Tables of Planetary Latitude in the *Huihui li* (I)*

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Professor Kiyosi Yabuuti was the first historian of astronomy who undertook scholarly surveys on the *Huihui li* 回回曆, a Chinese text with astronomical tables compiled at the Islamic Bureau of Astronomy in the Yuan-Ming period. It was because he was well aware of the crucial role of the Ptolemaic planetary system in the *Huihui li* that he read Halma's French translation of Ptolemy's *Almagest* and eventually translated it into Japanese. Without his pioneering work[1] our study would have been far more difficult.

Although Yabuuti has clarified almost all the topics in the *Huihui li* by comparing them with those in the *Almagest*, he left some room for more detailed analyses such as these attempted in our present research. An especially interesting and important problem is the search for a possible source of the *Huihui li* and a reconstruction of the original form of the Chinese translation which must have been made during the Yuan Dynasty. For this purpose we should be informed of Arabic and Persian texts on astronomy, especially those belonging to

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1 Originally published as "Kaikaireki kai" 回回曆解, *Tōhōgakuhō* 東方學報 36 (Kyoto, 1964), 611-32. Almost the same contents are found in his *Chūgoku no tenmon rekihō* 中國の天文曆法, pp. 215-34. See also K. Yabuuti, "The Influence of Islamic Astronomy in China," in D. King and G.A. Saliba (eds.), *From Deferent to Equant: A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honor of E.S. Kennedy. Annals of the New York Academy of Science*, vol. 500, pp. 547-59. Recently Chen Jiujin 陳久金 published a paper on the *Huihui li*: "Huili riyue weizhi di jisuan ji yundong di jihe muxing" 回曆日月位置的計算及運動的幾何模型 (The Calculation of Positions of the Sun and the Moon and Geometric Models of Their Motion in Hui Calendar), *Ziran kexueshi yanjiu* 自然科學史研究 (Studies in the History of Natural Sciences) 8(3) (1989), 219-29. For contacts between Chinese and Islamic exact sciences, see Jean-Claude Martzloff's brief summary in *A History of Chinese Mathematics* (Springer, 1997), pp. 101-105.

the category called *Zīj* (astronomical handbook with tables). No one before us, however, has taken the trouble of reading such sources and comparing them with the *Huihui li*. One of the reasons is that very few printed editions of *Zījes* are available, so that almost all the information we can expect is still in the form of manuscripts. The present study on the latitude tables offers an example of our new approach to the *Huihui li* which uses information from *Zījes*.

There are three different recensions of the *Huihui li*, namely, (1) that recorded in the official *Ming Dynastic History* (*Mingshi* 明史),[2] which was compiled during the Qing Dynasty, (2) the *Qizheng tuibu* 七政推步[3] compiled by Bei Lin 貝琳 in 1477, and (3) the Korean recension *Chilchŏng san* 七政算 which forms a part of the *Sejong sillok* 世宗實錄 compiled during the reign of King Sejong (r. 1419–50).[4] These recensions are considerably different, especially in the arrangement and order of the explanatory texts and tables.

The Latitude of the Five Planets

Determining the latitude of the planets was one of the most difficult problems in ancient astronomy. The difficulty was inevitable in the geocentric cosmology where the plane of a planet's orbit cannot have a fixed inclination to the plane of the ecliptic. This is why Ptolemy put the theory of planetary latitude in the last Book of the *Almagest*. In ancient Chinese astronomy, where no geometrical model was conceived, the problem should have been still more difficult, as is witnessed by the absence of a systematic discussion of this problem before the Yuan Dynasty.

In the introduction to the *Huihui li*, we read:

> In the autumn of the year 15 (A.D. 1382) emperor Taizu said: "The Western investigations of the heavenly phenomena are most refined. Moreover, their (theory of the) latitudes of the five planets is not available in China."[5]

Similar words are found[6] in the *Mingyi tianwen shu* 明譯天文書, which

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[2] We have used *Mingshi* 明史 "Lizhi" 曆志 contained in *Lidai tianwen lüli dengzhi huipian* 歷代天文律曆等志彙編 10.

[3] We have used the edition of the *Qizheng tuibu* contained in *Jinding siku quanshu* 欽定四庫全書, zibu 子部 6.

[4] We have used the reprint edition of the *Sejong sillok* published by the Oriental Institute of Gakushūin 學習院 University.

[5] 十五年秋, 太祖謂西域推測天象最精, 其五星緯度又中國所無.

