10
6	354	0;17	-1	0;19	0;19	0;19	0;19	51	309	2; 9	-2	2;12	2;12	2;12	2;12
7	353	0;20	-2	0;22	0;22	0;22	0;22	52	308	2;11	-2	2;13	2;13	2;13	2;13
8	352	0;23	-2	0;25	0;25	0;25	0;25	53	307	2;13	-2	2;15	2;15	2;15	2;15
9	351	0;26	-2	0;28	0;28	0;28	0;28	54	306	2;15	-1	2;16	2;16	2;16	2;16
10	350	0;28	-2	0;31	0;31	0;31	0;31	55	305	2;17	-1	2;18	2;17	2;18	2;18
11	349	0;31	-3	0;34	0;34	0;34	0;34	56	304	2;19	-1	2;19	2;19	2;19	2;19
12	348	0;34	-3	0;37	0;36	0;37	0;37	57	303	2;20	0	2;21	2;20	2;21	2;21
13	347	0;37	-3	0;40	0;39	0;40	0;40	58	302	2;22	0	2;22	2;22	2;22	2;22
14	346	0;40	-3	0;43	0;42	0;43	0;43	59	301	2;24	0	2;23	2;23	2;24	2;24
15	345	0;42	-3	0;46	0;46	0;46	0;46	60	300	2;25	1	2;25	2;25	2;25	2;25
16	344	0;45	-4	0;49	0;49	0;49	0;49	61	299	2;27	1	2;26	2;28	2;27	2;27
17	343	0;48	-4	0;52	0;52	0;52	0;52	62	298	2;28	1	2;27	2;29	2;29	2;29
18	342	0;51	-4	0;55	0;55	0;55	0;55	63	297	2;30	2	2;28	2;30	2;30	2;30
19	341	0;53	-4	0;57	0;58	0;58	0;58	64	296	2;31	2	2;29	2;31	2;31	2;31
20	340	0;56	-4	1; 0	1; 1	1; 1	1; 1	65	295	2;32	2	2;30	2;31	2;31	2;31
21	339	0;59	-4	1; 3	1; 4	1; 3	1; 4	66	294	2;34	3	2;31	2;32	2;32	2;32
22	338	1; 2	-4	1; 6	1; 7	1; 7	1; 7	67	293	2;35	3	2;32	2;33	2;33	2;33
23	337	1; 4	-5	1; 9	1;10	1;10	1;10	68	292	2;36	3	2;33	2;34	2;34	2;34
24	336	1; 7	-5	1;11	1;12	1;12	1;12	69	291	2;38	4	2;34	2;35	2;35	2;35
25	335	1; 9	-5	1;14	1;15	1;15	1;15	70	290	2;39	4	2;35	2;35	2;36	2;35
26	334	1;12	-5	1;17	1;17	1;17	1;17	71	289	2;40	4	2;36	2;36	2;36	2;36
27	333	1;15	-5	1;20	1;20	1;20	1;20	72	288	2;41	5	2;36	2;37	2;37	2;37
28	332	1;17	-5	1;22	1;23	1;23	1;23	73	287	2;42	5	2;37	2;37	2;37	2;37
29	331	1;20	-5	1;25	1;25	1;25	1;25	74	286	2;43	5	2;38	2;38	2;38	2;38
30	330	1;22	-5	1;27	1;27	1;27	1;27	75	285	2;44	6	2;38	2;39	2;39	2;39
31	329	1;25	-5	1;30	1;30	1;30	1;30	76	284	2;45	6	2;39	2;39	2;39	2;39
32	328	1;27	-5	1;32	1;32	1;32	1;32	77	283	2;45	6	2;39	2;40	2;40	2;40
33	327	1;30	-5	1;35	1;34	1;34	1;34	78	282	2;46	6	2;40	2;40	2;40	2;40
34	326	1;32	-5	1;37	1;37	1;37	1;37	79	281	2;47	7	2;40	2;41	2;41	2;41
35	325	1;35	-5	1;39	1;39	1;39	1;39	80	280	2;48	7	2;41	2;41	2;41	2;41
36	324	1;37	-5	1;42	1;41	1;41	1;41	81	279	2;48	7	2;41	2;42	2;42	2;42
37	323	1;39	-5	1;44	1;44	1;44	1;44	82	278	2;49	8	2;41	2;42	2;42	2;42
38	322	1;42	-5	1;46	1;46	1;46	1;46	83	277	2;49	8	2;42	2;42	2;42	2;42
39	321	1;44	-4	1;49	1;48	1;48	1;48	84	276	2;50	8	2;42	2;43	2;43	2;43
40	320	1;46	-4	1;51	1;51	1;51	1;51	85	275	2;50	8	2;42	2;43	2;43	2;43
41	319	1;49	-4	1;53	1;53	1;53	1;53	86	274	2;51	8	2;42	2;43	2;43	2;43
42	318	1;51	-4	1;55	1;55	1;55	1;55	87	273	2;51	9	2;42	2;43	2;43	2;43
43	317	1;53	-4	1;57	1;57	1;57	1;57	88	272	2;51	9	2;42	2;43	2;43	2;43
44	316	1;55	-4	1;59	1;59	1;59	1;59	89	271	2;52	9	2;42	2;43	2;43	2;43
45	315	1;57	-3	2; 1	2; 1	2; 1	2; 1	90	270	2;52	9	2;42	2;43	2;43	2;43

