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# DISTANCE OF PLANETS IN INDIAN ASTRONOMY

YANO Michio

Kyoto Sangyo University, Kamigamo Motoyama, Kita-ku, Kyoto 603, Japan.
e-mail: yanom@cc.kyoto-su.ac.jp

The principle of Indian planetary theory is based on the epicycle model which was transmitted from the Greek astronomy. The significant chronology here is that the Greek theory transmitted to India belongs to the time shortly before Ptolemy. Ptolemy tried to combine the two inequalities of planetary motion by using an eccenter for the eccentricity and an epicycle for the anomaly, thus representing the two effects by a single geometrical model. In doing so Ptolemy had to introduce a controversial point called 'equant'.

In Indian astronomy the both irregularities are represented by epicyles, i.e. the eccentricity by the *manda* ('slow') epicycle and the anomaly by the *śīghra* ('fast') epicycle. The geometrical equivalence of the eccentric and the epicyclic models was known to the Indian astronomers, but they had no need of keeping the eccentric model for the effect of the eccentricity. They computed the two effects separately as a function of mean motion and, after tabulating them, they made several attempts of combining two effects arithmetically so that the result of computation might agree with the observed longitude. It is therefore not easy for us to draw a cosmological picture which ancient Indian astronomers might have conceived.

Besides this, some modifications were made, for example, by hypothesizing the pulsation of the size of the epicycle according to the quadrants (*Āryabhaṭīya* and *Sūryasiddhānta*), or by introducing the iteration method which is discussed in this paper. The iteration is found in the computation of the planet's distance from the earth. It is to be noted that this method is used only in the *manda* correction.

Let us consider that the equation due to the *manda* epicycle is independent on that of the *śīghra* epicycle, as is the case with the Indian

[1] While preparing the first version of this paper three years ago, I got several important suggestions from Dr. Y. Ohashi. The first version (in a different title) was read at the International Colloquium of the Commission 41 of the IAU, Vienna 1990. I learned much from the advices given by Profs. D. Pingree and R. Mercier, and I have put a drastic change.

theory. In Figure 1 the center of the *manda* epicycle (M) rotates on the deferent with the constant angular velocity, while the planet (P) rotates on the epicycle with the same velocity but in the opposite direction, i.e. clockwise. The orbit thus described by the planet is an eccentric circle. The geometrically correct *manda* equation ($\angle$MOP $= \mu$), can be obtained in the following way.[2] Let $\alpha$ be anomaly and $r$ and $R$ be the radii of the *manda* epicycle and of the deferent respectively, then:

$$b = r \sin\alpha \quad (bhuj\bar{a}phala), \qquad k = r\cos\alpha \quad (ko\d{t}iphala) \tag{1}$$