[6] 爾來西域陰陽家, 推測天象, 至爲精密有驗. 其緯度之法, 又中國之所未備.

is a Chinese translation of Kūšyār ibn Labbān's book on astrology[7] completed in the same year by the same group of scholars.

The Chinese text on planetary latitude, in all three recensions, is very brief, just explaining how to use the tables. The tables, however, are extensive. It seems that the translators were less interested in the theoretical exposition of the problem than in the practical use of the tables.

Translation and Commentary

In what follows we shall give a translation of the Chinese text on planetary latitudes recorded in the *Ming Dynastic History*, vol. 37. The sentences in smaller point are commentaries in the original text, and the words within brackets are our additions. We have attached the Chinese text as Appendix 1. The tables of planetary latitude occupy a large part of vol. 39 of the *Ming Dynastic History*. We have shown an example of Saturn's table of latitude in Appendix 2. The original format is shown in Appendix 3.

The Latitude of the Five Planets.
We require the total apogee motion and the apogee position, the mean position, the anomaly and the centrum. We find them all according to the method of the longitude of the five planets.

Commentary
The latitudes will be determined as a function of the mean anomaly and mean centrum, computed in the preceding sections according to Ptolemy's longitude model. Unlike most Greek and Islamic planetary latitude tables, in the *Huihui li* the latitude can be taken directly from a table with double arguments (see the following paper by van Dalen).

The definite anomaly. We put down the signs, degrees and minutes of the anomaly. We multiply the signs by 10 and make degrees. For example, when we multiply 1 sign by 10 we obtain 10 degrees. This uses a simplifying method to reduce the calculation by which we construct the latitude table. We multiply the degrees by 20 and make minutes. In case sixties are filled we reduce them by division and make degrees. We also multiply the minutes by 20 and make seconds. In case sixties are filled we reduce them by division and make minutes. We add them, and thus we obtain it.

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[7] I have prepared an edition of the Arabic text of this book with an English translation and submitted it to Kyoto University as my D.Litt. dissertation. My study is to be published soon.

Commentary

The epicyclic (mean) anomaly (*zi xing gongdu fen* 自行宮度分), given by $\alpha = y_1^3 y_2^\circ y'_3$, is scaled down to one third:

$$y = \frac{1}{3}\alpha = \frac{y_1^s}{3}\frac{y_2^\circ}{3}\frac{y_3'}{3} = 10y_1^\circ 20y_2' 20y_3''.$$

Of course sexagesimal place value should be observed and therefore $20\,y_2'$ and $20\,y_3''$ should be reduced to suitable numbers if they exceed 60. The result (y) of this operation is called 'definite anomaly' (*zi xing dingdu* 自行定度). The terminology is confusing because the same term (自行定度) was already used (page 3762 line 11) to designate the 'true anomaly' and because no special marker was given to the 'degrees' etc. after this operation of scaling down.

> **The definite centrum.** We put down the signs, degrees and minutes of the centrum. We multiply the signs by 5 and make degrees. For example, when we multiply 1 sign by 5 we obtain 5 degrees. We multiply the degrees by 10 and make minutes. In case sixties are filled we reduce them by division and make degrees. We also multiply the minutes by 10 and make seconds. In case sixties are filled we reduce them by division and make minutes. We add them, and thus we obtain it.

Commentary

The centrum (*xiaolun xin gongdu fen* 小輪心宮度分), given by $\gamma = x_1^s x_2^\circ x_3'$, is scaled down to one sixth:

$$x = \frac{1}{6}\gamma = \frac{x_1^s}{6}\frac{x_2^\circ}{6}\frac{x_3'}{6} = 5x_1^\circ 10x_2' 10x_3''.$$

After taking care of sexagesimal place value as above, the result is called 'definite centrum' (*xiaolun xin dingdu* 小輪心定度). This is again misleading since the same term was already used (page 3762 line 11) to designate the 'true centrum.'

> **The latitude.** With the definite centrum (x) and the definite anomaly (y) we enter the latitude table for the present planet and we take (a value, indicated by β_{mn} in Figure 1) (according to) both. One vertically, one horizontally.
> We subtract the obtained number (β_{mn}) and (the number in) the following row ($\beta_{(m+1)n}$) from each other. If we happen to cross the ecliptic, we add (the obtained number) to (the number in) the following row.
> Then we subtract the definite centrum (x) and the definite centrum at the top of the table (c_m) from each other. The horizontal row at the top. We multiply the remainders of the two subtractions and divide it by the increment (Δc) of the centrum at the top of the table. For instance, for Saturn each partition of the centrum in the horizontal row at the top is 3 degrees, for Mars each partition is 2 dsgrees.

