The *Chiuchih-li* and the *Ārdharātrika-pakṣa*

——on the true daily motion of the moon——

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The *Chiuchih-li* (九執暦) is a book of Indian astronomical calculations (*karaṇa*) revised and translated into Chinese in A. D. 718 by an Indian, Chut'an Hsita (瞿曇悉達), who became naturalized in China. The original Sanskrit text is lost and we know little of the personal history of the translator. It was Prof. K. Yabuuti who first discovered the historical importance of this text and, with his edition and English translation[1], aroused the interest of readers of the history of Indian astronomy. He deciphered most of the difficult passages and demonstrated how this text depended[2] on the *Pañcasiddhāntikā* of Varāhamihira. Prof. D. Pingree, in the Introduction to his edition[3] of the *Pañcasiddhāntikā* (*PS*), made a list of the parallel passages of the two texts, but he admitted that Chut'an Hsita's source was based on other texts because he (Chut'an Hsita) used 3438 as the Radius of the Sine table instead of 120 employed in *PS*.

When we compare the parameters in the *Chiuchih-li* (*CL*) with those found in Sanskrit astronomical texts, it is clear that the *ārdharātrika-pakṣa*[4] (school of midnight epoch) was the main source of *CL*. As traditional Chinese calenders consistently used an epoch beginning with a midnight conjunction of the sun and moon, the *ārdharātrika-pakṣa* must have been most accessible to the Chinese. The majority of the parallels in *PS*, as identified by Pingree, are from the *Pauliśasiddhānta* and *Sūryasiddhānta*; the former was commented on and the latter was revised by Lāṭadeva (*ca.* 505) who belonged to the *ārdharātrika-pakṣa*[5]. It is not surprising, therefore, that one finds several passages in *CL* which can be explained very well with the help of Brahmagupta's *Khaṇḍakhādyaka*[6] (*Kh*: A. D. 665), a basic text representing this *pakṣa*.

In the following I will show an example which illustrates a relationship between *CL* and *Kh*, eventually commenting on the passage in *CL* which was not fully explained by Prof. Yabuuti.

In the chapter on the *yüeh-yü*[7] (月域, daily motion of the moon) *CL* gives two methods. The first is not uncommon in Sanskrit texts[8] and is easy to understand: ——

> Place the *ting-yüeh* (true longitude of the moon) for today, and subtract from it the *ting-yüeh* of yesterday. The remainder is reduced to minutes. Put it down as the *yüeh-yü*[9].

Following this is an 'alternative method' of our present concern: ——

> Place the number 790 in the proper place. Always take the number in the step table of the moon and multiply it——multiply it by 9. Immediately discard the number in the first (decimal) place. As for the remaining (figures), always note the moon's anomaly, and if (the anomaly is) in *hsieh-shou* (蟹首), the resut is added to the number in the proper place, if (the anomaly is) in *kuei-shou* (龜首), it is subtracted from the number of the proper place[10].

The number 790 is moon's mean velocity ($\bar{V}$) in minutes employed throughout the text. 'The number in the step table of the moon' is the difference ($\Delta\theta$) of the moon's equation of center (θ) tabulated for the interval 15° of anomaly (α). The six values in a quadrant are given in the chapter on the calculation of the *ting-yüeh*[11] (see Table). 'Multiplication by 9 and discard of the number in the first place' gives $\frac{9}{10}\times\Delta\theta$. This result is added to or subtracted from the number in the proper place ($\bar{V}=790$). Thus we have

$$V=\bar{V}\pm\frac{9}{10}\times\Delta\theta. \qquad \cdots\cdots (1)$$

Since *hsieh-shou* and *kuei-shou* are direct translations of *karkādi* (*-ṣaḍrāśayaḥ*) and *makarādi* (*-ṣaḍrāśayaḥ*) respectively, the application of (±) signs in the four quadrants from the apogee is negative, positive, positive, negative. The formula is crude and somewhat strange to Sanskrit texts, but it must be noted that a formula corresponding to it is found only (as far as I know) in *Kh*, without the help of which we can not explain it.

Table of equation

α°	θ'	$\Delta\theta'$
0–15	77	77
15–30	148	71
30–45	209	61
45–60	256	47
60–75	286	30
75–90	296	10

Brahmagupta, in *Kh* I, 17, gives exactly the same value of the equation of center (θ) as in the Table above. In *Kh* I, 19 he says: ——

One should divide the *Bhogyamānapiṇḍaka* ($\Delta\theta$), of the sun by 15, and 7 times that of the moon by 8. (The result is applied) to their respective (mean) motions, negatively, positively, positively, and negatively[12].

As the motion of the sun is discussed in the *Chiuchih-li* separately in different context[13], we are here concerned only in producing a formula for the moon;

$$V=\bar{V}\pm\frac{7}{8}\times\Delta\theta. \qquad \cdots\cdots (2)$$

When we compare the two formulae (1) and (2) we see that they are in principle the same differing only in the coefficient to $\Delta\theta$. How these coefficients were derived can be conjectured by Brahmagupta's next verse: ——

When the daily motion of the anomaly of the sun or moon is multiplied by the *Bhogyakhaṇḍa* ($\Delta\theta$), and the product is divided by 900, the result is the equation (of the motion) of the sun or moon[14].