Table 3 (Continued)

centrum		computed			texts			centrum		computed			texts		
		c_3°	c_4'	q_1	Qān.	Sanj.	Hui.			c_3°	c_4'	q_1	Qān.	Sanj.	Hui.
91	269	2;52	9	2;42	2;43	2;42	2;42	136	224	2; 4	5	1;59	1;59	1;59	1;59
92	268	2;52	10	2;42	2;42	2;42	2;42	137	223	2; 2	4	1;57	1;57	1;57	1;57
93	267	2;52	10	2;42	2;42	2;42	2;42	138	222	1;59	4	1;55	1;55	1;55	1;55
94	266	2;52	10	2;42	2;42	2;42	2;42	139	221	1;57	4	1;53	1;53	1;53	1;53
95	265	2;52	10	2;42	2;42	2;42	2;42	140	220	1;55	4	1;51	1;51	1;51	1;51
96	264	2;52	10	2;42	2;42	2;41	2;41	141	219	1;53	3	1;49	1;49	1;49	1;49
97	263	2;52	10	2;41	2;41	2;41	2;41	142	218	1;50	3	1;47	1;46	1;47	1;47
98	262	2;51	10	2;41	2;41	2;41	2;41	143	217	1;48	3	1;45	1;45	1;45	1;45
99	261	2;51	10	2;41	2;41	2;41	2;41	144	216	1;45	3	1;42	1;43	1;43	1;43
100	260	2;51	10	2;40	2;40	2;40	2;40	145	215	1;43	3	1;40	1;41	1;41	1;41
101	259	2;50	10	2;40	2;40	2;40	2;40	146	214	1;40	2	1;38	1;38	1;38	1;38
102	258	2;50	10	2;40	2;40	2;40	2;40	147	213	1;38	2	1;35	1;35	1;35	1;35
103	257	2;49	10	2;39	2;39	2;39	2;39	148	212	1;35	2	1;33	1;33	1;33	1;33
104	256	2;49	10	2;38	2;39	2;39	2;39	149	211	1;32	2	1;31	1;31	1;31	1;31
105	255	2;48	10	2;38	2;38	2;38	2;38	150	210	1;30	2	1;28	1;28	1;28	1;28
106	254	2;47	10	2;37	2;38	2;38	2;38	151	209	1;27	2	1;26	1;26	1;26	1;26
107	253	2;47	10	2;37	2;37	2;37	2;37	152	208	1;24	1	1;23	1;23	1;23	1;23
108	252	2;46	10	2;36	2;36	2;36	2;36	153	207	1;22	1	1;20	1;20	1;20	1;20
109	251	2;45	10	2;35	2;36	2;36	2;36	154	206	1;19	1	1;18	1;17	1;17	1;17
110	250	2;44	10	2;34	2;35	2;35	2;35	155	205	1;16	1	1;15	1;14	1;14	1;14
111	249	2;43	10	2;34	2;35	2;35	2;35	156	204	1;13	1	1;12	1;11	1;11	1;11
112	248	2;42	10	2;33	2;34	2;34	2;34	157	203	1;10	1	1;10	1; 9	1; 9	1; 9
113	247	2;41	9	2;32	2;33	2;33	2;33	158	202	1; 8	1	1; 7	1; 6	1; 6	1; 6
114	246	2;40	9	2;31	2;32	2;32	2;32	159	201	1; 5	1	1; 4	1; 4	1; 4	1; 4
115	245	2;39	9	2;30	2;31	2;31	2;31	160	200	1; 2	1	1; 1	1; 1	1; 1	1; 1
116	244	2;38	9	2;29	2;30	2;30	2;30	161	199	0;59	0	0;58	0;58	0;58	0;58
117	243	2;37	9	2;28	2;28	2;28	2;28	162	198	0;56	0	0;55	0;55	0;55	0;55
118	242	2;35	9	2;27	2;27	2;27	2;27	163	197	0;53	0	0;52	0;52	0;52	0;52
119	241	2;34	8	2;26	2;26	2;26	2;26	164	196	0;50	0	0;49	0;48	0;49	0;49
120	240	2;33	8	2;24	2;25	2;25	2;25	165	195	0;47	0	0;46	0;45	0;46	0;46
121	239	2;31	8	2;23	2;23	2;23	2;23	166	194	0;44	0	0;43	0;43	0;43	0;43
122	238	2;30	8	2;22	2;22	2;22	2;22	167	193	0;41	0	0;40	0;40	0;40	0;40
123	237	2;28	8	2;20	2;21	2;21	2;21	168	192	0;38	0	0;37	0;37	0;37	0;37
124	236	2;27	7	2;19	2;19	2;19	2;19	169	191	0;34	0	0;34	0;34	0;34	0;34
125	235	2;25	7	2;18	2;18	2;18	2;18	170	190	0;31	0	0;31	0;31	0;31	0;31
126	234	2;23	7	2;16	2;16	2;16	2;16	171	189	0;28	0	0;28	0;28	0;28	0;28
127	233	2;21	7	2;15	2;15	2;15	2;15	172	188	0;25	0	0;25	0;25	0;25	0;25
128	232	2;20	6	2;13	2;14	2;14	2;14	173	187	0;22	0	0;22	0;22	0;22	0;22
129	231	2;18	6	2;12	2;12	2;12	2;12	174	186	0;19	0	0;19	0;19	0;19	0;19
130	230	2;16	6	2;10	2;10	2;10	2;10	175	185	0;16	0	0;16	0;16	0;16	0;16
131	229	2;14	6	2; 8	2; 8	2; 8	2; 8	176	184	0;13	0	0;13	0;13	0;13	0;13
132	228	2;12	6	2; 7	2; 6	2; 6	2; 6	177	183	0; 9	0	0; 9	0; 9	0; 9	0; 9
133	227	2;10	5	2; 5	2; 5	2; 5	2; 5	178	182	0; 6	0	0; 6	0; 6	0; 6	0; 6
134	226	2; 8	5	2; 3	2; 3	2; 3	2; 3	179	181	0; 3	0	0; 3	0; 3	0; 3	0; 3
135	225	2; 6	5	2; 1	1; 1	2; 1	2; 1	180	180	0; 0	0	0; 0	0; 0	0; 0	0; 0