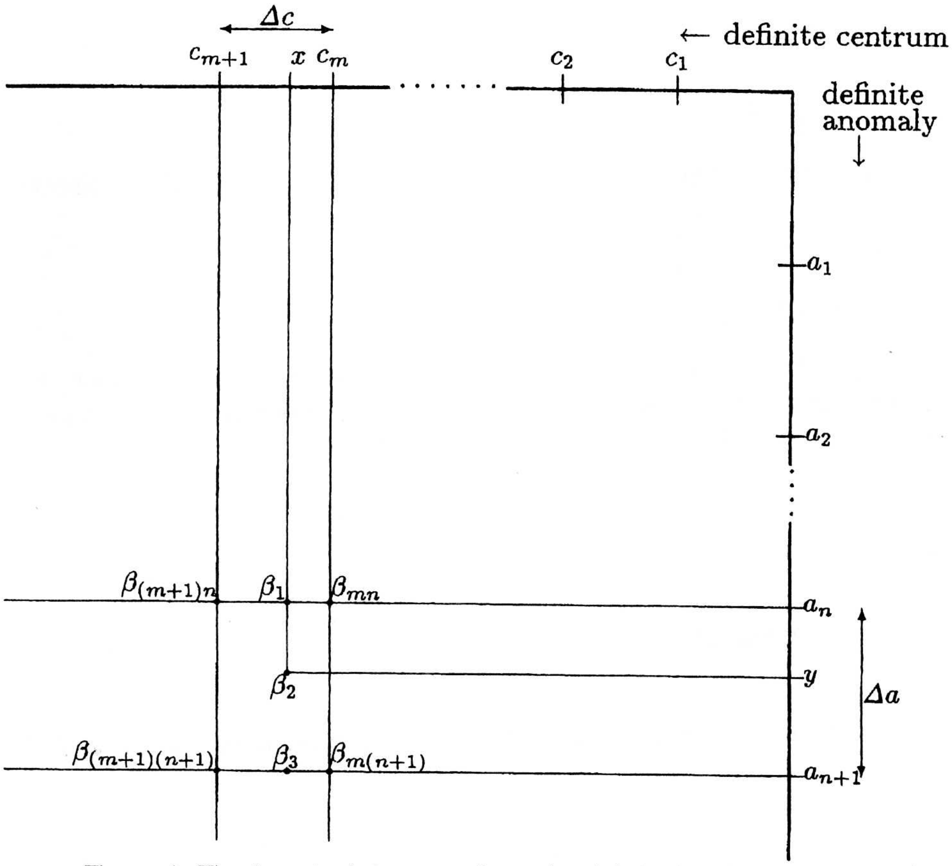

Figure 1. The layout of the two dimensional latitude table of the *Huihui li* and its interpolation scheme.

In case sixties are filled we take them together and make minutes. We add it to or subtract it from the number taken with both (arguments, i.e. β_{mn}). If it is more than (the number in) the following row we subtract, if less we add. In case we happen to cross the ecliptic, then if the number in the following row is more, we also subtract. We place it on the left.

Furthermore, we subtract the definite anomaly (y) and the definite anomaly at the top of the table (a_n) from each other. The first vertical row. Then we subtract the number taken with both (arguments, i.e. β_{mn}) and (the number in) the row below it ($\beta_{m(n+1)}$) from each other. If we happen to cross the ecliptic, we add it and the row below it. Then we multiply the remainders of the two subtractions. We divide it by the increment of the anomaly (Δa) at the top (i.e. the right) of the table. For instance, for Saturn each partition of the anomaly in the vertical row is 10 degrees, for Mars each partition is 4 degrees. We take it together and make minutes. These and the number previously placed on the left we add to or subtract from each other. If the number taken with both (arguments) is more than (the number in) the row below it we subtract, if less we add. If we happen to cross the ecliptic and the obtained minutes are more than the number placed on the left, we put down the obtained minutes and subtract the number placed on the left. The remainder is the minutes north or south that the ecliptic has been passed. Thus we obtain the definite ecliptical latitude.