Both Pṛthūdaka (9th cent.) and Utpala (10th cent.), commenting on this verse, say that this is 'another' method of calculating the true motion, but this verse is simply the basis on which formula (2) rests. When we denote by V_A daily motion of moon's apogee, then the daily motion in anomaly of the moon is $\bar{V}-V_A$, and thus the equation given is $(\bar{V}-V_A)\times\frac{\Delta\theta}{900}$ which is applied to the mean motion as before. We have thus

$$V=\bar{V}\pm(\bar{V}-V_A)\times\frac{\Delta\theta}{900}\text{[15]}. \qquad \cdots\cdots (3)$$

Now this last formula reminds us of the formula given in *PS* IX, 12–13[16]:

$$V=\bar{V}\pm(\bar{V}-V_A)\times\frac{\Delta Sin\alpha}{225}\times\frac{c}{360} \qquad \cdots\cdots (4)$$

where c is the circumference of the epicycle expressed in "degrees". In fact, (3) can be derived from (4) ——we can replace 225 ($=\Delta\alpha$) by 900 because α is tabulated at the interval of 15° ($=900'$), and $\Delta Sin\alpha\,\frac{c}{360}$ by $\Delta\theta$ (cf. *PS* IX, 7–8 and Neugebauer-Pingree's commentary). Further, if we use, as the commentators do, 790;34 and 6;40 as $\bar{V}$ and V_A respectively, $\frac{\bar{V}-V_A}{900}$ comes to be $\frac{783;54}{900}$ (= 0.871......) which seems to have been replaced by $\frac{7}{8}$ (=0.875) in the *Khaṇḍakhādyaka*. Likewise the coefficient $\frac{9}{10}$ in the *Chiuchih-li* seems to be a result of cruder approximation using 790 and 6;41.

Lāṭadeva's formula (4) allows us to reflect on the process of its formulation and on the underlying theory as well, but as we proceed to (3), (2) and (1), we miss the theory and at the same time lose exactness. It is a general tendency of *karaṇa*-texts of a technical nature that they simplify sophisticated rules, sometimes at the cost of theory and exactitude[17].

We can not tell which of the formulae (1) or (2) did first exist, but we can say at least that the *Chiuchih-li* is on the same line of the *ārdharātrika* tradition that leads from Lāṭadeva to the *Khaṇḍakhādyaka*[17].

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1) K. Yabuuti, *Zui-tō Reki-hō-shi no Kenkyu* (Researches in the History of the Astronomical Tables of Sui-T'ang Period), Tokyo 1944, Chap. 6 (study) and Chap. 7 (text). English tr. was published in *History of Chinese Science and Technology in the Middle Ages*, Tokyo 1963. A revised translation will appear in *Acta Asiatica*.

2) Especially convincing is the fact that the *Chiuchih-li*'s epoch is A. D. 657 March 20, exactly 152 years (8 Metonic Cycles) later than *PS*'s epoch of A. D. 505 March 21.

3) O. Neugebauer and D. Pingree, *The Pañcasiddhāntikā of Varāhamihira*, Kopenhagen Part I (1970), II (1971).

4) For the classification of Indian *pakṣas*, see D. Pingree, *JHA* i (1970), p. 95ff.

5) Neugebauer-Pingree, Part I, pp. 12–13.

6) *Khaṇḍakhādyakaṃ Karaṇam* ed. by P. C. Senupta, Calcutta 1941. I have also used B. Chatterji's ed. and tr., 2 vols., Calcutta 1970.

7) p. 188, Eng. tr. p. 514 (Chap. 12 in the revised translation).

8) See *Paitāmahasiddhānta of the Viṣṇudharmottarapurāṇa* (tr. by D. Pingree, *Adyar Lib. Bul.* 31/32, pp. 472–510) II, 29 and IV, 14ab; *BSS* II, 29; *Mbh* IV, 18.

9) 置今日定月。以昨日定月減之。余通作分。凡置為月域位。

10) 置七百九十為本位。又取通乘月段。以九乘之訖。直弃一位。余者恆視月藏。蟹首益本位。亀首損本位。即是月域。
The third character of the 2nd sentence, 通 (*t'ung*) is not a technical term but an adverb meaning 'usually', 'generally' etc. I have translated it as 'always'.

11) p. 185, Eng. tr. p. 507 (Chap. 10 in the revised translation).

12) *pañcadaśakena vibhajet bhānumato bhogyamānakaṃ piṇḍam/*
śaśino 'gaguṇaṃ vasubhiḥ kṣayadhanadhanahānayaḥ svagatau//

13) See Eng. tr. p. 515 where there is given a table closely related to *PS* III, 17.

14) *gatibhogyakhaṇḍakavadhāl labdhaṃ navabhiḥ śatai ravīnduphalam/* (*Kh* I, 20ab)

15) The same rule is found in *Paitāmaha°* IV, 14cd.

16) A similar rule is found in *BSS* II, 41.

17) The *Chiuchih-li* shows this characteristics throughout the text.