I could not find out on which theory this peculiarity was based, nor was I sure whether it was based on any theory at all or whether it was a result of a sheer mistake. Therefore, I investigated the first equation table for Mercury in the several *zījes* which were at hand. The crucial point of checking was, of course, whether q_1 of each text was the result of adding c_4 to c_3 or subtracting the former from the latter. As a result, I found that only the two texts, *Sanjufīnī Zīj* and al-Bīrūnī's *al-Qānūn al-Mas'ūdī*, share this particular feature with the *Huihui li*.

It is understandable that the table of the *Sanjufīnī Zīj* (column 7 of Table 3) is almost idential to that of the *Huihui li* (column 8),[19] since these two texts show a very close relationship to each other in the other respects, too.

I was quite surprised, however, to find that al-Bīrūnī's table (column 6 of Table 3) was so close to that of the *Huihui li* and the *Sanjufīnī Zīj*[20] that there was no doubt that these three tables belonged to the same tradition. Since all the predecessors of al-Bīrūnī, including his elder contemporary Kūšyār ibn Labbān,[21] added c_4 to c_3, al-Bīrūnī seems to be the first person who subtracted c_4 from c_3 and started this strange tradition. In order to find out whether this peculiarity was based on any theoretical ground, I read the 10th part (*maqāla*) of *al-Qānūn al-Mas'ūdī*,[22] but I found nothing special concerning the particularity of Mercury's first equation table. What al-Bīrūnī says in this context was simply a summary of the *Almagest*, and no attempt at innovation could be found.