Commentary

First we carry out a linear interpolation in the horizontal direction:

$$\beta_h = \frac{|\beta_{mn} - \beta_{(m+1)n}|(x - c_m)}{\Delta c}$$

$$\beta_1 = \beta_{mn} \pm \beta_h .$$

Then another interpolation has to be made in the vertical direction:

$$\beta_v = \frac{(y - a_n)|\beta_{mn} - \beta_{m(n+1)}|}{\Delta a}$$

$$\beta_2 \approx \beta_1 \pm \beta_v .$$

This is approximate because the correct vertical interpolation should be made between β_1 and β_3 (See Figure 1), instead of between β_{mn} and $\beta_{m(n+1)}$, and β_3 is to be computed by

$$\beta_3 = \beta_{m(n+1)} \pm \beta'_h$$

where

$$\beta'_h = \frac{|\beta_{m(n+1)} - \beta_{(m+1)(n+1)}|(x - c_m)}{\Delta c}$$

Now β'_v would be found by linear interpolation between β_1 and β_3 from:

$$\beta'_v = \frac{(y - a_n)|\beta_1 - \beta_3|}{\Delta a}$$

and finally we would get the correct value:

$$\beta_2 = \beta_1 \pm \beta'_v$$

From an inspection of the tabular values it can be seen that for Saturn, Jupiter and Mercury the maximum error which can be produced in the interpolation described in the *Huihui li* is smaller than 10′ and hence could hardly be observed. For Mars and Venus, however the error would reach half a degree in exceptional cases.

Detailed positions in latitude. We put down the latitude of the given planet in the section preceding (the desired latitude) and subtract it and the latitude at the following section from each other. We divide the remainder by the distance in days and make the day difference. We put down the latitude at the preceding section and we increase it by the day difference in the normal order or decrease it by it in the reverse order. The result is the latitude on successive days. We examine the latitude: if (the value at) the preceding section is less than (that at) the following section, we increase by the day difference in the normal order and decrease by it in the reverse order. If (the value at) the preceding section is more than (that at) the following section, we should decrease by the day difference in the normal order and increase by it in the reverse order. We cannot have one rule. In case (the values at) the preceding and following sections are not the same as far as north and south are concerned, we put down the latitude of the given planet at the preceding and the following section and add them. We divide it by the distance in days and make the day difference. We put down the latitude at the preceding section and repeatedly decrease it by the day difference until we cannot decrease it (any more). We subtract it from the day difference and repeatedly increase the remainder by the day difference. Thus we obtain the latitudes on successive days.

Commentary

The term 'detailed position' (*xi xing fen* 細行分) was already used in the section of the planetary longitude. In general, this is a simple, one-dimensional, linear interpolation. In this particular case, when the latitude β_n of a planet on one day and that on another day β_{n+d} after an interval of d days are given (called "sections" in our text), we first compute the 'day difference' (*richa* 日差) by

$$\Delta\beta = \frac{|\beta_n - \beta_{n+d}|}{d}$$

and, according to our main text, the result is successively added to or subtracted from β_n as the case may be, i.e., depending on whether $\beta_n \leq \beta_{n+d}$ or $\beta_n > \beta_{n+d}$. A special rule is also given for the cases when the directions of the two latitudes are different.

Appendix 1. Chinese Text[8]

五星緯度 求最高總行度，中心行度，自行度，小輪心度，並依五星經度術求之.

求自行定度 置自行宮度分，其宮以十乘之爲度. 如一宮，以二十乘之得十度，此用約法折算，以造緯度立成. 其度以二十乘之爲分，滿六十約之爲度. 其分亦以二十乘之爲秒，滿六十約之爲分. 併之卽得.

求小輪心定度 置小輪心宮度分，其宮以五乘之爲度. 如一宮以五乘之，得五度. 其度以一十乘之爲分，滿六十約之爲度. 其分亦以一十乘之爲秒，滿六十約之爲分. 併之卽得.

求緯度 以小輪心定度及自行定度，入本星緯度立成內兩取，一縱一橫.

得數與後行相減. 若遇交黃道者，與後行相併.

又以小輪心定度，與立成上小輪心定度相減，上橫行.

兩減餘相乘，以立成上小輪心度累加數除之. 如土星上橫行小輪心度每隔三度，火星每隔二度之類.

滿六十收之爲分，用加減兩取數，多於後行減，少加. 若遇交黃道者，卽後行數多亦減. 寄左.

復以自行定度與立成上自行定度相減，首直行.

又以兩取數，與下行相減，若遇交黃道者，與下行併. 兩減餘相乘，以立成上自行度累加數除之，如土星直行，自行度每隔十度，火星每隔四度之類.