$$\rho = \sqrt{b^2 + (R \pm k)^2} \quad (mandakar\d{n}a) \tag{2}$$

$$\mu = \sin^{-1}(\frac{b}{\rho}). \tag{3}$$

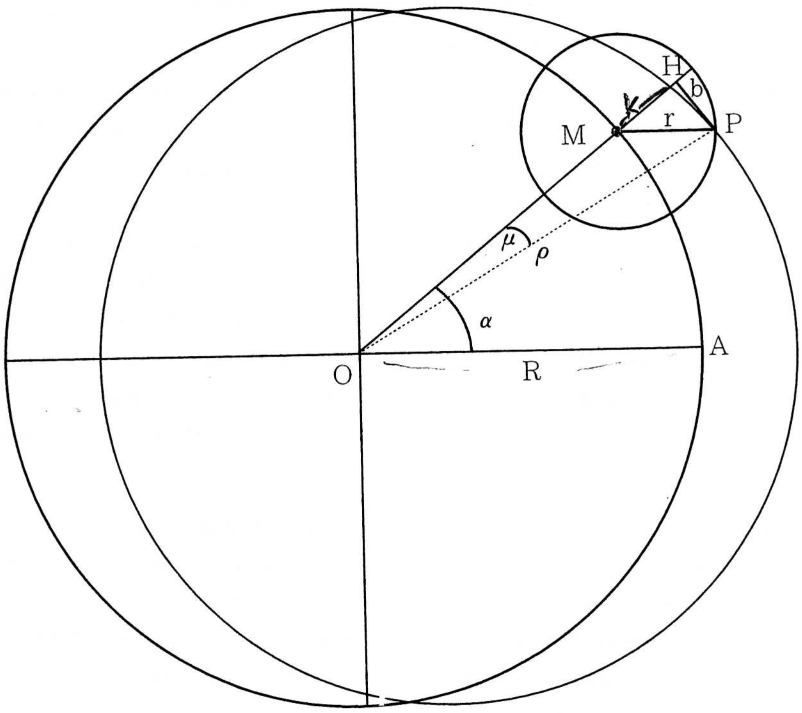


Figure 1. Manda epicycle and equation

[2] This process was known to the *Paitāmahāsiddhānta* (4.10) of the *Viṣṇudharmottarapurāṇa*. Cf. Pingree [1967/8], p.494.

Usually, however, the following 'approximation' is used: [3]

$$\mu \approx b. \tag{4}$$

The earliest Sanskrit text which mentions the iteration in this context is Brahmagupta's *Brāhmasphuṭasiddhānta* (A.D. 628), Chapter 21 verse 29, which says quite succinctly:

> The circumference [of the epicycle] multiplied by the hypothenuse and devided by the Radius is the multiplier of the *bhujā* and *koṭi*. In the *manda* [equation] [the process is] repeated. Its equation is the same as the first [equation], [but] the hypothenuse here is not [the same] as that [first one].[4]

In this case the size of the epicycle is given by the linear "degrees" of its circumference ($c_0$) as relative to the 360 degree circumference of the deferent. Thus the above statement is a brief sketch of the following process:

$$c_1 = c_0 \frac{\rho_0}{R}$$

$$b_1 = \frac{c_1}{360} R \sin\alpha, \qquad k_1 = \frac{c_1}{360} R \cos\alpha$$

$$\rho_1 = \sqrt{b_1^2 + (R \pm k_1)^2}$$

$$\vdots$$

$$c_i = c_{i-1} \frac{\rho_{i-1}}{R}$$

$$b_i = \frac{c_i}{360} R \sin\alpha, \qquad k_i = \frac{c_i}{360} R \cos\alpha$$

[3] For the significance of this so-called 'approximation', see the discussion below. It is because 'minutes' is used here as the units of linear length, with the Radius of 3438 minutes, that this approximation holds.

[4] *trijyābhaktaḥ paridhiḥ karṇaguṇo bāhukoṭiguṇakāraḥ/*
*asakṛt mande tatphalaṃ adyasamaṃ nātra karṇo 'smāt//*
S. Dvivedin's reading *karṇaḥ paridhiguṇo* ('the hypothenuse multiplied by the circumference') in the first line does not make sense. I followed the reading in R.S. Sarma's edition.

$$\rho_i = \sqrt{b_i^2 + (R \pm k_i)^2}$$

until $\rho_i \approx \rho_{i-1}$

What Brahumagupta intended to say here is that the size of the epicycle and the length of the hypothenuse (*mandakarṇa*) are mutually dependent but that the result of the equation is not affected by their change. The more detailed statement is found in the *Mahābhāskarīya* [5] of Bhāskara I, the contemporary of Brahmagupta. Here the problem is found in the context of finding the true distance of the sun and moon. The same method was applied to the five planets too. The process in the *Mahābhāskarīya* 4.9-12 can be expressed as:

$$b_1 = b_0 \frac{\rho_0}{R}, \qquad k_1 = k_0 \frac{\rho_0}{R} \tag{5}$$

$$\rho_1 = \sqrt{b_1^2 + (R \pm k_1)^2} \tag{6}$$

$$b_2 = b_0 \frac{\rho_1}{R}, \qquad k_2 = k_0 \frac{\rho_1}{R}$$

$$\rho_2 = \sqrt{b_2^2 + (R \pm k_2)^2}$$

$$\vdots$$

$$b_i = b_0 \frac{\rho_{i-1}}{R}, \qquad k_i = k_0 \frac{\rho_{i-1}}{R} \tag{7}$$

$$\rho_i = \sqrt{b_i^2 + (R \pm k_i^2)}, \tag{8}$$

until $\rho_i \approx \rho_{i-1}$.