For the moment, until counter-evidence is offered to disprove my conjecture, I would call this al-Bīrūnī's mistake. It is not strange that this kind of mistake should have happened, since, in the case of the other planets, c_4 is positive when the centrum is in the first and fourth quadrants, while in the case of Mercury it is negative when the centrum is less than 60° and greater than 300°. As is seen from Table 2 and Table 3, the difference due to the difference of adding and subtracting c_4 is not so remarkable – at most 20 minutes. Observations of Mercury could not have been used for the examination of the accuracy of the table.

It is historically interesting to note that, if al-Bīrūnī made such a mistake, he must have had a separate set of tables for c_3 and c_4 at hand and simply subtracted c_4 from c_3. Then what was the case with the other Arabic/Islamic authors of astronomical texts? It is likely that some of them also had separate tables for c_3 and c_4 and simply added them together, without computing q_1 anew by the direct method which I mentioned above. Another question then arises – who was the first person to prepare a separate set of tables for c_3 and c_4 at the interval of each degree?

I have investigated the following sources in order to check the method of obtaining the first equation table for Mercury. This list is in chronological order.[23]

Appendix

List of Sources

- Abū Manṣūr: *al-Mumtahan Zīj* (*c.* 830), Escorial arabe 927.
- al-Ḥabash al-Ḥāsib: *K. al-Ḥabash al-Ḥāsib* (*c.* 850), Berlin 5750.
- al-Battānī: *al-Zīj al-Ṣābi'* (*c.* 900), Nallino's edition. Cf. Endnote 16 above.
- Ibn Yūnus: *al-Zīj al-Ḥākimī* (990), Leiden Or. 143.

- Kūshyār ibn Labbān: *al-Zīj al-Jāmiʿ* (*c.* 1000), Istanbul, Vehbi 893, Fatih 3418, Berlin, Staatbibliothek, Ahlwardt 5751, etc.
- al-Bīrūnī: *al-Qānūn al-Masʿūdī* (1030), Hyderabad ed. and British Library Or. 1997.
- al-Khāzinī: *al-Zīj al-Sanjarī* (*c.* 1120), BM Or. 6669.
- al-Ṭabarī: *Zīj-i Mufrad* (*c.* 1230), Cambridge Browne O.1.
- al-Ṭūsī: *al-Zīj-i Īlkhānī* (after 1260), Cairo DMF 1.
- al-Maghribī: *Adwār al-Anwār* (*c.* 1280), Chester Beatty 3665.
- al-Baghdādī (*c.* 1285): Paris ms., Paris arabe 2486.
- Sanjar al-Kamālī: *Zīj-i Ashrafī* (*c.* 1310), Paris Suppl. Pers. 1488.
- al-Sanjufīnī: *Sanjufīnī Zīj* (1366), Paris arabe 6040.
- Ibn Isḥāq al-Tamīmī: *Tunisian Zīj* (14th c.), Hyderabad 298.
- al-Kāshī: *Zīj-i Khāqānī* (*c.* 1420), India Office Library 430.