收之爲分. 與前寄左數相加減，如兩取數多於下行者減，少加. 若遇交黃道者，所得分多於寄左數，置所得分內，減寄左數，餘爲交過黃道南北分也.

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8 *Lidai tianwen luli dengzhi huipian* 歷代天文律曆等志彙編 10, pp. 2764-65.

卽得黃道南北緯定分

求緯度細行分 置其星前段緯度, 與後段緯度相減, 餘以相距日除之, 爲日差.

置前段緯度, 以日差順加退減, 卽逐日緯度分. 按緯度前段少於後段者, 以日差順加退減. 若前段多於後段者, 宜以日差順減退加. 非可一例也.

若前後段南北不同者, 置其星前後段緯度倂之, 以相距日除之, 爲日差.

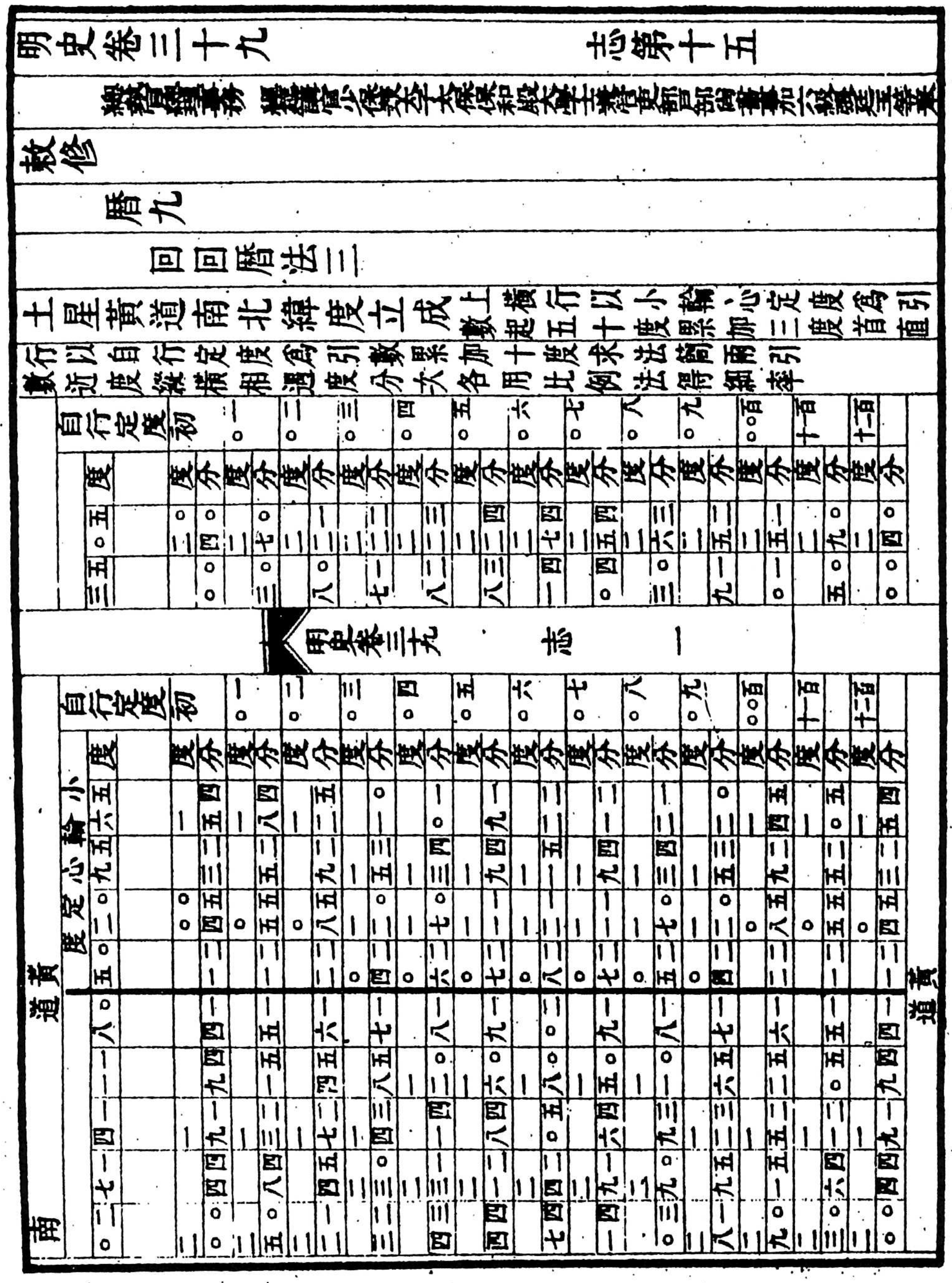
置前段緯度, 以日差累減之, 至不及減者, 於日差內減之, 餘以日差累加之, 卽得逐日緯度.