Since Bhāskara I's aim is to find the true distance of the sun and moon, he does not go further. But Nīlakaṇṭha [1931, page 36], the 15 century commentator on the *Āryabhaṭīya*, obtains the final equation by:

$$\mu_i = \sin^{-1}(\frac{b_i}{\rho_i}). \tag{9}$$

Bhāskara I also explains the case of the eccentric model in the verses 4.19-20, where the process is almost the same as above, the only difference

[5] Pingree wrote a paper [1974] on the *Mahābhāskarīya* 4.19-21 in order to give a textual evidence of the equant model in Indian astronomy which had been proposed in 1952 and later abandoned by van der Waerden [1956].

being that the eccentricity, instead of the epicyle radius, varies.[6]

With a computer program I have computed the true distance with iteration and found how many iterations were necessary before the difference disappears. If one is satisfied with the precision down to the unit of minutes, the iteration required is only four or five times around the beginning of the odd quadrants and two or three times around the beginning of the even quadrants (cf. Shukla, p.118). Thus this process is called *aviśeṣakarma* (computation without remainder) or *asakṛtkarma* (not-once-computation). The orbit described by the planet is no more a circle (as is dotted in the Figure 2). The deviation from circularity is not remarkable in the case of the sun and moon whose epicycles are relatively small. In the case of Mars, however, the deviation is significant.