Notes

1. There are three different recensions of the *Huihui li* – (1) that recorded in the official *Ming Dynastic History* which was compiled during the Qing Dynasty, (2) the *Qizheng tuibu* compiled by Bei Lin in A.D. 1477, and (3) the Korean recension *Chiljong san* which forms a part of the *Sejong sillok* compiled during the reign of King Sejong (1419–1450). These recensions are considerably different, especially in the arrangement and order of the explanatory texts and tables. For the difference, see the article by Benno van Dalen in this Volume.
2. See Part 2, Chapter 3 of Yabuuti's *Chinese Astronomy and Calendrical Sciences* (*Chûgoku no tenmon rekihô* in Japanese), Tokyo (Heibonsha) 1969, 2nd ed. 1990, which was a revision of his earlier paper, published in the *Tôhô Gakuhô*, Vol. 36 (1964), pp. 611–632 with the title 'Kaikai reki kai'. This work was recently translated, with some improvements, into English by Benno van Dalen as 'Islamic Astronomy in China during the Yuan and Ming Dynasties', *Historia Scientiarum*, Vol. 7, No. 1 (1997), pp. 11–43.
3. Recently van Dalen informed me of a very interesting paper which had escaped scholarly attention for long time: A. Wagner, 'Ueber ein altes Manuscript der Pulkowaer Sternwarte', *Copernicus*, Vol. II (1882), pp. 123–129. The author of this paper happened to examine an Arabic manuscript which was brought to the library of the Pulkova observatory by a consul in China. A mere glance at the table of contents and some parameters used in this manuscript is enough to say that this text was the best candidate for the source of the *Huihui li*. Needham (*Science and Civilisation in China*, Vol. 3, 1959, p. 372, footnote e), briefly referring to Wagner's paper, just hoped that 'they were not destroyed when the Obervatory was burnt during the second world war'. It is regrettable that no historian of astronomy tried to get access to the Pulkova manuscript. Let us hope that the manuscript survived the recent fire, too.
4. I thank the Japan Society for Promotion of Science for offering scholarship to Dr. van Dalen and thus making possible our joint project.
5. My contribution was published as 'Tables of Planetary Latitute in the *Huihui li* (I)', *Current Perspectives in the History of Science in East Asia*, ed. by Yung Sik Kim and Francesca Bray, Seoul National University, 1999 (June 30), pp. 307–315, which was followed by van Dalen's paper.
6. The unique manuscript is extant in the Bibliothèque Nationale, Paris, arabe 6040. I thank van Dalen who brought a photocopy of this manuscript for my use.
7. For this very interesting *zīj*, see Herbert Franke 'Mittelmongolische Glossen in einer arabischen astronomischen Handschrift', *Oriens* 31 (1988), pp. 95–118. See also Edward S. Kennedy, 'Eclipse Predictions in Arabic Astronomical Tables Prepared for the Mongol Viceroy of Tibet', *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften* 4 (1987/88), pp. 60–80 and Edward S. Kennedy and Jan Hogendijk 'Two Tables from an Arabic Astronomical Handbook for the Mongol Viceroy of Tibet', *A Scientific Humanist, Studies in Memory of Abraham Sachs*, ed. by Erle Leichty et al., Occasional Publications of the Samuel Noah Kramer Fund, 9, Philadelphia: The University Museum, 1988, pp. 233–242.
8. I have used the printed edition in 3 vols., Osmania Oriental Publications Bureau, 1956 and a copy of the manuscript from British Library Or. 1997.

9. See Gerald J. Toomer, *Ptolemy's Almagest*, London/New York, 1984.
10. Otto Neugebauer, *Exact Sciences in Antiquity*, New York, 1969, p. 200 ff.
11. In modern expression, when the eccentricity (e) is given, q_1 is a function of centrum (γ):

$$q_1 = \arcsin\left(\frac{2e\sin\gamma}{\rho}\right)$$

where

$$\rho = \sqrt{(2e\sin\gamma)^2 + (e\cos\gamma + \sqrt{R^2 - (e\sin\gamma)^2})^2}.$$

12. Neugebauer, *op. cit.*, p. 201. See also Toomer's translation of the *Almagest*, p. 546 and footnote 48.
13. I have used the Ph.D. dissertation of Willium D. Stahlman, *The Astronomical Tables of Codex Vaticanus Graecus 1291*, submitted to Brown University in 1959.
14. See Olaf Pedersen, *A Survey of the Almagest*, Odense University Press, 1974, p. 320.
15. The angles are so small that I want to make them clear here: D is at the intersection of EC and the dotted circle of which the centre is E and the radius is R. $q_1 = \angle$OCE, $c_3 = \angle$ODE, and $c_4 = \angle$COD.
16. Carolo A. Nallino ed. *Al-Battani sive Albatenii: Opus astronomicum*, 3 vols., Milano, 1903, 1907, 1899. Reprinted from Georg Olms Verlag, Hildesheim·New York, 1977. The first equation for Mercury is in vol.II, pp. 132–137. Al-Battānī's table for $q_1$is virtually identical to that in Theon's *Handy Tables*. The difference is only 2;4 instead of 2;5 for centrum 138/222 and 1;35 instead of 1;34 for centrum 149/211.
17. I have used the facsimile edition of Escorial arabe 927 published by F. Sezgin.
18. Since the tabular values of c_3 and c_4 are rounded to the unit of minutes, some values of q_1 are different from the sum of c_3 and c_4 of this table by one minute.
19. Only two out of 180 values are different, i.e., for centrum 21/339 and 70/290.
20. Out of 180, only 11 values are different.
21. The equation tables of Kūšyār ibn Labbān have another special feature of 'displacement', in order to avoid negative values.
22. I thank Toshiaki Kashino for reading this text with me.
23. For the date of the texts, see Kennedy, *A Survey of Islamic Astronomical Tables*, Transactions of the American Philosophical Society, Vol. 46, Part 2 (1956). For the date of *al-Mumtaḥan*, al-Baghdādī, and *Īlkhānī*, I acknowledge to Benno van Dalen's personal communication.