Appendix 2. Saturn's Latitude Table*

	definite anomaly (1/3)	0	10	20	30	40	50	60	70	80	90	100	110	120	
	deg.	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	
north	50	2 04	2 07	2 12	2 22[1]	2 32	2 42	2 47	2 45	2 36	2 25	2 15	2 09	2 04	north
	53	2 00	2 03	2 08	2 17	2 28	2 38	2 41	2 40	2 30[2]	2 19	2 10	2 05	2 00	
	56	1 45	1 48	1 52	2 01	2 10	2 19	2 22	2 21	2 12	2 02	1 54	1 50	1 45	
	59	1 23	1 25	1 29	1 35	1 43	1 49	1 51	1 49	1 43	1 35	1 29	1 25	1 23	
	definite centrum (1/6) 02	0 54	0 55	0 58	1 02	1 07	1 11	1 12	1 11	1 07	1 02	0 58	0 55	0 54	
north ecl. south	05	0 21	0 21	0 22	0 24	0 26	0 27	0 28	0 27	0 25	0 24	0 22	0 21	0 21	north ecl. south
	08	0 14	0 15	0 16	0 17	0 18	0 19	0 20	0 19	0 18	0 17	0 16	0 15	0 14	
	11	0 49[3]	0 51[4]	0 54	0 58	1 02	1 06	1 08	1 05	1 01	0 56	0 52	0 50	0 49	
	14	1 19	1 23	1 27	1 34	1 41	1 48	1 50	1 46	1 39	1 32[5]	1 25	1 21	1 19	
	17	1 44	1 48	1 54	2 03	2 13	2 21	2 24	2 19	2 09	1 59	1 51	1 46	1 44	
	20	2 00	2 05	2 12	2 23	2 34	2 44	2 47	2 41	2 30	2 18	2 09	2 03	2 00	
	23	2 04	2 09	2 14	2 24	2 35	2 45	2 47	2 43	2 31	2 22[6]	2 12	2 07	2 04	
	26	1 50	1 53	1 58	2 07	2 17	2 25	2 28	2 24	2 15	2 05	1 57	1 53	1 50	
	29	1 25	1 27	1 31	1 38	1 45	1 52	1 54	1 52	1 45	1 38	1 31	1 27	1 25	
	32	0 48	0 49	0 51	0 55	0 59	1 03	1 04	1 03	0 59	0 55	0 51	0 49	0 48	
south ecl. north	35	0 06	0 06	0 07	0 07	0 07	0 08	0 08	0 08	0 08	0 07	0 07	0 06	0 06	south ecl. north
	38	0 33	0 35	0 38	0 43	0 49	0 55	0 58	0 57	0 51	0 45	0 39	0 36	0 33	
	41	1 09	1 11	1 14	1 19	1 26	1 32	1 36	1 34	1 28	1 22	1 16	1 12	1 09	
	44	1 38[7]	1 40	1 45	1 52	2 02	2 10	2 15[8]	2 13	2 05	1 56	1 47	1 42	1 38	
	47	1 56	1 59	2 04	2 13	2 24	2 35	2 41	2 38	2 28	2 18	2 07	2 01	1 56	
	50	2 02	2 05	2 11	2 20	2 32	2 43	2 49	2 46	2 36	2 25	2 14	2 07	2 02	

* pp. 3819-22. The horizontal and vertical directions of the original table in the Chinese text (as seen in Fig.1 and Appendix 3) have been interchanged in this table. The abbreviations used in the following footnotes: C: 世宗實錄七政算 H: 明史回回曆 Q: 七政推步. [1] CQ 21; [2] H 03; [3] C 94; [4] H 15; [5] CQ 31; [6] C 21; [7] H 28; [8] C 25.

Appendix 3.



明史卷三十九　志第十五

總裁官經筵講官少保兼太子太保保和殿大學士兼管吏部戶部尚書事加六級臣張廷玉等奉

敕修

曆九

回回曆法三

土星黃道南北緯度立成上橫行以小輪心定度爲引數起五十度累加三度首直行以自行定度爲引數累加十度求法簡兩引數近度縱橫相遇度分次各用比例法得細率

明史卷三十九　志　一