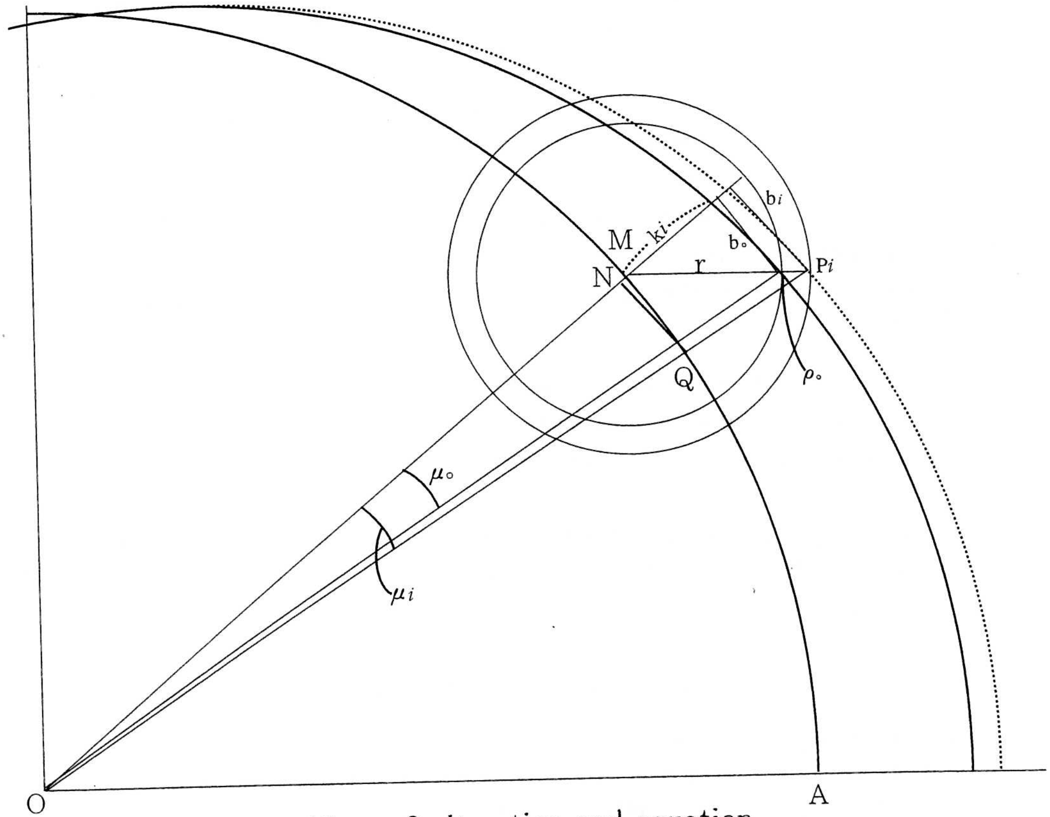


Figure 2. Iteration and equation

[6] Pingree [1974] interpretes this model of varyng eccentricity as an 'equant' model and suggests its 'Periparetic origin'. I hesitate to support this view because of lack of evidence.

While drawing Figure 2 to the scale of Mars I have used the ratio $r/R = c/C = 70/360$, the ratio which is used by Brahmagupta in his *Khaṇḍa-khādyaka* (AD 665). When Brahmagpta tabulates the *manda* equation[7] he uses the 'approximatin' (4) mentioned above. He knew that the two results, one with iteration and the other without iteration but *with approximation*, were the same. (cf. the verse of *BSS* quoted above) This fact was more clearly stated in Bhāskara II's *Siddhāntaśiromaṇi*, *Golādhyāya* 6.36–37. The equivalence of the two can be easily proved by dropping the perpendicular QN from the intersection of $OP_i$ and the deferent to OM:

$$\mathrm{QN} = \frac{b_i R}{\rho_i} = \frac{b_0 \frac{\rho_{i-1}}{R} R}{\rho_i} \approx b_0 \quad \text{(because of (7) and } \rho_i \approx \rho_{i-1}). \qquad (10)$$

It is here that one must pay attention. If the aim of the computation were only to get the true longitude of planets, Indian astronomers should have avoided taking the trouble of iteration method. They had another aim, i.e. that of computing the true distance of the planet from the earth. In the process of 'trial and error' they found that the *manda* equation after the iteration was same that of the 'approximation' and that by this approximation they could be closer to the observed reality than by the mathematically correct process ((1)–(3) above). [8] This explains why the iteration, as well as the 'approximation', is applied only to the *manda* equation.

The concern with the distance of planets is already found in the earliest stage of the classical Indian astronomy, i.e. in the *Āryabhaṭīya* 3.25ab. The very enigmatic half verse runs:

> The mutual multiplication of own [two] hypothenuses devided by the Radius is the distance between the earth and the planet.[9]

This very concern was long preserved in Indian astronomy, as is witnessed

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[7] *KhKh.*1.16 (sun), 17 (moon), and 2.6–7 (five planets) .

[8] Thus we should reconsider Neugebauer's view: "There is no compelling reason to treat the effect of the eccentricity with so much less accuracy than the effect of anomaly,..." (Neugebauer[1956, p.16]). This approximation is found already in the *Pañcasiddhāntikā* (9.7–8) of Varāhamihira (mid 6th century).

[9] *bhūtārāgrahavivaraṃ vyāsārdhahṛtaḥ svakarṇasaṃvargaḥ*

in Nīlakaṇṭha's commentary on the *Āryabhaṭīya* mentioned above. He even tries to combine the *manda* and *sighra* epicycles in a unified model[10] and to interprete the half verse cited above from his own point of view.

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[10] Cf. Nīlakaṇṭha, pp. 51–54. This part is very diificult but interesting. In any case, van der Waerden [1988, p.231] is wrong when he says " ... was auch die Inder nicht tun, ist, die beiden Anomalien in einer geometrisch korrekten Weise miteinander zu verbinden."

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